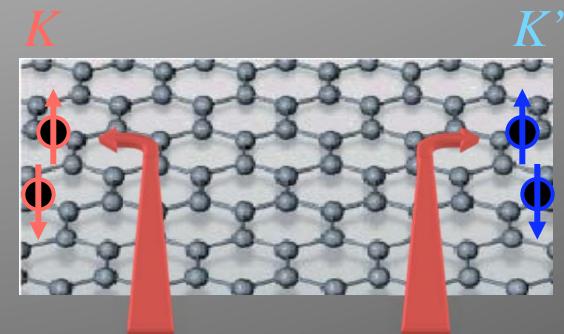


Other quantum Hall effects

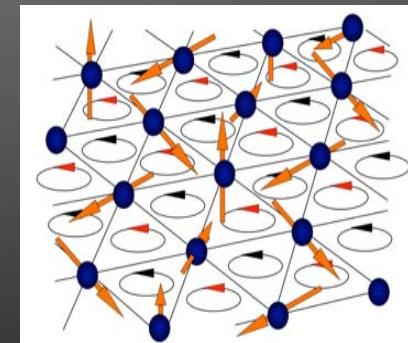
- Part 1: Quantum Valley Hall
 - No interactions
 - Valley = “emergent” spin
 - “Edge” states



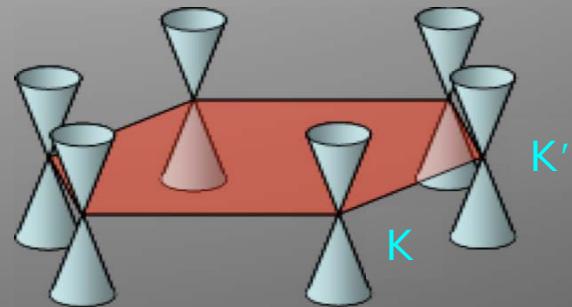
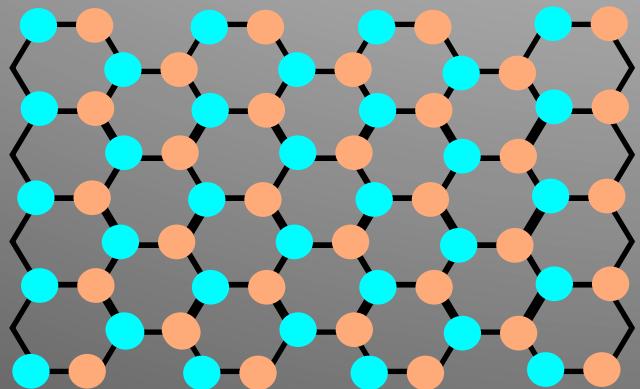
Ivar Martin, Ya. M. Blanter, A. F. Morpurgo, PRL 100, 036804 (2008)

- Part 2: Spontaneous Hall effect
 - Interactions – yes (**weak coupling**)
 - Broken Continuous and Ising symmetries

Ivar Martin and C. D. Batista, PRL 101, 156402 (2008)



Single layer graphene



Linear (Dirac) dispersion

$$\hat{H} = \begin{pmatrix} 0 & v_F(p_x - ip_y) & 0 & 0 \\ v_F(p_x + ip_y) & 0 & 0 & 0 \\ 0 & 0 & 0 & v_F(-p_x - ip_y) \\ 0 & 0 & v_F(-p_x + ip_y) & 0 \end{pmatrix}$$

$= p_x \sigma_x \tau_z + p_y \sigma_y$

$E = \pm v_F p$

- Valley K
- Valley K'

σ	: sublattice	$(T = C)$
τ	: valley	$(T = i\tau_y C)$

Graphene with “mass”

$E_A - E_B = 2m \neq 0$ (G. Semenoff 1984)

$$\hat{H} = \begin{pmatrix} m & v_F(p_x - ip_y) & 0 & 0 \\ v_F(p_x + ip_y) & -m & 0 & 0 \\ 0 & 0 & m & v_F(-p_x - ip_y) \\ 0 & 0 & v_F(-p_x - ip_y) & -m \end{pmatrix} \quad \begin{array}{l} \text{Valley K} \\ \text{Valley } K' \end{array}$$

$$= p_x \sigma_x \tau_z + p_y \sigma_y + m \sigma_z \equiv \mathbf{g}_\tau \cdot \boldsymbol{\sigma} \quad E = \pm \sqrt{p^2 + m^2}$$

How about quantum Hall?

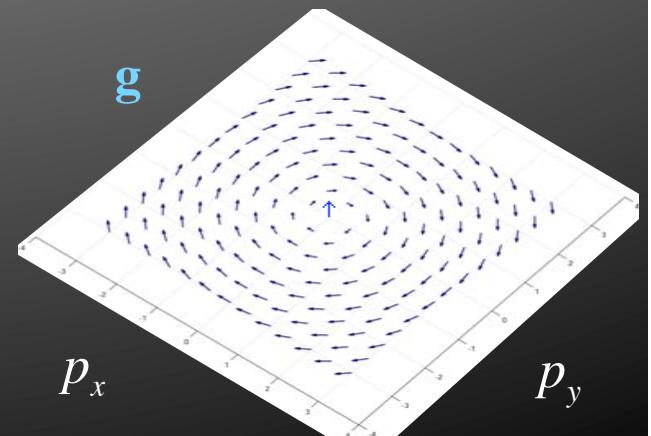
$$\sigma_{xy} = \frac{e^2}{4\pi h} \int dp_x dp_y \hat{\mathbf{g}} \cdot [\partial_{p_x} \hat{\mathbf{g}} \times \partial_{p_y} \hat{\mathbf{g}}] = \frac{e^2}{h} \frac{1}{2} \text{sgn}(m) \tau_z$$

There is quantum *Valley* Hall effect!

BUT!

1. $\frac{1}{2}$ is suspicious

2. Need sublattice imbalance, $E_A - E_B = 2m$



Quantum spin hall effect (Kane + Mele, 2005)

- Another possibility (T-invariant) for the mass term:

$$\hat{H} = \begin{pmatrix} s_z m & v_F(p_x - ip_y) & 0 & 0 \\ v_F(p_x + ip_y) & -s_z m & 0 & 0 \\ 0 & 0 & -s_z m & v_F(p_x - ip_y) \\ 0 & 0 & v_F(p_x - ip_y) & s_z m \end{pmatrix}$$

$$= p_x \sigma_x \tau_z + p_y \sigma_y + \boxed{m \sigma_z s_z \tau_z}$$

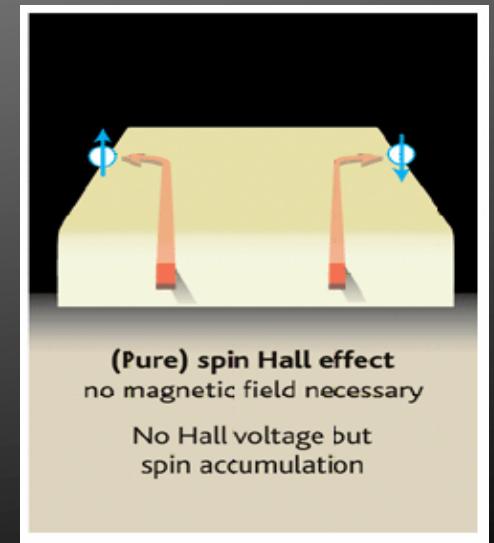
T-inv – spin-orbit

Still, dispersion

$$E = \pm \sqrt{p^2 + m^2} \quad \text{in every valley}$$

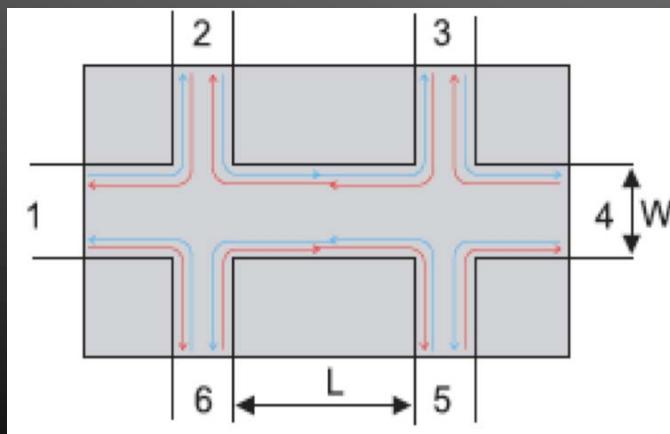
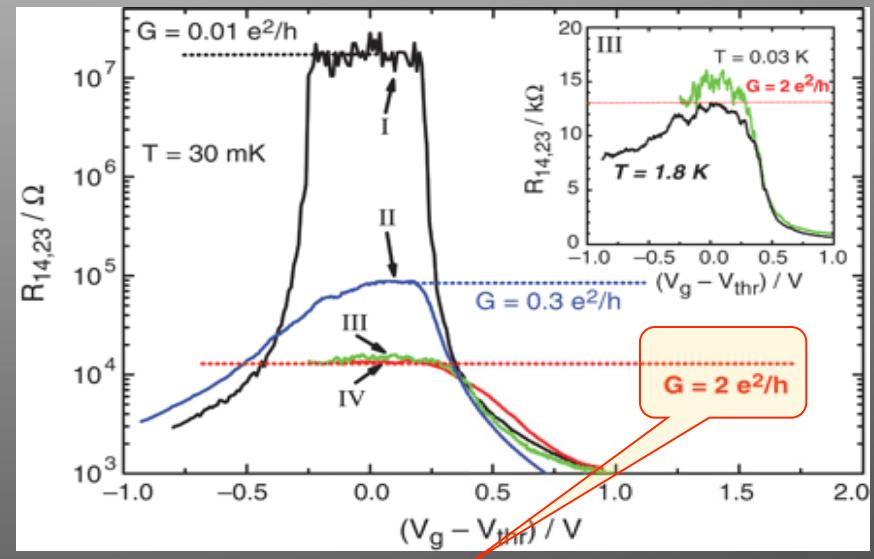
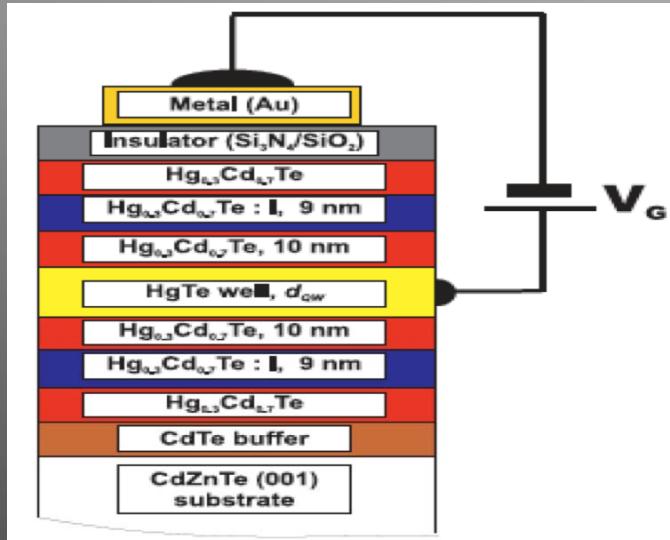
But instead of valley quantum Hall
obtain Quantum Spin Hall

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{2} \operatorname{sgn}(m) \tau_z \times (\tau_z s_z) = \frac{e^2}{h} \operatorname{sgn}(m) s_z$$



Quantum spin Hall effect:

(Molenkamp, SC Zhang, et al, 2007)



Quantized conductance!

- I: trivial insulator
- II, III, IV: quantum spin hall insulators
- I & II – $(20 \times 13) \mu\text{m}^2$
- III – $(1 \times 1) \mu\text{m}^2$ & IV $(1 \times 0.5) \mu\text{m}^2$

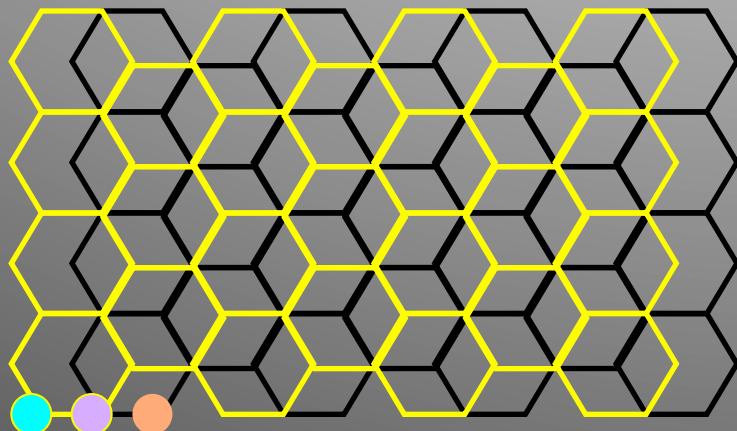
M. König et al Science 318, 766 (2007)

What about valley Hall effect?

Possible to observe in the conventional
Bilayer graphene!

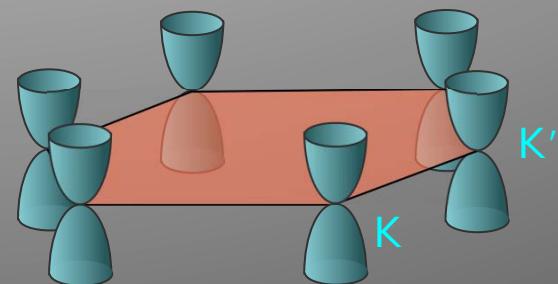
1. Can generate mass term
2. Can create a smooth "edge"

Bilayer graphene



t

t_⊥ Interlayer coupling



Quadratic dispersion!

Bilayer, one of the valleys (e.g. K):

$$\hat{H} = \begin{pmatrix} 0 & v_F(p_x - ip_y) & 0 & 0 \\ v_F(p_x + ip_y) & 0 & \textcolor{red}{t_\perp} & 0 \\ 0 & \textcolor{red}{t_\perp} & 0 & v_F(p_x - ip_y) \\ 0 & 0 & v_F(p_x - ip_y) & 0 \end{pmatrix}$$

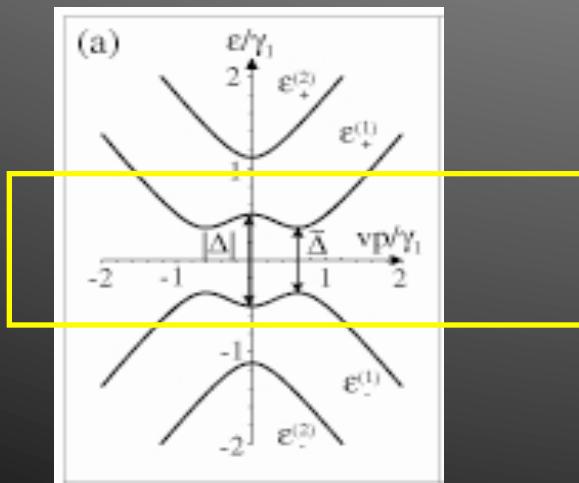
$|E| < t_\perp$ $v_F = 3ta/2$
 $\approx -\frac{v_F^2}{t_\perp} \begin{pmatrix} 0 & (p_x - ip_y)^2 \\ (p_x + ip_y)^2 & 0 \end{pmatrix}$
● Layer 1 ● Layer 2
 $E \propto \pm p^2$

Biased graphene bilayers

Field-induced gap in the spectrum

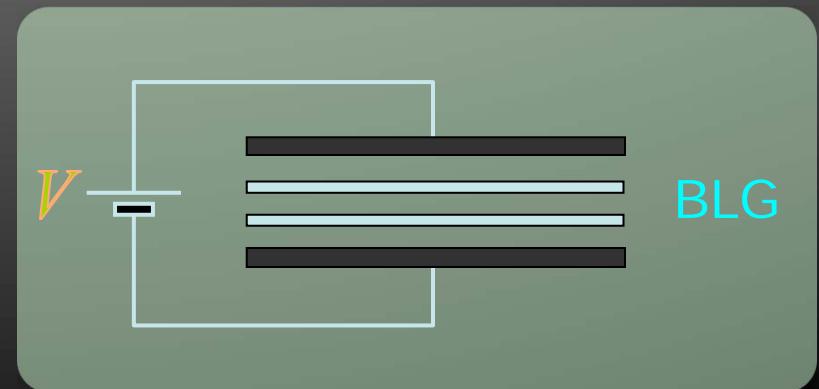
McCann '06

$$\hat{H} = -\frac{v_F^2}{t_\perp} \begin{pmatrix} \varphi & (p_x - ip_y)^2 \\ (p_x + ip_y)^2 & -\varphi \end{pmatrix} \equiv \mathbf{g}(p_x, p_y) \cdot \boldsymbol{\sigma}$$

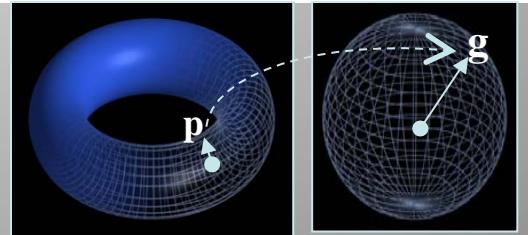


$$E = \pm \sqrt{p^4 + \varphi^2}$$

$$\varphi(x) = \frac{V(x)t_\perp a^2}{2v_F^2}$$



Topology of insulating BLG

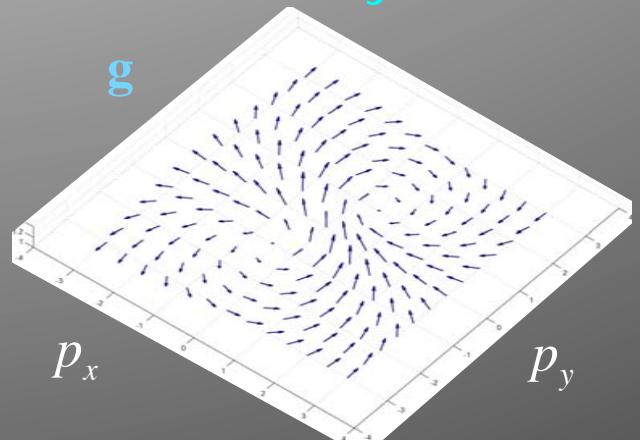


$$\hat{H} = -\frac{v_F^2}{t_\perp} \begin{pmatrix} \varphi & (p_x - ip_y)^2 \\ (p_x + ip_y)^2 & -\varphi \end{pmatrix} \equiv \mathbf{g}(p_x, p_y) \cdot \boldsymbol{\sigma}$$

Mass !

$$\mathbf{g} = (p_y^2 - p_x^2, 2p_x p_y, \varphi)$$

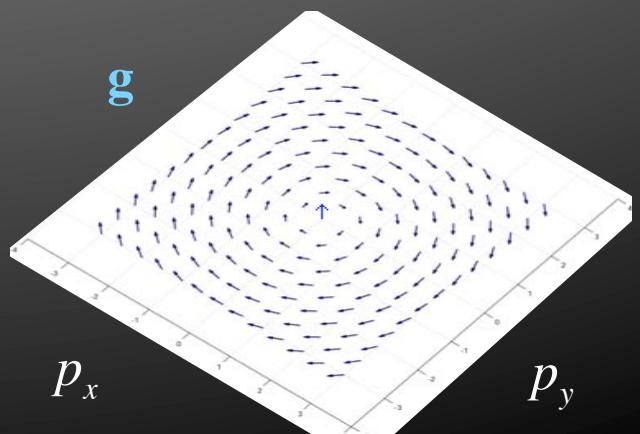
Biased bilayer:



Quantum Valley Hall effect?

$$\sigma_{xy} = \frac{e^2}{4\pi h} \int dp_x dp_y \hat{\mathbf{g}} \cdot [\partial_{p_x} \hat{\mathbf{g}} \times \partial_{p_y} \hat{\mathbf{g}}] = \frac{e^2}{h} \text{sgn}(\varphi) \tau_z$$

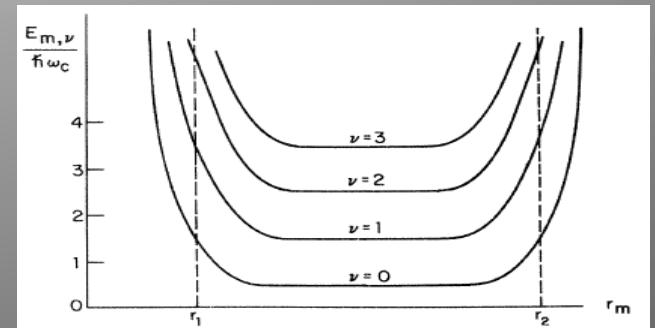
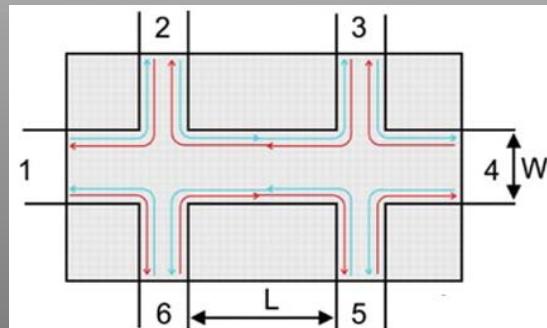
Single layer with mass:



Quantized Valley Conductance,
similar to charge and spin Hall effects

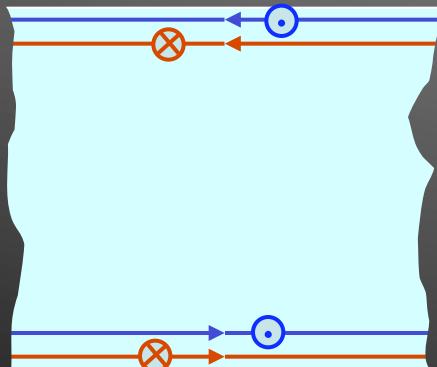
How about the quantum Valley Hall effect? Is it observable?

Edge states



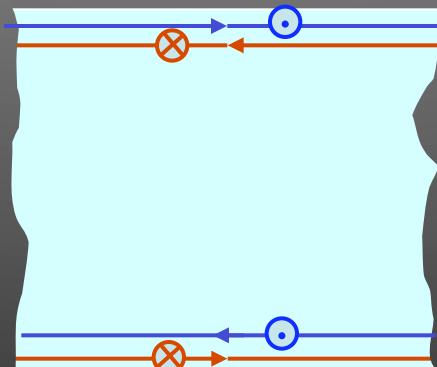
QH edge states, Halperin '82

QHE (charge)



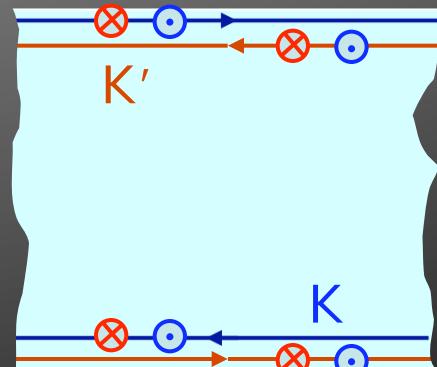
OK: any disorder
(V , V_{so} , S)

QSHE (spin)



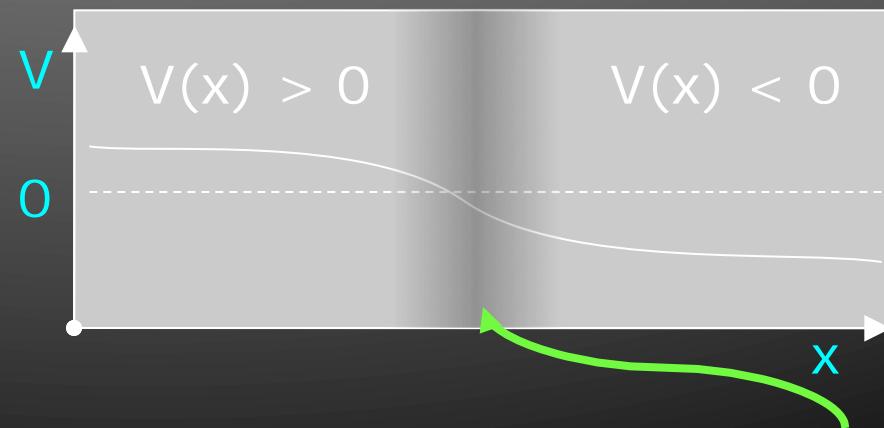
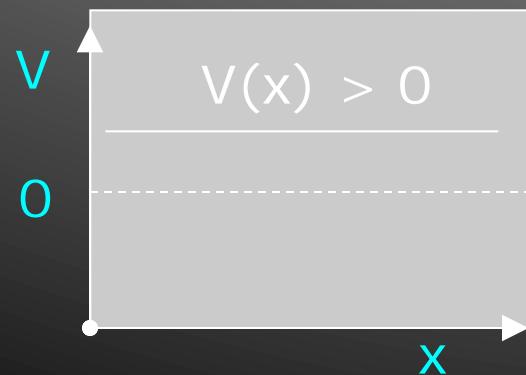
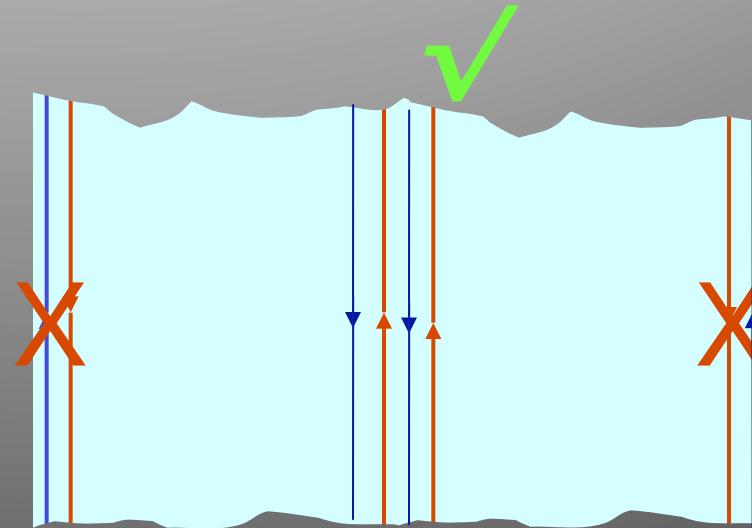
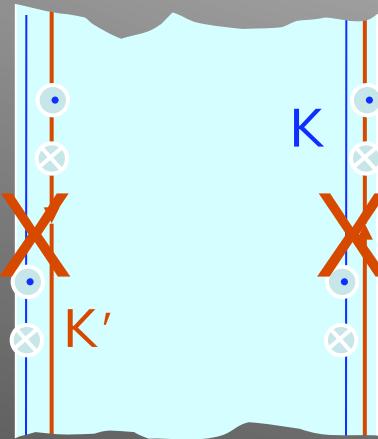
OK: V and V_{so}
X: S - magnetic

QVHE (valley)



OK: any,
Unless it scatters
X: $K \rightarrow K'$

Observation of Valley Hall “Internal” edges



Usual edge states are destroyed by disorder. Not the internal ones!

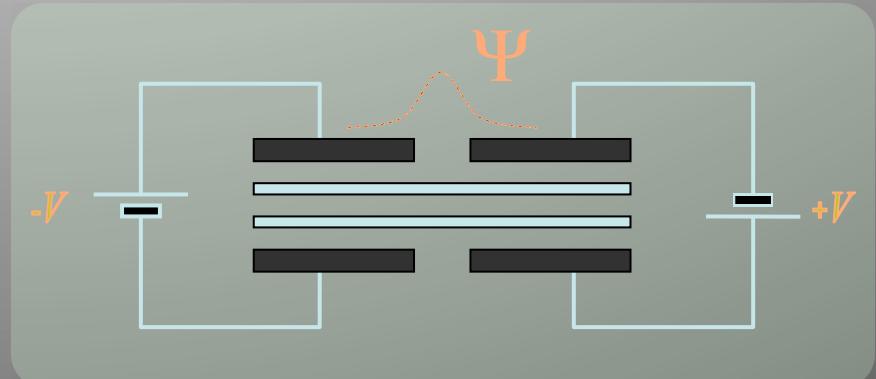
KITP, Dec 2008

Topological confinement

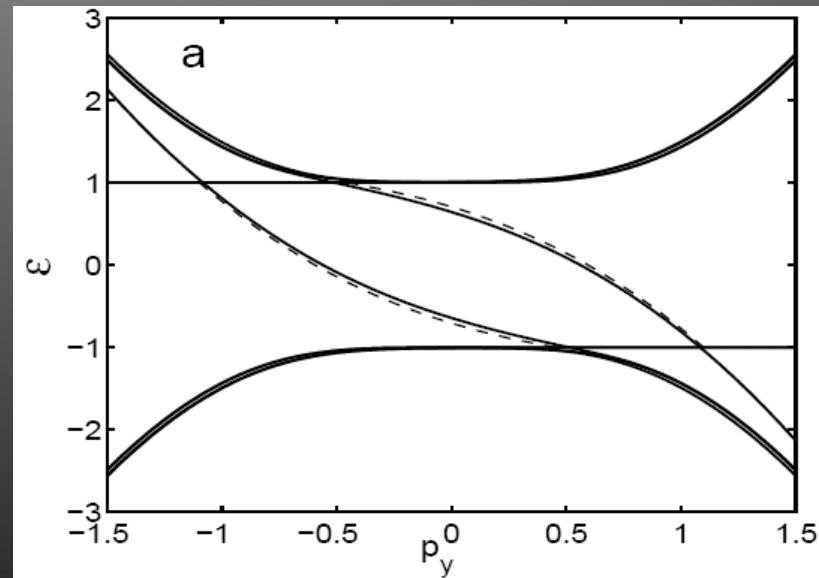
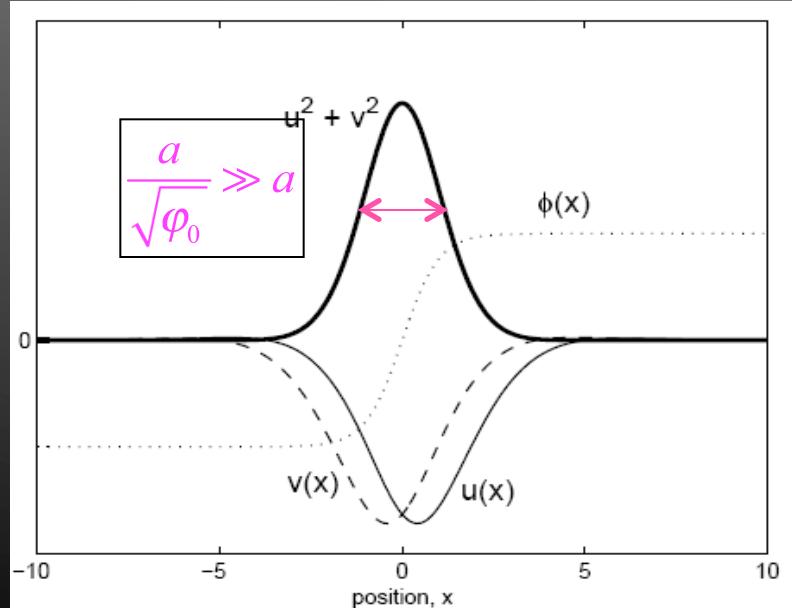
Equations of motion for $V(x)$

$$-\varphi(x)u + (\partial_x + p_y)^2 v = \varepsilon u$$

$$\varphi(x)v + (\partial_x - p_y)^2 u = \varepsilon v$$



Martin, Blanter, Morpurgo '07



Valleytronics

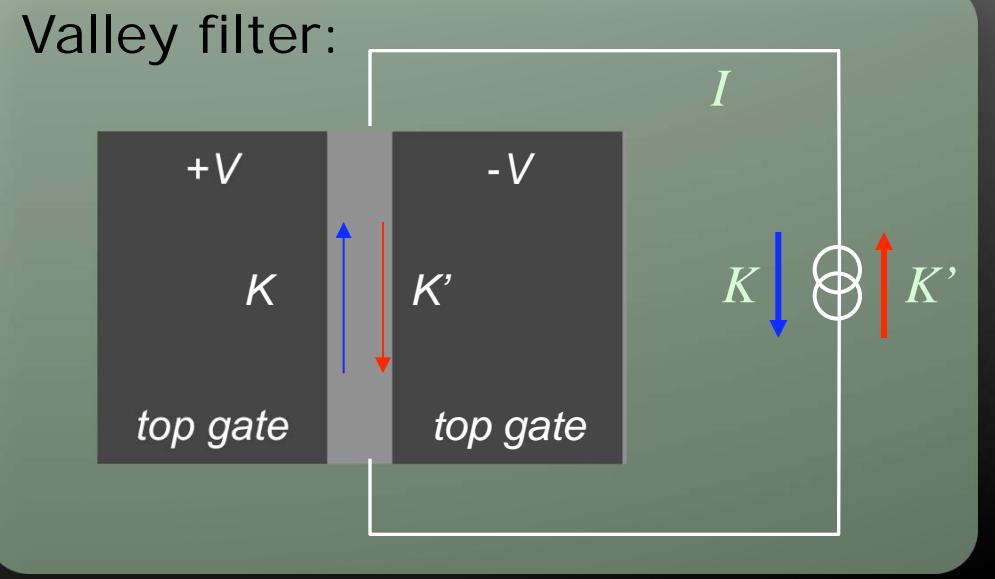
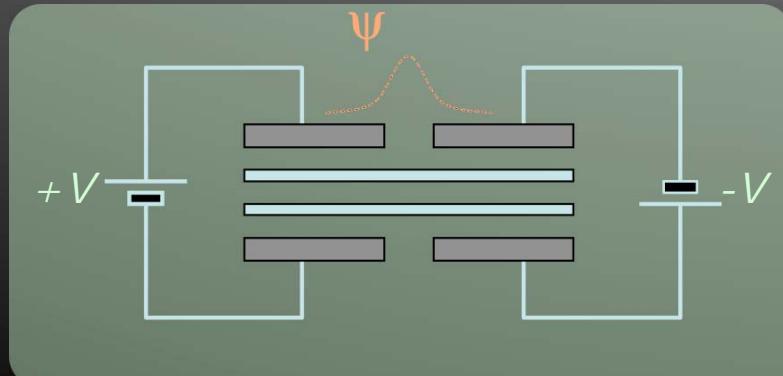
Proposals to use the valley degree of freedom for classical or quantum manipulation (intervalley relaxation can be slow)

Use zig-zag ribbon edge state or Aharonov-Bohm

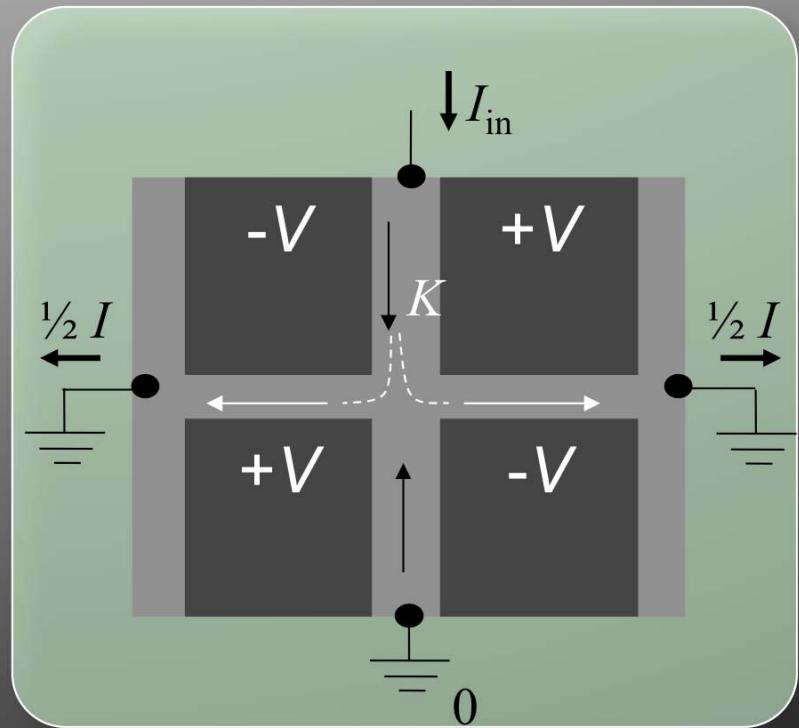
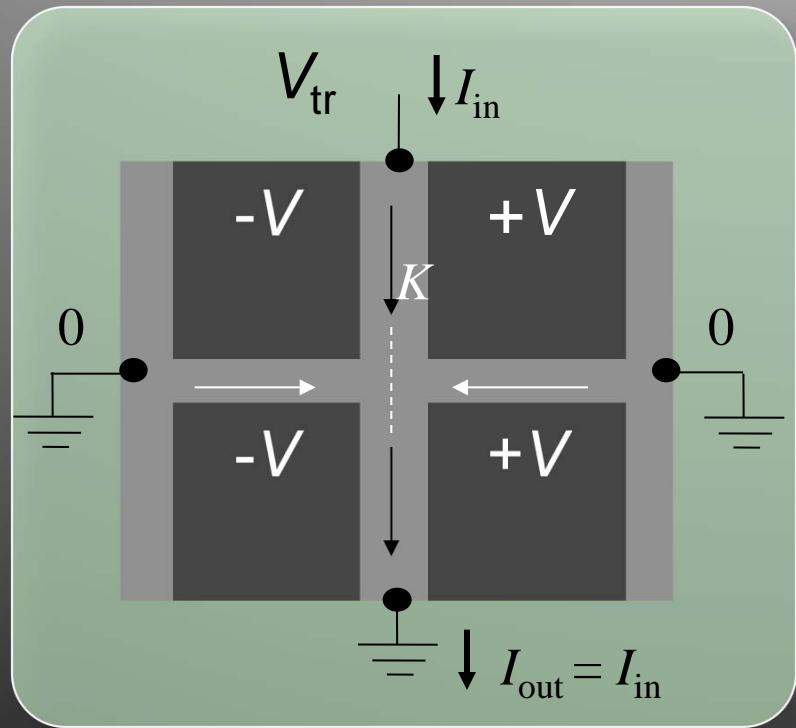
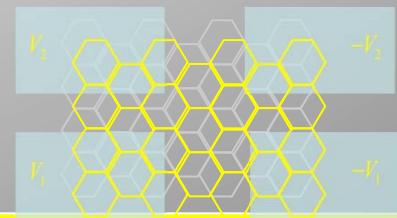
(Rycerz, Tworzydlo, Beenakker '07, Recher et al '07)

Both require ideal interfaces: Hard with the existing technology

Alternative: use INTERNAL valley-polarized states:



Valley Valve



→ Direction of propagation of K-valley states

Open questions

- Interesting phenomena in more complicated structures?
Aharonov-Bohm, non-local transport,
+ Superconductivity, + Magnetism?
- Interesting physics upon inclusion of interactions? Luttinger liquid?
- Valley Hall vs. Spin Hall: Spin \leftrightarrow Valley

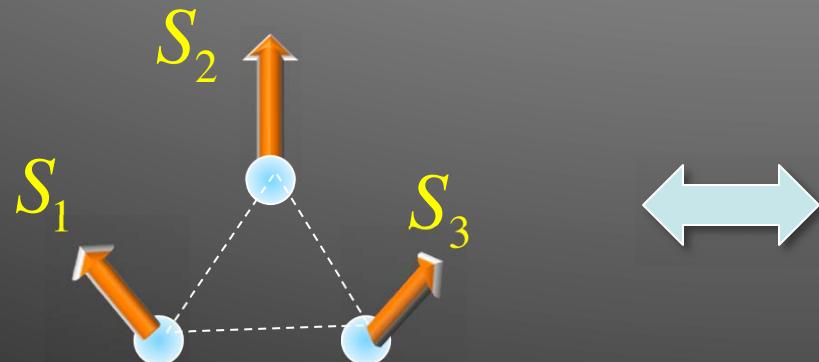
Spontaneous CHARGE quantum Hall effect

$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} - J_H \sum_i \vec{S}_i \cdot c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

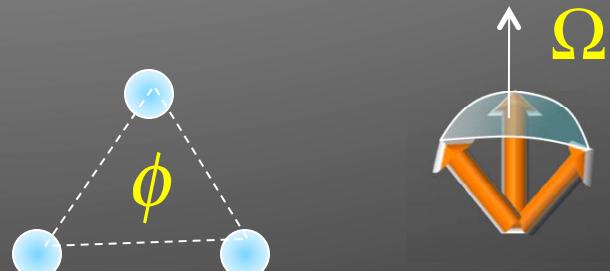
$c_{i\alpha}^\dagger$ – *itinerant electrons*

\vec{S}_i – *local moments (classical)*

Hopping in a texture



Hopping in an orbital field

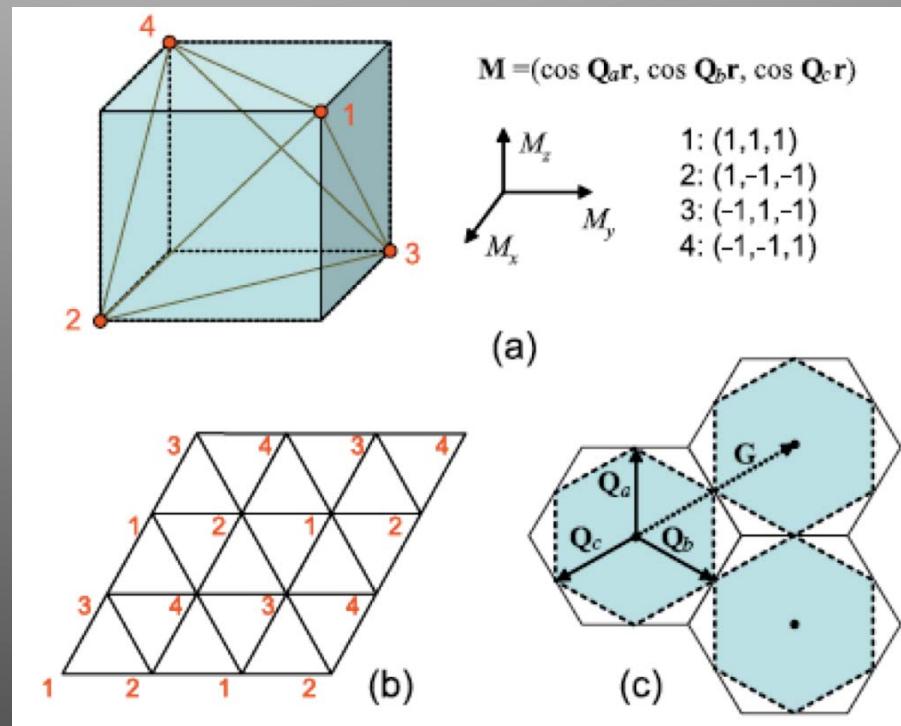
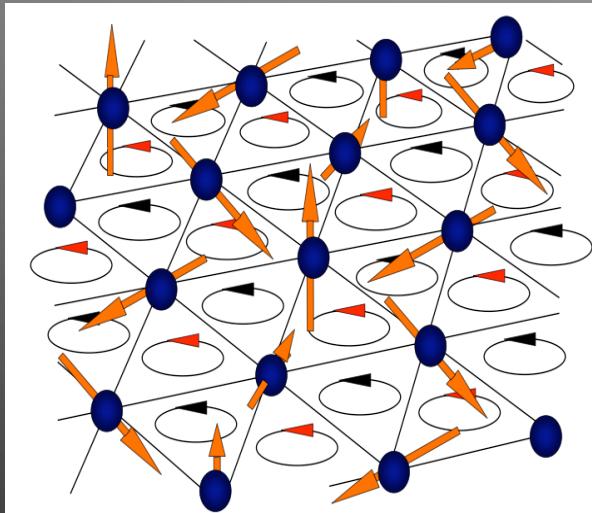


$$\Omega \approx S_1 \cdot [S_2 \times S_3]$$

$$\phi = \Omega / 2$$

Weak-coupling instability

- Chiral spin ordering:

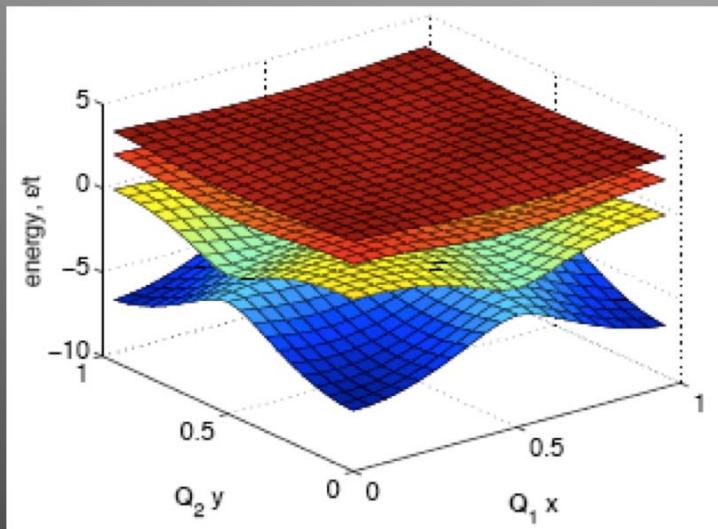


At $n = 1.5$ el/site

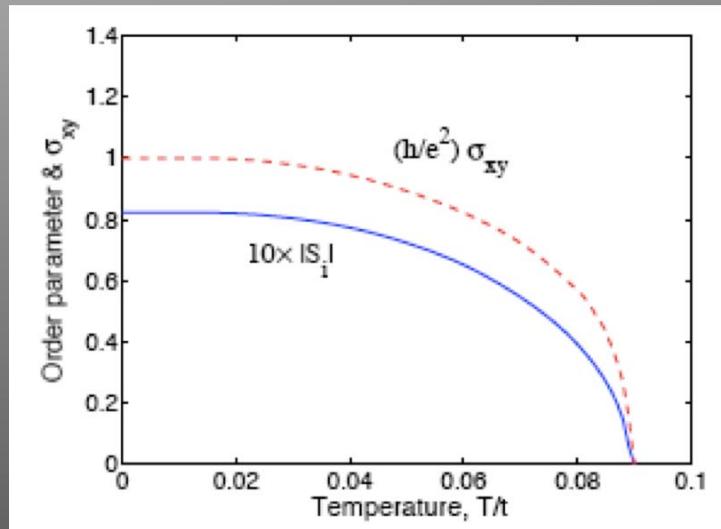
- Ordering with \mathbf{Q}_1 & \mathbf{Q}_2 & \mathbf{Q}_3 fully gaps Fermi surface
- for classical spins “tetrahedral” order is the best

Answers and Questions

Band structure



Observable: Hall conductivity (mean field Hubbard)



$$L = [-r + \omega^2 - (q - Q_\alpha)^2] |S_\alpha|^2 - u |S_\alpha|^4 + v [S_a \cdot (S_b \times S_c)]^2$$

Questions:

- role of fluctuations?
- topological defects (Z2 vortices, domain walls)?
- topological doping, superconductivity?
- connection to quantum limit ($S \gg 1 \rightarrow S \sim 1$)?