Fluctuation theorems for non-equilibrium, strongly-coupled environments

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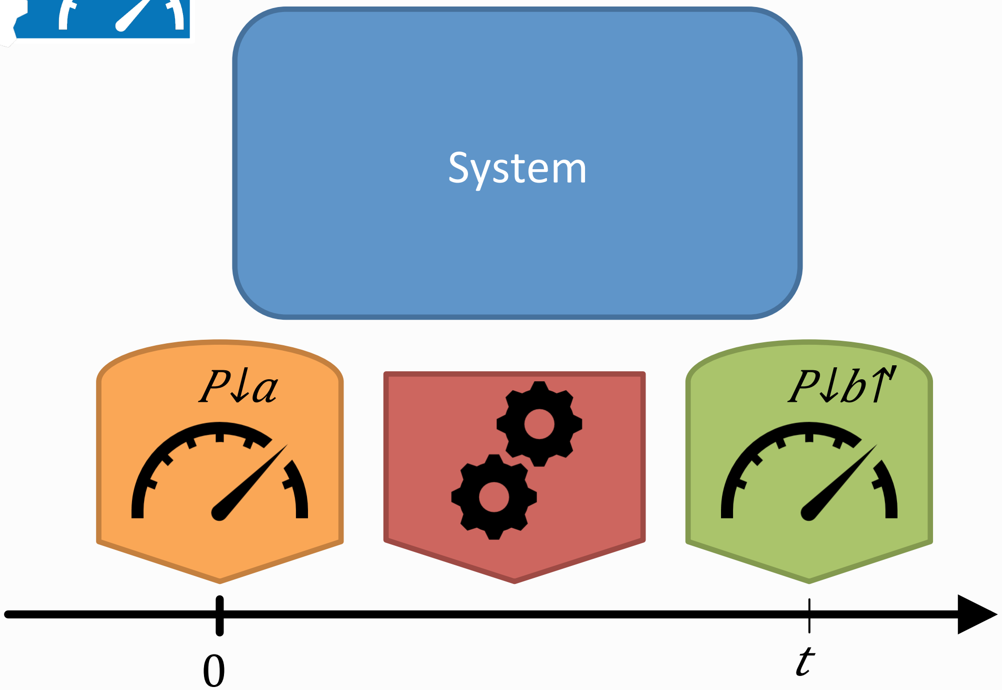
Outline

- Symmetry of generating functions
- Fluctuation theorems and open quantum systems
- Properties
- Applications



State

$$U(t) P_a \rho(0) P_a U^\dagger(t) \quad P_b\, U(t) P_a \rho(0) P_a U^\dagger(t) P_b$$

1. Projective measurement $A = \sum_a a P_a$
2. Non-equilibrium protocol
3. Projective meaurement $B = \sum_b b P_b$

- Measurement of the system energy: work fluctuations
  $A = H_0$
  $B = H_t$
- Initially thermal distribution

Probability distribution

$$p(b,a,t) = Tr[P_b U(t) P_a \rho(0) P_a U^\dagger(t) P_b]$$

Generating function

$$G(\chi_b, \chi_a, t) = Tr[e^{i\chi_b B} U(t) e^{i\chi_a/2 A} \rho(0) e^{i\chi_a/2 A} U^\dagger(t)] \qquad \rho(0) = e^{-\beta H_0} / Z_0(\beta)$$

$$G(\chi_b, \chi_a, t, \beta)$$

Symmetries of the work statistics

Generating function

$G(\chi_b,\chi_a,t,\beta)=Tr[e^{i\chi_b H_t}\, U(t) e^{i\chi_a H_0}\, e^{-\beta H_0}/Z_0(\beta)\, U^\dagger(t)] = \langle e^{i\chi_b H_t}(t)\, e^{i\chi_a H_0}\rangle_\beta$

1. $\chi_b = i\beta$ and $\chi_a = -i\beta$

$G(i\beta,-i\beta,t,\beta)=Tr[e^{-\beta H_t}\, U(t) e^{\beta H_0}\, e^{-\beta H_0}/Z_0(\beta)\, U^\dagger(t)]$

Jarzynski equality

$\langle e^{-\beta W}\rangle_\beta = Z_t(\beta)/Z_0(\beta)$

2. $\chi_b = -\chi_a = \chi$ and time reversed $\chi = -\chi + i\beta$

$G(\chi,-\chi,t,\beta)=Tr[e^{i\chi H_t}\, U(t) e^{-i\chi H_0}\, e^{-\beta H_0}/Z_0(\beta)\, U^\dagger(t)]$

Fluctuation theorem

$\ldots = Z_t(\beta)/Z_0(\beta) \langle e \ldots$

3. $\chi_b = -\chi_a = \chi$ and $\beta = \beta - i\chi$

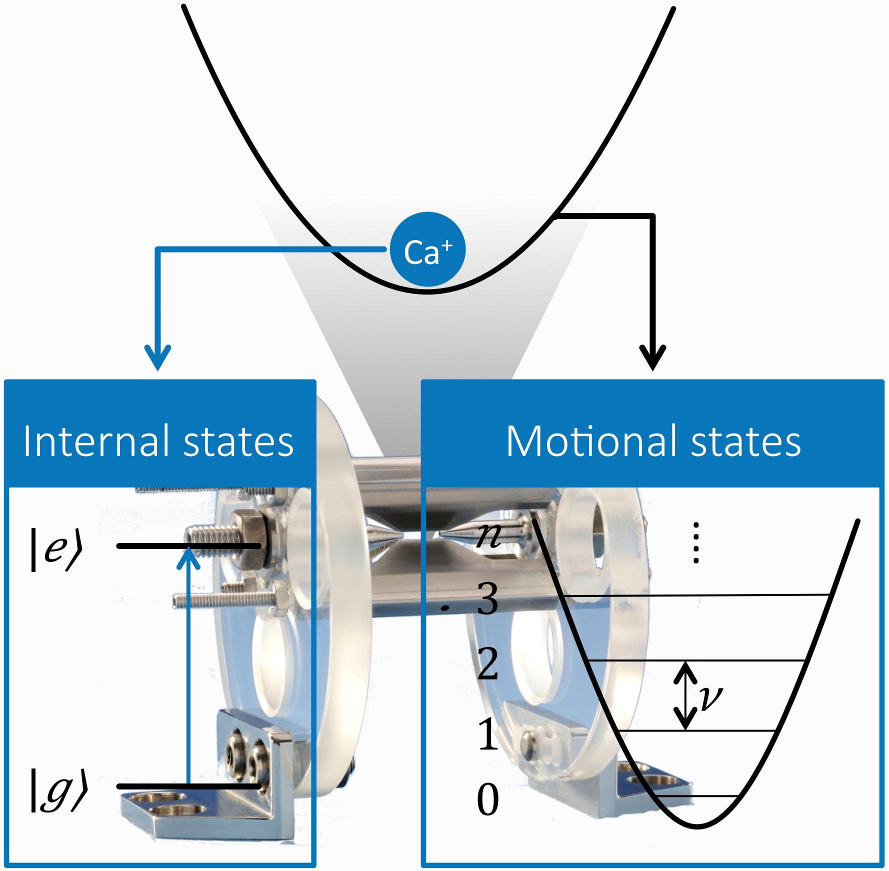
$G(\chi,-\chi,t,\beta-i\chi)=Tr[e^{i\chi H_t}\, U(t) e^{-i\chi H_0}\, e^{-(\beta-i\chi) H_0} \ldots$

Proposal

$\langle e^{i\chi W}\rangle_{\beta-i\chi} \approx Z_0(\beta)/Z_0(\beta-i\chi)$

$\mathbf{G(\chi, \ldots, \beta - i\chi) \ldots G(\chi, 0, t, \beta)/ \ldots}$

Trapped ions



Motional states as an isolated quantum system

✓ Employing trapped cold ions to verify the quantum Jarzynski equality.

G. Huber, F. Schmidt-Kaler, S. Deffner and E. Lutz, *PRL 101*, 070403 (2008).

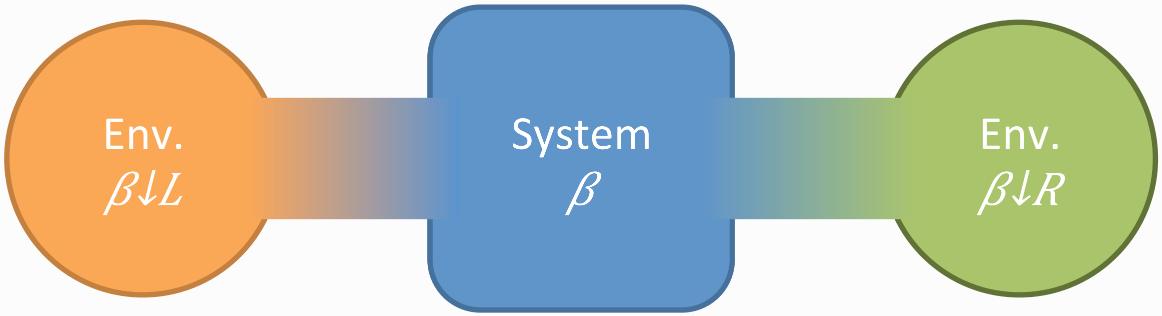
✓ Experimental test of the quantum Jarzynski equality with a trapped-ion system.

S. An, … K. Kim, *Nature Physics 11*, 193 (2015).

Motional states as an open quantum system?

General open quantum systems

Strong coupling to non-Markovian baths



Jarzynski equality for initial global equilibrium ($\beta_R = \beta_L = \beta$) and driving only on the system

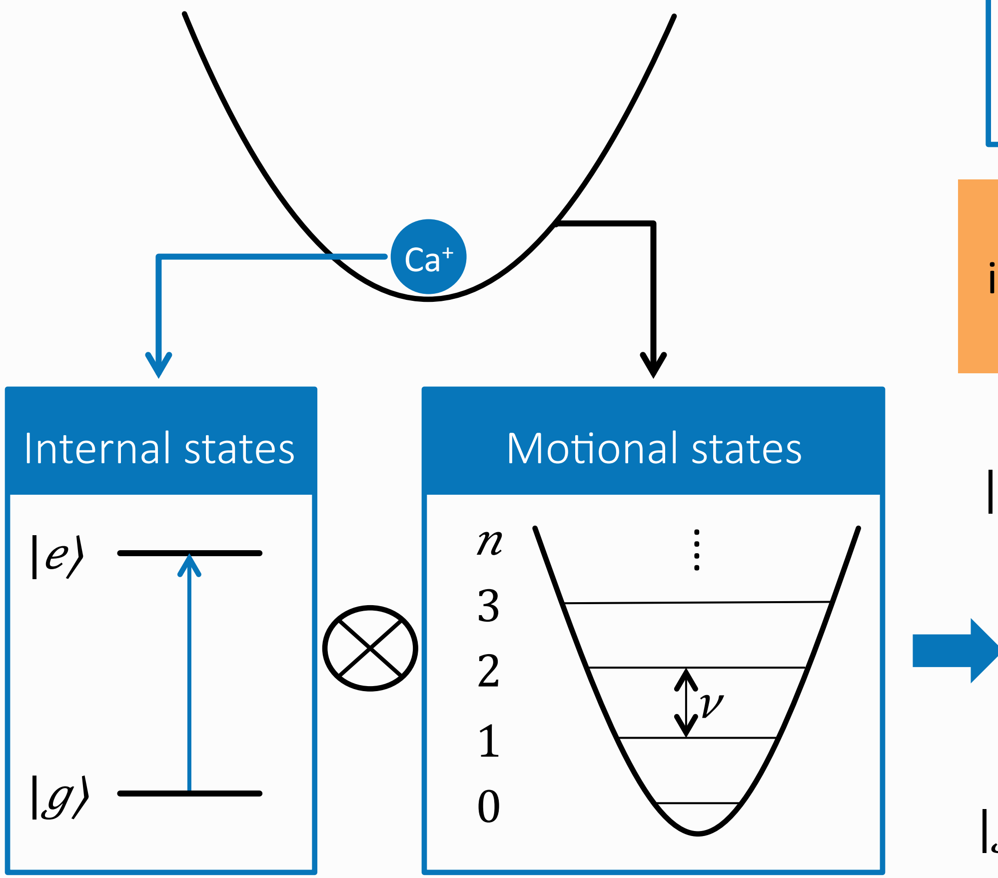
M. Campisi, P. Talkner and P. Hänggi, *PRL 102*, 210401 (2009).

Steady state fluctuation theorem for Markovian thermal environments (Non-equilibrium N-spin-boson model) and weaker form for non-Markovian

L. Nicolin and D. Segal, *PRB 84*, 161414(R) (2011).
L. Nicolin and D. Segal, *JCP 135*, 164106 (2011).

Transient fluctuation theorem for arbitrary driving (and arbitrary environmental state)

Sideband cooling



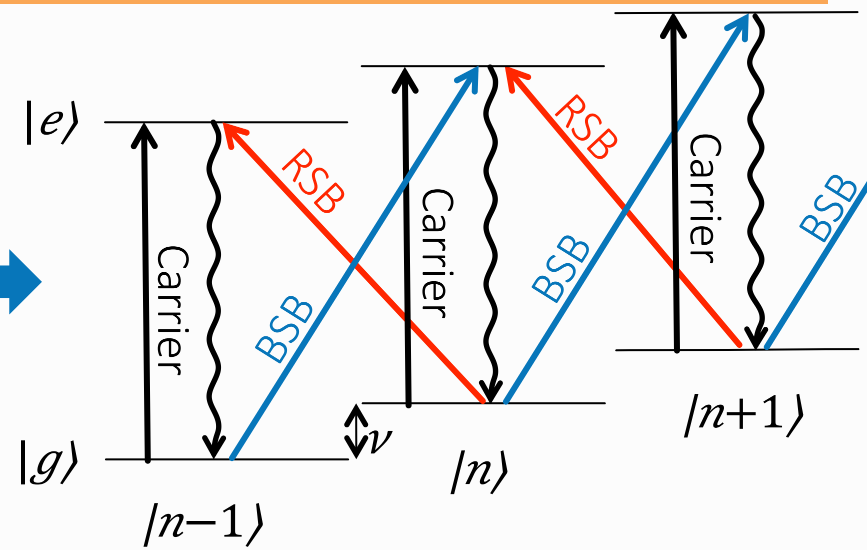
Cooling rate

$R \propto \eta^2 \Omega^2$

Phonon number

$\langle n \rangle \propto \Omega^2$

Environment of the motional states involves the „internal states“ and it is in a non-equilibrium state



C. Monroe, D. M. Meekhof, B. E. King, S. R. Jefferts, W. M. Itano, and D. J. Wineland, PRL **75**, 4011 (1995).

Properties

$$\langle e^{i\chi w}\rangle_{\beta-i\chi}=\frac{Z_0(\beta)}{Z_0(\beta-i\chi)}\langle e^{i\chi H_t(t)}\rangle_{\beta}$$

- Relates two non-equilibrium distributions
  - statistics of energy transfer at an initial temperature $\langle e^{i\chi w}\rangle_{\beta-i\chi}$ to
  - final energy distribution with a different temperature $\langle e^{i\chi H_t(t)}\rangle_{\beta}$.
- Relation holds for
  - strong coupling to non-Markovian environments
  - non-thermal states in the environment ρ_{Env}

since $\langle e^{i\chi H_t(t)}\rangle_{\beta}=Tr[e^{i\chi H_t}\,U(t)\,e^{-\beta H_0}/Z_0(\beta)\otimes\rho_{Env}\,U^{\dagger}(t)]$

- Jarzynski-like limit

$$\langle e^{\beta w}\rangle_{\beta}=Z_0(2\beta)\langle e^{\beta H_t(t)}\rangle_{2\beta}/Z_0(\beta)$$

(note sign change of β, compare to Hamiltonian of mean force)

- Fluctuation-theorem-like limit

$$C_{TPM}(\chi,\beta,t)=C_{OPM}(\chi,\beta+i\chi,t)-C_{OPM}(\chi,\beta+i\chi,0)$$

Cumulant generating functions, compare with $C(\Delta\chi)=C^{tr}(-\Delta\chi+i\Delta\beta)$

- Non-linear response-like limit (propagator of energy distribution).

Propagator of energy distribution

$\langle e^{i\chi w}\rangle_{\beta-i\chi} = Z_0(\beta)/Z_0(\beta-i\chi)\,\langle e^{i\chi H_t(t)}\rangle_{\beta}{}^{\uparrow}$

$P(\chi,\beta,t)=\langle e^{i\chi w}\rangle_{\beta-i\chi}$

$p(E,t)=\int dE'\,P(E-E',\beta,t)\,p(E',0)$

- When is $p(E,t)$ compatible with a thermal distribution β?

$\langle e^{i\chi w}\rangle_{\beta-i\chi} = Z_0(\beta)/Z_0(\beta-i\chi)\; Z_t(\beta_t-i\chi)/Z_t($

Open bosonic mode of frequency Ω, $Z(\beta)=1/1-e^{-\beta\Omega}$

$$\lim_{t\to 0} d/dt\,\langle e^{i\chi w}\rangle_{\beta-i\chi} = d\beta/dt\,[Z(\beta-i\chi)-Z(\beta)]$$

Born Markov Secular master equation reproduces that limit

$$d/dt\,\rho(t)=-i\Omega[a^{\dagger}a,\rho(t)]+\gamma(n+1)[2a\rho(t)a^{\dagger}-a^{\dagger}a\rho(t)-\rho(t)a^{\dagger}a]+\gamma n\,[2a^{\dagger}\rho(t)a-aa^{\dagger}\rho(t)-\rho(t)aa^{\dagger}]$$

and a cooling rate $R=\gamma$ is well defined.

Note: In general slow thermalization does not imply thermal evolution (large bias).

Non-linear response

- Linear response: fluctuation dissipation theorem

$$R \propto \lim_{t\to\infty} \int e^{i\omega\tau} \langle n(t+\tau)n(t)\rangle_{\beta} \, d\tau.$$

- The rate is also valid for high bias.
- Otherwise we choose the definition

$$R(\beta_0, t) = \partial\langle n \rangle / \partial\langle n \rangle = \frac{\partial n / \partial\beta_0}{\partial n / \partial\beta_0} = \langle n^2(t)\rangle_{\beta_0} / \langle n^2(t)\rangle_{\beta_0}.$$

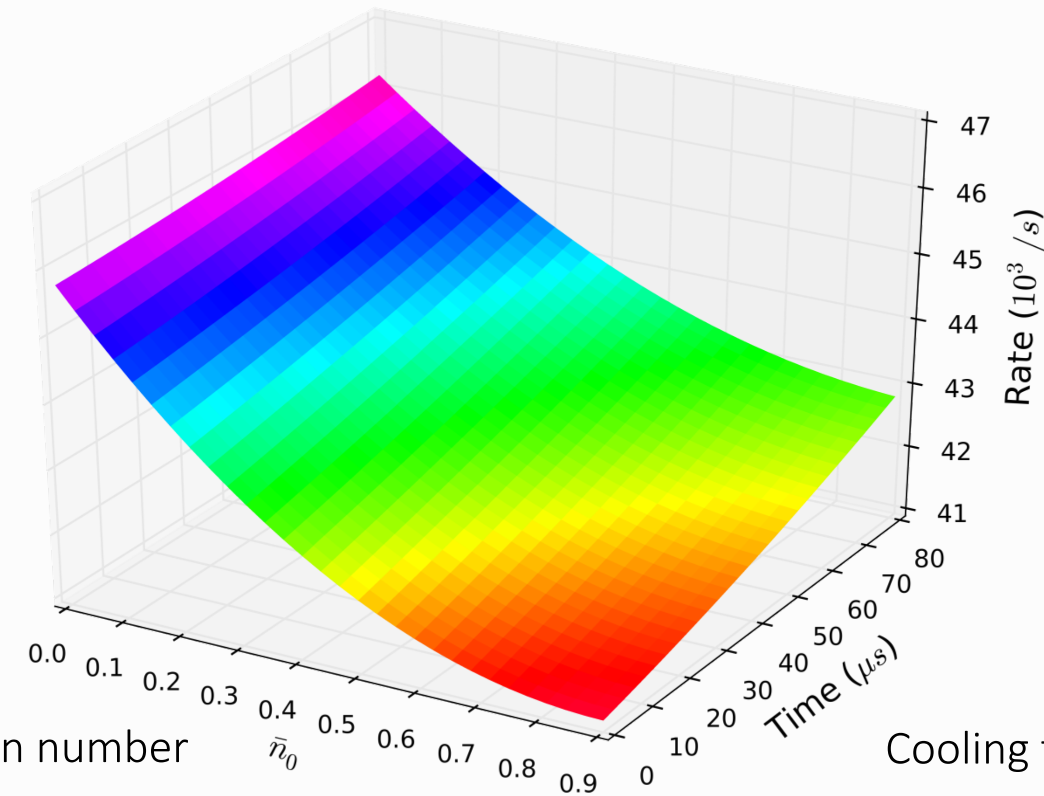
Since $C_{TPM}(\chi,\beta,t) = C_{OPM}(\chi,\beta+i\chi,t) - C_{OPM}(\chi,\beta+i\chi,0)$

- The cooling rate of an ion is the coefficient of the linearized differential equation associated to the energy current.
- In general, the cooling speed will depend on the initial temperature and time, since evolution is not anymore within a thermal manifold of states.

N. Scharnhorst, JC et al., Multi-mode double-bright EIT cooling, arXiv:1711.00738
N. Scharnhorst, JC et al., Exp. and theo. aspects of double-bright EIT cooling, arXiv:1711.00732

Full master prediction for cooling rate

Time dependent axial single EIT cooling rate R(t, n_0) at $\Delta=3\Gamma$



Initial occupation number

Cooling time

Lamb-Dicke theory: $52.2\times10^{3}\ s^{-1}$
Measured: $38.2\times10^{3}\ s^{-1}$

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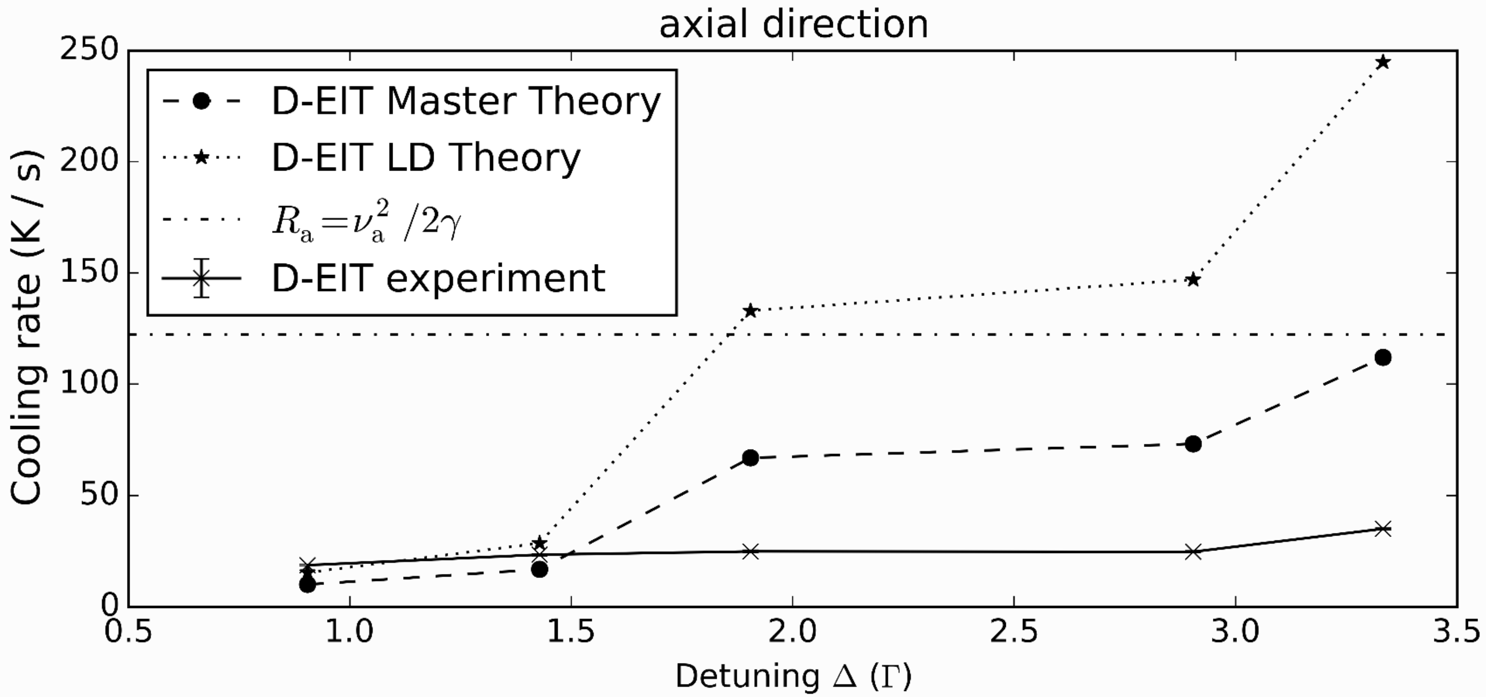
Full master prediction for cooling rate

Lamb Dicke, Master equation and Experimental cooling rates



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Summary

$$\langle e^{i\chi w}\rangle_{\beta-i\chi} = Z_0(\beta)/Z_0(\beta-i\chi)\,\langle e^{i\chi H_t(t)}\rangle_{\beta}$$

- Work statistics and non-equilibrium energy distribution are related by temperature shifts regardless of the nature of the environment.
- Only requirement is a thermal state of the system of interest.
- It produces a Jarzynski-like, FT-like and non-linear-response like relations.
- Rigorous definition and characterization of rate of cooling for laser control of trapped ions.
- Outlook
  - More measurements? Non-commuting (coherent)? simultaneous observables?
  - Characterization of non-adiabatic protocols?
  - Generating-function based inequalities?

J. Cerrillo, M. Buser and T. Brandes, PRB 94, 214308 (2016).

N. Scharnhorst, J. Cerrillo et al., Multi-mode double-bright EIT cooling, arXiv:1711.00738

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