Shortcut-to-adiabaticity machines: A new twist for finite-time thermodynamics?

Obinna Abah

Queen's University Belfast, United Kingdom

Quantum Thermodynamics Program

KITP, Santa Barbara 2018

News..

Sales of inefficient vacuum cleaners banned
*By **Euronews*** · last updated: 01/09/2017

Powerful vacuum cleaners are to be banned from today after the European Union introduced new rules which aim to improve energy efficiency across the continent.

'Widespread misconception'

The European Environment Bureau (EEB) said: "Power doesn't always equal performance, though the misconception has become widespread."

"Some efficient models maintained high standards of dust pick-up while using significantly less energy - due to design innovation."

From **BBC News** 01/09/2017

Outline

Introduction: Miniaturization

"There is plenty of room at the bottom": Feynman 1959

"Consider any machine – for example, an automobile – and ask about the problems of making an infinitesimal machine like it"

Mobile phone:

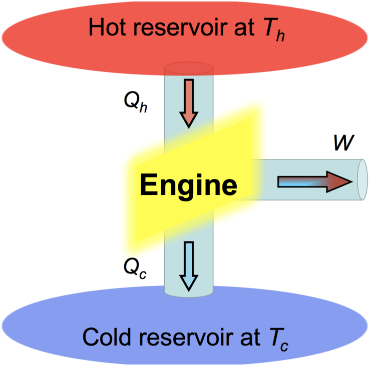
1973/1983

Today

➔ weight 1.1kg, 30min talk time, 10h charge time, price 4000$

Macroscopic heat engine

$\rightarrow$ convert thermal energy into mechanical work $=$ motion



Carnot efficiency:

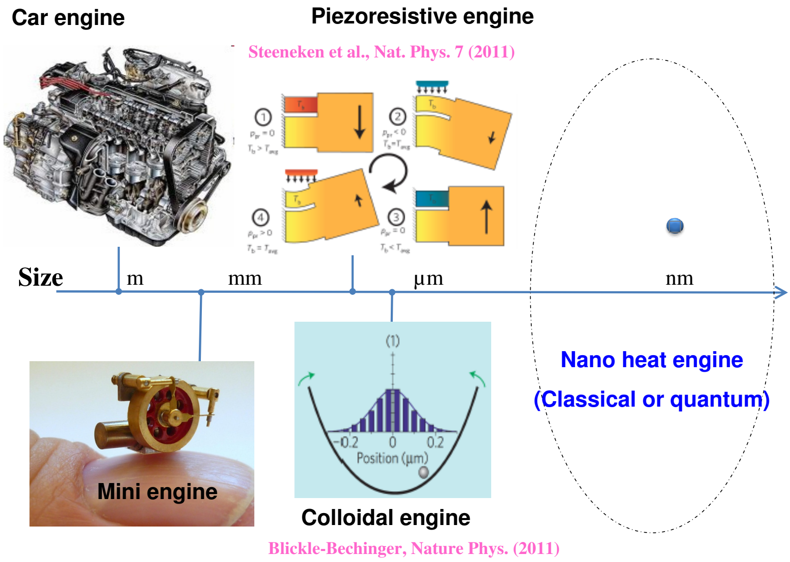
$$\eta = \frac{\textit{Work produced}}{\textit{Heat absorbed}} \leq 1 - \frac{\beta_h}{\beta_c} = 1 - \frac{T_c}{T_h}$$

(James Watt 1783: $\eta \sim 5 - 7$ %)

$\rightarrow$ maximum efficiency

Today's gasoline engines: $\eta \sim 25 - 30$ %

Downscaling of heat engines

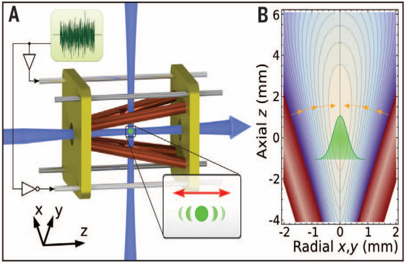


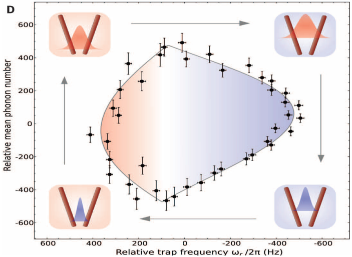
Single atom heat engine

Rossnagel et al., Science 352, 325 (2016)

Reservoir engineering:

- Cold reservoir: laser (Doppler) cooling (always on)
- Hot reservoir: electrode noise (switched on/off)

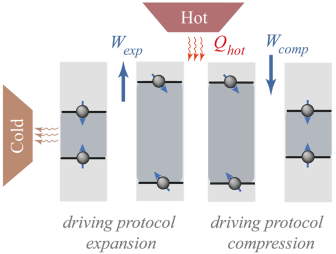


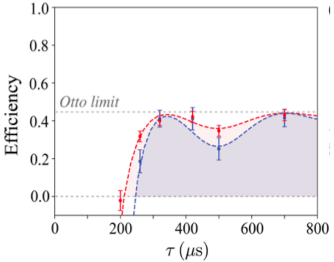


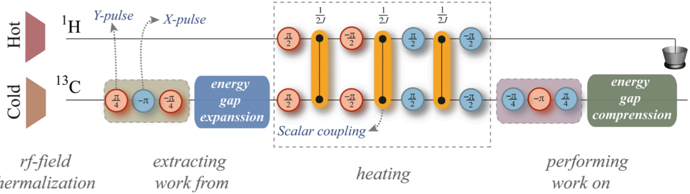
Spin quantum heat engine

Technique: Nuclear Magnetic Resonance

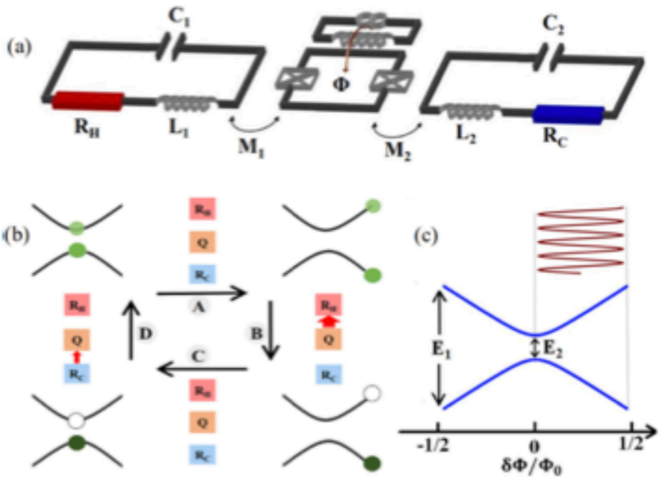
Peterson et al., arXiv. 1803.06021 (2018)





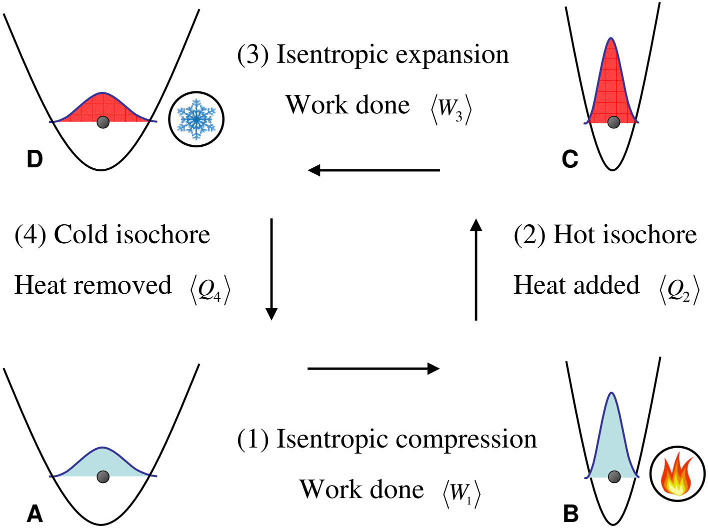


Otto heat engine: Based on superconducting qubit



Karimi and Pekola, PRB (2016)

Otto heat engine



Quantum Oscillator: Thermodynamic analysis I

Gaussian wave function ansatz,

$$\psi_t(x) = \exp \frac{i}{2\,\hbar}[a_t\,x^2 + 2\,b_t\,x + c_t], \tag{1}$$

Using $a_t = \dot{X}_t/X_t$, it can be mapped to the equation of motion of a classical time-dependent harmonic oscillator,

$$\ddot{X}_t + \omega_t^2\,X_t = 0. \tag{2}$$

The dynamics is fully characterized by the adiabaticity quantity

$$Q^*(t) = \frac{1}{2\omega_i\,\omega_t}\left\{\omega_i^2\left[\omega_t^2\,X_t^2 + \dot{X}_t^2\right] + \left[\omega_t^2\,Y_t^2 + \dot{Y}_t^2\right]\right\}. \tag{3}$$

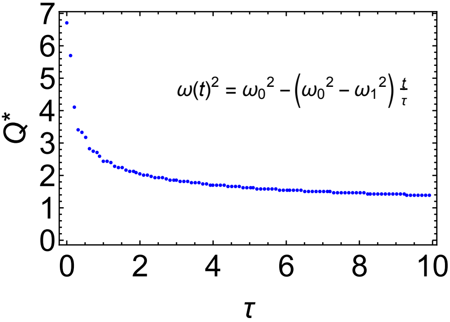
X_t and Y_t are solutions of Eq. (2) satisfying the boundary conditions $X_0 = 0, \dot{X}_0 = 1$ and $Y_0 = 1,\ \dot{Y}_0 = 0$.

Husimi (1953); Deffner, Abah, & Lutz, (2010)

Quantum Oscillator: Thermodynamic analysis II

Adiabaticity parameter $Q^* = \langle H \rangle_{NA} \,/\, \langle H \rangle_{AD}$ Husimi 1953

$\rightarrow$ depends on the driving



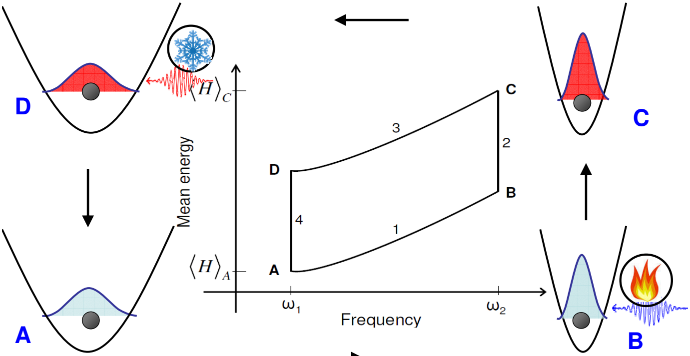
For adiabatic process: $Q^*_{1,2} = 1$

For sudden process: $Q^*_{1,2} = (\omega_1^2 + \omega_2^2)/(2\,\omega_1\,\omega_2)$

Quantum Otto heat engine: theory

$$\langle H \rangle_D = \frac{\hbar\omega_1}{2} Q_2^* \coth\left(\frac{\beta_2 \hbar\omega_2}{2}\right)$$

$$\langle H \rangle_C = \frac{\hbar\omega_2}{2} \coth\left(\frac{\beta_2 \hbar\omega_2}{2}\right)$$



$$\langle H \rangle_A = \frac{\hbar\omega_1}{2} \coth\left(\frac{\beta_1 \hbar\omega_1}{2}\right)$$

$$\langle H \rangle_B = \frac{\hbar\omega_2}{2} Q_1^* \coth\left(\frac{\beta_1 \hbar\omega_1}{2}\right)$$

Quantum Otto cycle: theory

Efficiency:

O. Abah et al., Phys. Rev. Lett. **109**, 203006 (2012).

$$\begin{aligned}\eta &= -\frac{\langle W_1\rangle + \langle W_3\rangle}{\langle Q_2\rangle} \\ &= 1 - \frac{\omega_1}{\omega_2}\frac{\coth(\beta_1\hbar\omega_1/2) - Q_2^*\coth(\beta_2\hbar\omega_2/2)}{Q_1^*\coth(\beta_1\hbar\omega_1/2) - \coth(\beta_2\hbar\omega_2/2)}\end{aligned}$$

Power output:

$$P = \frac{\text{work done per cycle}}{\text{duration of a cycle}} = \frac{-(\langle W_1\rangle + \langle W_3\rangle)}{t_{cycle}}$$

$$= \frac{\hbar(\omega_2 Q_1^* - \omega_1)\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right) + \hbar(\omega_1 Q_3^* - \omega_2)\coth\left(\frac{\beta_2\hbar\omega_2}{2}\right)}{2t_{cycle}}$$

→ exact expressions

Efficiency at maximal power ($\sim$ work)

Power output:

$$P = \frac{\hbar(\omega_2 Q_1^* - \omega_1)\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right) + \hbar(\omega_1 Q_3^* - \omega_2)\coth\left(\frac{\beta_2\hbar\omega_2}{2}\right)}{2t_{cycle}}$$

Maximization: for a given ω_1 & cycle time, maximize with respect to ω_2

High temperature (classical) regime:

- adiabatic process: Curzon-Ahlborn, AJP (1975)

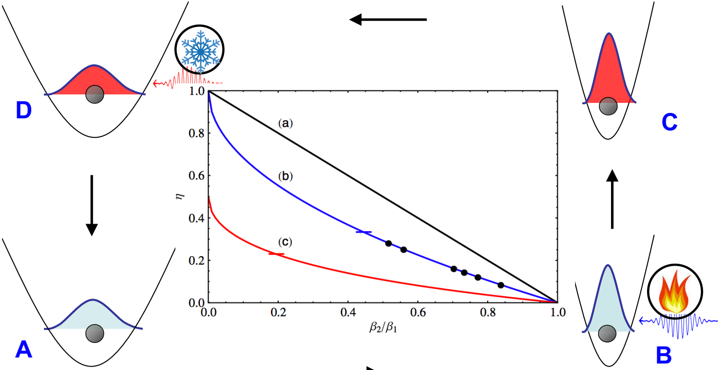
$$\eta = 1 - \sqrt{\frac{\beta_2}{\beta_1}} = 1 - \sqrt{\frac{k_\mathrm{B}\,T_1}{k_\mathrm{B}\,T_2}} \leq 1$$

- sudden frequency switch: Rezek and Kosloff, NJP **8** (2006)

$$\eta_{ss} = \frac{1 - \sqrt{\beta_2/\beta_1}}{2 + \sqrt{\beta_2/\beta_1}} \leq 0.5$$

Abah et al, PRL **112**, 030602 (2012)

$$\langle H \rangle_D = \frac{\hbar\omega_1}{2} Q_2^* \coth\left(\frac{\beta_2 \hbar\omega_2}{2}\right) \qquad \langle H \rangle_C = \frac{\hbar\omega_2}{2} \coth\left(\frac{\beta_2 \hbar\omega_2}{2}\right)$$



$$\langle H \rangle_A = \frac{\hbar\omega_1}{2} \coth\left(\frac{\beta_1 \hbar\omega_1}{2}\right) \qquad \langle H \rangle_B = \frac{\hbar\omega_2}{2} Q_1^* \coth\left(\frac{\beta_1 \hbar\omega_1}{2}\right)$$

Question How can we speed up the heat engine?

Outline

National waiters day

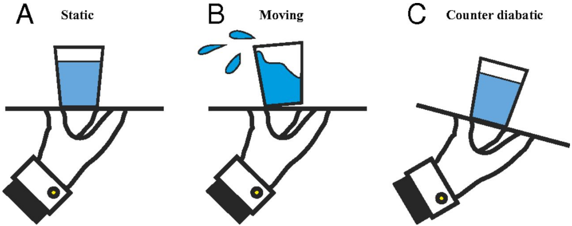


gadgets – shoes, tray, ...

Waiters race, fast service is a priority!

waiters day - Insight..

Counteradiabatic driving (CD) $\rightarrow$ tilting of tray



Sels & Polkovnikov, PNAS (2017)

... inducing a "fast motion video of the adiabatic dynamics."

Chen et al, PRL **109**, 100403 (2010)

Shortcut-to-adiabaticity (STA)

Effective Hamiltonian: $H_{\text{eff}}(t) = H_0(t) + H^i_{\text{STA}}(t)$

$H_{\text{STA}}(t)$ - STA driving Hamiltonian

➔ fast and reduces irreversible losses

Boundary conditions:

$$\begin{aligned} \omega(0) &= \omega_i, \quad \dot{\omega}(0) = 0, \quad \ddot{\omega}(0) = 0, \\ \omega(\tau) &= \omega_f, \quad \dot{\omega}(\tau) = 0, \quad \ddot{\omega}(\tau) = 0, \end{aligned}$$

Counterdiabatic driving method

$$H_{\text{CD}}(t) = H_0(t) + i\hbar \sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle\, |n\rangle\langle n| \right)$$

$|n\rangle = |n(t)\rangle$ denotes the n'th eigenstate of the original Hamiltonian, $H_0(t)$

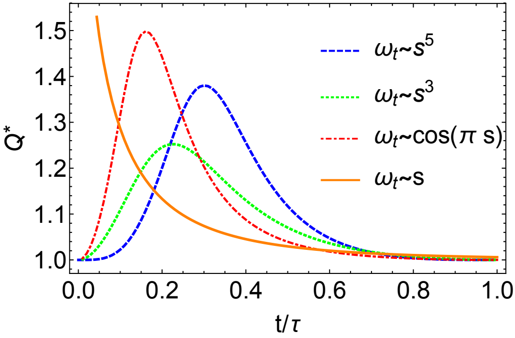
$$H^{\text{CD}}_{\text{STA}}(t) = -\frac{\dot{\omega}_t}{4\,\omega_t}(xp + px)$$

Demirplak and Rice, JPC A **107**, 9937 (2003); Berry, JPA **42**, 365303 (2009);Chen et al, PRL **109**, 100403 (2010)

Counterdiabatic driving method contd...

the adiabaticity parameter,

$$Q^*_{\rm CD}(t) = \frac{\omega_t}{\Omega_t}, \qquad \Omega_t = \omega_t\sqrt{1 - \dot{\omega}_t^2/(4\omega_t^4)}$$



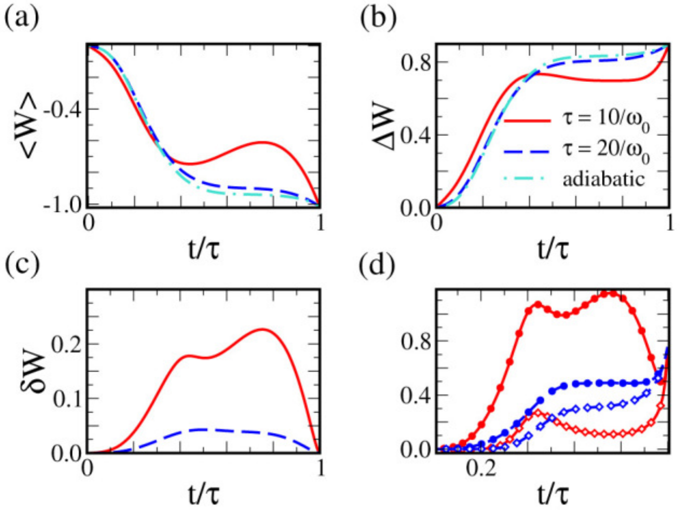
validity: $\Omega_t > 0$

STA protocol on compression/expansion steps

More bang for your buck:
Super-adiabatic quantum engines

A. del Campo[1,2], J. Goold[3] & M. Paternostro[4]

Scientific Report 4 : 6208 (2014)



Question: What is the cost of shortcut driving? **Any free lunch?**

On STA cost...

Cost of counterdiabatic driving and work output
Yuanjian Zheng, Steve Campbell, Gabriele De Chiara, and Dario Poletti
Phys. Rev. A 94 042132 (2016)

Energetic Cost of Superadiabatic Quantum Computation
Ivan B. Coulamy, Alan C. Santos, Itay Hen, and Marcelo S. Sarandy
Front. ICT 3 (2016)

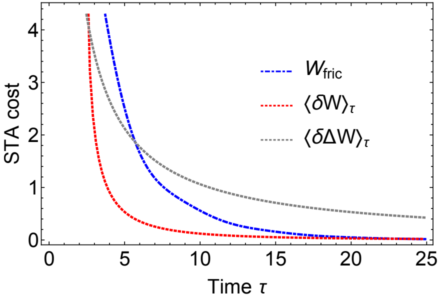
Trade-Off Between Speed and Cost in Shortcuts to Adiabaticity
Steve Campbell and Sebastian Deffner
Phys. Rev. Lett. 118 100601 (2017)

Universal Work Fluctuations During Shortcuts to Adiabaticity by Counterdiabatic Driving
Ken Funo, Jing-Ning Zhang, Cyril Chatou, Kihwan Kim, Masahito Ueda, and Adolfo del Campo
Phys. Rev. Lett. 118 100602 (2017)

Energy efficient quantum machines
Obinna Abah and Eric Lutz
EPL 118 40005 (2017)

Energy consumption for shortcuts to adiabaticity
E. Torrontegui, I. Lizuain, S. González-Resines, A. Tobalina, A. Ruschhaupt, R. Kosloff, and J. G. Muga
Phys. Rev. A 96 022133 (2017)

Energetic cost of the shortcut driving



Work difference

$$\delta W = \langle H_{eff}(t) \rangle - \langle H_0(t) \rangle$$

Work variance

$$\delta(\Delta W)^2 = \langle \Delta W_{eff}^2 \rangle - \langle \Delta W_{AD}^2 \rangle$$

Work friction

$$W_{fric} = \langle W \rangle_{\text{NA}} - \langle W \rangle_{\text{AD}}$$

Time-averaged: $\langle \cdot \rangle_\tau = 1/\tau \int_0^\tau (\cdot) dt$

On STA cost

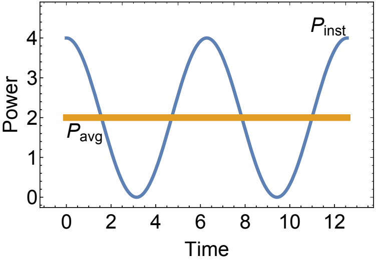
Zheng et al, PRA (2016); Abah and Lutz, EPL (2017)
Campbell and Deffner, PRL (2017); Funo, et al, PRL (2017)

Torrentegui, et al, PRA (2017); Bravetti and Tapias, PRE (2017)

Energetic cost of the shortcut driving

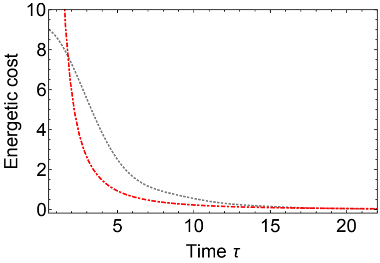
Elementary power analysis

$$P_{inst} = 2 + 2\cos\omega t$$



$$P_{avg} = (1/T)\int_0^T P_{inst}\,dt$$

Abah and Lutz, EPL **118**, 40005 (2017)



Cost of the driving:

$$\langle H^i_{\mathrm{STA}}\rangle_\tau = (1/\tau)\int_0^\tau dt \langle H^i_{\mathrm{SA}}(t)\rangle$$

Nonadiabatic work (friction):

$$\langle W_i\rangle_{\mathrm{NA}} = \langle W_i\rangle - \langle W_i\rangle_{\mathrm{AD}}$$

- the actual and the adiabatic work

Performance of STA quantum engines I

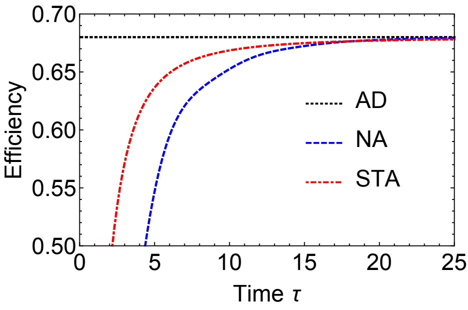
Efficiency:

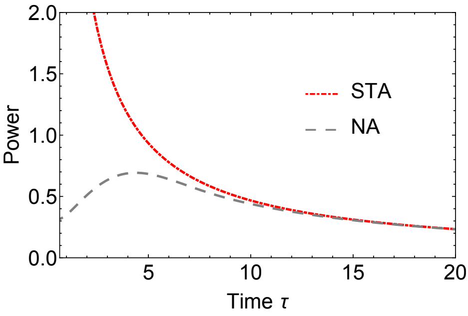
$\langle W \rangle_{\text{STA}} = \langle W \rangle_{\text{AD}}$

$$\eta_{\text{STA}} = \frac{\text{energy output}}{\text{energy input}} = \frac{-(\langle W_1 \rangle_{\text{STA}} + \langle W_3 \rangle_{\text{STA}})}{\langle Q_2 \rangle + \left\langle H^1_{\text{STA}} \right\rangle_\tau + \left\langle H^3_{\text{STA}} \right\rangle_\tau}$$

Power:

$$P_{\text{STA}} = \frac{\text{energy output}}{\text{Cycle time}} = -\frac{\langle W_1 \rangle_{\text{STA}} + \langle W_3 \rangle_{\text{STA}}}{\tau_{\text{cycle}}}.$$



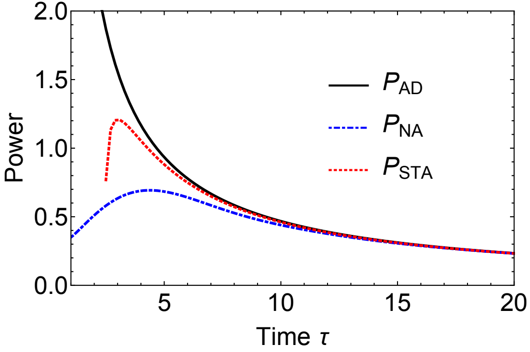


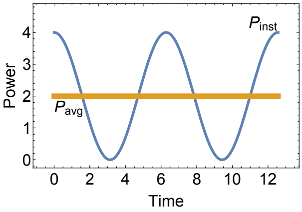
Performance of STA quantum engines II

Power:

Li et al, NJP (2018)

$$P_{\text{STA}} = -\frac{\langle W_1 \rangle_{\text{STA}} + \langle W_3 \rangle_{\text{STA}} - \left\langle H^1_{\text{STA}} \right\rangle_\tau - \left\langle H^3_{\text{STA}} \right\rangle_\tau}{\tau_{\text{cycle}}}$$





"*true*" performance = Time-averaged performance

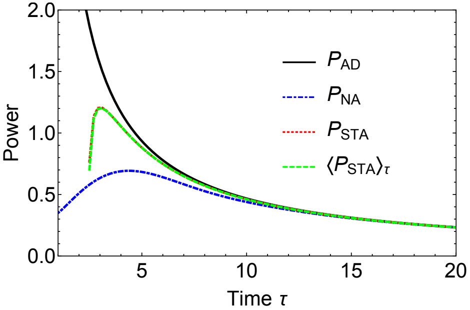
Recall: Power

$$P = \frac{\text{work done per cycle}}{\text{duration of a cycle}} = \frac{-(\langle W_1 \rangle + \langle W_3 \rangle)}{t_{cycle}}$$

$$= \frac{\hbar(\omega_2 Q_1^* - \omega_1)\coth\left(\frac{\beta_1 \hbar \omega_1}{2}\right) + \hbar(\omega_1 Q_3^* - \omega_2)\coth\left(\frac{\beta_2 \hbar \omega_2}{2}\right)}{2 t_{cycle}}$$

Time-averaged power:

$$\langle P_{\text{STA}} \rangle_\tau = -\frac{\langle (\langle W_1 \rangle + \langle W_3 \rangle) \rangle_\tau}{\tau_{\text{cycle}}}$$



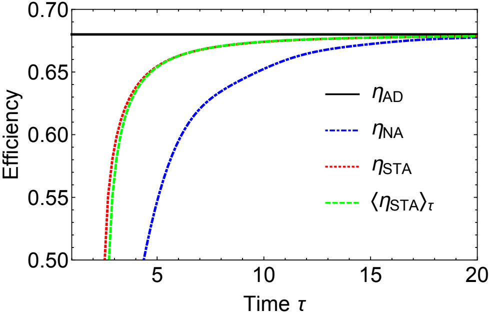
"*true*" performance = Time-averaged performance

Recall: Efficiency

$$\eta = -\frac{\langle W_1\rangle + \langle W_3\rangle}{\langle Q_2\rangle} = 1 - \frac{\omega_1}{\omega_2}\frac{\coth(\beta_1\hbar\omega_1/2) - Q_2^* \coth(\beta_2\hbar\omega_2/2)}{Q_1^*\coth(\beta_1\hbar\omega_1/2) - \coth(\beta_2\hbar\omega_2/2)}$$

Time-averaged efficiency:

$$\langle \eta_{\mathrm{STA}}\rangle_\tau = \left\langle \frac{-(\langle W_1\rangle + \langle W_3\rangle)}{\langle Q_2\rangle}\right\rangle_\tau$$



Outline

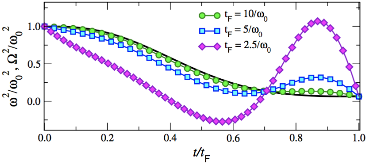
Local counteradiabatic driving (LCD)

Recall: $H^{\rm CD}_{\rm STA}(t) = -\frac{\dot\omega_t}{4\,\omega_t}(xp+px)$ using $U_x = \exp\left(im\dot\omega_t x^2/4\hbar\omega\right)$

$$\begin{aligned} H_{\rm LCD}(t) &= U_x^\dagger(H^{\rm CD}_{\rm STA}(t) - i\hbar\dot U_x U_x^\dagger)U_x \\ &= \frac{p^2}{2m} + \frac{m\tilde\Omega_t^2 x^2}{2} \end{aligned}$$

the adiabaticity parameter,

$$Q^*_{\rm LCD}(t) = 1 - \frac{\dot\omega_t^2}{4\omega_t^4} + \frac{\ddot\omega_t}{4\omega_t^3}.$$



For harmonic oscillator:

$$\begin{aligned} H_{\rm STA} &= \frac{m}{2}\left(\tilde\Omega_t^2 - \omega_t^2\right)x^2 \\ &= \frac{m}{2}\left(-\frac{3\dot\omega_t^2}{4\omega_t^2} + \frac{\ddot\omega_t}{2\omega_t}\right)x^2 \end{aligned}$$

Chen et al, PRL **109**, 100403 (2010)
del Campo, PRL **111**, 100502 (2013)

Inverse engineering (IE)

Based on Lewis-Riesenfeld invariants of motion

$$I(t) = \frac{1}{2}\left(\frac{x^2}{\bar{b}^2}m\omega_0^2 + \frac{1}{m}\pi^2\right),$$

where $\pi = \bar{b}p - m\dot{\bar{b}}x$ plays the role of a momentum conjugate to $x/\bar{b}$

The dimensionless scaling function $\bar{b}_t = \bar{b}(t)$ satisfies the Ermakov equation,

$$\ddot{\bar{b}}_t + \omega_t^2\bar{b}_t = \omega_0^2/\bar{b}_t^3.$$

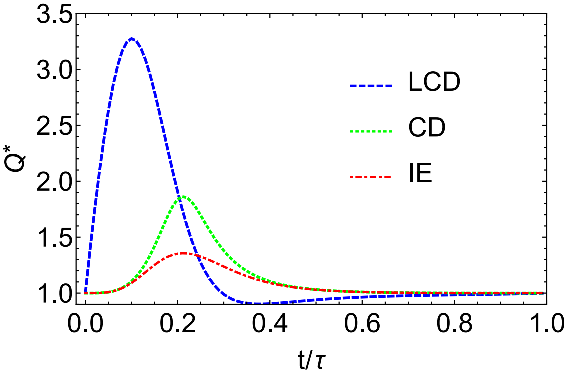
boundary conditions:

$$\begin{aligned}
&\bar{b}(0) = 1, && \dot{\bar{b}}(0) = 0, && \ddot{\bar{b}}(0) = 0,\\
&\bar{b}(\tau) = \sqrt{\omega_0/\omega_f}, && \dot{\bar{b}}(\tau) = 0, && \ddot{\bar{b}}(\tau) = 0,
\end{aligned}$$

Hierarchy of STA methods

IE contd... adiabaticity parameter reads

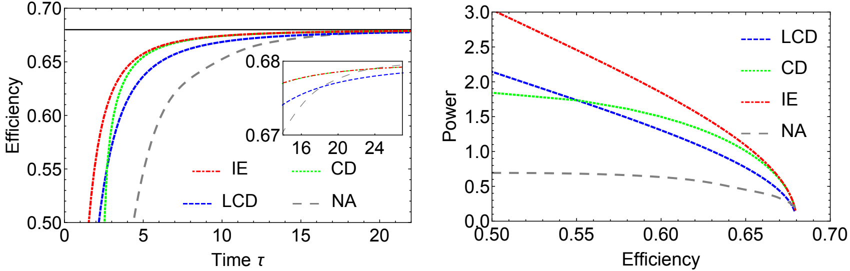
$$Q^*_{\mathrm{IE}}(t) = 1 + \frac{\dot{\omega}_t^2}{8\omega_t^4}$$



Q^* - Adiabaticity parameter of different methods

Abah and Lutz, arXiv. 1707.09963 (2017)

Fast and efficient engines



➔ simultaneous increase of efficiency and power for fast cycles

Outline

Bounds on efficiency at maximum power ($Q^* = 1$)

Validity of STA, $t_f \geq 1/(2\omega_f)$

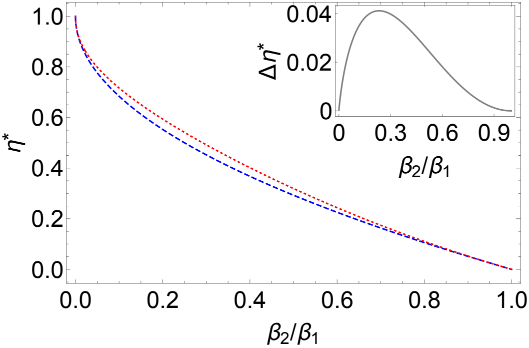
$\tau_{cycle} = \tau_1 + \tau_3 \simeq 1/\omega_1 + 1/\omega_2$

Power, $P = -\langle W_{\text{tot}} \rangle / \tau_{cycle}$

$$\eta^*_{STA} = 1 - \frac{\gamma_\beta + \sqrt{2\gamma_\beta(1+\gamma_\beta)}}{2+\gamma_\beta}.$$

$\Delta\eta^* = \eta^*_{STA} - \eta_{CA}$

$\gamma_\beta = \langle H \rangle_A / \langle H \rangle_C$



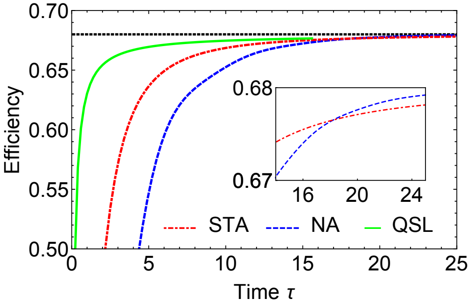
Generic bounds: quantum speed limit (QSL)

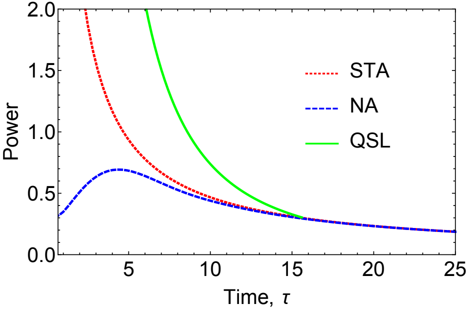
Quantum: limits the speed of evolution of a system   Anandan and Aharonov, PRL (1990)

QSL time: $\tau_{\text{QSL}} = \frac{\hbar \mathcal{L}(\rho_i,\rho_f)}{\langle H_{\text{STA}} \rangle_\tau} \leq \tau$ $\mathcal{L}(\rho_i, \rho_f)$ - the Bures angle between density operators

Efficiency: $\eta_{\text{STA}} \leq \eta_{\text{STA}}^{\text{QSL}} = -\frac{\langle W_1 \rangle_{\text{AD}} + \langle W_3 \rangle_{\text{AD}}}{\langle Q_2 \rangle + \hbar(\mathcal{L}_1 + \mathcal{L}_3)/\tau}$

Power: $P_{\text{STA}} \leq P_{\text{STA}}^{\text{QSL}} = -\frac{\langle W_1 \rangle_{\text{AD}} + \langle W_3 \rangle_{\text{AD}}}{\tau_{\text{QSL}}^1 + \tau_{\text{QSL}}^3}$ Abah and Lutz, EPL 118, 40005 (2017)



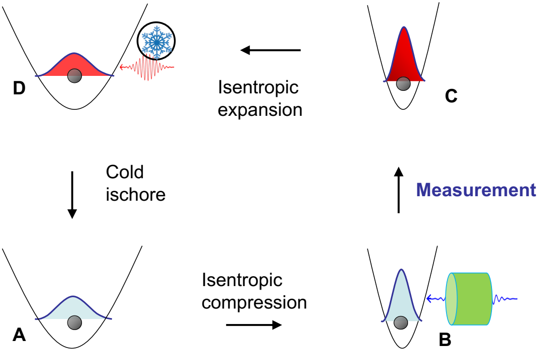


Outline

Outlook:

- Measurement driven single temperature engine

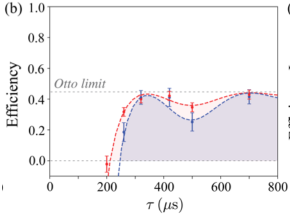
Ding et al, 1803.07621 (2018)



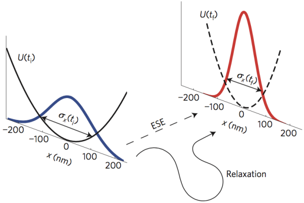
$$\eta = -\langle W \rangle \, / \, \langle E_m \rangle$$

Outlook/Future direction...

- spin quantum heat engine   Peterson et al, arXiv. (2018)



- system + reservoir   Martinez et al, Nature Phys. (2016)



- many-body, ... , quantumness, more...

Take-Home message

- STA engine are energy efficient machines
  - ➔ outperform their convention counterpart
- Power doesn't always equal performance
  - ➔ overall efficiency is important quantity
- Quantum speed limit impose bounds to performance
  - ➔ fundamental limit for quantum machines

Thanks - for - your - attention!!

References

⋆ *Energy efficient quantum machines*
O. Abah and E. Lutz

EPL (Europhys. Lett.) **118**, 40005 (2017)

⋆ *Performance of shortcut-to-adiabaticity quantum engines*
O. Abah and E. Lutz

arxiv: 1707.09963 (2017)

⋆ *Shortcut-to-adiabaticity quantum refrigerator*
O. Abah, M. Paternostro, and E. Lutz

(*to appear "soon"*)

⋆ *STA Otto engine: A new twist to finite-time thermodynamics*

O. Abah and M. Paternostro (*in preparation*)

⋆ *STA of a spin quantum heat engine*

⋆ ...

Thanks - for - your - attention!!