Autonomous thermal rotor in the quantum regime

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In collaboration with

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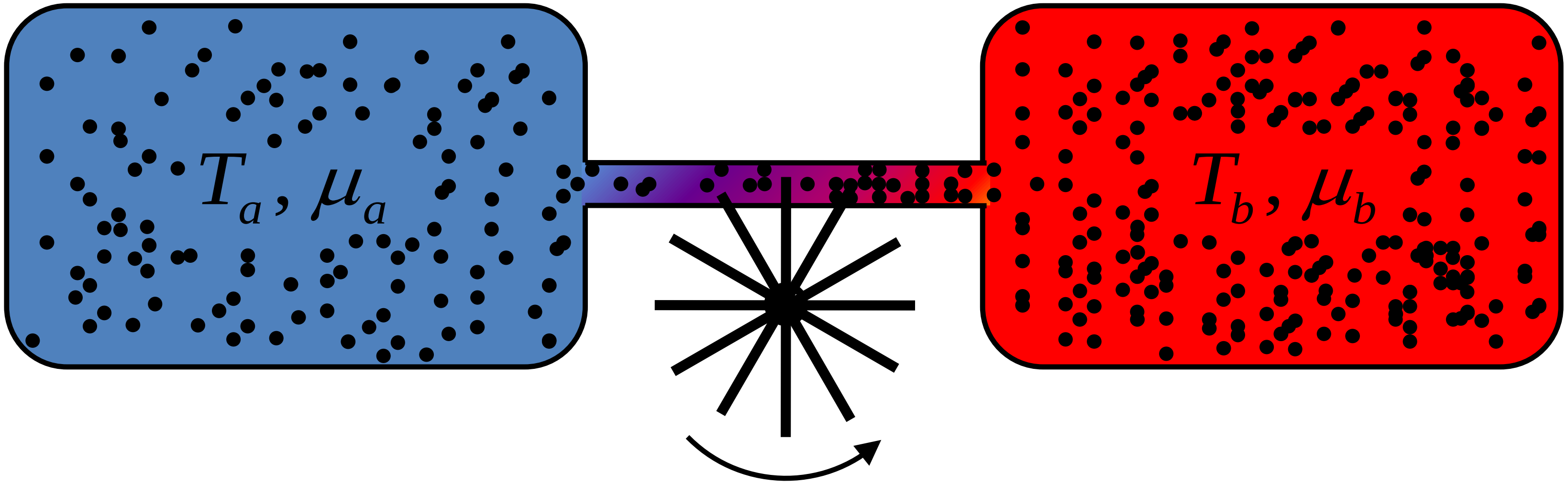
Plan of the talk

- Motivation
- Defining particle current
- Symmetries and physical properties

Introduction: Thermal → Mechanical



Introduction: Thermal → Mechanical

Smoluchowski-Feynman ratchet

broken spatial symmetry

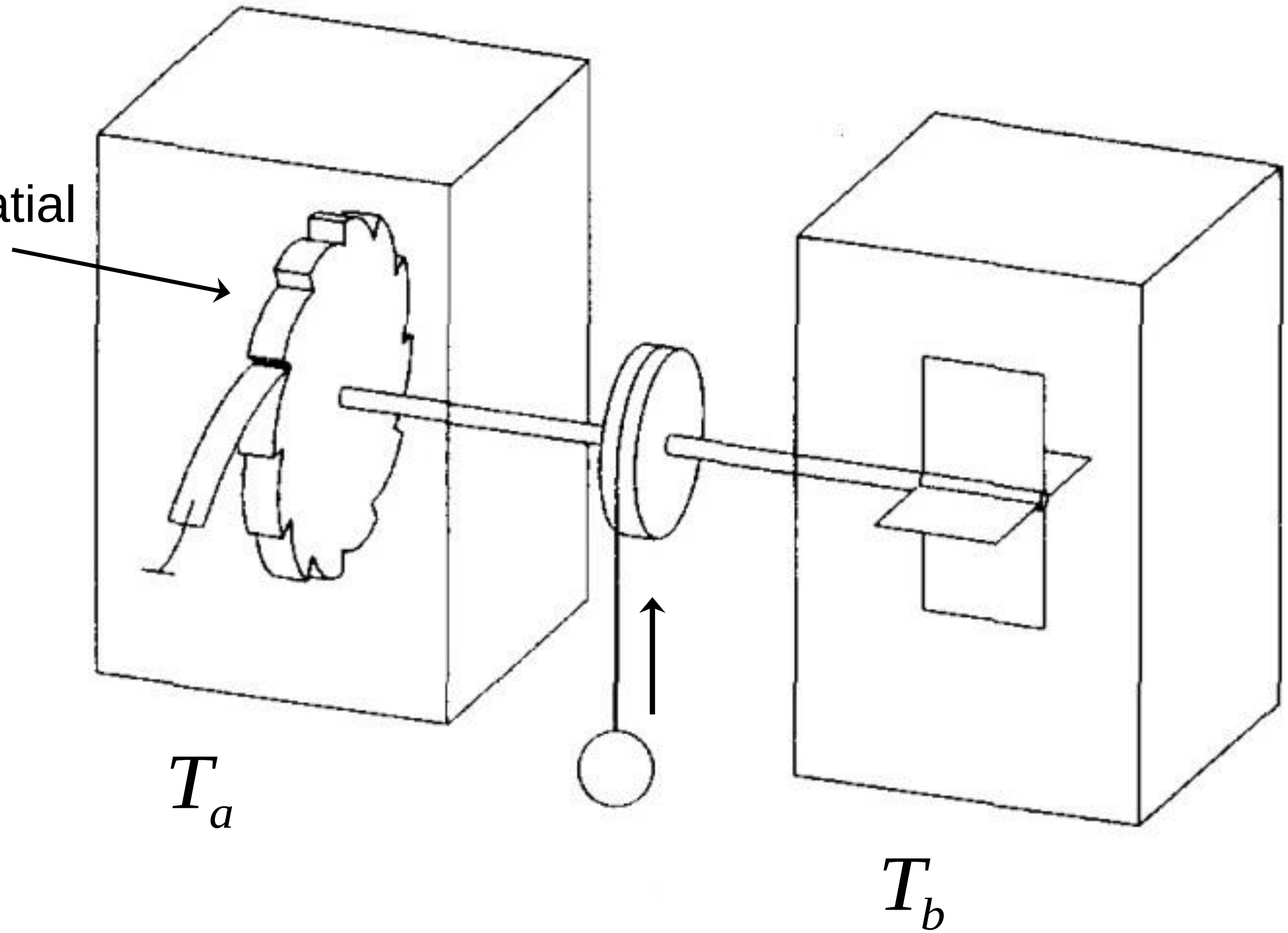
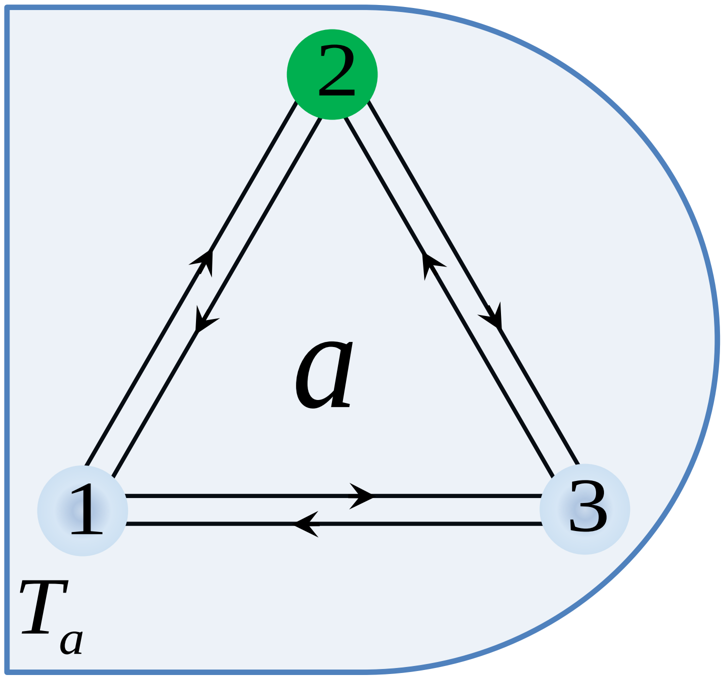


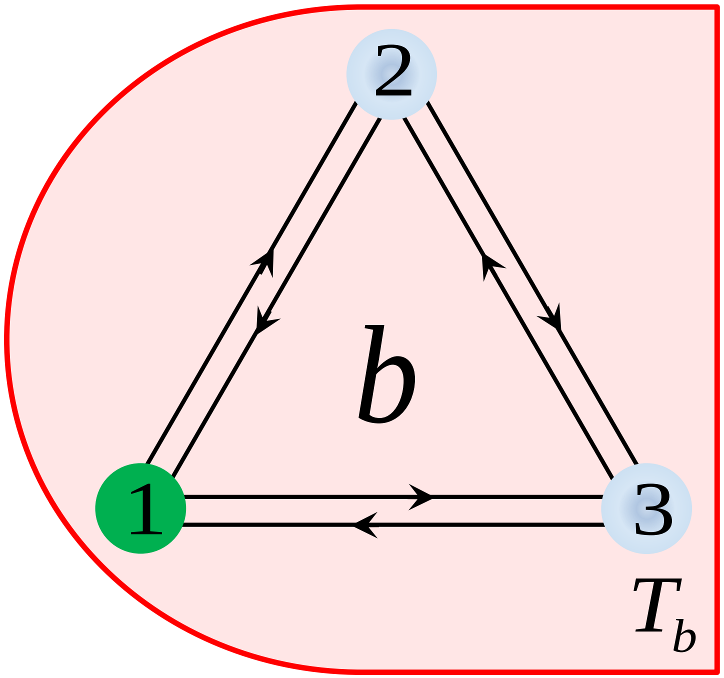
Figure taken from *Parrondo & Español, Am. J. Phys. 64, 1125 (1996)*
Reimann, Phys. Rep. 361, 57 (2002)

The model

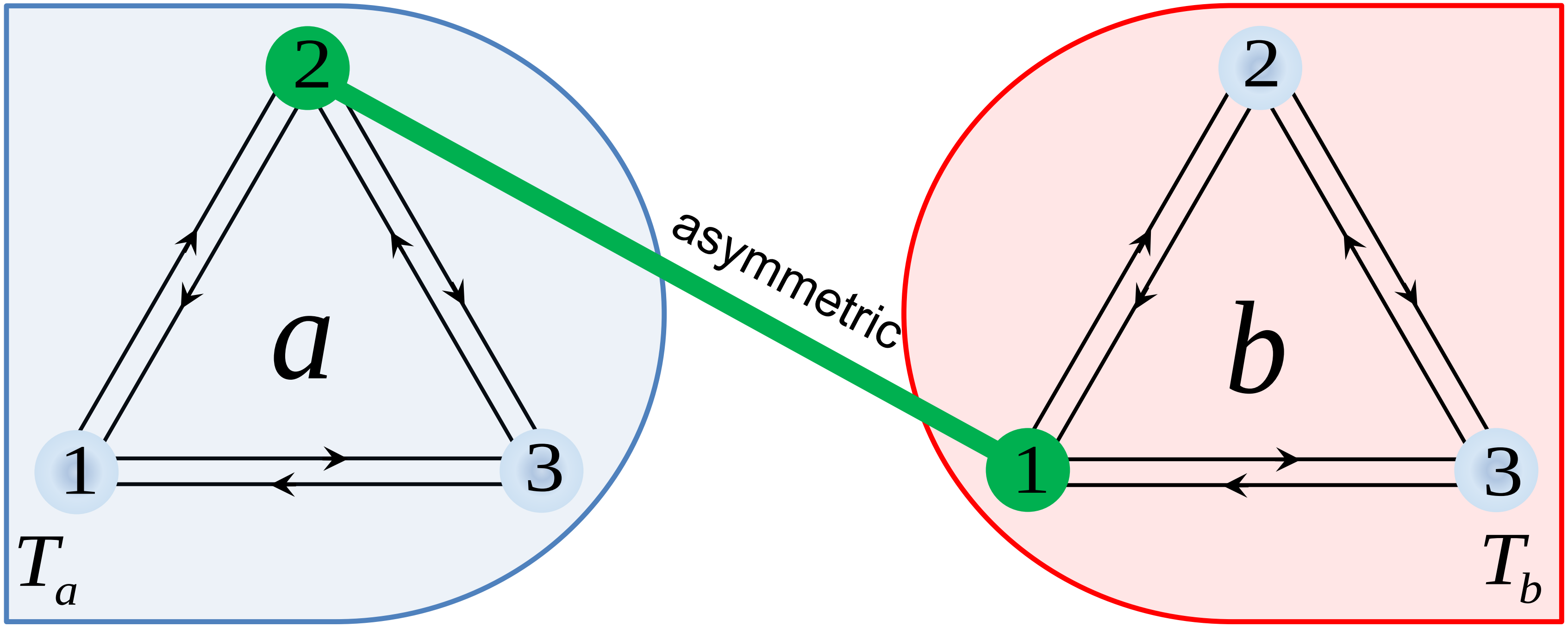
symmetric



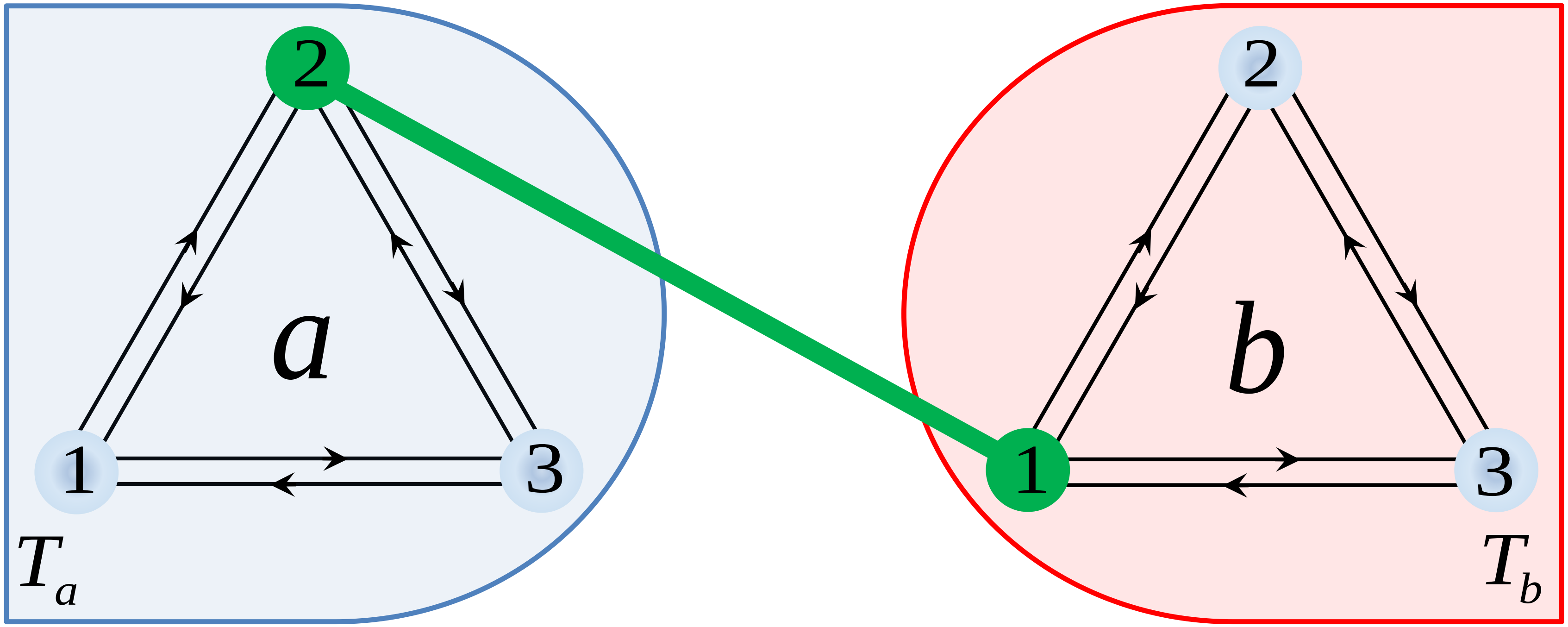
symmetric



The model



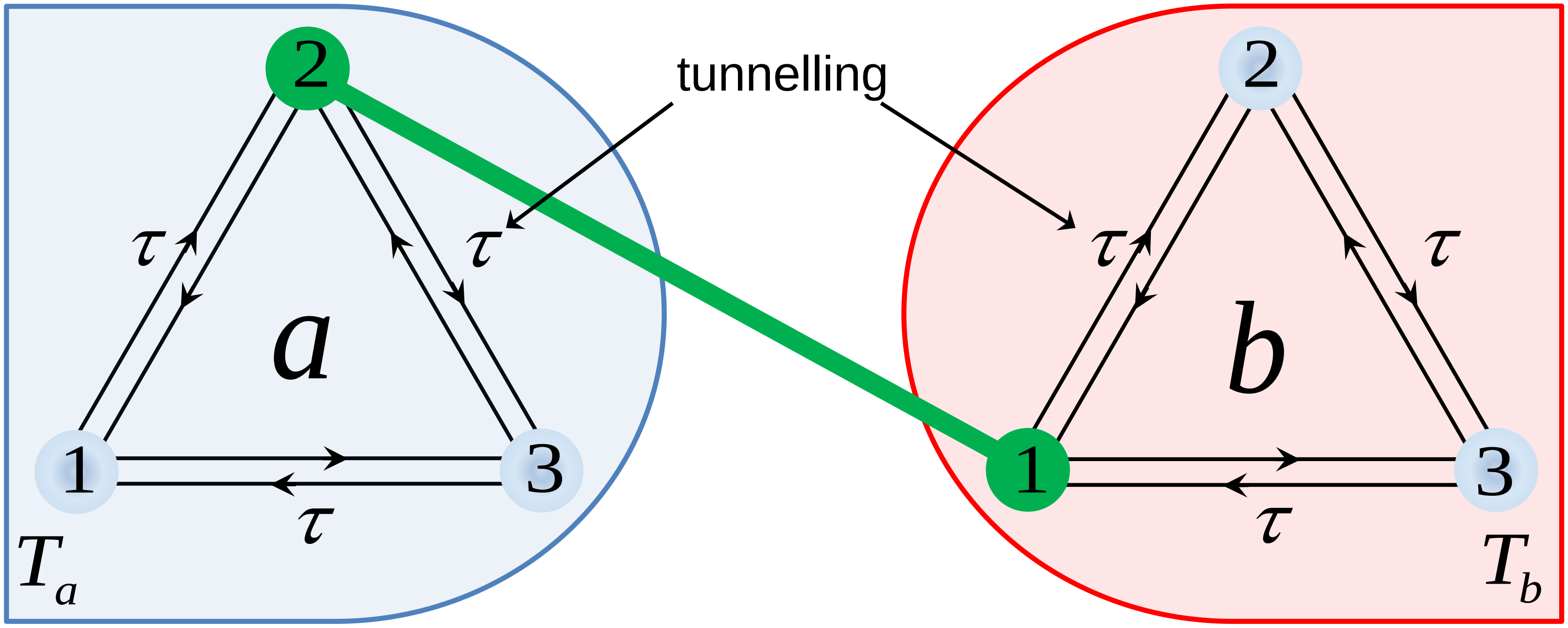
The model



$$U(j_a, j_b) = \frac{K}{2}\cos\left[\frac{2\pi}{3}(j_a - j_b) + \phi\right]$$

Potts model. Imitates dipole-dipole interaction

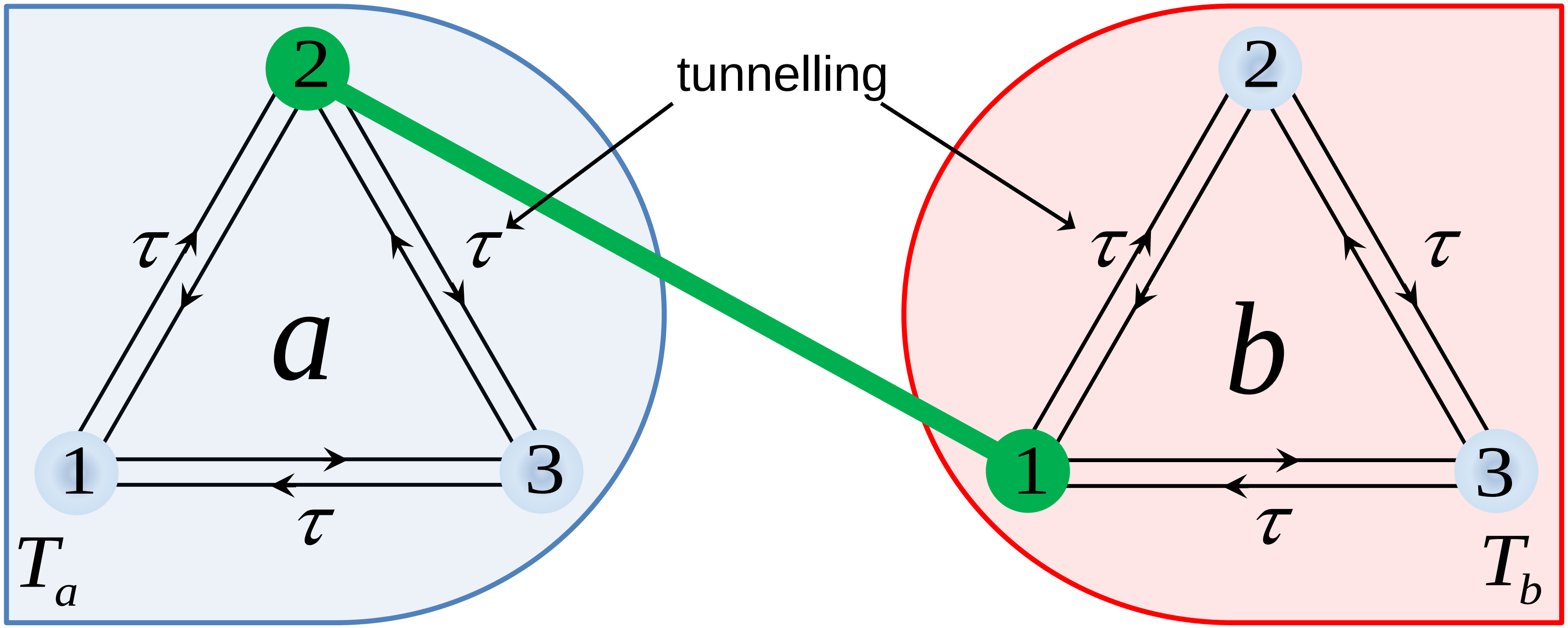
The quantum model



$$H = H_{cl} + \tau X \otimes I_b + \tau I_a \otimes X$$

$$\langle j_a j_b | H_{cl} | j_a j_b \rangle = U(j_a, j_b) \qquad X = \sum_{j=1}^{3} |k\rangle\langle k+1| + |k+1\rangle\langle k|$$

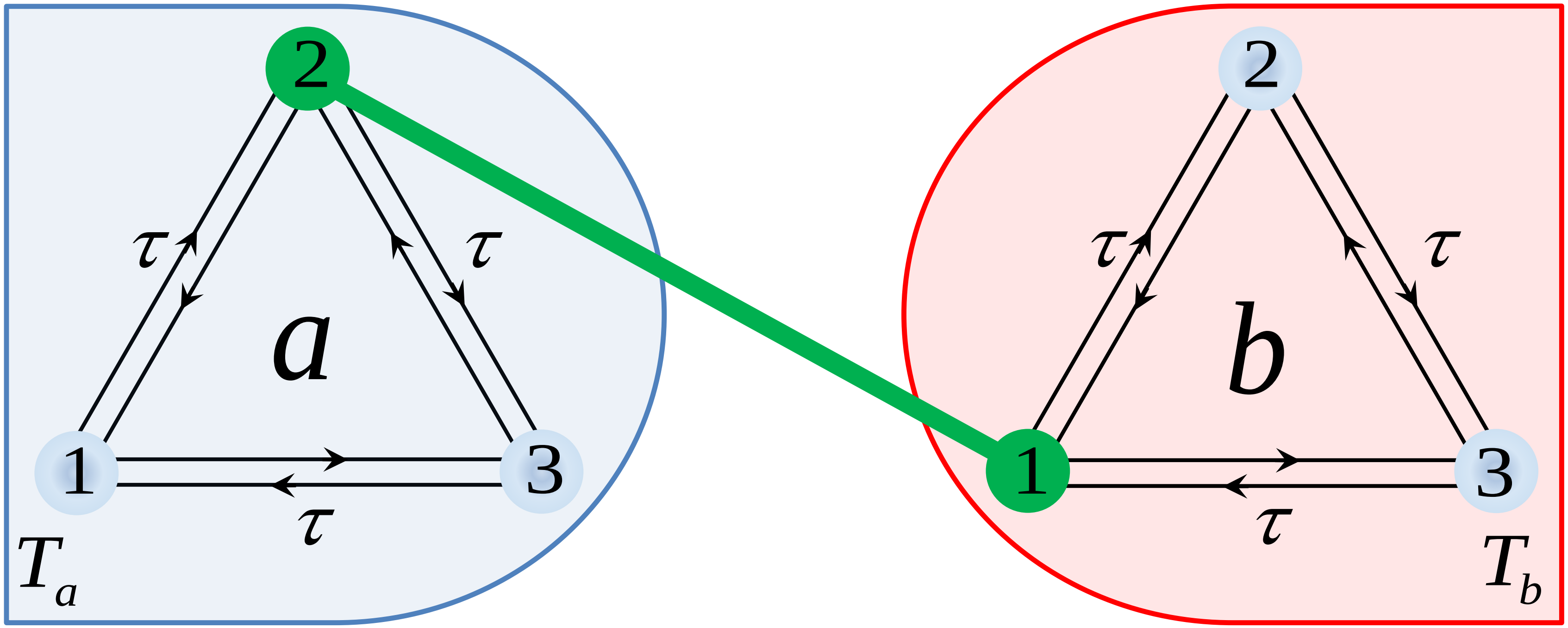
The quantum model



$$H = H_{cl} + \tau X \otimes I_b + \tau I_a \otimes X$$

$$H_{tot} = H + A_a \otimes I_b \otimes B_a + I_a \otimes A_b \otimes B_b$$

The quantum model

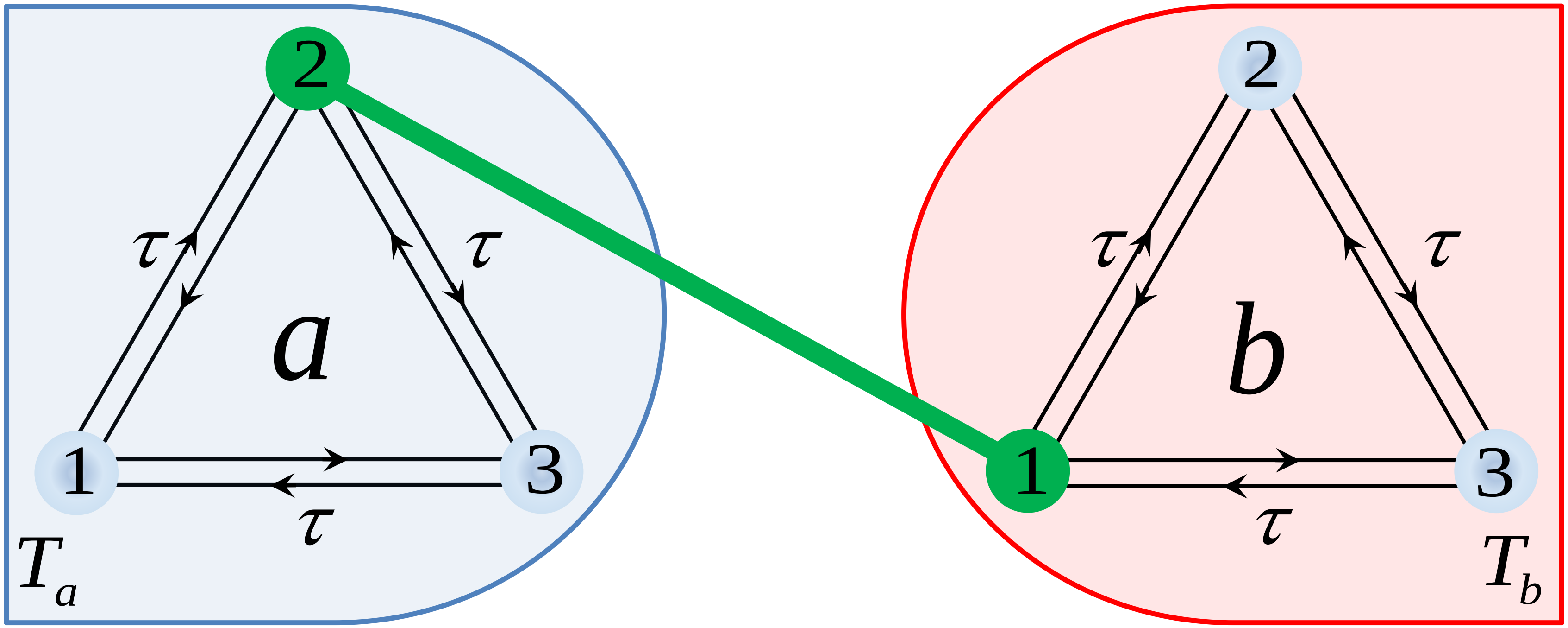


$$\frac{d\rho_{ab}}{dt} = -i[H, \rho_{ab}] + \sum_{\substack{\omega \\ \alpha=a,b}} \gamma_\alpha(\omega)\Xi[\Lambda_\alpha(\omega)][\rho_{ab}]$$

$$\Xi[\Lambda][\rho_{ab}] = \Lambda\rho_{ab}\Lambda^+ - \tfrac{1}{2}\{\Lambda^+\Lambda, \rho_{ab}\}$$

Particle current

Particle Current

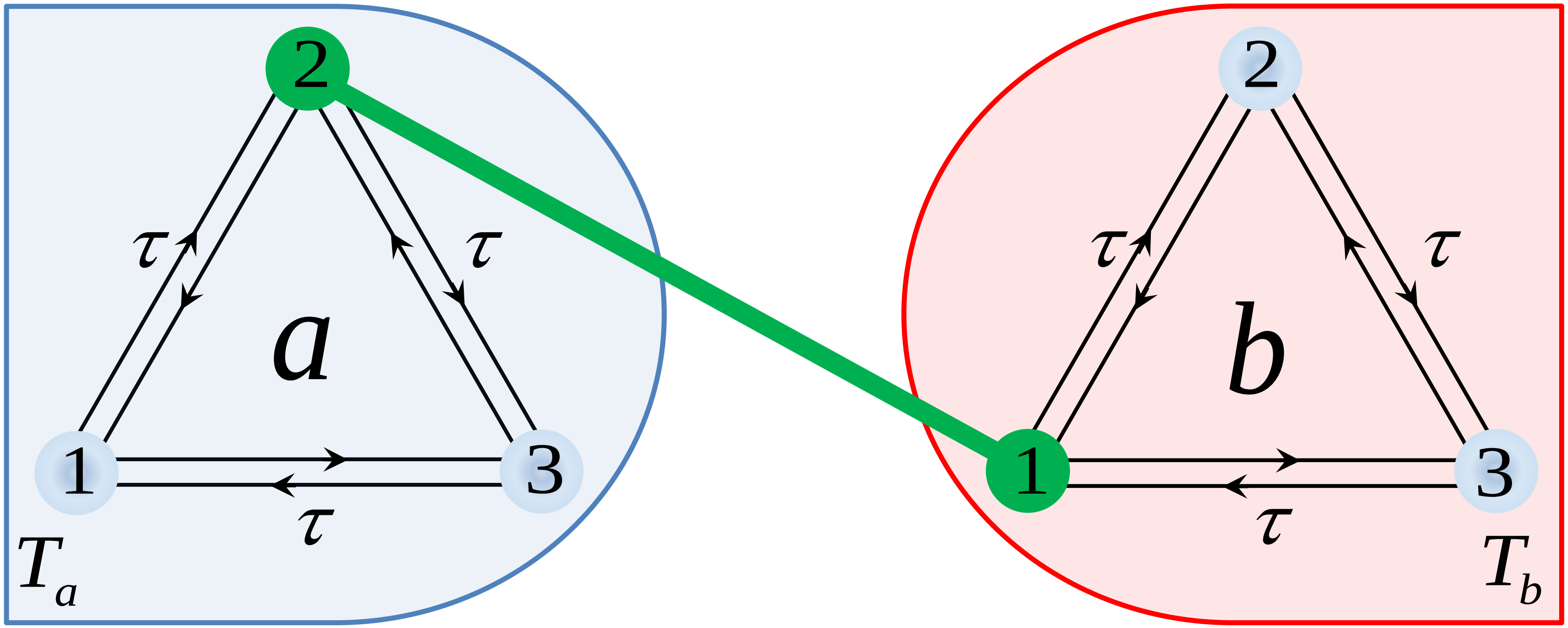


$$x_j = |j\rangle\langle j| \otimes I_b \longrightarrow$$ Position operator
Number operator

$$\frac{dx_j}{dt} = -\mathrm{div}\, J\,|_j = J_{j-1\to j} - J_{j\to j+1}$$

Caroli, Combescot, Nozieres & Saint-James, J. Phys. C 4, 916 (1971)

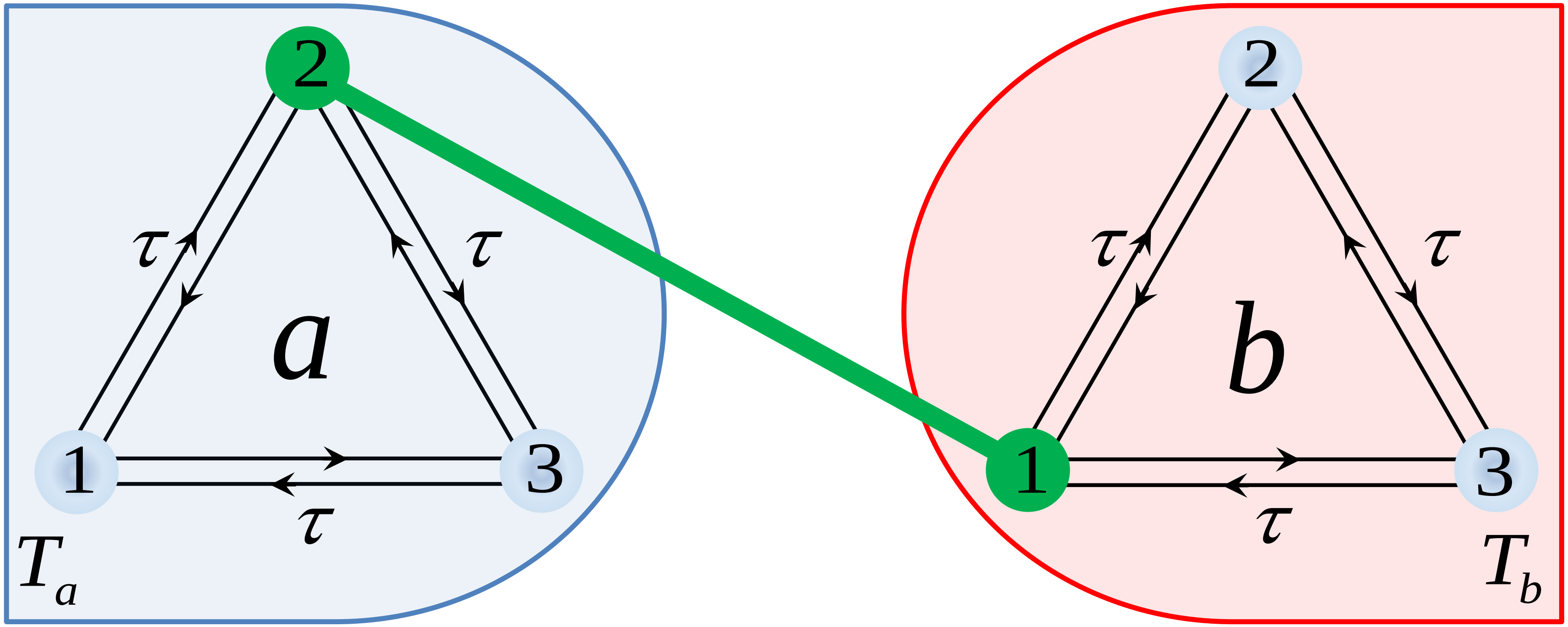
Particle Current



$$J_{j\to j+1} = \frac{1}{2}\left\{\frac{dx_{j+1}}{dt}, x_j\right\} - \frac{1}{2}\left\{\frac{dx_j}{dt}, x_{j+1}\right\}$$

$$J_{j\to j+1} = J^{(\mathrm{tunn})}_{j\to j+1} + J^{(\mathrm{ther})}_{j\to j+1}$$

Particle Current



$$J_{j\to j+1}^{(\mathrm{tunn})} = i\tau\left[\left|j\right\rangle\left\langle j+1\right| - \left|j+1\right\rangle\left\langle j\right|\right]\otimes I_b$$

$$J_{j\to j+1}^{(\mathrm{ther})} = \frac{1}{2}\sum_{\alpha,\omega}\gamma_\alpha(\omega)\left[\left\{\Lambda_\alpha^+(\omega)x_{j+1}\Lambda_\alpha(\omega), x_j\right\} - \left\{\Lambda_\alpha^+(\omega)x_j\Lambda_\alpha(\omega), x_{j+1}\right\}\right]$$

Particle Current

$$J_{j\to j'} = \frac{1}{2}\left\{\frac{dx_{j'}}{dt}, x_j\right\} - \frac{1}{2}\left\{\frac{dx_j}{dt}, x_{j'}\right\}$$

Holds whenever the evolution is trace preserving.
All other definitions are special cases of this expression.

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$$\left\langle J_{j\to j'}\right\rangle = \lim_{\varepsilon\to 0}\frac{P[x_j(t)\,|\,x_{j'}(t+\varepsilon)]\,p[x_{j'}(t+\varepsilon)] - (j \leftrightarrow j')}{\varepsilon}$$

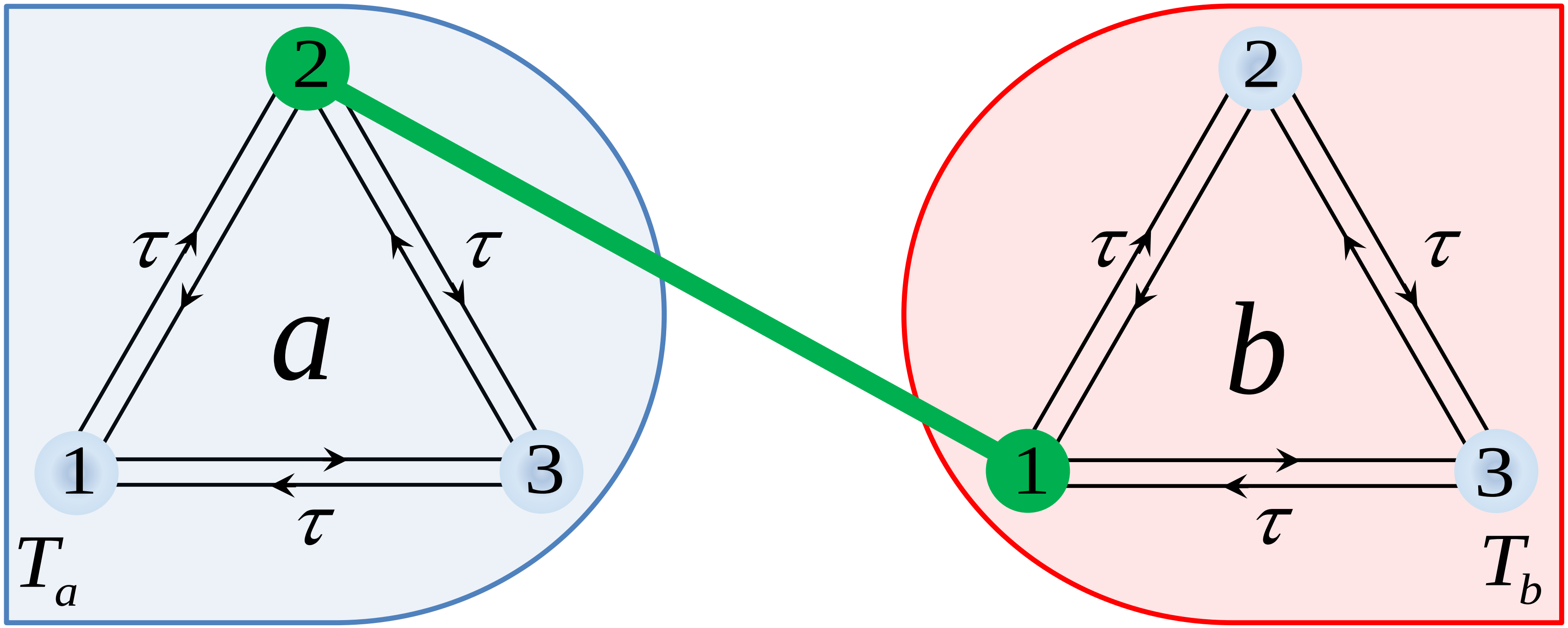
$$P[x_j(t)\,|\,x_{j'}(t+\varepsilon)] = \mathrm{Re}\,\frac{\mathrm{Tr}[\rho_{ab}x_{j'}(t+\varepsilon)x_j(t)]}{\mathrm{Tr}[\rho_{ab}x_{j'}(t+\varepsilon)]}$$

Weakly measured average of x_j at moment t conditioned on the strong measurement outcome j' at moment $t+\varepsilon$.

Aharonov, Albert & Vaidman, Phys. Rev. Lett. 60, 1351 (1988)
Dressel, Agarwal & Jordan Phys. Rev. Lett. 104, 240401 (2010)

Symmetries and transport

Symmetries of the model



Global rotation: $(j_a, j_b) \rightarrow (j_a + 1, j_b + 1)$ — ALWAYS symmetric

Particle swap: $(j_a, j_b) \rightarrow (j_b, j_a)$ — Symmetric ONLY for $\phi = k\pi / 3$

Symmetry breaking

Classical regime:

Particle swap symmetry breaking is necessary for non-zero current.

Direction of current determined by ϕ.

Quantum regime:

Particle swap symmetry breaking is necessary for non-zero thermal current.

Total current can be non-zero even if particle swap symmetry holds.

Direction of current can change depending also on τ .

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Direction of current can change depending also on τ.

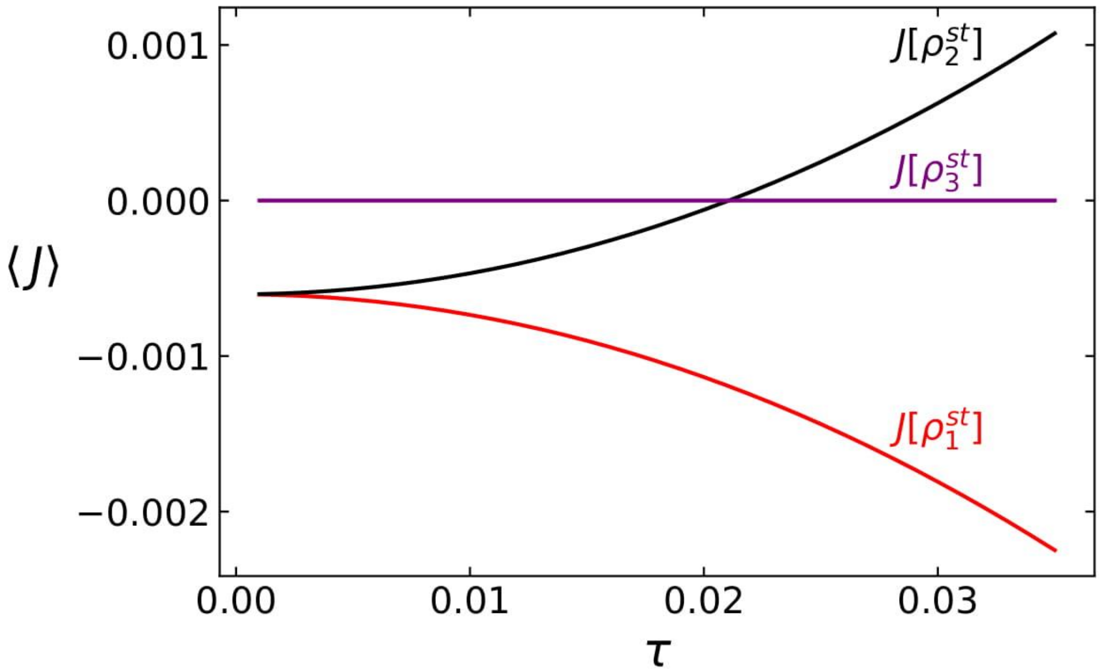
In both cases, symmetry breaking has no effect on heat flux

Current inversion

Due to global rotation symmetry, there are 3 linearly-independent steady states: $\rho^{st}_{1,2,3}$.

Buca and Prosen, New J. Phys. 14, 073007 (2012)

Total current can change direction as the tunnelling rate is varied:



Entanglement generation

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We can use this to entangle uncorrelated states:

$$\rho_a \otimes \rho_b \to \rho_{ab}^{st} = \lambda_1 \rho_1^{st} + \lambda_2 \rho_2^{st} + \lambda_3 \rho_3^{st}$$

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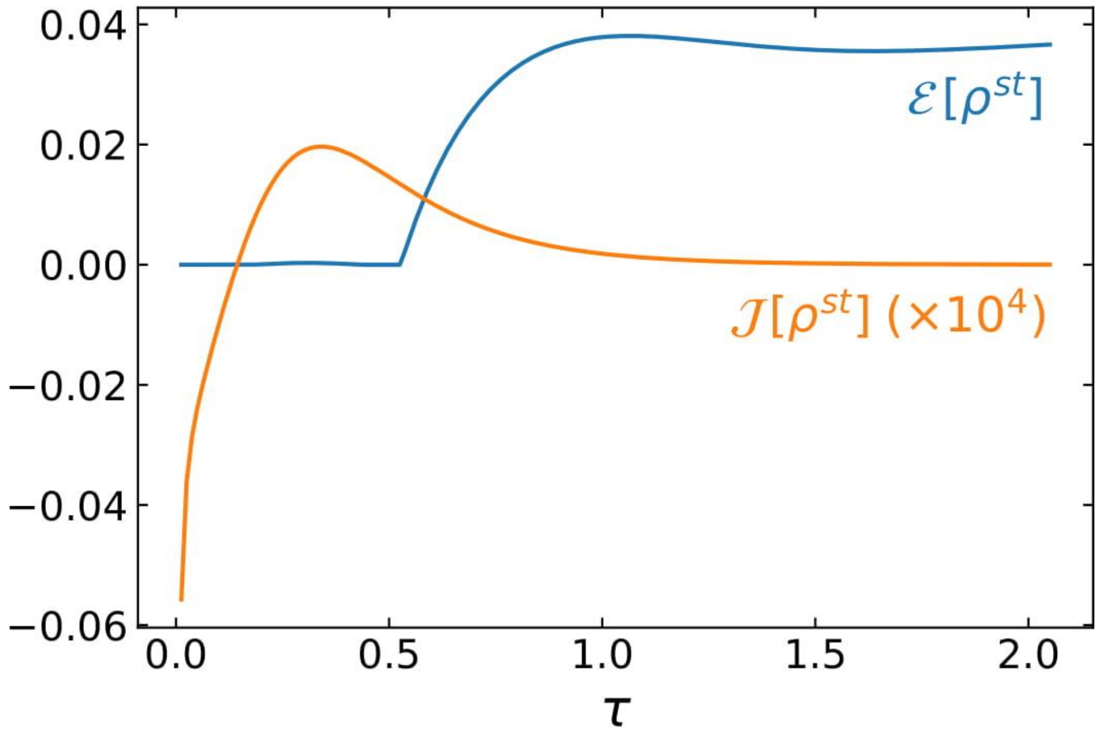
The higher the T_a, T_b the more coherent ρ_a, ρ_b need to be.

The machine converts local coherence into entanglement.

Ergotropy generation

$\frac{1}{Z}e^{-H/T_a}$ is free but useless for work extraction.

The machine can convert $\frac{1}{Z}e^{-H/T_a}$ into a non-passive steady state.



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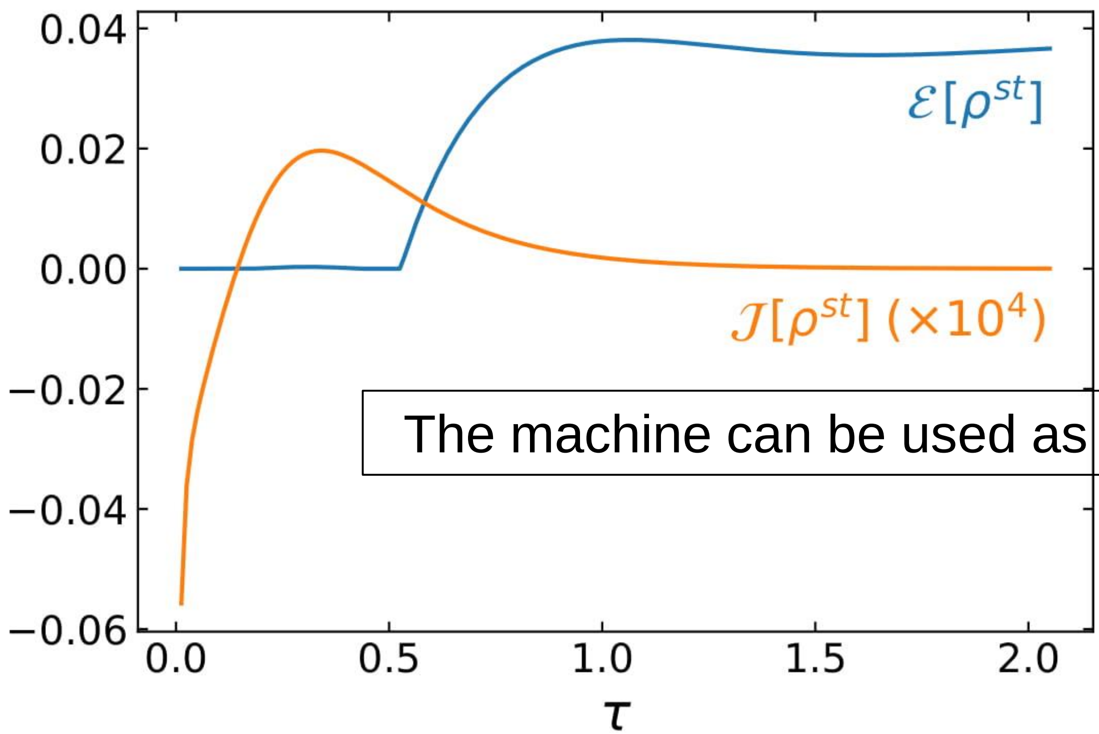

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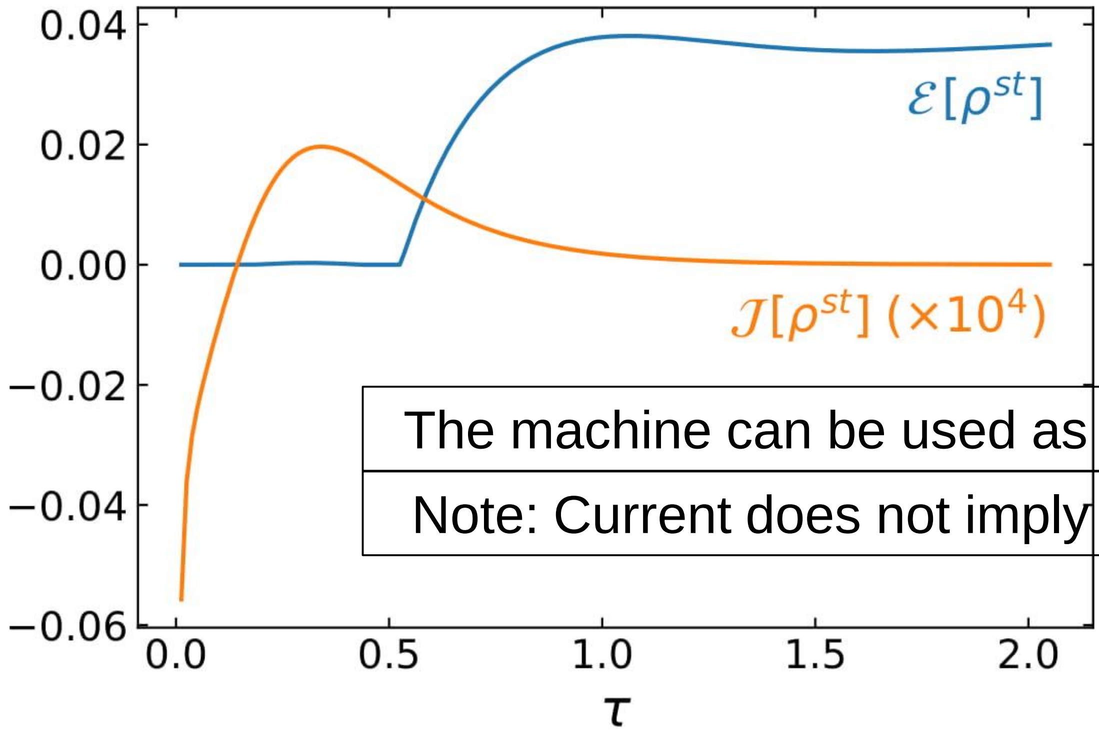

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Note: Current does not imply ergotropy.

Lastly…

Global GKSL	Local GKSL
Current is via 1 weak and 1 strong measurement	Current is via 2 strong measurements
$\dot{Q} = 0$ when $T_a = T_b$	$\dot{Q} \propto \tau^2$ when $T_a = T_b$
Current is inverted	Current is inverted

Summary

- We derived a universal expression for current operator.
- Symmetry breaking is beneficial for current.
- Symmetry is beneficial for entanglement and ergotropy.