Partial Thermalizations Allow for Optimal Thermodynamic Processes

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Joint work with…

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Szilard Engine

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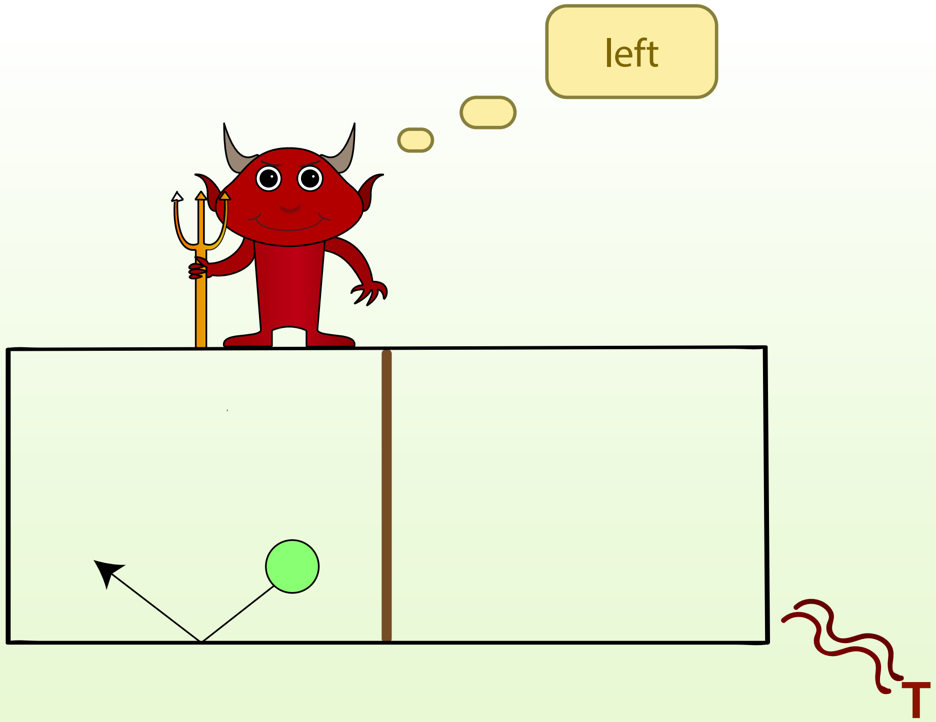

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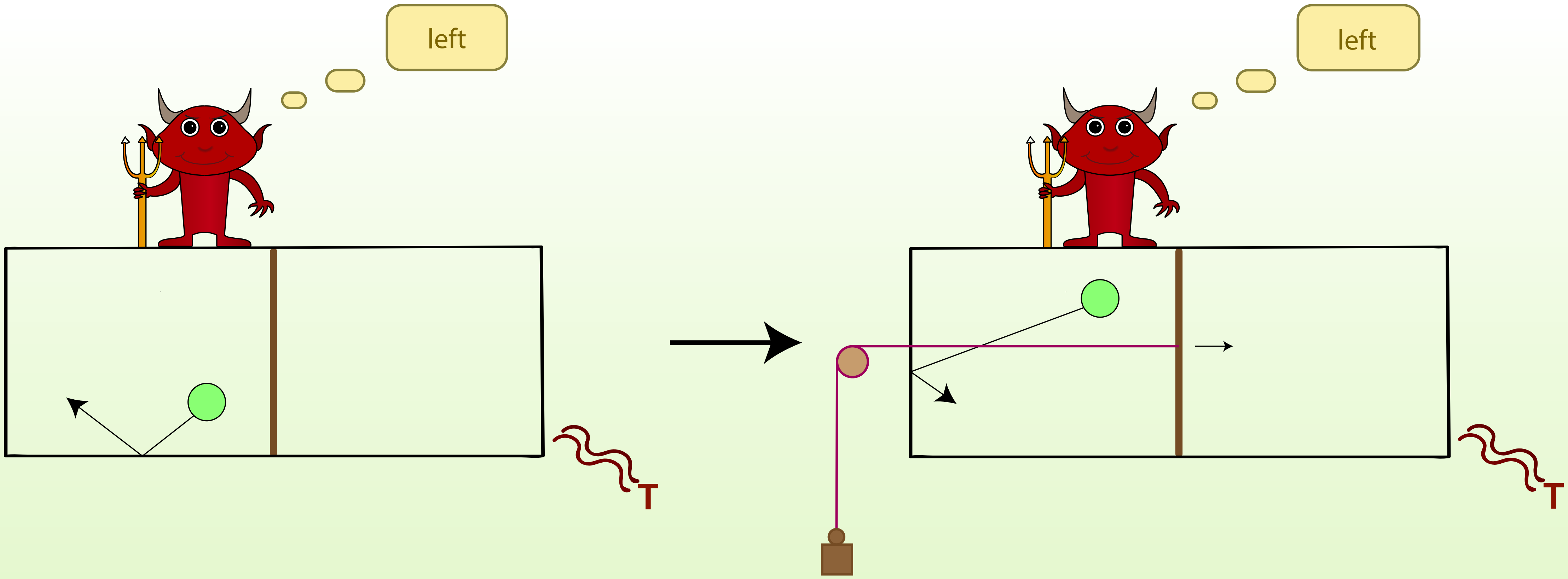

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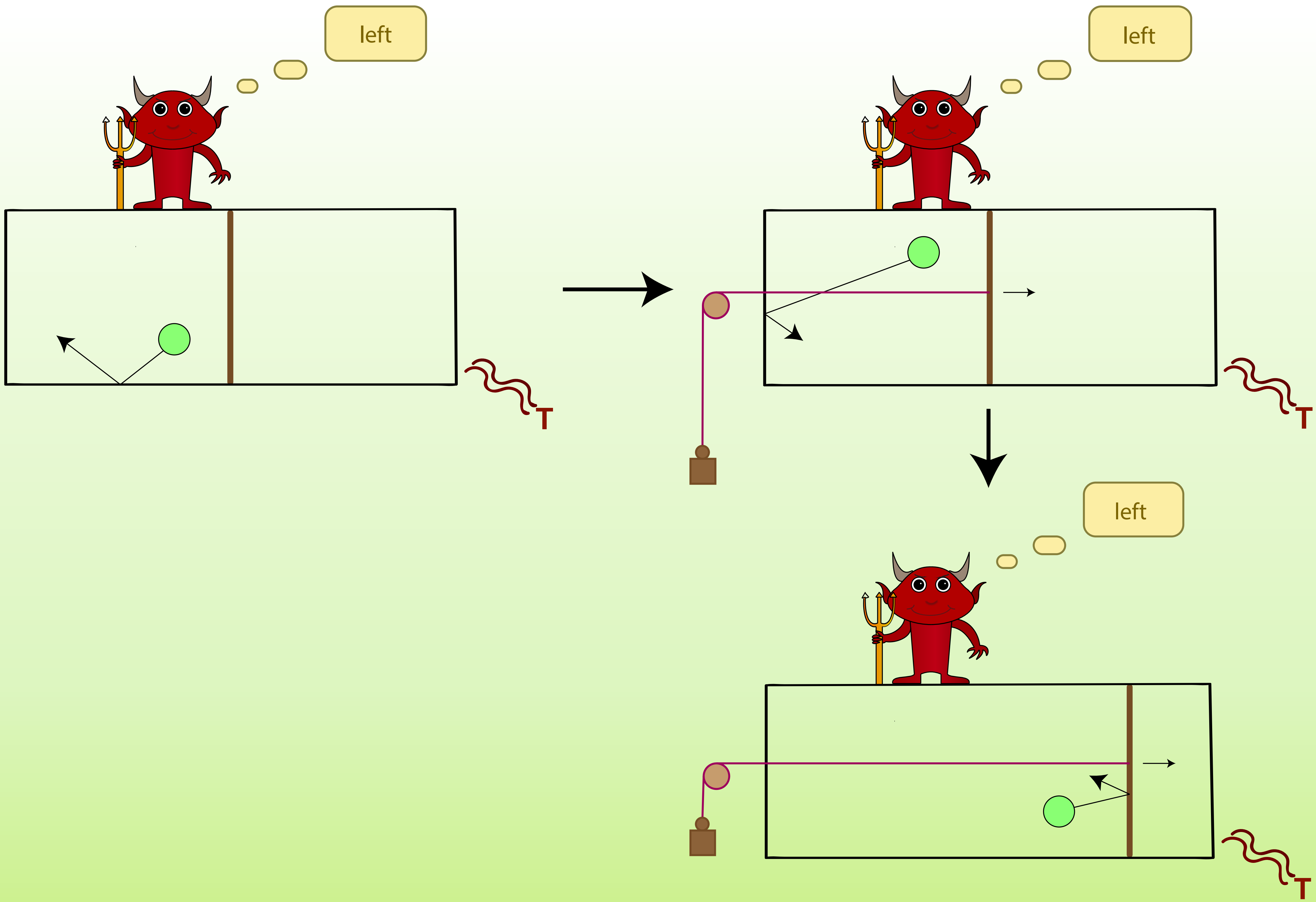

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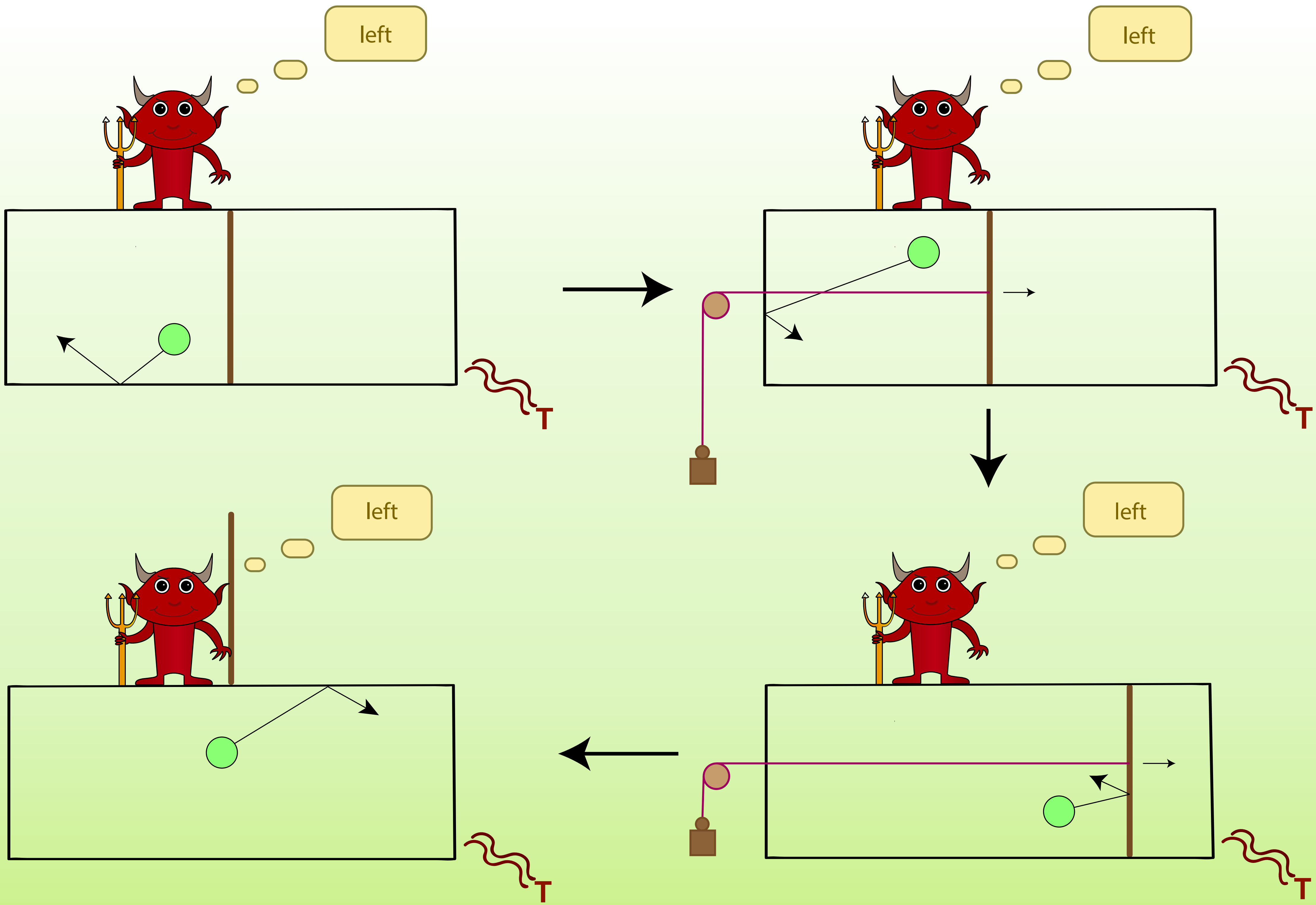

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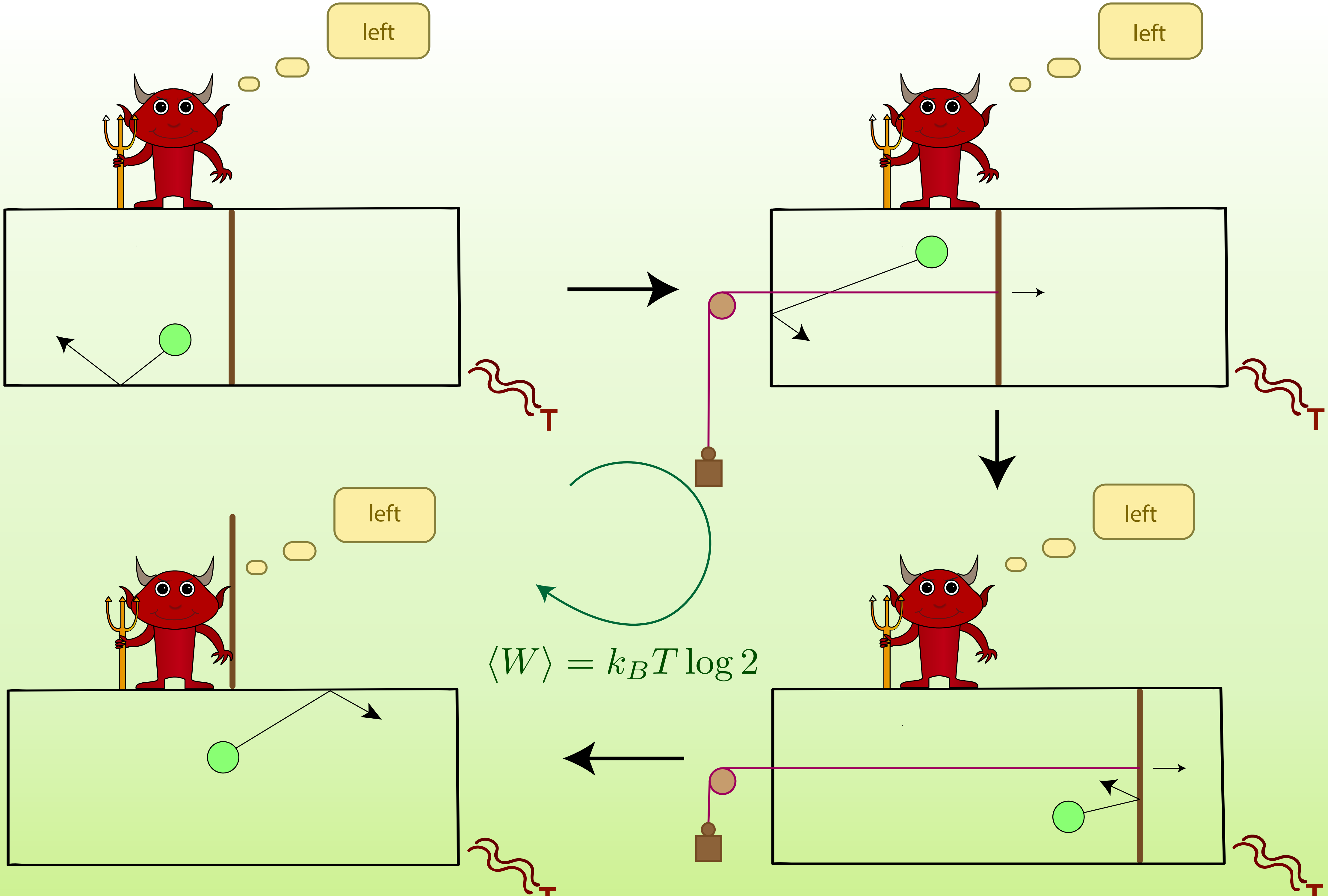

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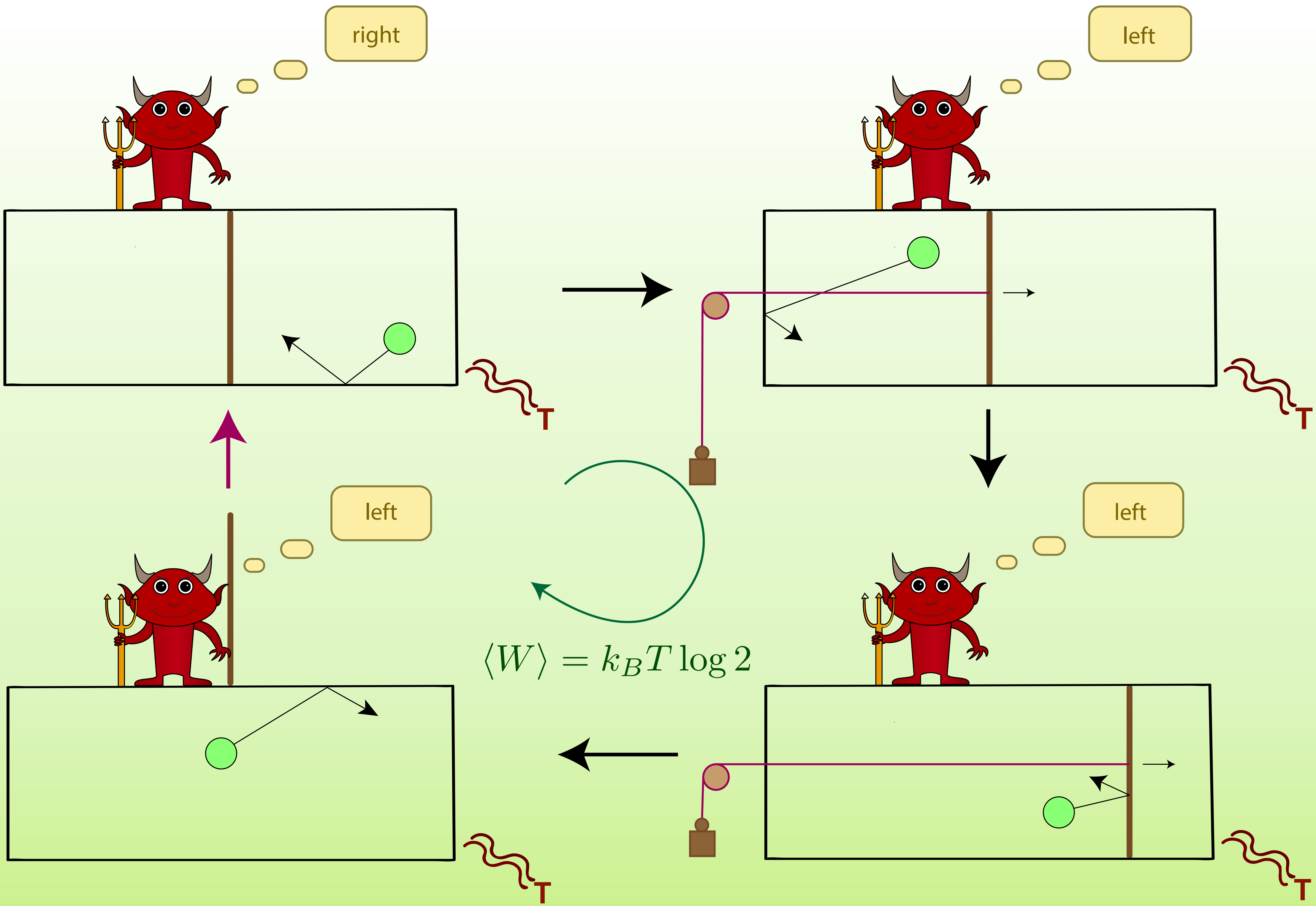

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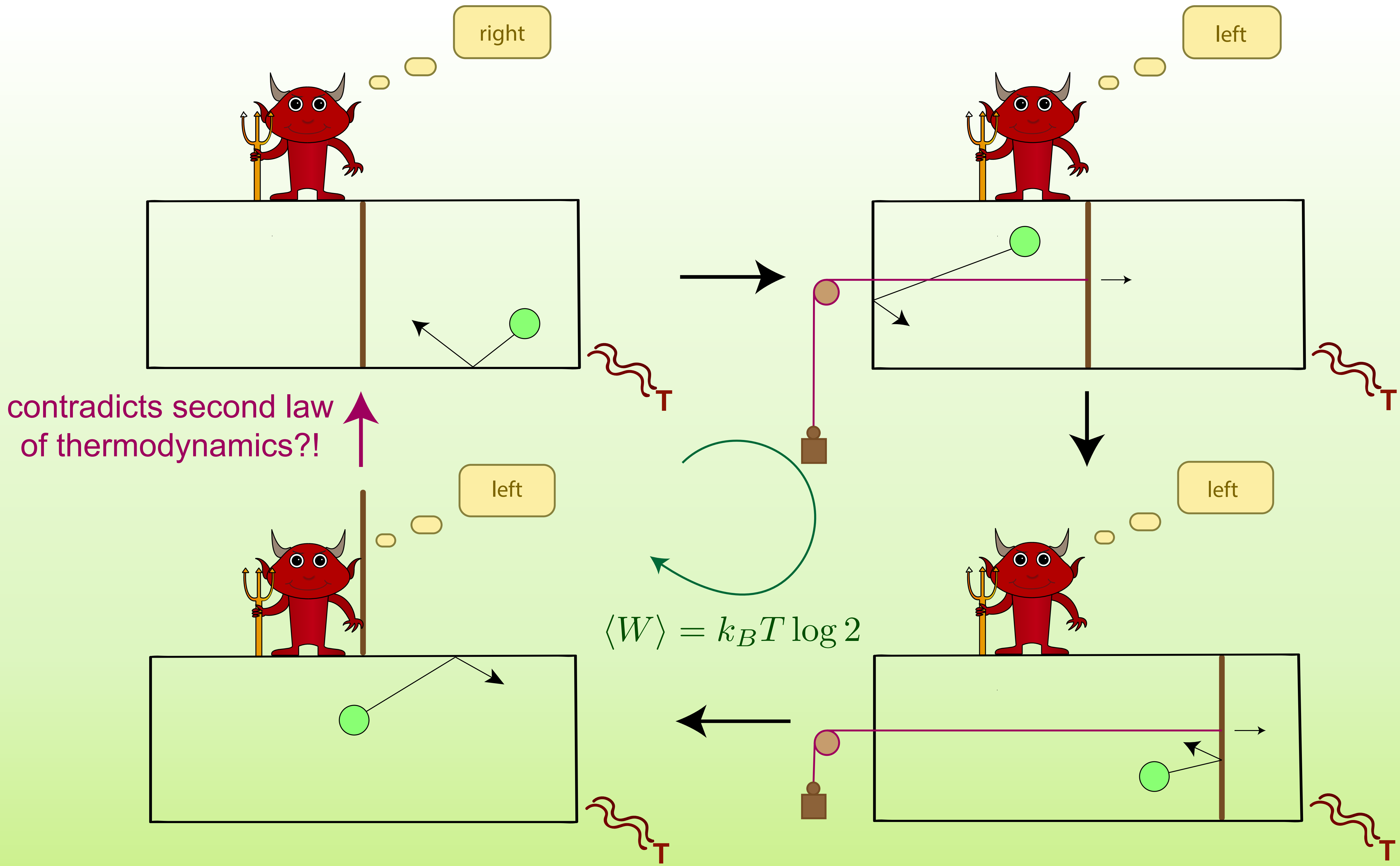

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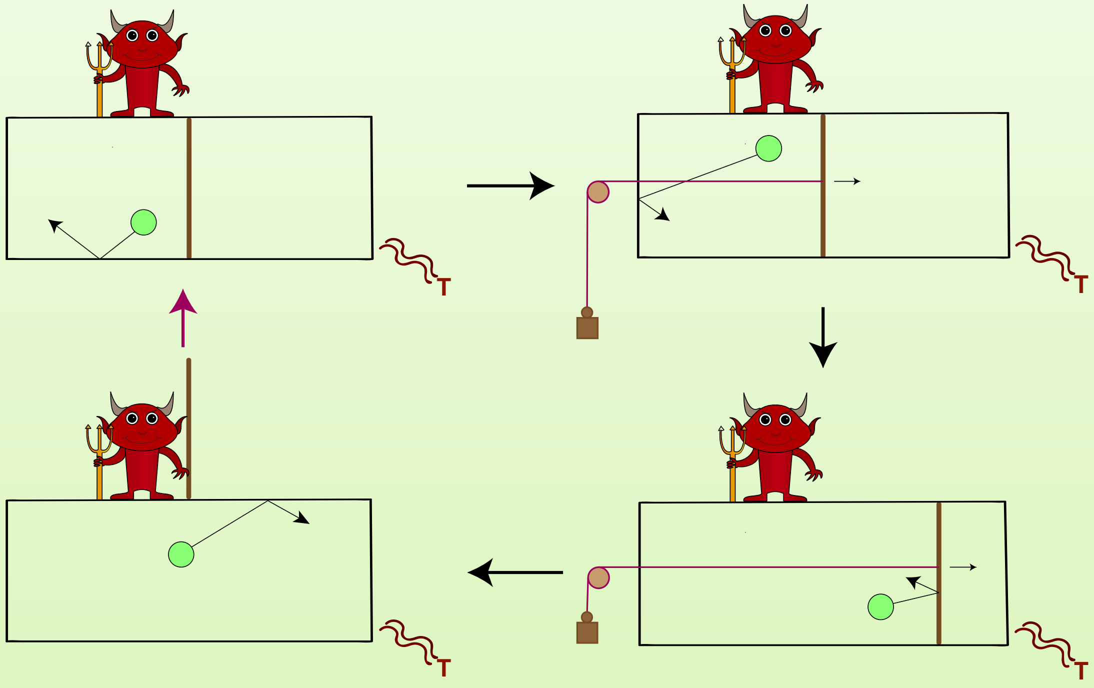
  - ✴ Box and demon viewed together from the outside: whole cycle can be described ➡ here the demon's bit is unknown

Szilard Engine
(view from outside)



Szilard Engine (view from outside)

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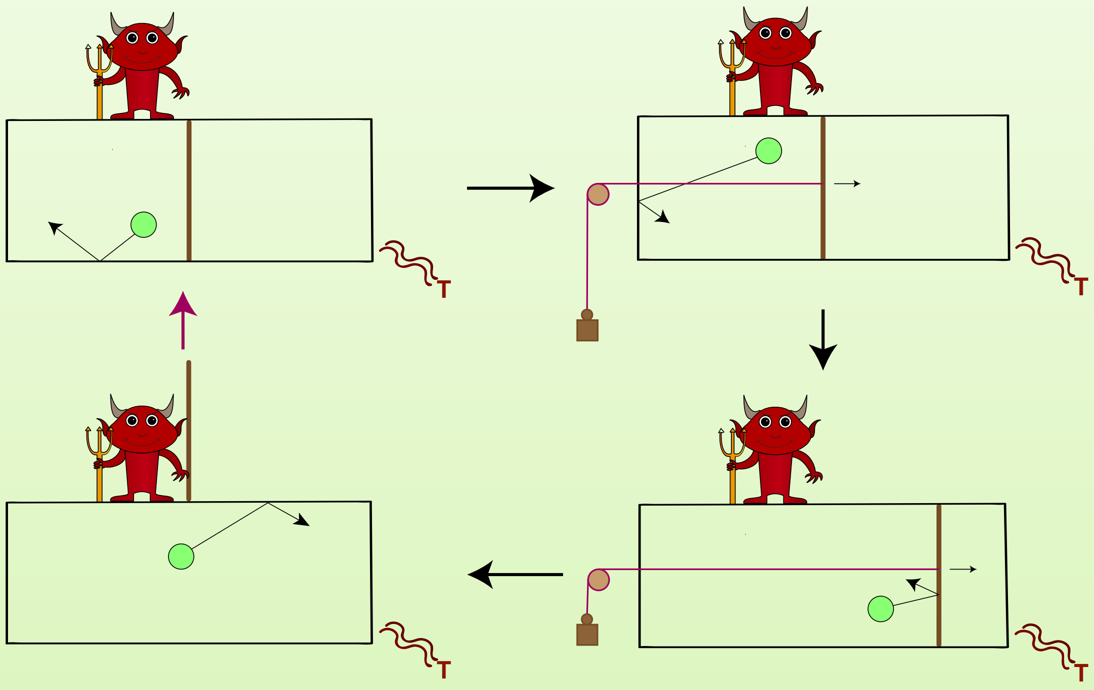

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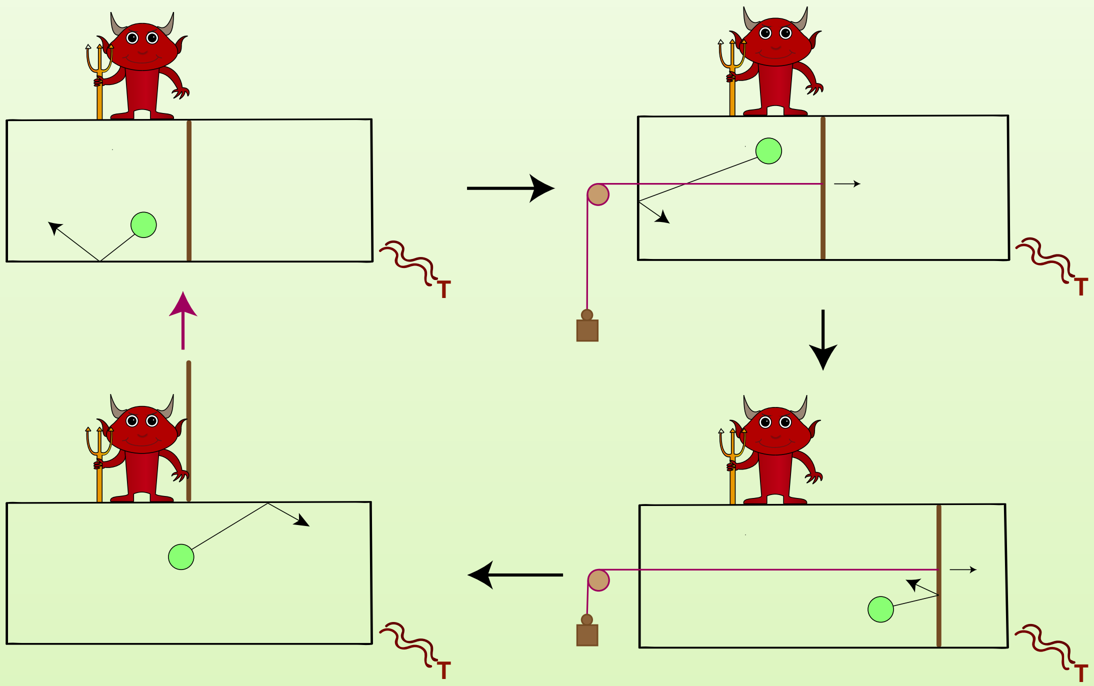

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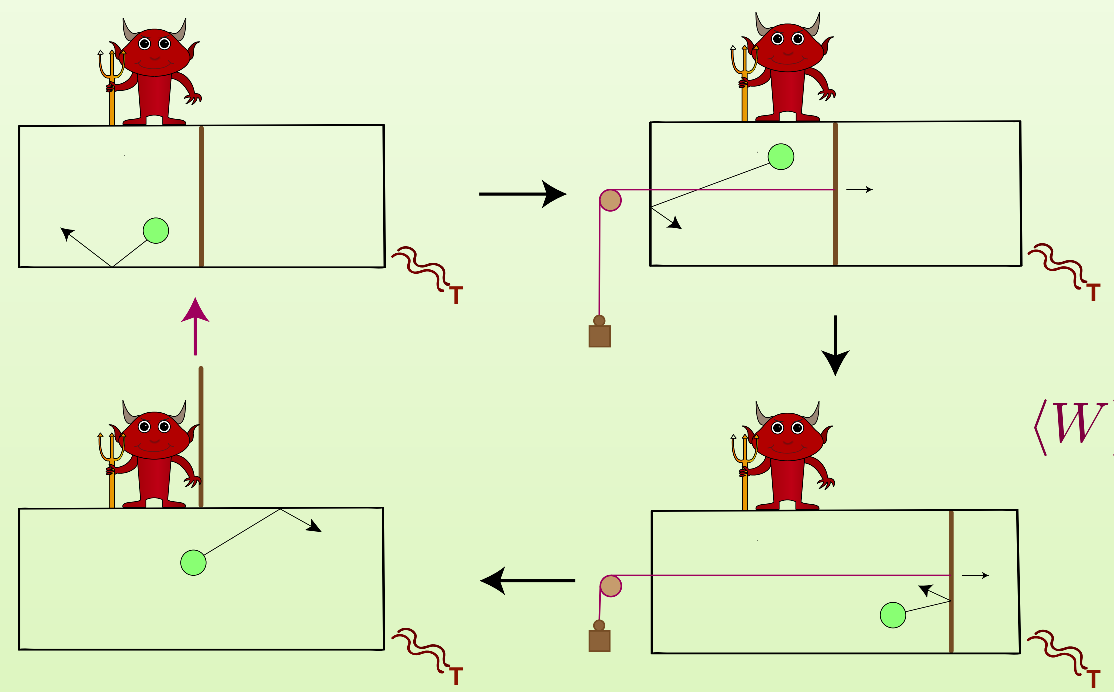

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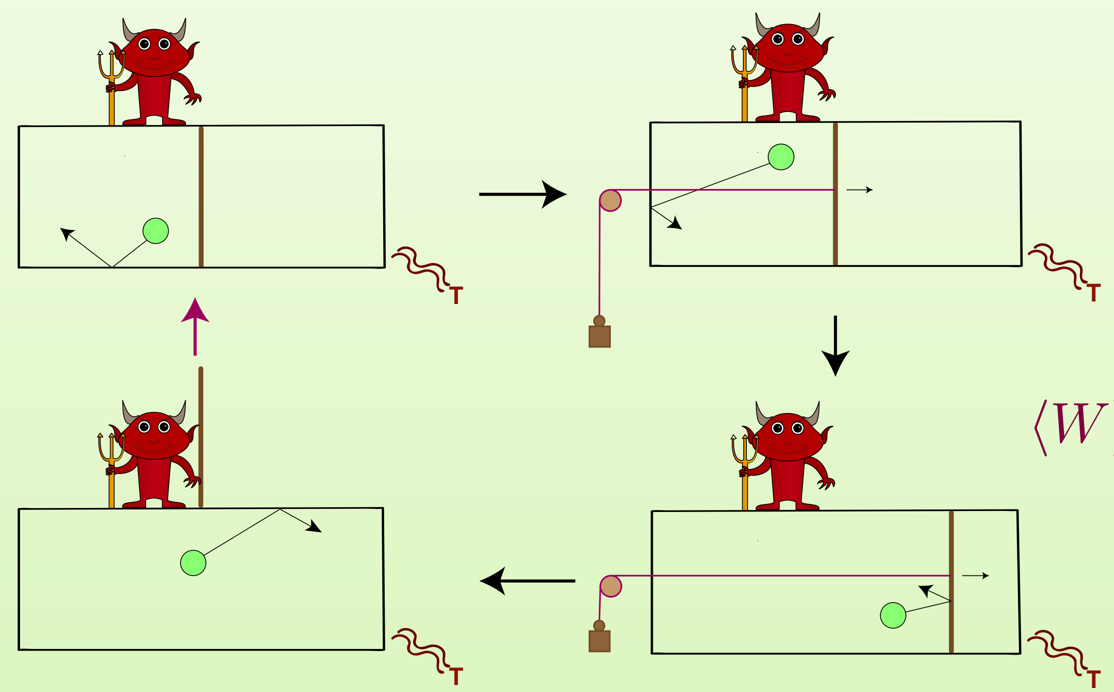

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- **Main result:** Optimal isothermal processes are possible for any $\alpha < 1$

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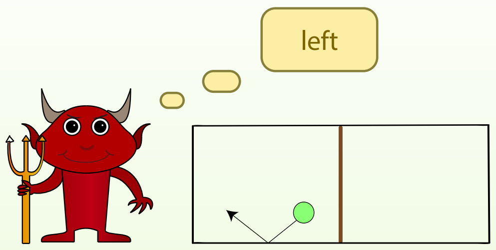

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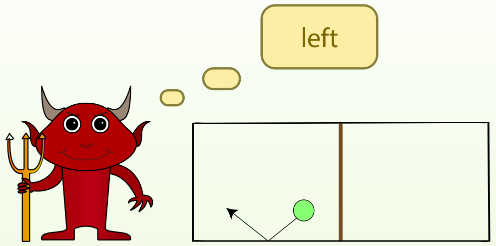
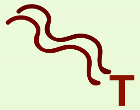


  - ✴ Thermal bath B at fixed temperature T: N thermal states (free resource) with different Hamiltonians $H_B^{(k)},\ k=0,...,N$



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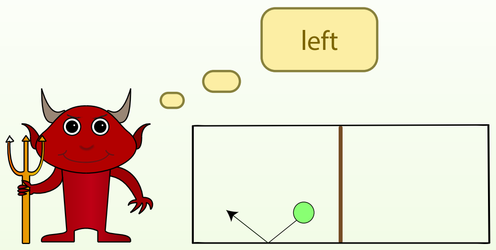
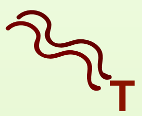
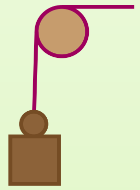
  - ✳ System S of one information qubit 

  - ✳ Thermal bath B at fixed temperature T: N thermal states (free resource) with different Hamiltonians $H_B^{(k)},\ k = 0, ..., N$ 

  - ✳ Work storage system W 

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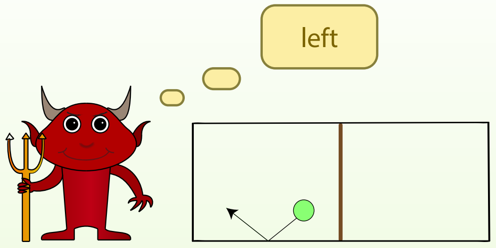
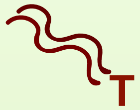
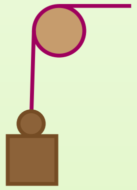
  - ✳ System S of one information qubit 

  - ✳ Thermal bath B at fixed temperature T: N thermal states (free resource) with different Hamiltonians $H_B^{(k)},\ k = 0, ..., N$ 

  - ✳ Work storage system W 

➡ Using the information of the system qubit, we apply N thermal operations to convert heat from the coupled thermal bath B into work stored in system W:

In the k^{th} interaction step the energy-conserving unitary $U_{\mathrm{SBW}}^{(k)}$ acts on S, W and the k^{th} bath qubit

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➡ Partial thermalization, where the degree of thermalization is quantified by α :

  - For $\alpha = 0$: standard case of full thermalization
  - For $\alpha = 1$: no interaction between S, B, W

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$\ast\ \gamma = \mathcal{O}\left(\frac{1}{N}\right)$: Error due to finite number of steps

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- Proof can be extended to qudits

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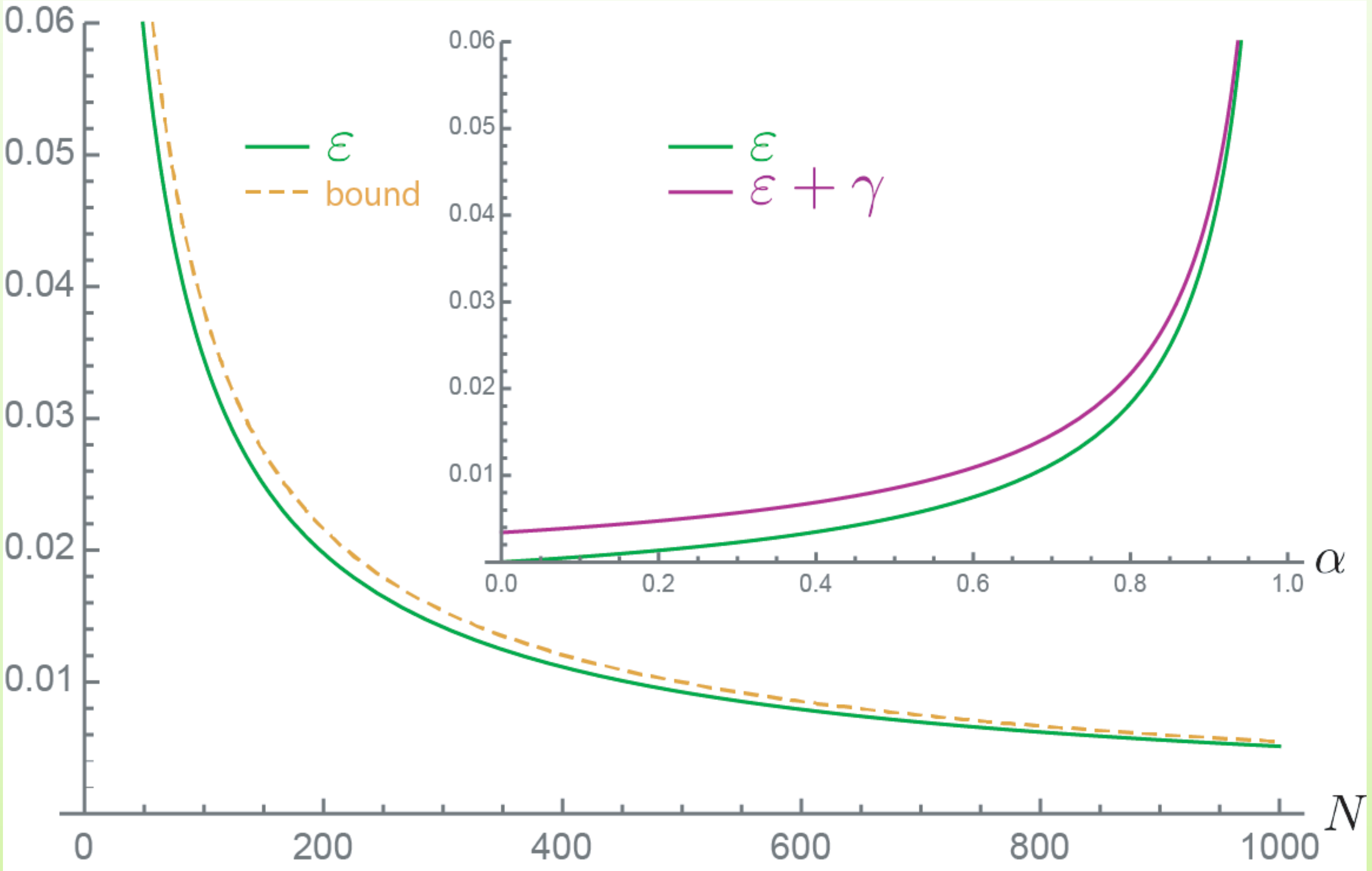
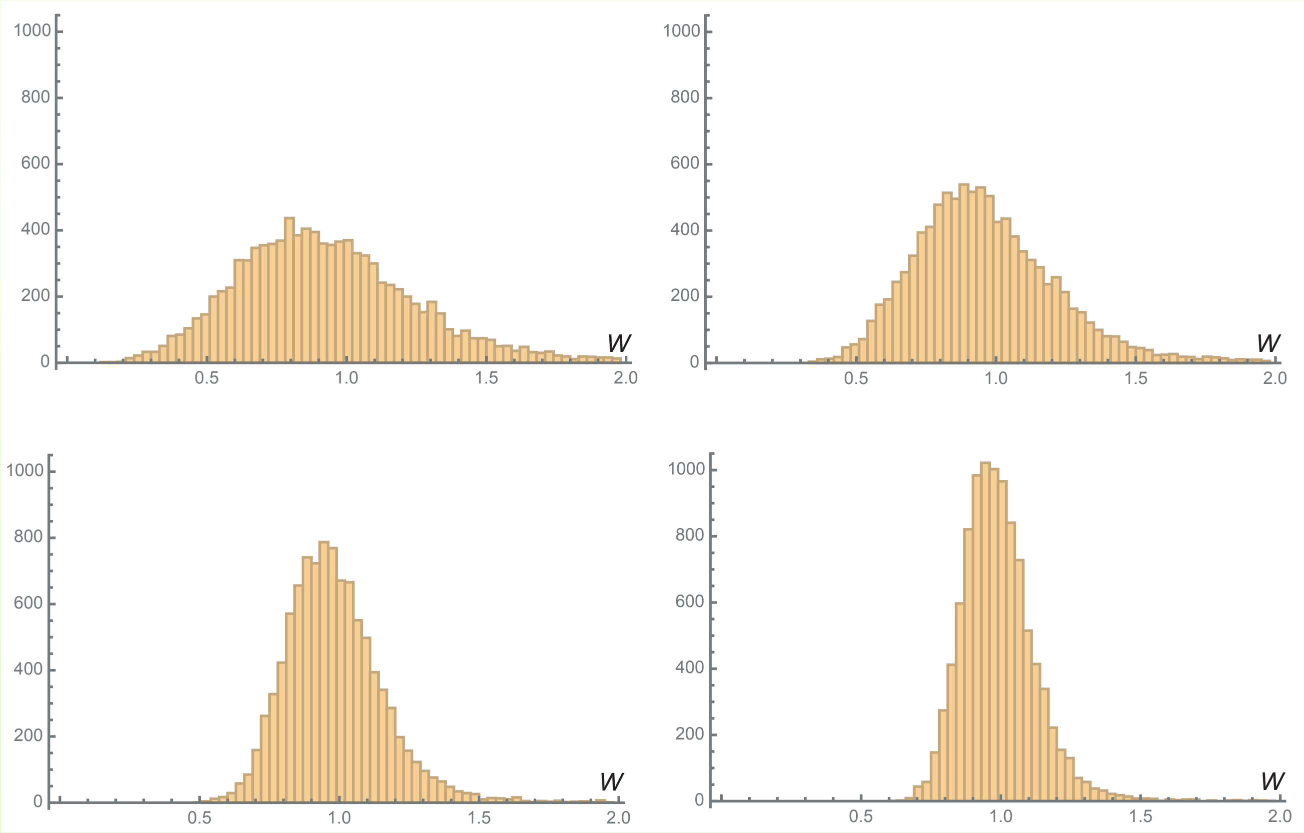
- Determined an almost tight upper bound for a specific example



error as a function of N for $\alpha = 1/2$ and of α for $N = 1000$, respectively, with $k_B T \log 2 = 1$, $p_k = k/2N$

Results

- Characterized the work fluctuations which decrease for large N



histrograms showing the fluctuations for $N = 100$, $N = 200$, $N = 500$ and $N = 1000$

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- Evolution described by N Gibbs preserving maps G_k :

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➡ Optimal processes are much more common than previously expected in small quantum systems