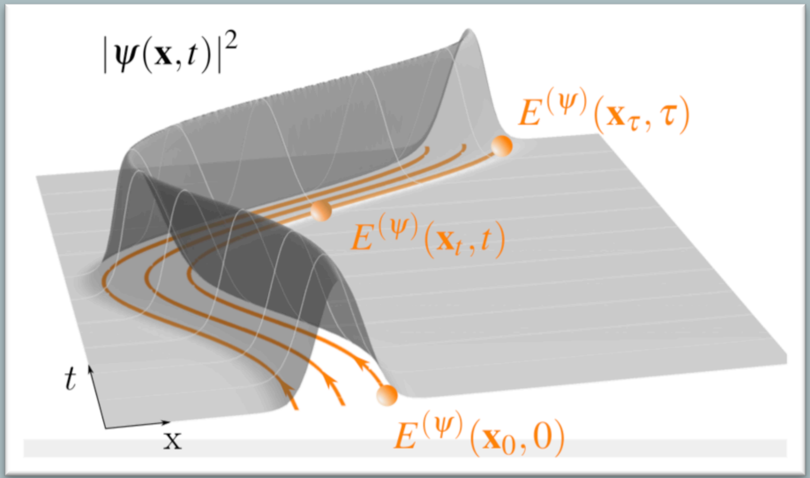


THE IMPOSSIBLE QUANTUM WORK DISTRIBUTION

Rui Sampaio

QTF, Center of Excellence, Aalto University, Finland

In collaboration with:

Tapio Ala-Nissilä, Janet Anders, Thomas Philbin and Samu Suomela

@ Quantum Thermodynamics Conferece
KITP, UCSB | USA 2018

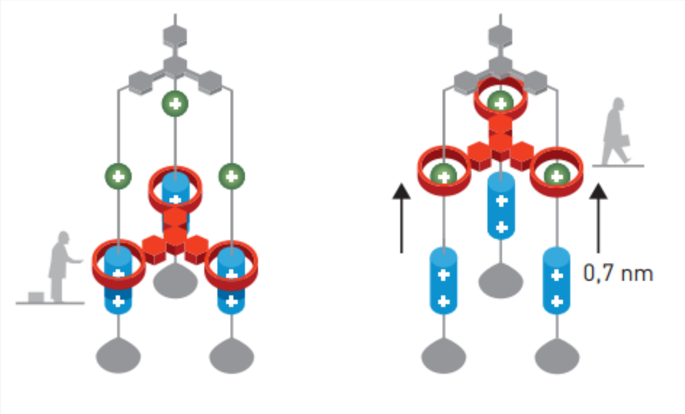
Phys. Rev. A 97, 012131 (2018)

OVERVIEW

- Motivation and the problem
- Quantum Work in the phase-space using Bohmian trajectories
- Conclusions

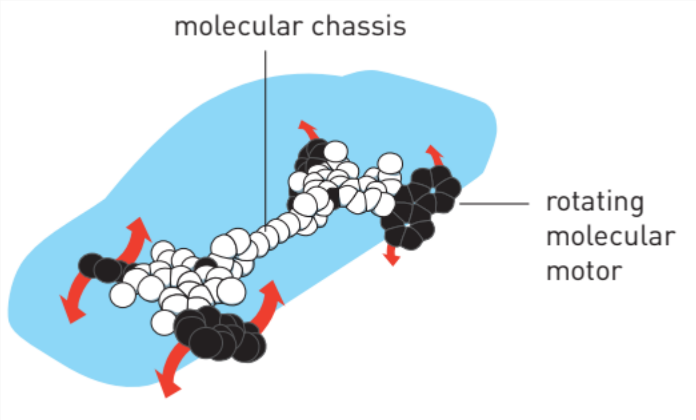
MOTIVATION

Molecular lift

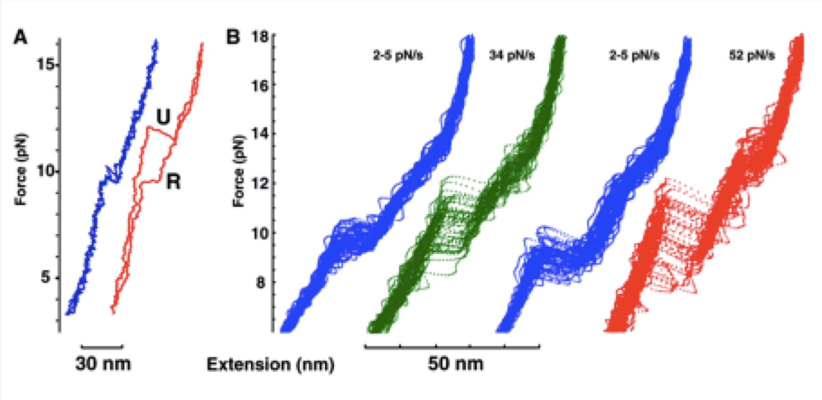


[Badjic, J. et al., Science, 2004]

Molecular car



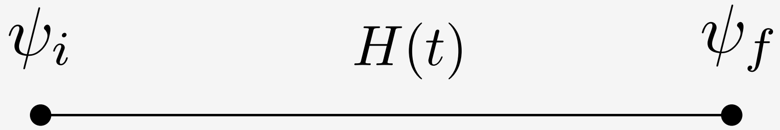
[Kudernac, T et al., Nature, 2011]



Classically we define work along trajectories in phase-space

$$W=\int_{x_t} dt\, \partial_t H|_{x_t}$$

MOTIVATION

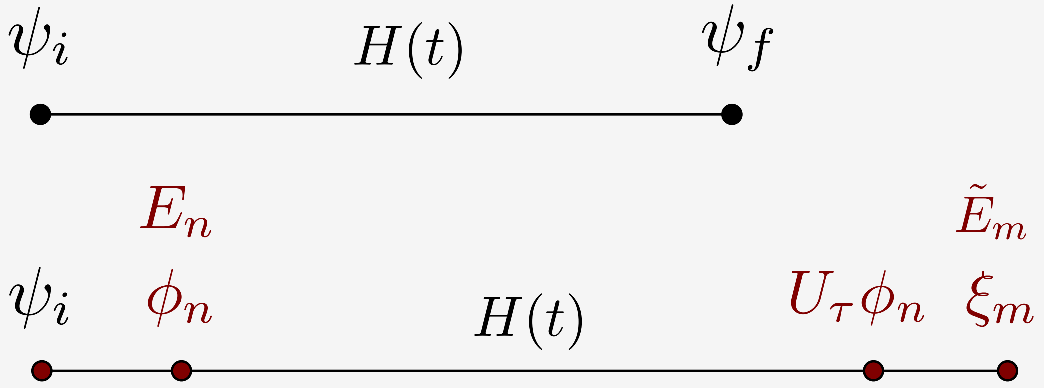


Objective:
To assign a (stochastic) work value to any given a process described by some Hamiltonian $H(t)$ and state $\psi(t)$ for closed systems.

Problem:
$\psi(t)$ not eigenstate of $H(t)$:

☞ Energy not well-defined!

TPM PROTOCOL

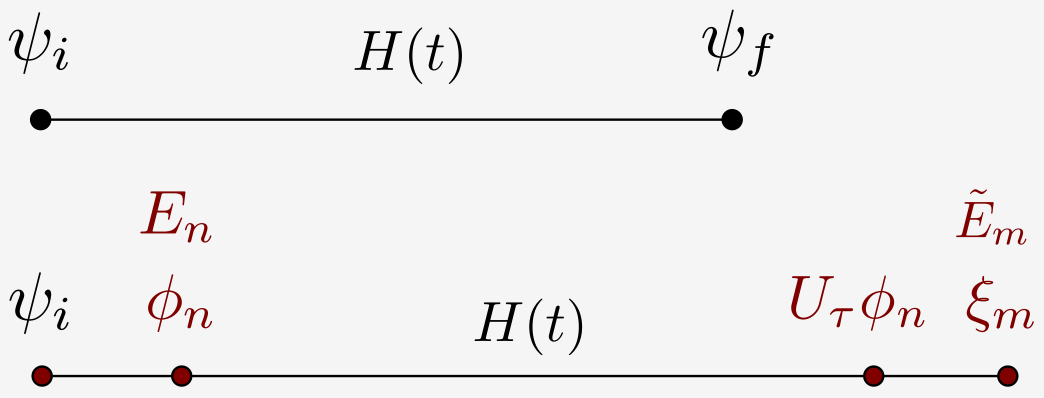


$$W_{\mathrm{TPM}} = \tilde{E}_m - E_n$$

- Applied in many different contexts:
  - **Fluctuation relations** [Campisi et al, Rev. Mod. Phys. 83 771 (2011)]
  - **Open systems** [Campisi et al, J. Phys. A: Math. Theor., 39, 392002 (2009)]
  - **Continuously measure systems** [Campisi et al, Phys. Rev. Lett. 105, 140601 (2010)]
  - **Dirac particles** [Deffner et al, Phys. Rev. E 92, 032137 (2015)]

- **Implemented experimentally** [An et al, Nature Physics 11 193 (2014), Batalhao et al, Phys. Rev. Lett. 14 113, 140601 (2014)]

TPM PROTOCOL



$$W_{\mathrm{TPM}} = \tilde{E}_m - E_n$$

Not exactly what we want

- The measurements generate a different process.
- In general,

$$\langle W_{\mathrm{TPM}} \rangle = \langle H(t) \rangle_{\psi(t)} - \langle H(0) \rangle_{\psi(0)}$$

- The system becomes open during the measurement steps.
- Equating work with change in energy of the system no longer obvious [Kammerlander et al, Scientific reports 6, 22174 (2016), Solinas et al, Phys. Rev. E 92, 042150 (2015), Elouard et al, NPJ Q. Info. 3, 9 (2017)]

OTHER APPROACHES

- Operator of work [Allahverdyan et al., Phys. Rev. E 71 066102 (2005)]
- Quasi-probabilities
  - Full counting statistics [Solinas et al., Phys. Rev. A 94 052103 (2016)]
  - Wigner distributions [Deffner et al., EPL 103 30001 (2013)]
  - Consistent histories [Miller et al., New J. Phys 19, 062001 (2017)]
- …

NO-GO THEOREM

PRL **118,** 070601 (2017) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2017

No-Go Theorem for the Characterization of Work Fluctuations in Coherent Quantum Systems

Martí Perarnau-Llobet,[1,*] Elisa Bäumer,[1,2,†] Karen V. Hovhannisyan,[1,‡] Marcus Huber,[3,4,§] and Antonio Acin[1,5,¶]
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(Received 7 July 2016; revised manuscript received 16 December 2016; published 14 February 2017)

No work definition can fulfill three distinct properties:

- P(W) is described by a POVM
- For initial diagonal states, P(W) ☞ P_{TPM}(W)
- Average work = Change in expectation value of Hamiltonian

QUANTUM WORK IN THE PHASE-SPACE USING BOHMIAN TRAJECTORIES

Phys. Rev. A 97, 012131 (2018)

BOHMIAN MECHANICS

- Bohmian mechanics (BM) is an alternative description of quantum mechanical phenomena fully consistent with experimental observations.
- It's a theory about point particles and how they move in space.
- It allows a phase space and configuration space description such that quantities and concepts from classical statistical physics can be naturally generalized.
- All the machinery of operators, observables, projections, observation, "colapse", etc can be recovered and interpreted from these trajectories.

Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer

Sacha Kocsis,[1,2]* Boris Braverman,[1]* Sylvain Ravets,[3]* Martin J. Stevens,[4] Richard P. Mirin,[4] L. Krister Shalm,[1,5] Aephraim M. Steinberg[1]†

A consequence of the quantum mechanical uncertainty principle is that one may not discuss the path or "trajectory" that a quantum particle takes, because any measurement of position irrevocably disturbs the momentum, and vice versa. Using weak measurements, however, it is possible to operationally define a set of trajectories for an ensemble of quantum particles. We sent single photons emitted by a quantum dot through a double-slit interferometer and reconstructed these trajectories by performing a weak measurement of the photon momentum, postselected according to the result of a strong measurement of photon position in a series of planes. The results provide an observationally grounded description of the propagation of subensembles of quantum particles in a two-slit interferometer.

Steinberg et al, Science 332, 6034 (2011)

RESEARCH ARTICLE

QUANTUM MECHANICS

Experimental nonlocal and surreal Bohmian trajectories

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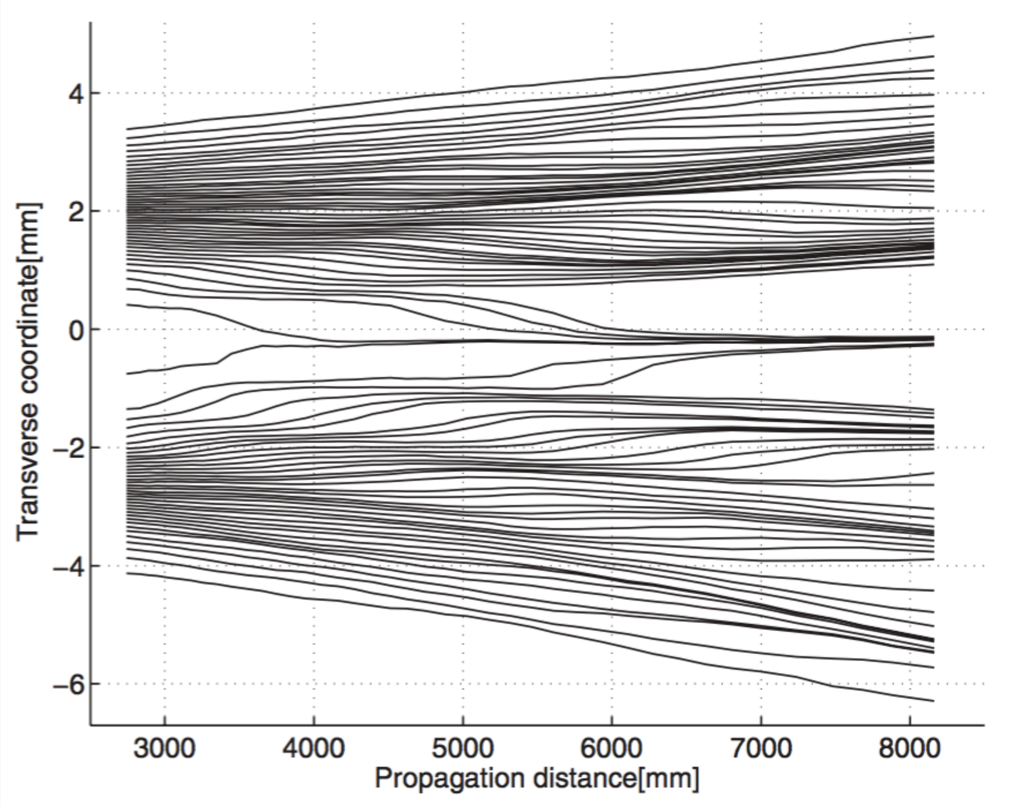
Dylan H. Mahler,[1,2]* Lee Rozema,[1,2] Kent Fisher,[3] Lydia Vermeyden,[3] Kevin J. Resch,[3] Howard M. Wiseman,[4]* Aephraim Steinberg[1,2]

Weak measurement allows one to empirically determine a set of average trajectories for an ensemble of quantum particles. However, when two particles are entangled, the trajectories of the first particle can depend nonlocally on the position of the second particle. Moreover, the theory describing these trajectories, called Bohmian mechanics, predicts trajectories that were at first deemed "surreal" when the second particle is used to probe the position of the first particle. We entangle two photons and determine a set of Bohmian trajectories for one of them using weak measurements and postselection. We show that the trajectories seem surreal only if one ignores their manifest nonlocality.

Steinberg et al, Science advances 2, e1501466 (2016)

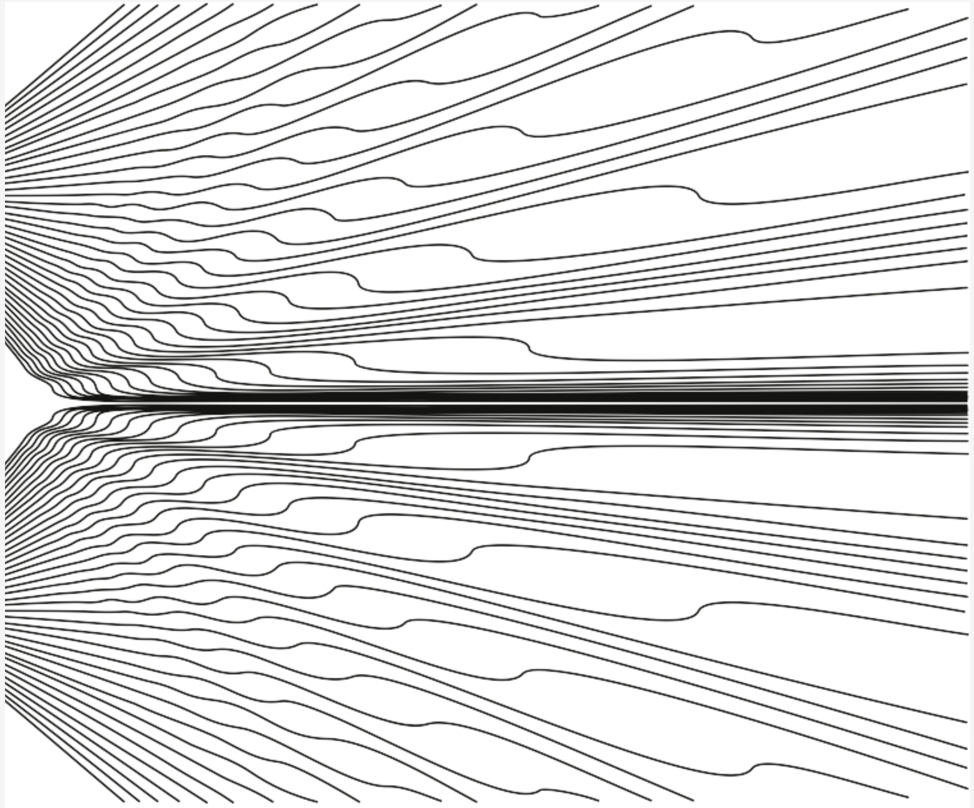
BOHMIAN MECHANICS

Experimental reconstructions of the trajectories



Steinberg et al, Science 332, 6034 (2011)

Numeric simulation



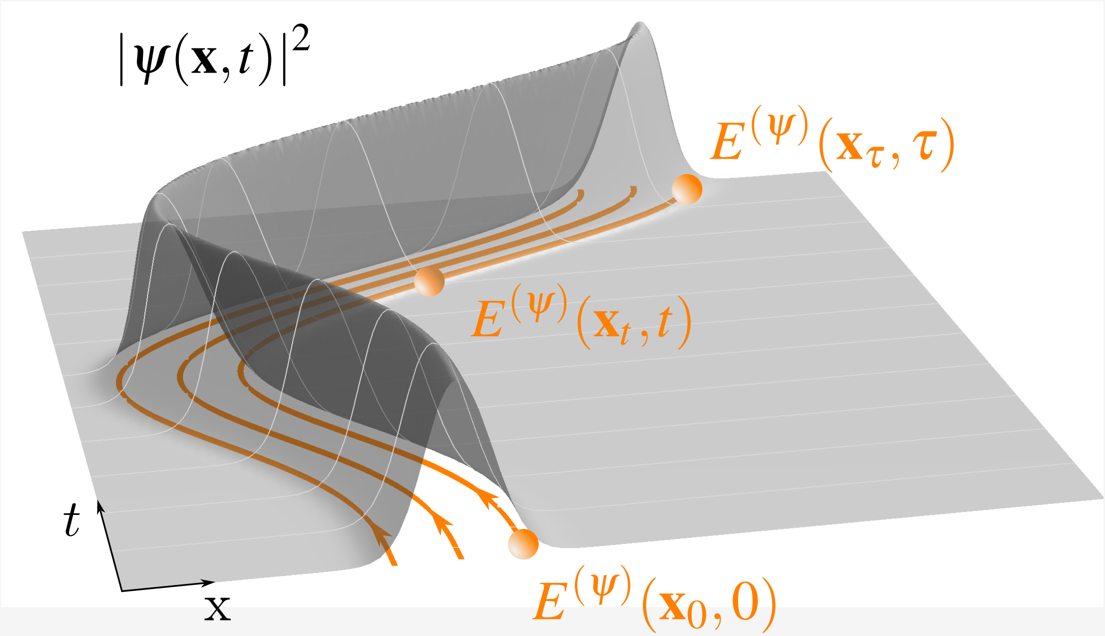
BOHMIAN MECHANICS (IN A NUTSHELL)

- The particles have well-defined energy and position at all times.
- The initial positions cannot be known with absolute certainty - a distribution $\rho(x,0)$ must be provided.
- The initial momenta is fixed by a field $S(x,0)$.
- The evolution of $\rho(x,t)$ and $S(x,t)$ is compactly given by the Schrödinger equation (or Dirac, KG, Pauli, etc)

$$i\hbar \partial_t \psi(x,t) = (H\psi)(x,t) \text{ with } \psi = Re^{iS/\hbar}$$

- Once $\psi(x,0)$ and an initial position x_0 are given, evolution is completely deterministic.

Classical	Quantum
(X, P)	(X, ψ)



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QUANTUM HAMILTON-JACOBI

- Consider a system of N particles with Hamiltonian $H=\sum_i \left(P_i^2/2m_i + V_i \right) + V_{int}$.
- Write the wave function in the hydrodynamic representation $\psi(x,t)=R(x,t)\exp(iS(x,t)/\hbar)$
- Take the real part of the Schrödinger Eq.

$$-\partial_t S(x,t)=\sum_i \left((\nabla_i S)^2/2m_i + V_i(x,t) \right) + V_{int} + Q^{(\psi)}(x,t) = E^{(\psi)}(x,t)$$

Quantum potential: $Q^{(\psi)}(x,t) = -\hbar^2 \sum_i \nabla_i^2 R(x,t)/2m_i R(x,t)$

QUANTUM WORK

Hamiltonian that generates the traj. in phase-space:

$$H^{(\psi)}(x,p,t)=p^2/2m+V(x,t)+V_{int}(x,t)+Q^{(\psi)}(x,t) \qquad \dot{x}=\nabla_p H^{(\psi)}, \quad \dot{p}=-\nabla_x H^{(\psi)}$$

Work definition:

$$W^{(\psi)}(x_0,\tau)=\int_0^\tau dt\, \partial_t H^{(\psi)}\big|_{x_t}=E^{(\psi)}(x_\tau,\tau)-E^{(\psi)}(x_0,0)$$

Work distribution:

$$P_\tau(W;\psi)=\int dx_0\, |\psi(x_0,0)|^2\, \delta(W-W^{(\psi)}(x_0,\tau))$$

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DISTRIBUTION PROPERTIES

- $P^{(\psi)}(W) \geq 0,$
- $\int dW\, P^{(\psi)}(W) = 1,$
- $\langle\langle W \rangle\rangle = \int dW\, W P^{(\psi)}(W) = \langle H(\tau)\rangle_{\psi(\tau)} - \langle H(0)\rangle_{\psi(0)}$

- For a proper statistical mixture of wave functions $\{\psi^{(j)}\}$, the distribution

  $P_\tau^{\{j\}}(W) = \sum_j p_j\, P_\tau(W; \psi^{(j)})$

  is mixture-dependent!

TPM

✓ Positive
✘ $\langle W \rangle = \langle \Delta H \rangle$
✓ POVM

Q-P

✘ Positive
✓ $\langle W \rangle = \langle \Delta H \rangle$
✘ POVM

Bohmian

✓ Positive
✓ $\langle W \rangle = \langle \Delta H \rangle$
✘ POVM

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EXAMPLE – DRIVEN 1D HO

Hamiltonian:

$$H(X,P,t)=\frac{P^2}{2m}+\frac{1}{2}m\omega^2X^2+XA\sin\omega t+PA\cos\omega t$$

Thermal state:

$$\rho_\beta=\frac{e^{-\beta H}}{Z_\beta}=\sum_n \frac{e^{-\beta E_n}}{Z_\beta}|n\rangle\langle n|=\frac{1}{\pi n}\sum_n e^{-|\eta|^2/n}|\eta\rangle\langle\eta|$$

Can be described as a mixture of eigenstates $\{n\}$ or coherent states $\{\eta\}$.

Average work:

$$\langle\langle W\rangle\rangle_{\{n\}}=\langle\langle W\rangle\rangle_{\{\eta\}}=Tr\rho\Delta H=\frac{A\tau^2}{2m}$$

Average exponentiated work:

$$\langle\langle e^{-\beta W}\rangle\rangle_{\{n\}}=1-\frac{\beta(A\tau)^2}{2m}\sin^2(\omega t)+O(\beta^2)$$

$$\langle\langle e^{-\beta W}\rangle\rangle_{\{\eta\}}=1+\beta\hbar\omega\sin^2(\omega t/2)+O(\beta^2)$$

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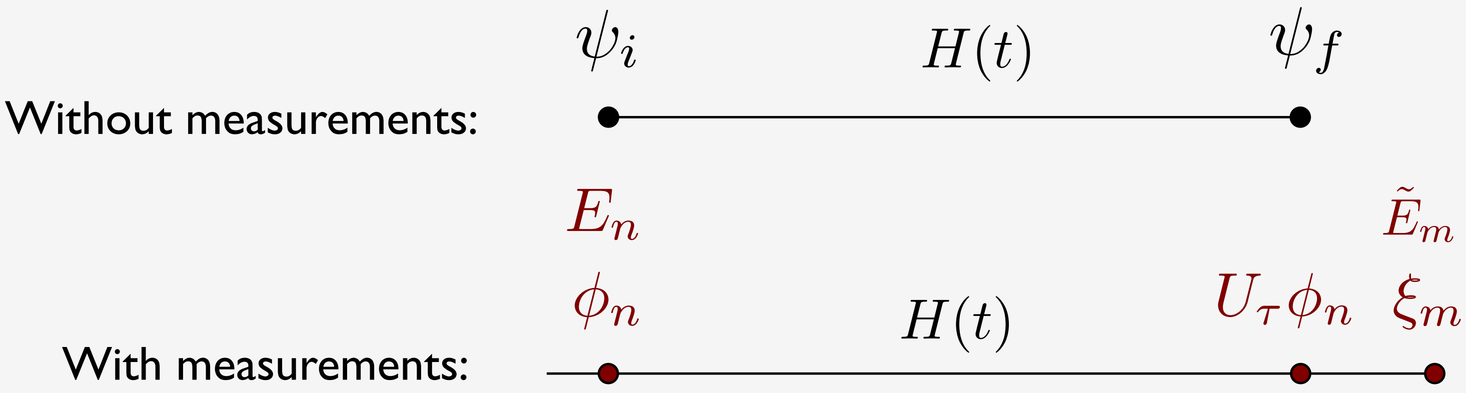
EXAMPLE – DRIVEN 1D HO



If work defined as change in energy: $W := \Delta E = E(x_\tau, \tau) - E(x_0, 0)$

And **if** the meas. are performed: $P_{Bohm}(W) = P_{TPM}(W)$

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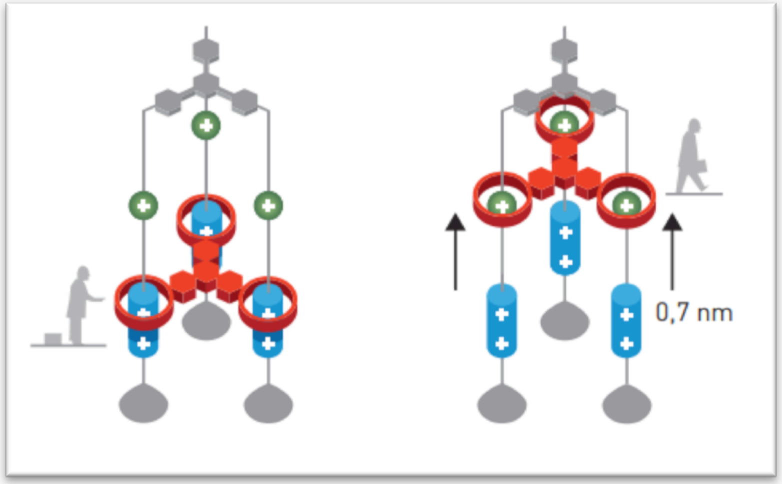
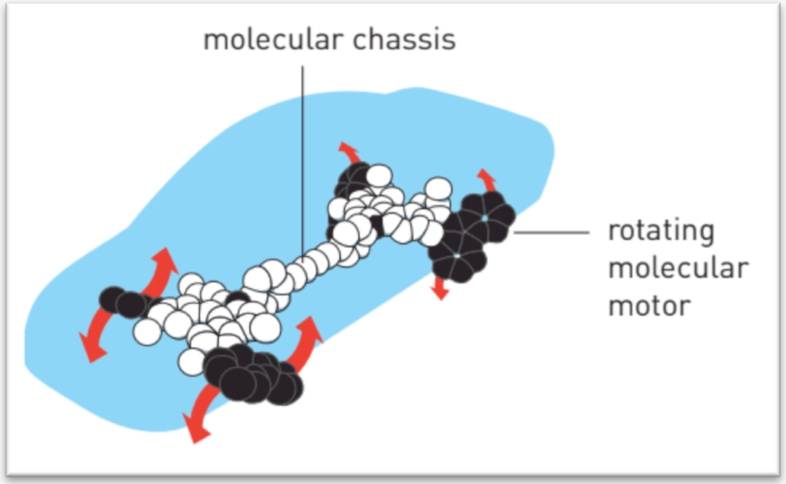
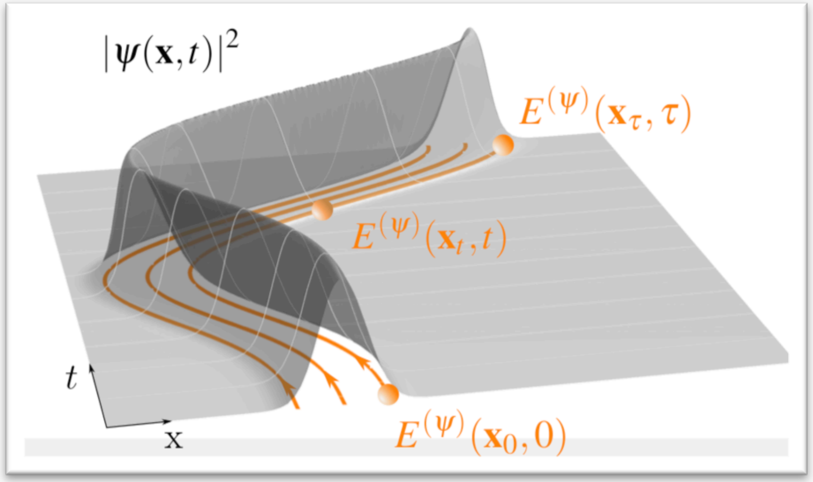
CONCLUSIONS

- We have taken the classical work definition along the trajectories generated by the Hamiltonian and applied it to the Bohmian trajectories.
- The resulting work distribution is positive, normalized and respects the change in the expectation value of the Hamiltonian
- No conflict with no-go theorem because it's not given by a POVM.
- It seems particularly suited to tackle problems in continuous Hilbert spaces.







THANK YOU!

In collaboration with:

Dr. Janet Anders
Dr. Thomas Philbin
Dr. Samu Suomela
Prof. Tapio Ala-Nissilä

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