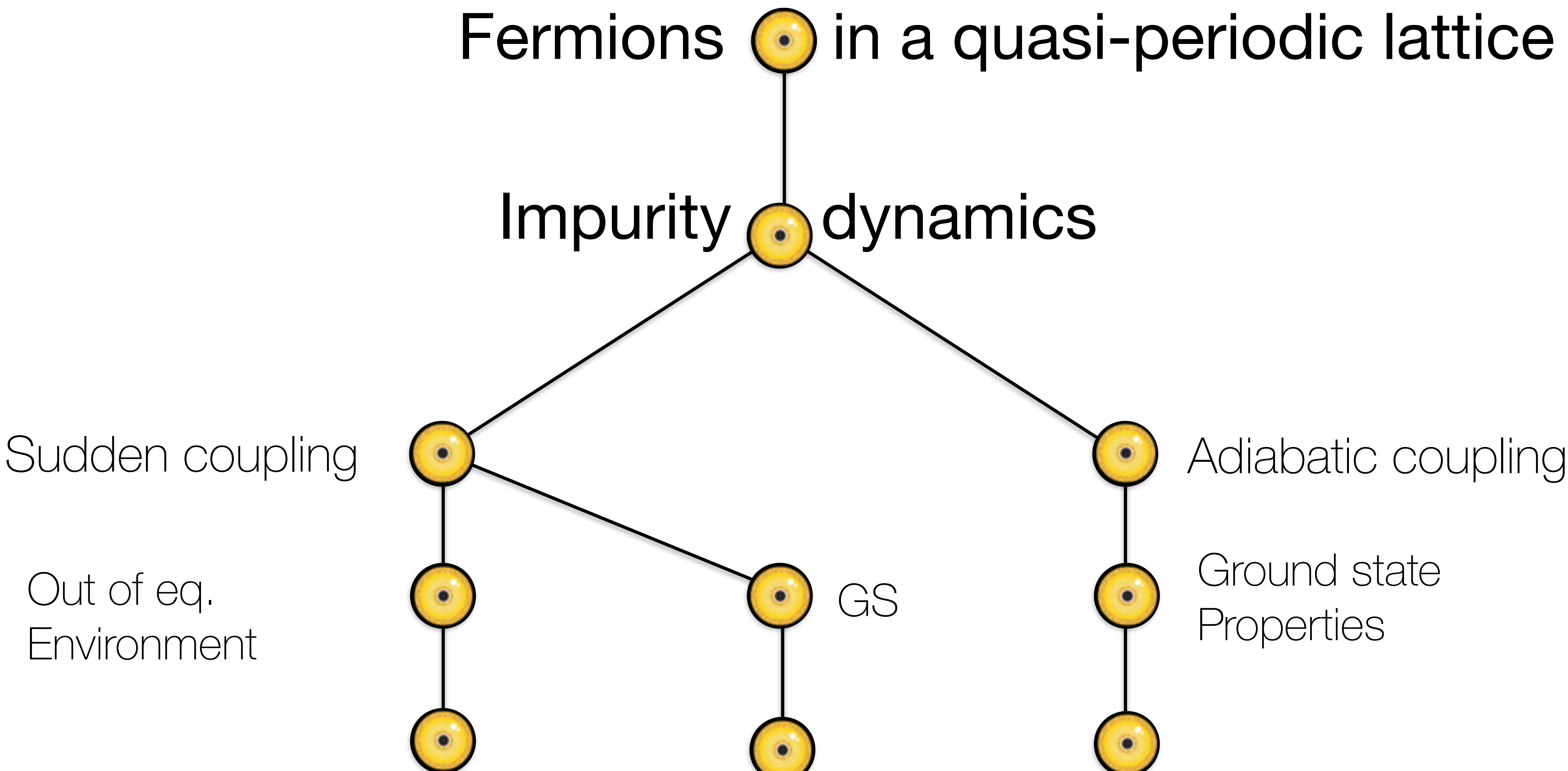
Quasi-periodic Lattices: memory effects and work statistics

Francesco Cosco
QTF centre of excellence, Turku Centre for Quantum Physics, University of Turku

21/06/2018

KITP

OUTLINE



- Information backflow
- Memory effects

In preparation

- Work statistics
- *Anomalies* in WS

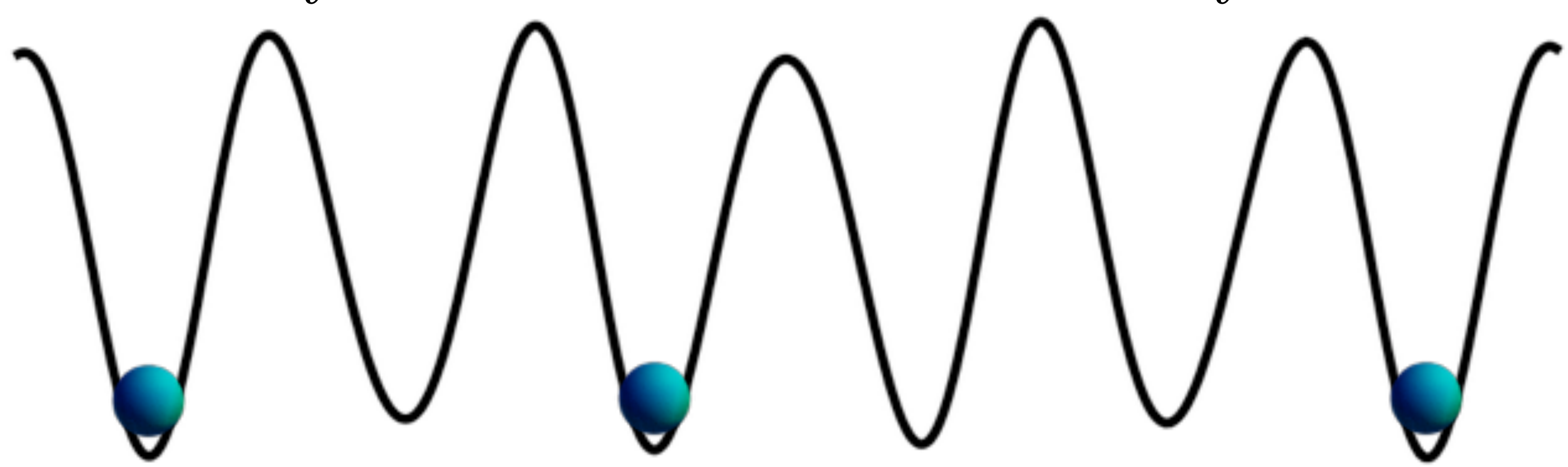
- Orthogonality Catastrophe
- Density fluctuations (GS)

arXiv:1803.04382

THE ENVIRONMENT

Fermi-Hubbard model with a quasi periodic on-site potential

$$\hat{H} = -J\sum_i (\hat{a}^\dagger_{i+1}\hat{a}_i + \hat{a}^\dagger_i \hat{a}_{i+1}) + \sum_i V_i \hat{n}_i$$



$V_i = \Delta \cos(2\pi\beta i + \phi)$

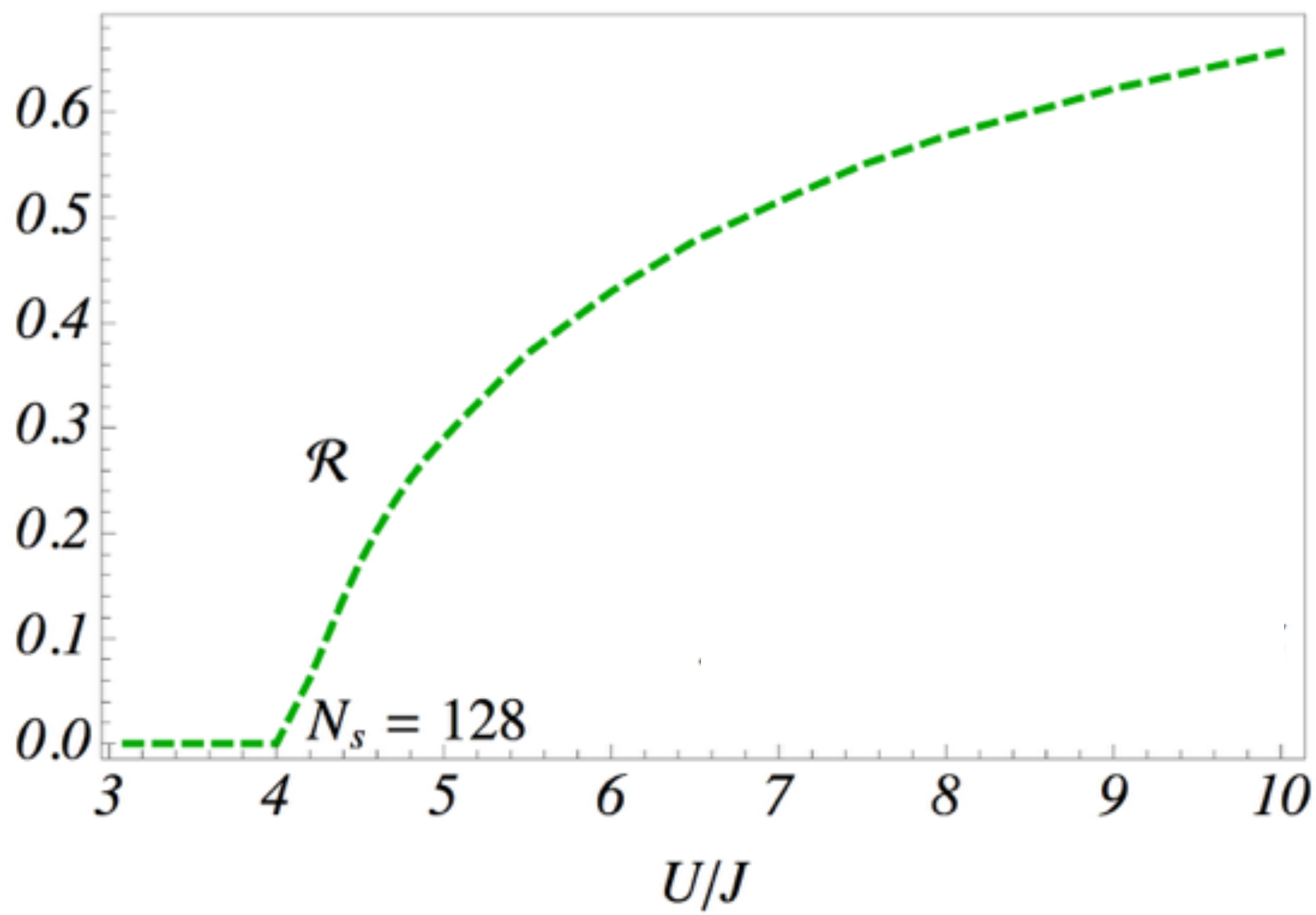
β Irrational

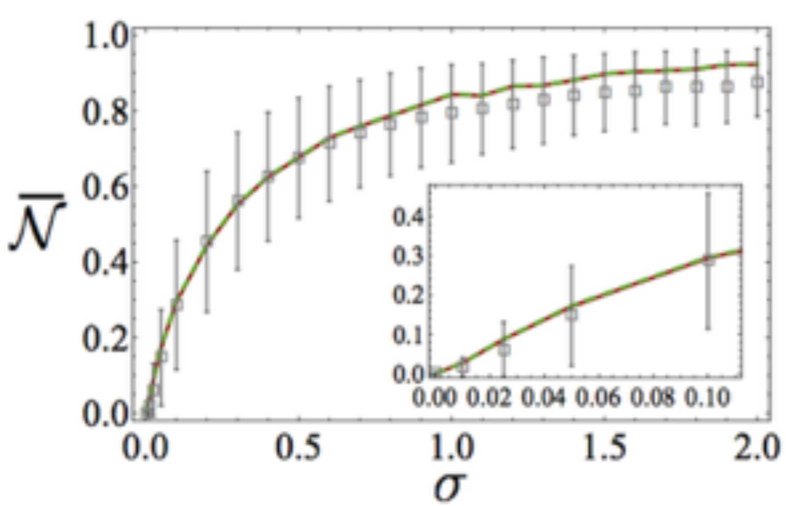
Interplay **quasi-periodic** potential

and **kinetic** term

N-M AND LOCALISATION

Bose lattice as controllable environment





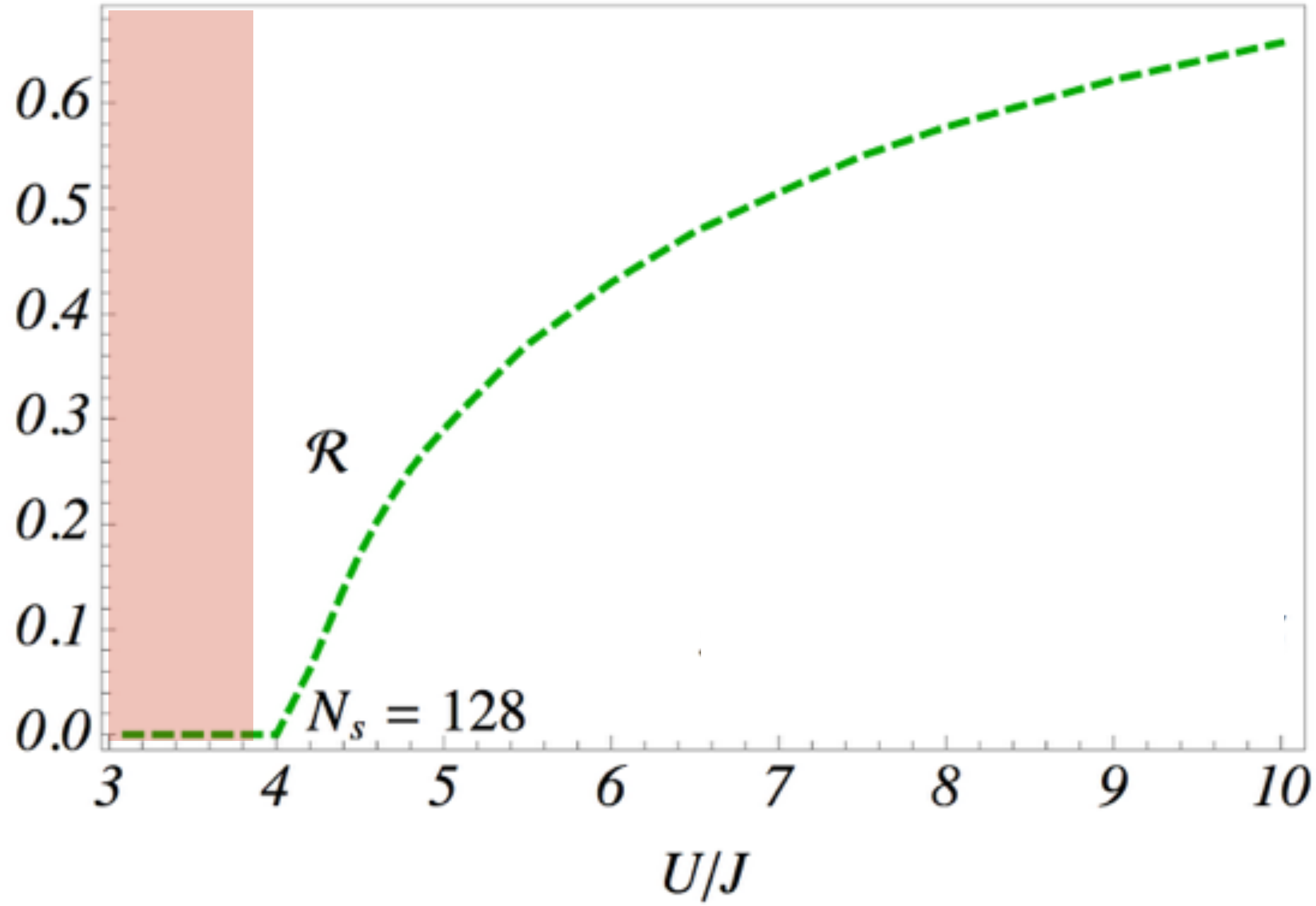
Quantum non-Markovianity induced by Anderson localization

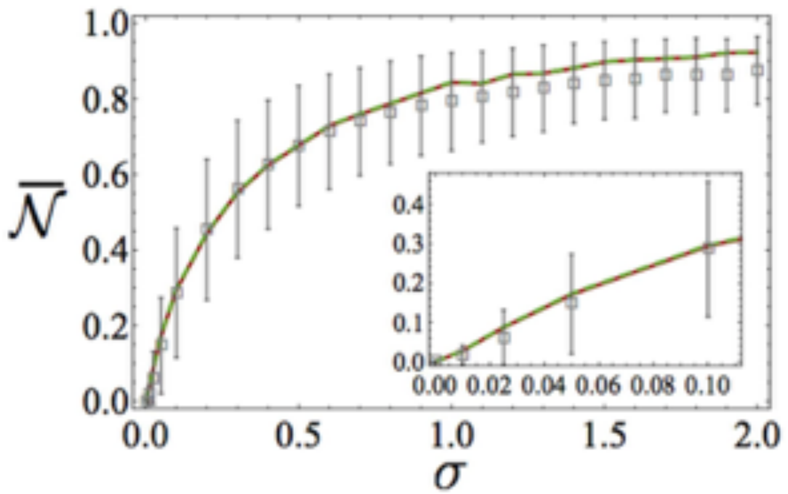
S. Lorenzo, F. Lombardo, F. Ciccarello, and G. M. Palma, *Scientific Reports* **7**, 42729 (2017)

N-M AND LOCALISATION

Bose lattice as controllable environment

superfluid - Markovian





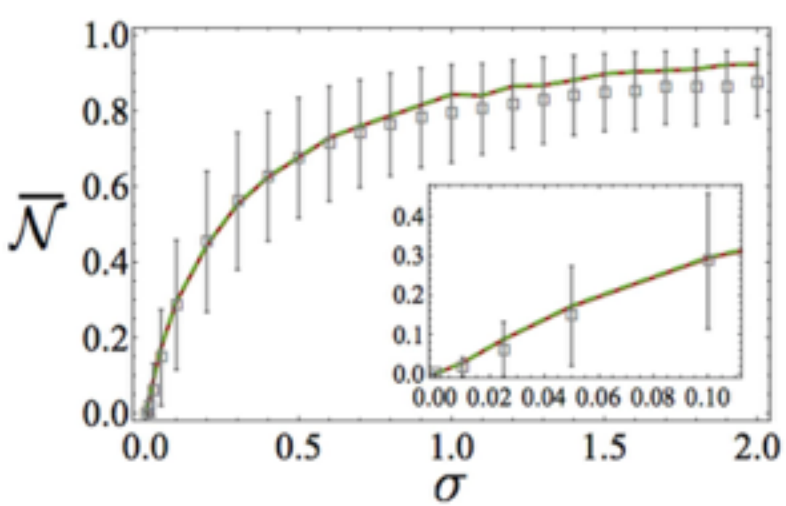
Quantum non-Markovianity induced by Anderson localization

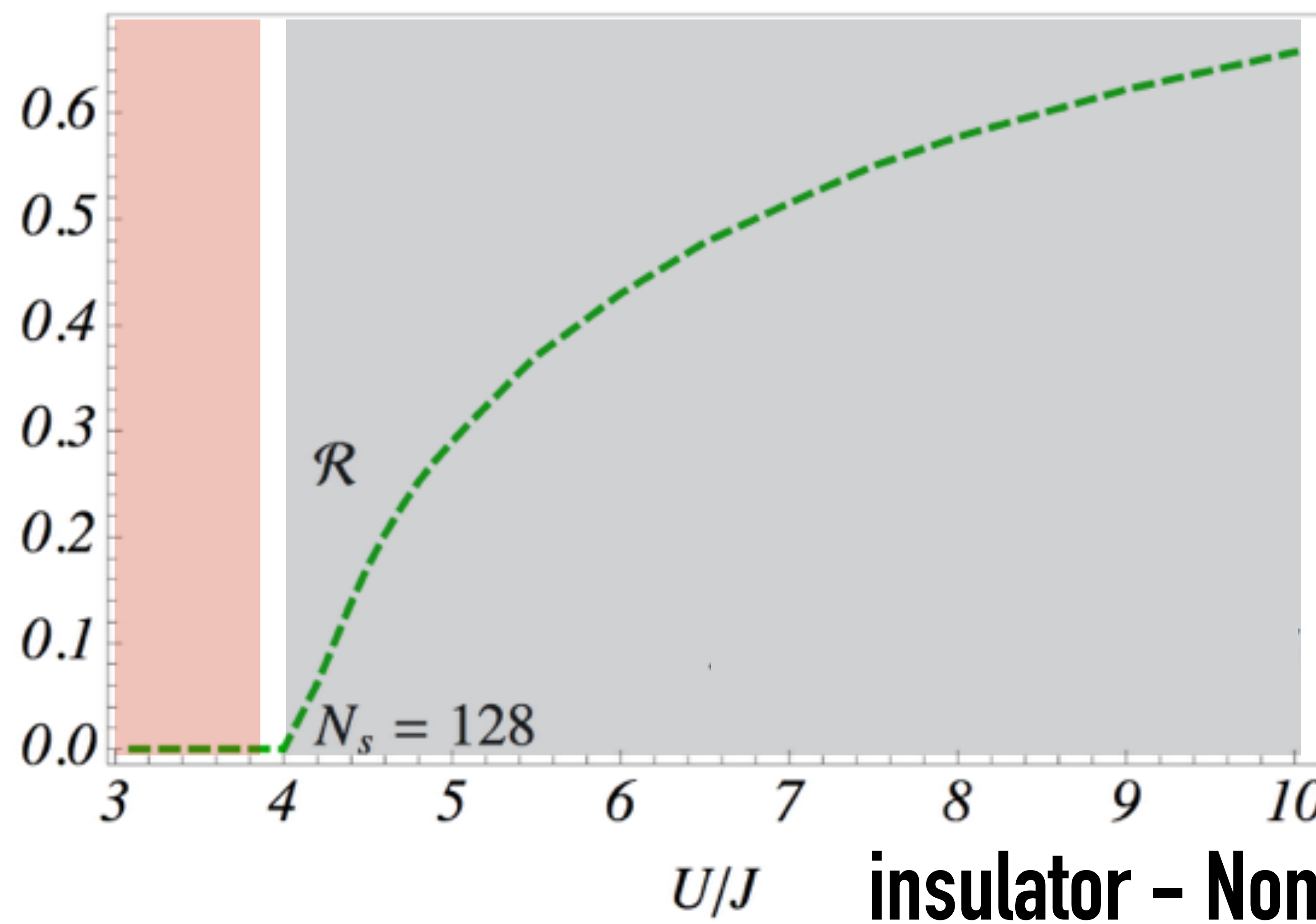
S. Lorenzo, F. Lombardo, F. Ciccarello, and G. M. Palma, *Scientific Reports* **7**, 42729 (2017)

N-M AND LOCALISATION

Bose lattice as controllable environment

superfluid – Markovian





insulator – NonMarkovian

Quantum non-Markovianity induced by Anderson localization

S. Lorenzo, F. Lombardo, F. Ciccarello, and G. M. Palma, *Scientific Reports* **7**, 42729 (2017)

THE ENVIRONMENT

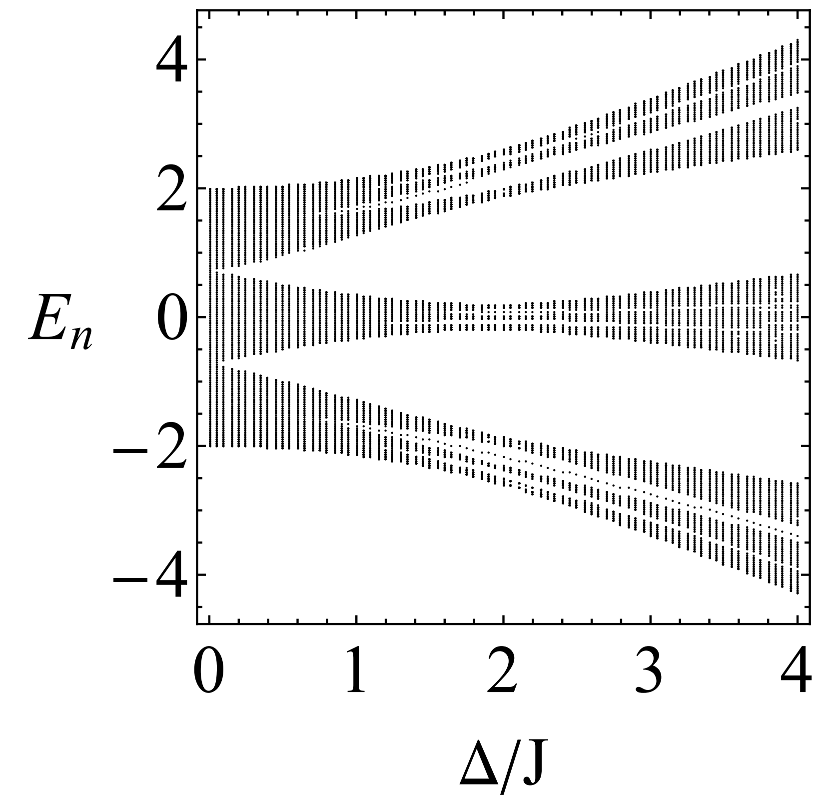
Aubry-André model

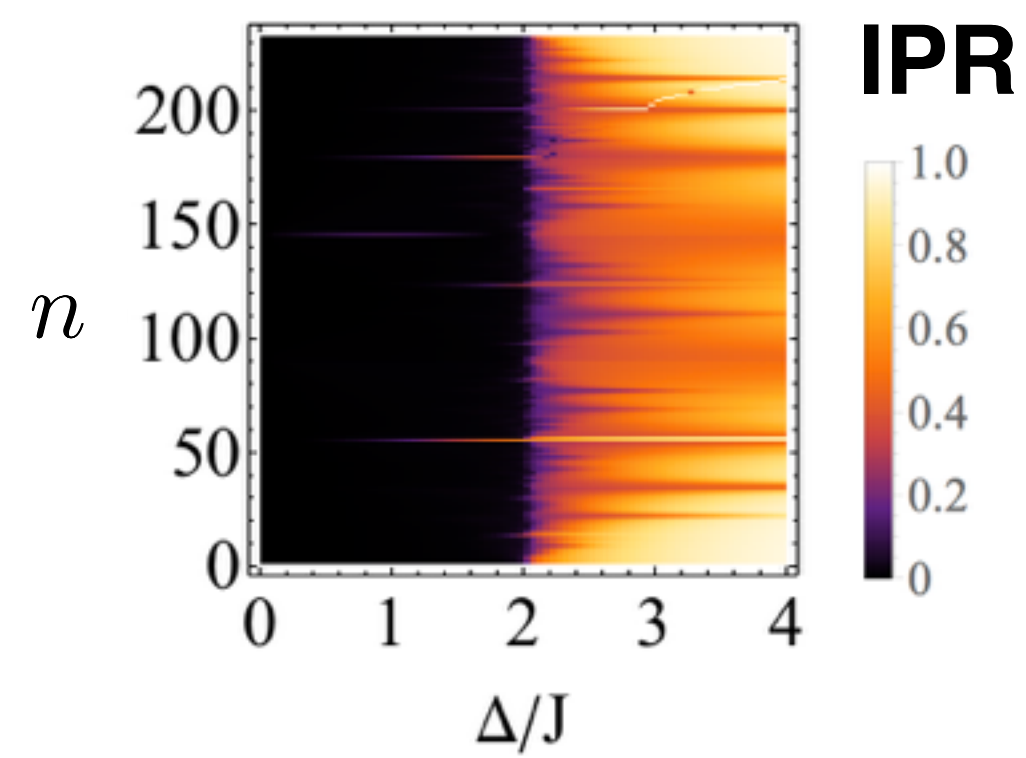
$$V_i = \Delta \cos(2\pi\beta i + \phi)$$

β Irrational

Eigenstates localisation

$$\Delta / J \geq 2$$





THE ENVIRONMENT

Aubry-André model

$$V_i = \Delta \cos(2\pi\beta i + \phi)$$

β Irrational

Eigenstates localisation

$$\Delta / J \geq 2$$

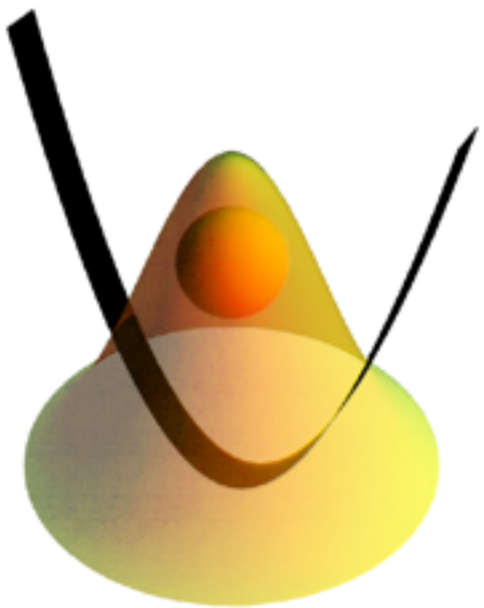
Often compared

Anderson insulator

$$V_i \in [-\Delta, +\Delta]$$

Eigenstates localisation $\Delta > 0$

THE SYSTEM: IMPURITY

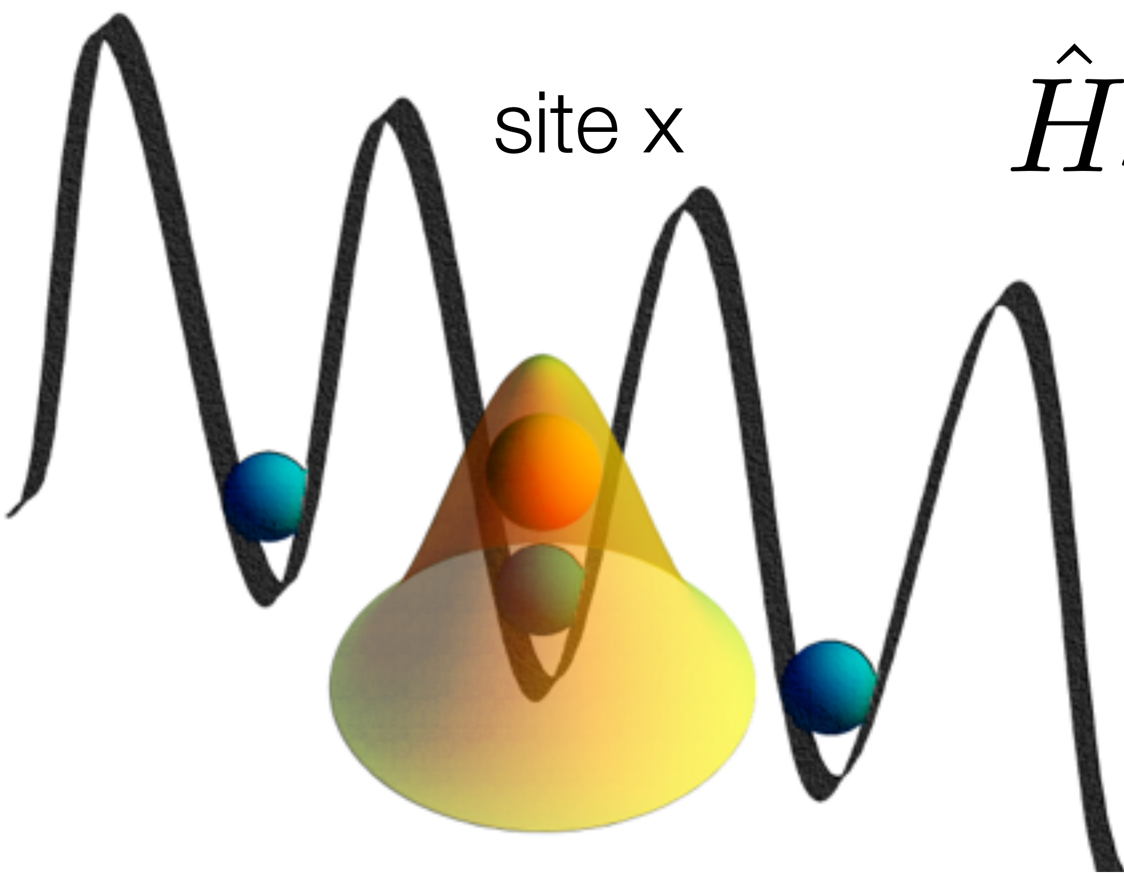


Impurity atom

- Motional ground state $\psi_0(x)$
- two lowest internal states $|e\rangle\ |g\rangle$

$$\hat{H}_S = \frac{\omega_0}{2}\hat{\sigma}_z$$

THE S-E INTERACTION



$$\hat{H}_{int} = \epsilon(t)|e\rangle\langle e| \otimes \hat{a}_0^\dagger \hat{a}_0$$

local number operator $\hat{n}_0$

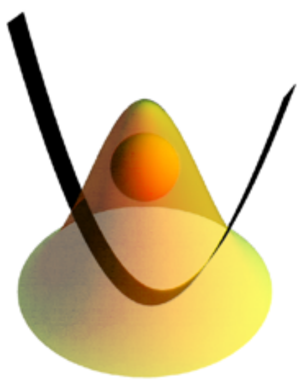
$$\epsilon \propto \int dx|\psi_0(x)\omega_0(x)|^2$$

density-density interaction

LOSCHMIDT ECHO OF THE FERMI LATTICE

$$|\rho_{eg}(t)/\rho_{eg}(0)| = |\langle\Psi_0|e^{i\hat{H}_g t}e^{-i\hat{H}_e t}|\Psi_0\rangle| = \sqrt{L(t)}$$

$$\hat{H}_g = \langle g|\,\hat{H}\,|g\rangle \quad \hat{H}_e = \langle e|\,\hat{H}\,|e\rangle$$

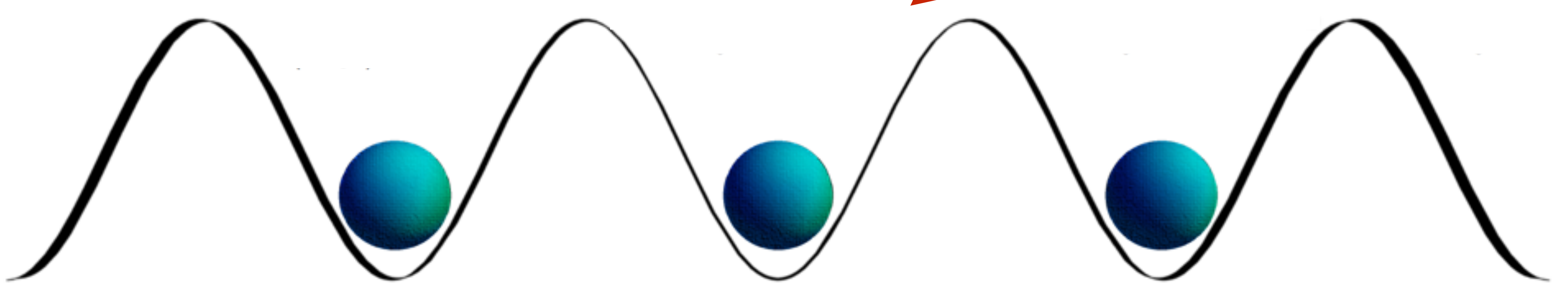


probe

$$\rho(0) = |\varphi\rangle\langle\varphi| \otimes |\Psi_0\rangle\langle\Psi_0|$$

$$|\varphi\rangle = \frac{1}{2}(|g\rangle + |e\rangle)$$

environment



IMPURITY DYNAMICS

No initial correlations

$$\hat{\rho}_S(t) = \Lambda_t[\hat{\rho}_S(0)] = \mathrm{Tr}[\hat{U}(t)\rho_S(0) \otimes \rho_E(0)\hat{U}^\dagger(t)]$$

$$= \begin{pmatrix} \rho_{gg}(0) & \chi^*(t)\rho_{ge}(0) \\ \chi(t)\rho_{eg}(0) & \rho_{ee}(0) \end{pmatrix}$$

Decoherence function

$$\chi(t) = \langle \Psi_0 | e^{i\hat{H}_g t} e^{-i\hat{H}_e t} | \Psi_0 \rangle$$

IMPURITY DYNAMICS

No initial correlations

$$\hat{\rho}_S(t) = \Lambda_t[\hat{\rho}_S(0)] = \mathrm{Tr}[\hat{U}(t)\rho_S(0) \otimes \rho_E(0)\hat{U}^\dagger(t)]$$

$$= \begin{pmatrix} \rho_{gg}(0) & \chi^*(t)\rho_{ge}(0) \\ \chi(t)\rho_{eg}(0) & \rho_{ee}(0) \end{pmatrix}$$

Decoherence function

$$\chi(t) = \det(1 - \hat{r} + \hat{r}e^{-i\hat{h}_e t}e^{i\hat{h}_g t})$$

Klich I 2003 An Elementary Derivation of Levitov's Formula BT - Quantum Noise in Mesoscopic Physics ed Nazarov Y V

IMPURITY DYNAMICS

Uncorrelated initial state

$$\hat{\rho}_S(t) = \Lambda_t[\hat{\rho}_S(0)] = \mathrm{Tr}[\hat{U}(t)\rho_S(0) \otimes \rho_E(0)\hat{U}^\dagger(t)]$$
$$= \begin{pmatrix} \rho_{gg}(0) & \chi^*(t)\rho_{ge}(0) \\ \chi(t)\rho_{eg}(0) & \rho_{ee}(0) \end{pmatrix}$$

Decoherence function

$$\chi(t) = \det(1 - \hat{r} + \hat{r}e^{-i\hat{h}_e t}e^{i\hat{h}_g t})$$

$\hat{r}$ → Environment

$e^{i\hat{h}_g t}$ → Single particle counterpart of

$$\hat{H}_{e/g} = \langle e/g|\hat{H}_{AA} + \hat{H}_{int}|e/g\rangle$$

Klich I 2003 An Elementary Derivation of Levitov's Formula BT - Quantum Noise in Mesoscopic Physics ed Nazarov Y V

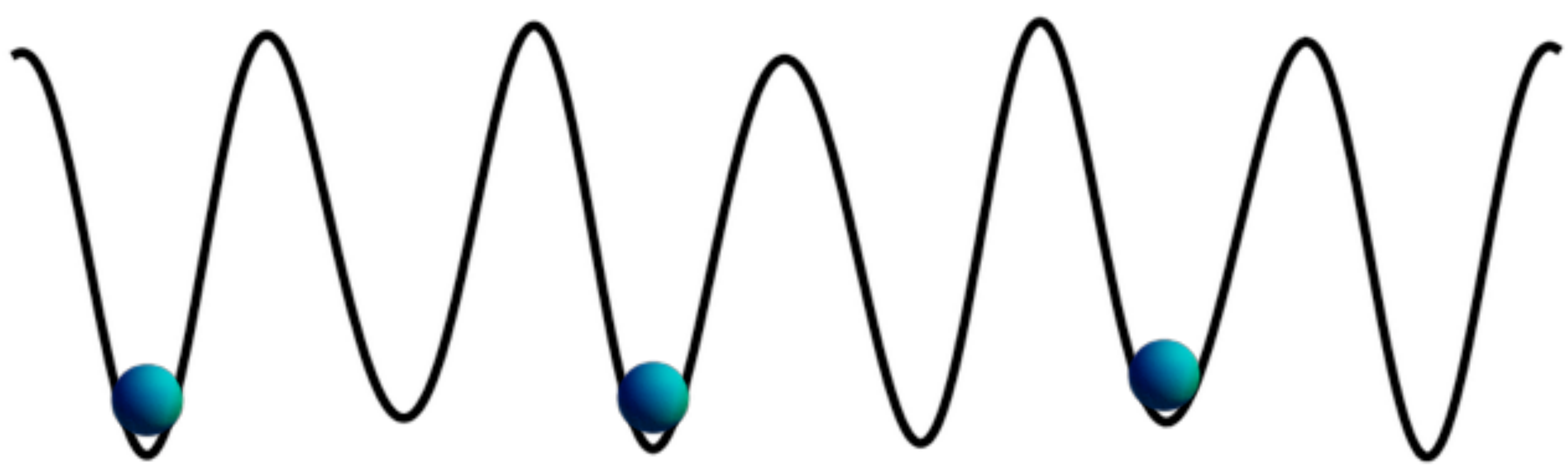
ENVIRONMENT

Initial state of the environment

$$|\Psi_0\rangle = \prod_{i\,\mathrm{odd}} \hat{a}_i^\dagger |0\rangle \quad \Rightarrow \quad \chi(t) = \det(1 - \hat{n}_{cdw} + \hat{n}_{cdw} e^{-i\hat{h}_e t} e^{i\hat{h}_g t})$$

$$\hat{n}_{cdw} = \sum_{i\,\mathrm{odd}} |i\rangle\langle i|$$

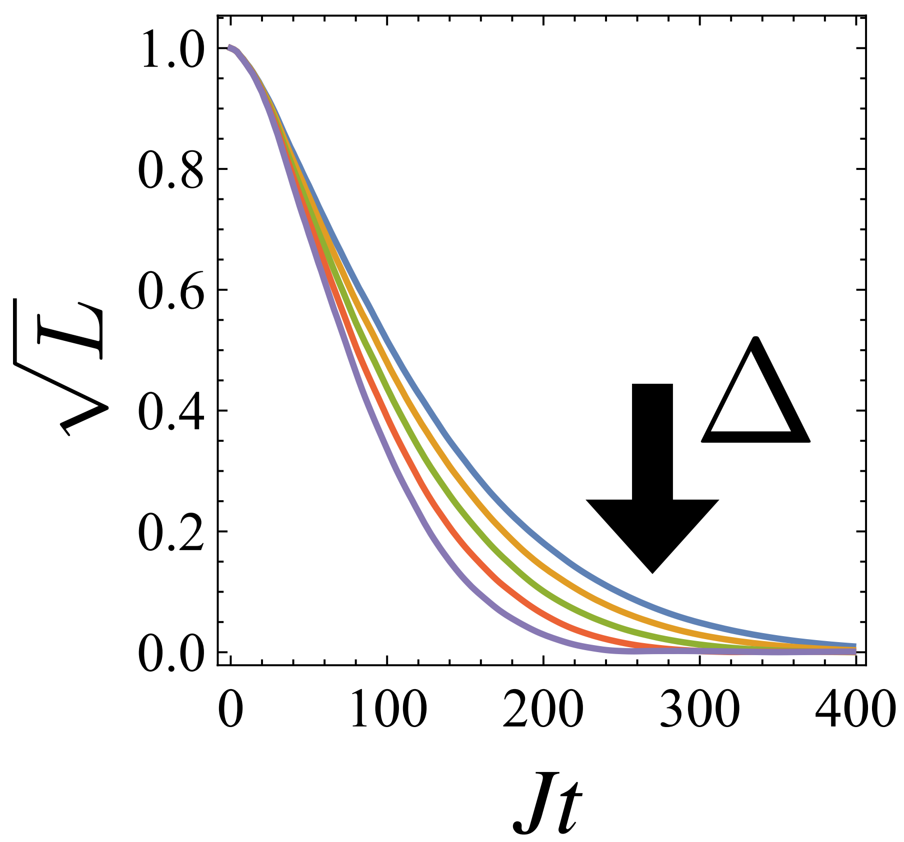
Charge density wave state (**CDW**)



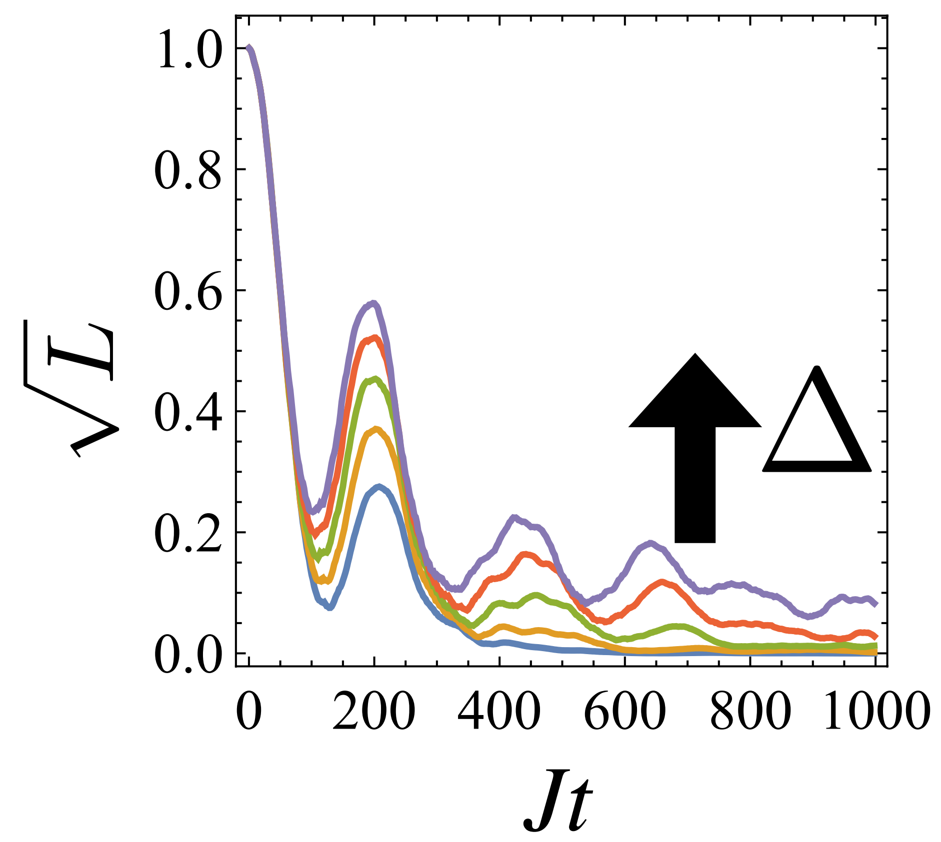
Out of equilibrium environment

H. P. Lüschen, P. Bordia, S. Scherg, F. Alet, E. Altman, U. Schneider, and I Bloch, Phys. Rev. Lett. **119**, 260401 (2017)
S. Vardhan, G. De Tomasi, M. Heyl, E. J. Heller, and F. Pollmann, Phys. Rev. Lett. **119**, 016802 (2017)

DELOCALISED PHASE

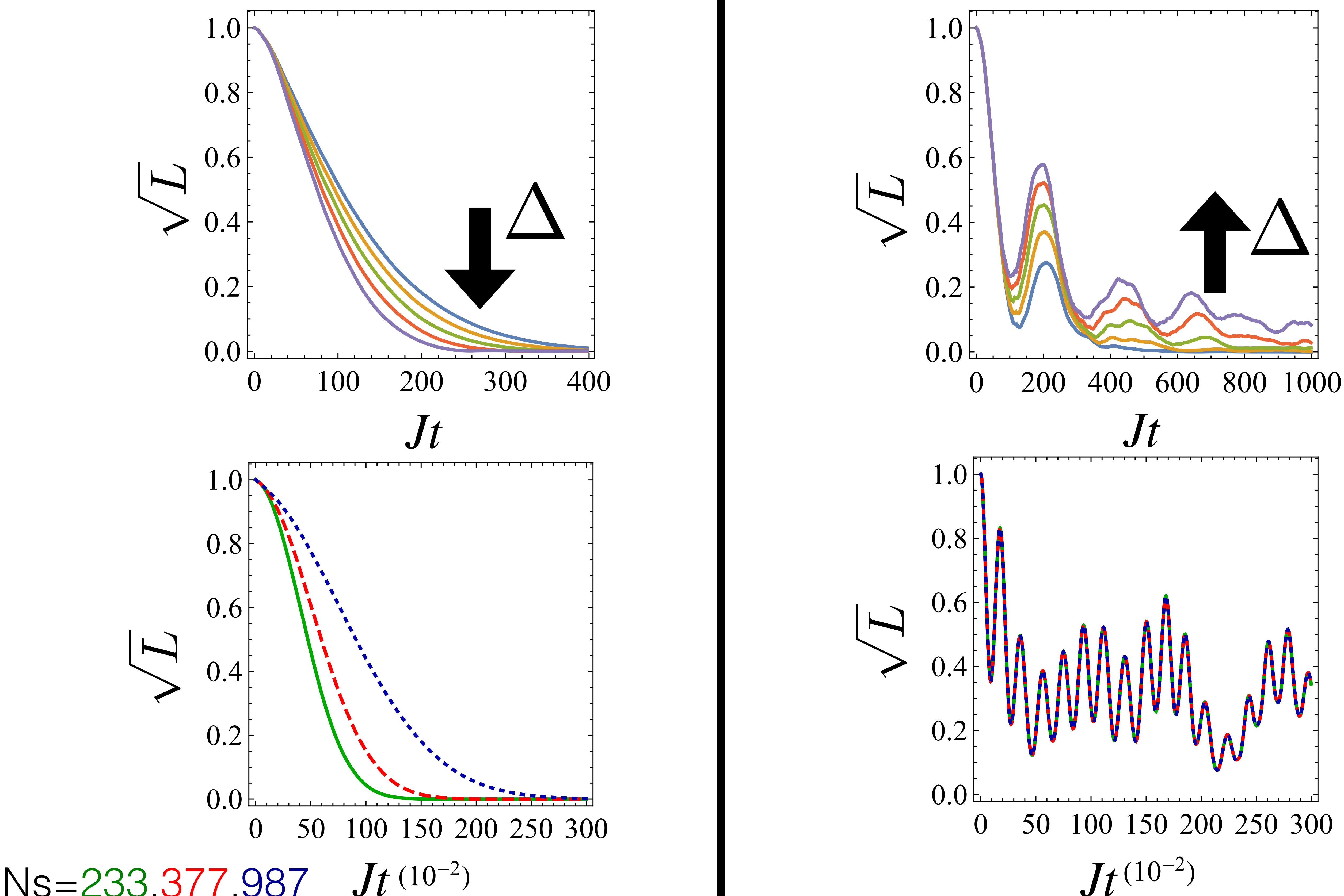


LOCALISED PHASE



DELOCALISED PHASE

LOCALISED PHASE



NON-MARKOVIANITY MEASURE

- Information Flow

$$\sigma(t) \equiv \frac{d}{dt} D(\Phi_t \rho_S^1, \Phi_t \rho_S^2)$$

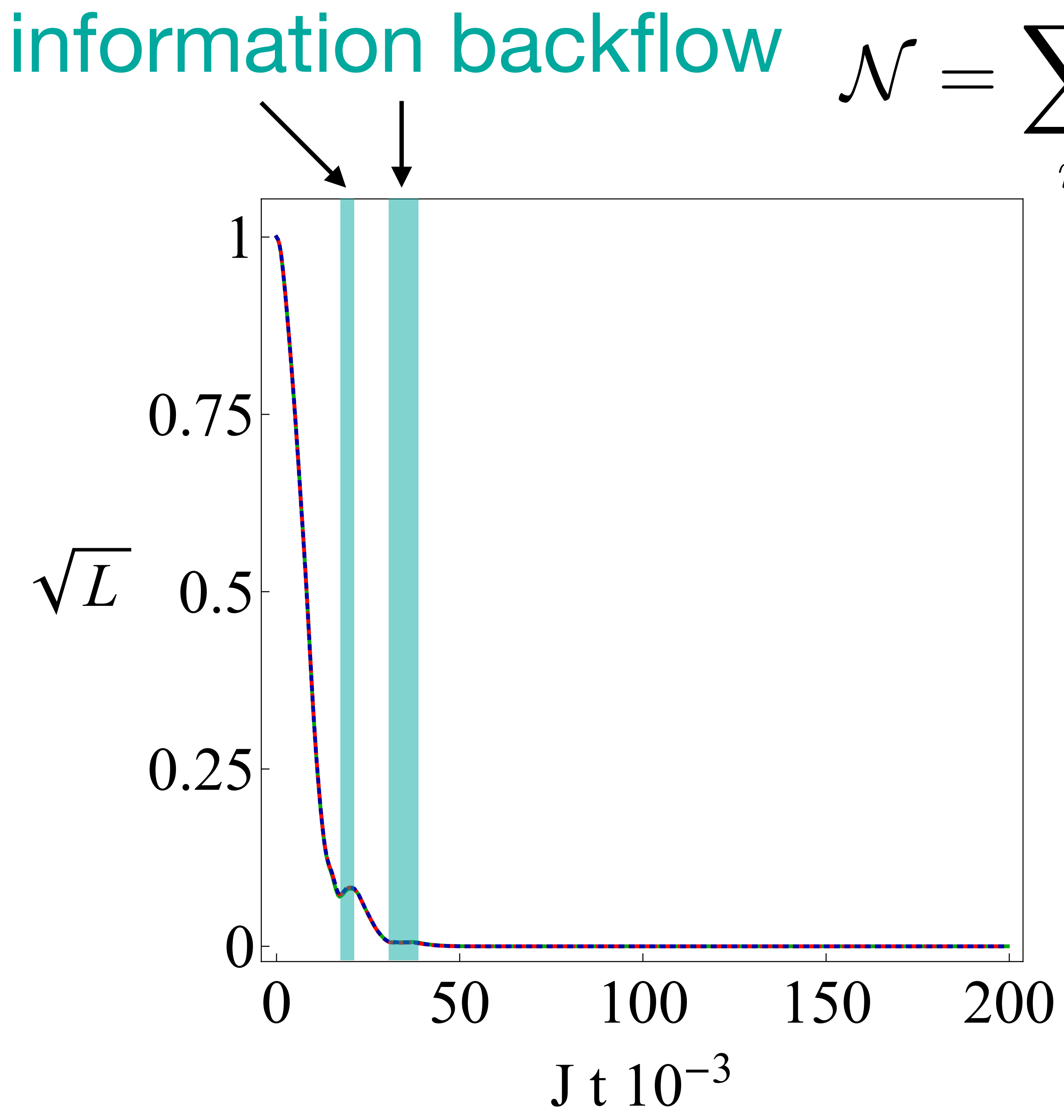
- BLP-measure

$$\mathcal{N}(\Phi) = \max_{\rho_S^{1,2}} \int_{\sigma>0} dt\, \sigma(t)$$

Heinz-Peter Breuer, Elsi-Mari Laine, and Jyrki Piilo
Phys. Rev. Lett. **103**, 210401 (2009)

NON-MARKOVIANITY MEASURE

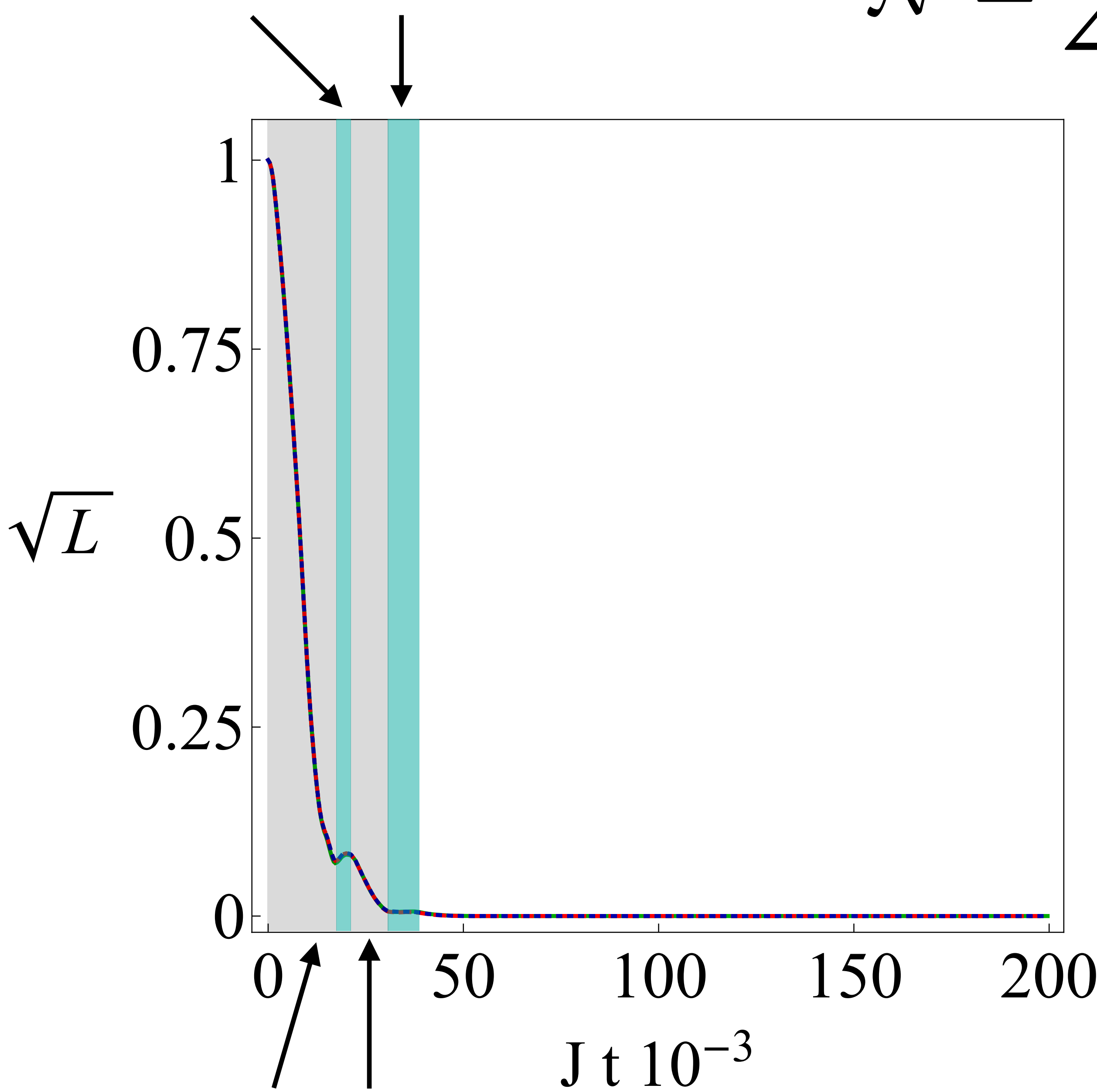
$$\mathcal{N} = \sum_n \sqrt{L(t_{n+1})} - \sqrt{L(t_n)}$$



NON-MARKOVIANITY MEASURE

information backflow

$$\mathcal{N} = \sum_n \sqrt{L(t_{n+1})} - \sqrt{L(t_n)}$$

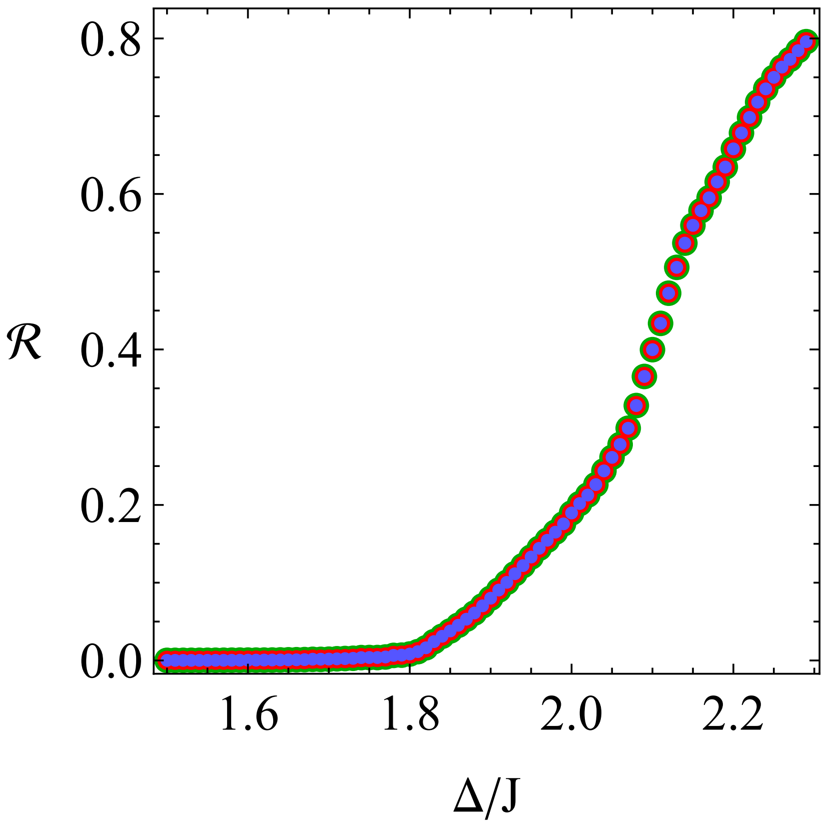


information outflow

$$\mathcal{R} = \text{backflow}/\text{outflow}$$

NON-MARKOVIANITY MEASURE

$$\mathcal{R} = \text{backflow}/\text{outflow}$$

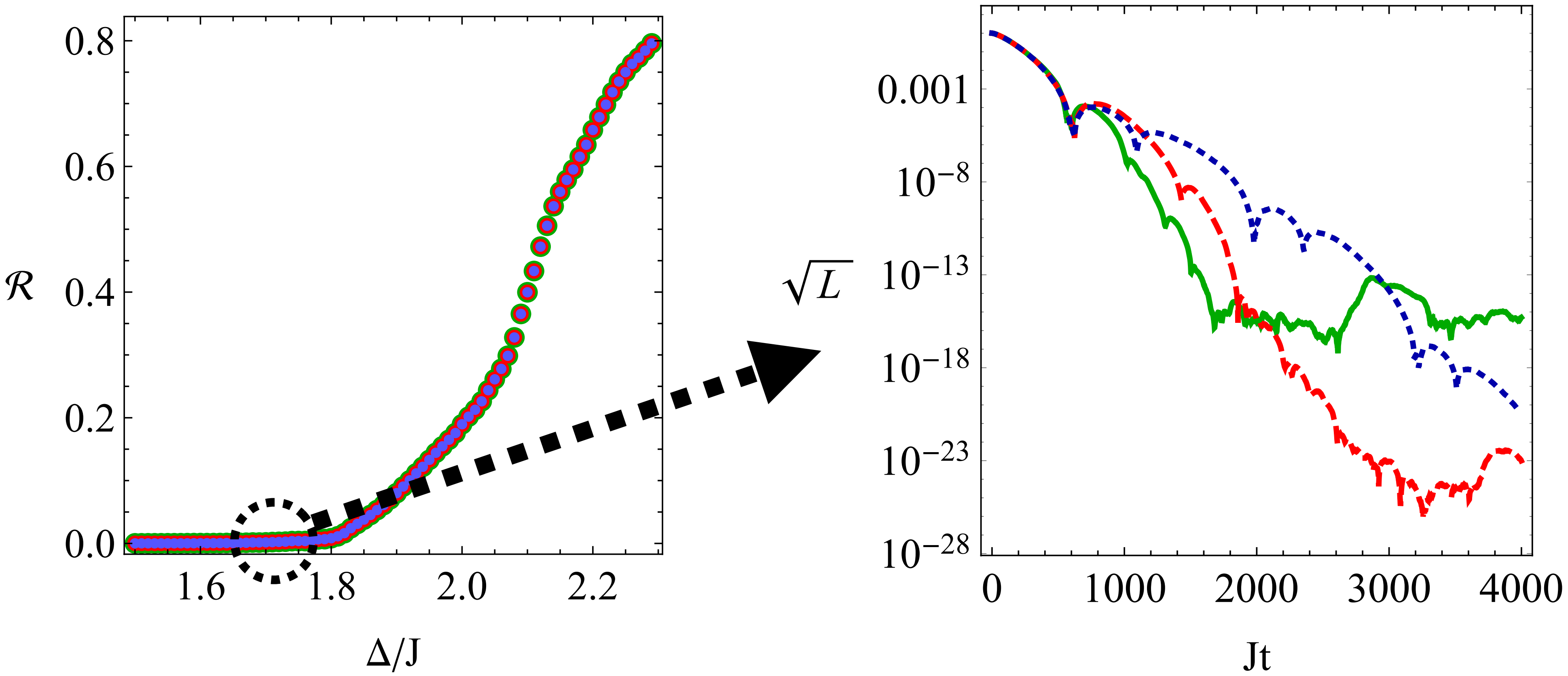


Lattice size

$$N_s = 233, 377, 987$$

NON-MARKOVIANITY MEASURE

$\mathcal{R}$ = backflow/outflow

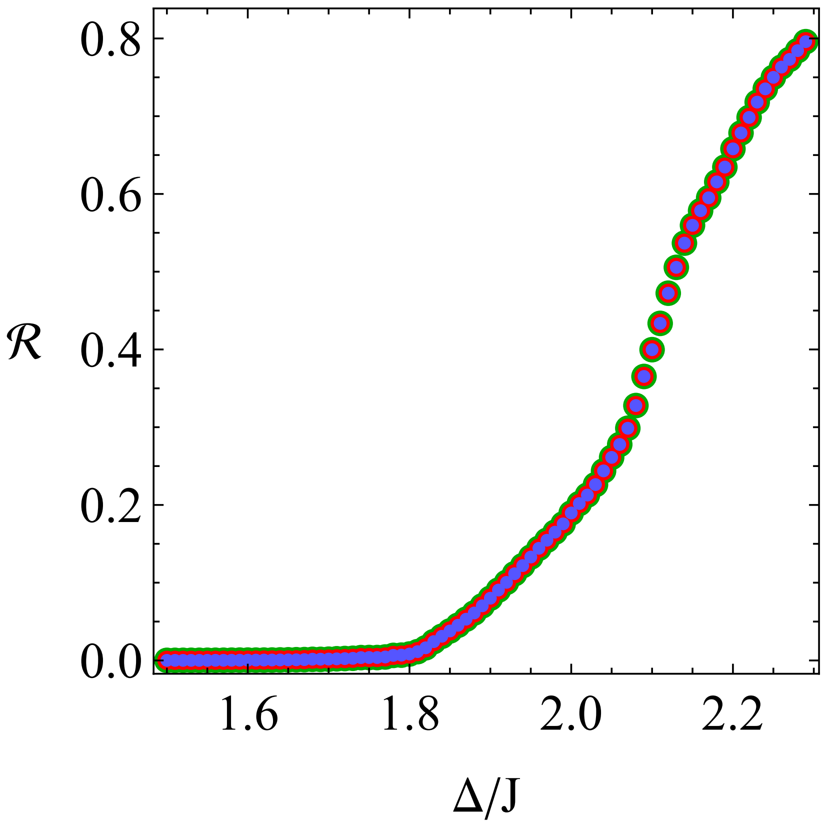


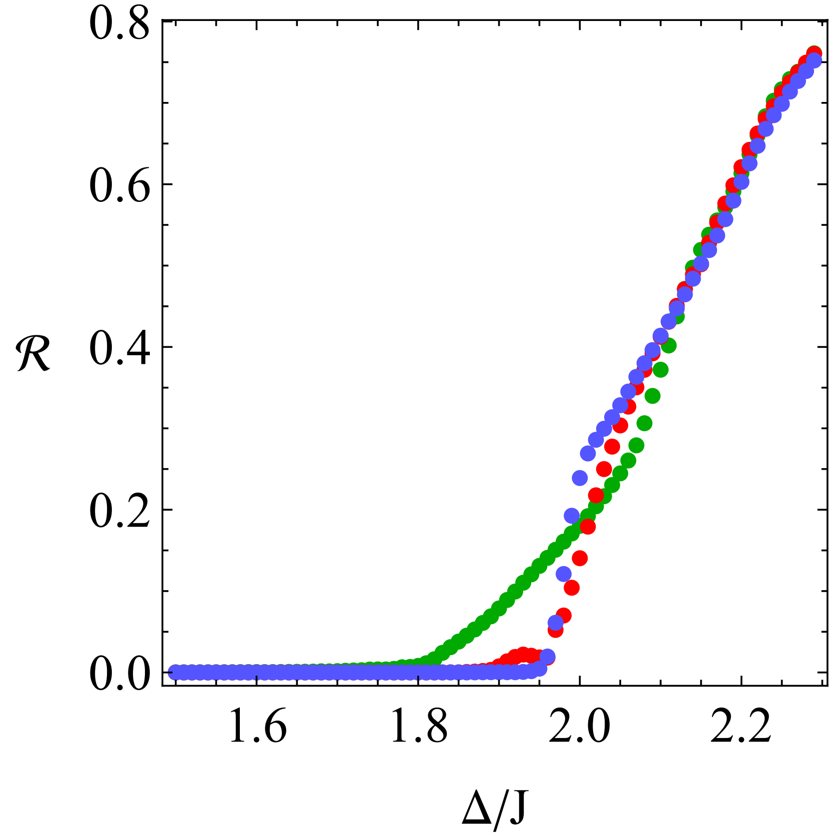
Lattice size

$N_s = 233, 377, 987$

NON-MARKOVIANITY MEASURE

$$\mathcal{R} = \text{backflow/outflow}$$





Lattice size

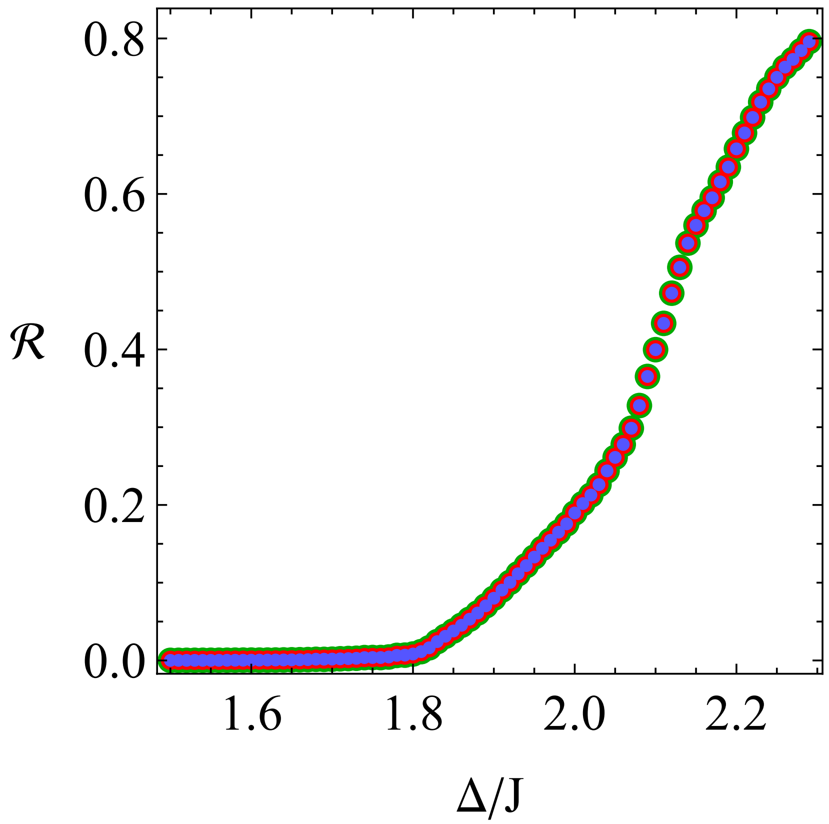
Impurity strength

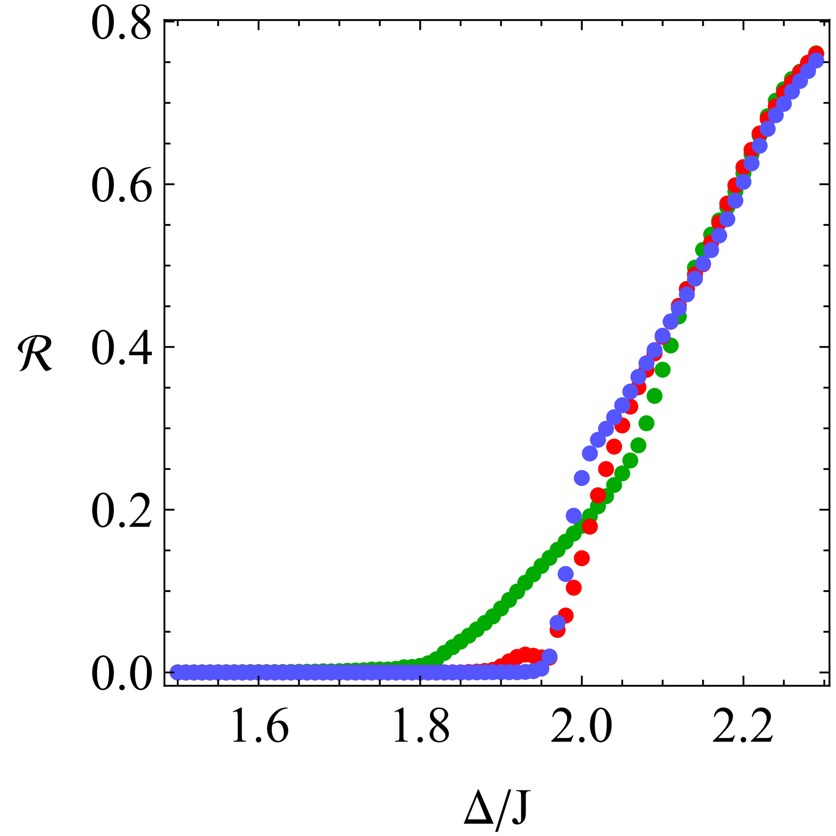
$$N_s = 233, 377, 987$$

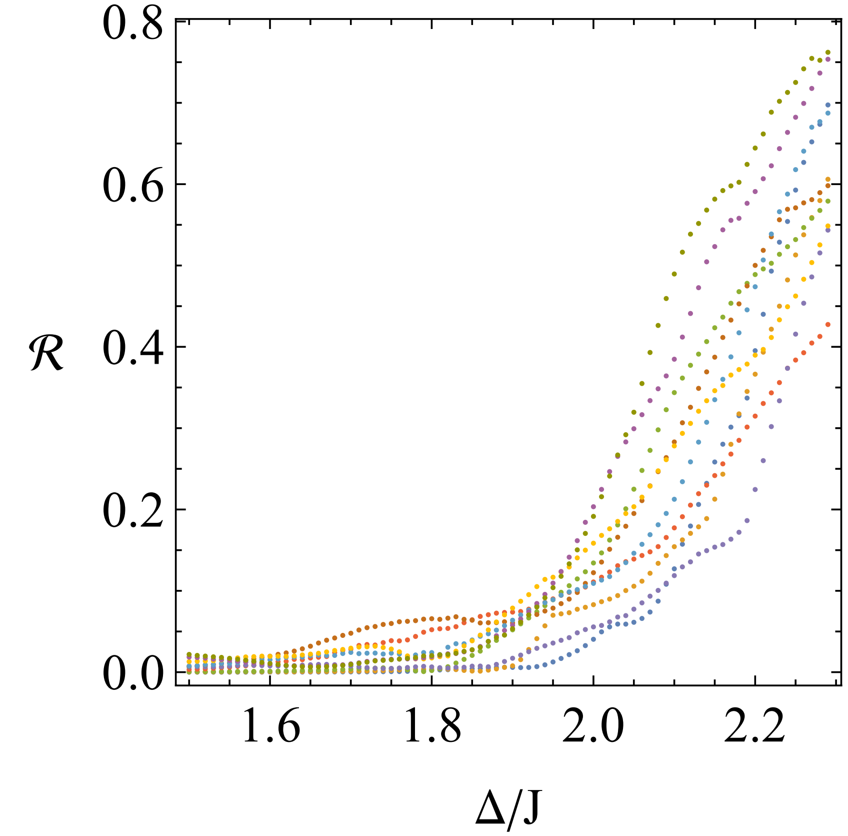
$$\epsilon/J = 10^{-1}, 10^{-2}, 10^{-3}$$

NON-MARKOVIANITY MEASURE

$$\mathcal{R} = \text{backflow}/\text{outflow}$$







Lattice size

$$N_s = 233, 377, 987$$

Impurity strength

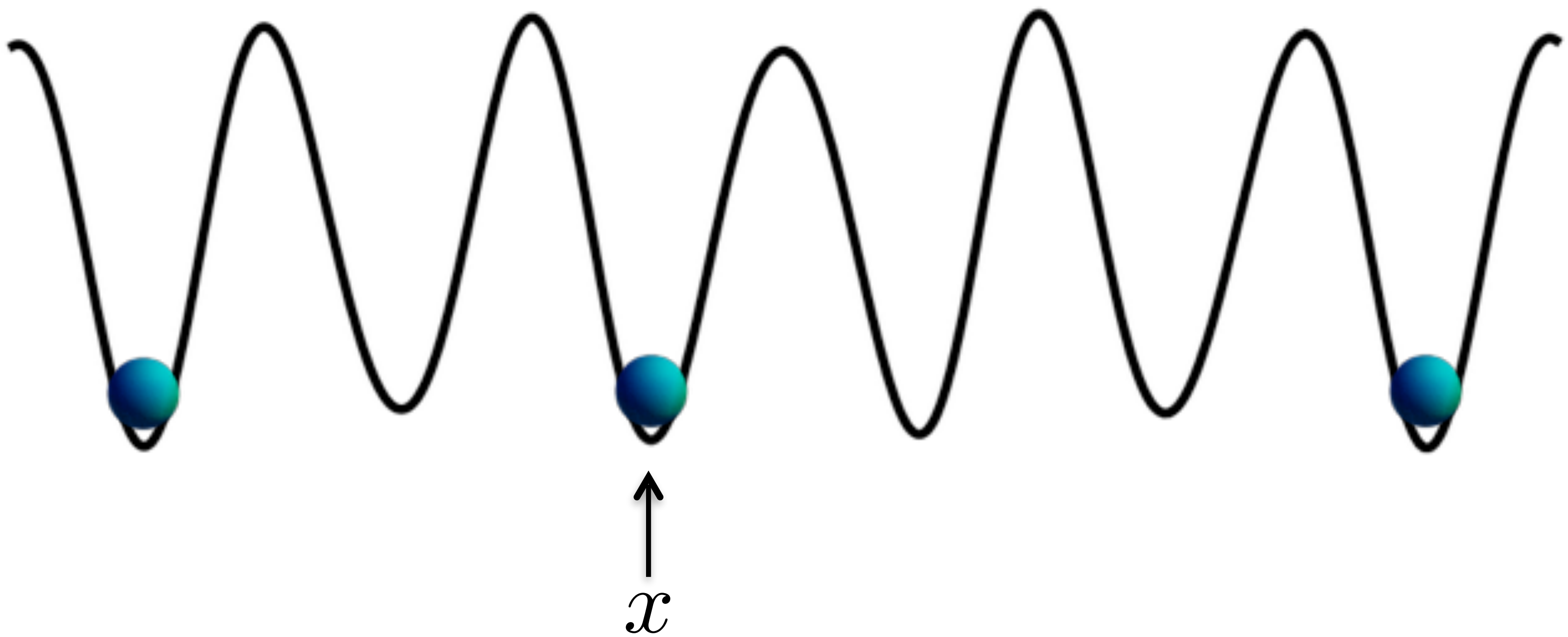
$$\epsilon/J = 10^{-1}, 10^{-2}, 10^{-3}$$

Random Phase

$$\phi$$

WORK STATISTICS

$$\hat{H} = -J\sum_i(\hat{a}^\dagger_{i+1}\hat{a}_i + \hat{a}^\dagger_i\hat{a}_{i+1}) + \sum_i V_i\hat{n}_i + \epsilon\hat{n}_x$$

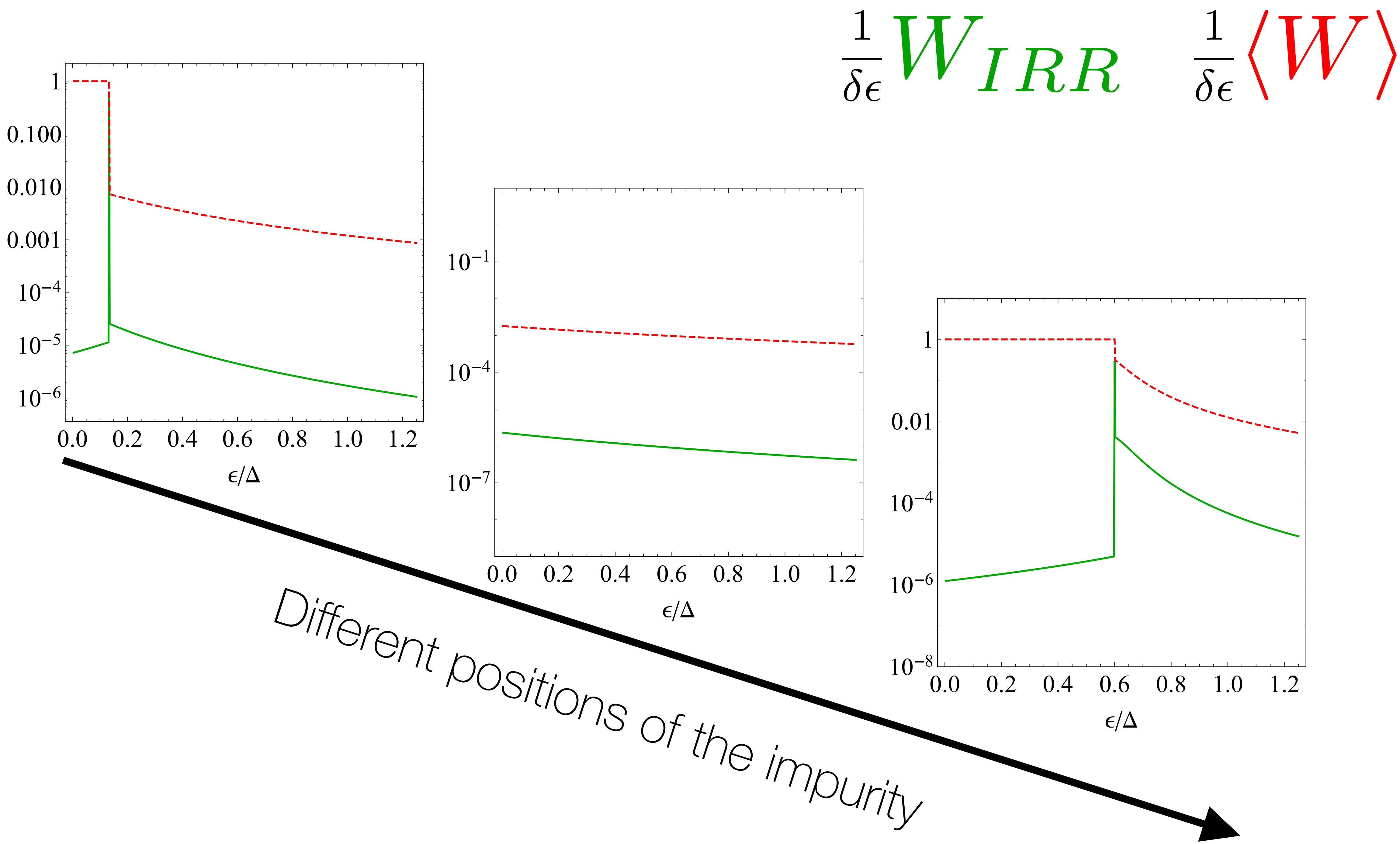


Infinitesimal quench $\epsilon \rightarrow \epsilon + \delta\epsilon$

Average work $\langle W \rangle = \langle \delta\hat{H} \rangle$

Irreversible work $W_{IRR} = \langle W \rangle - \Delta F$

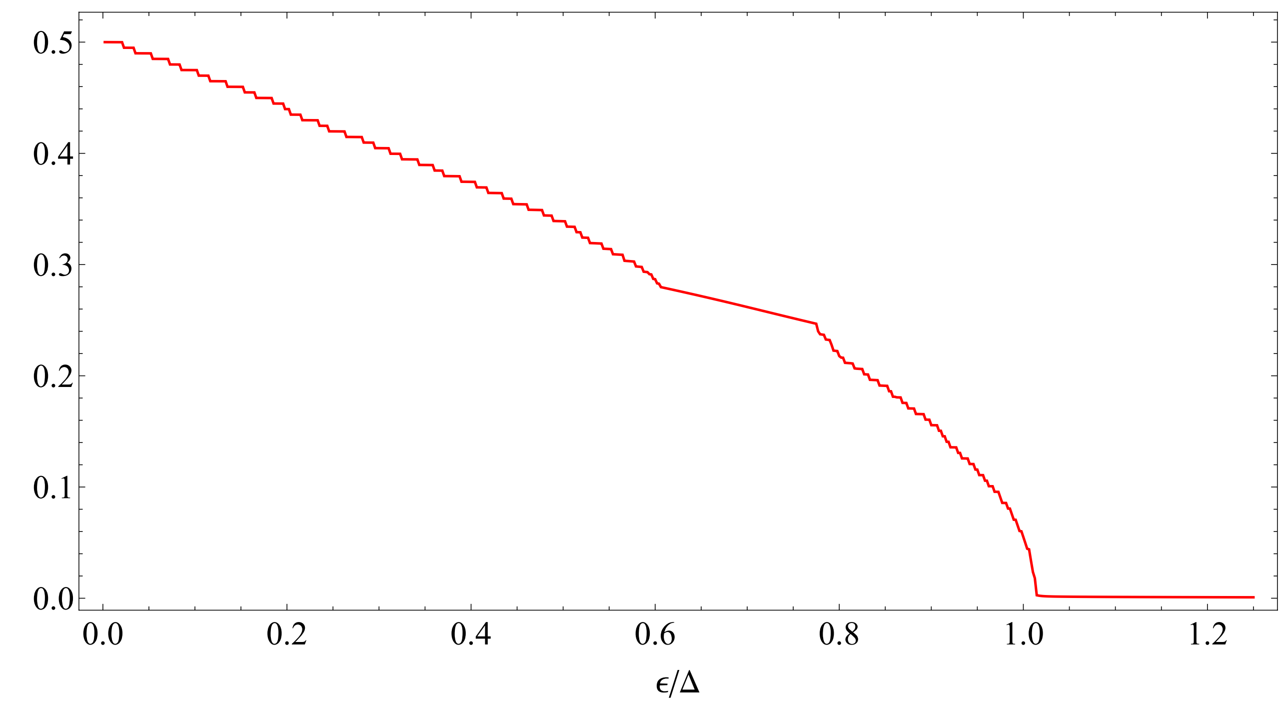
WORK STATISTICS



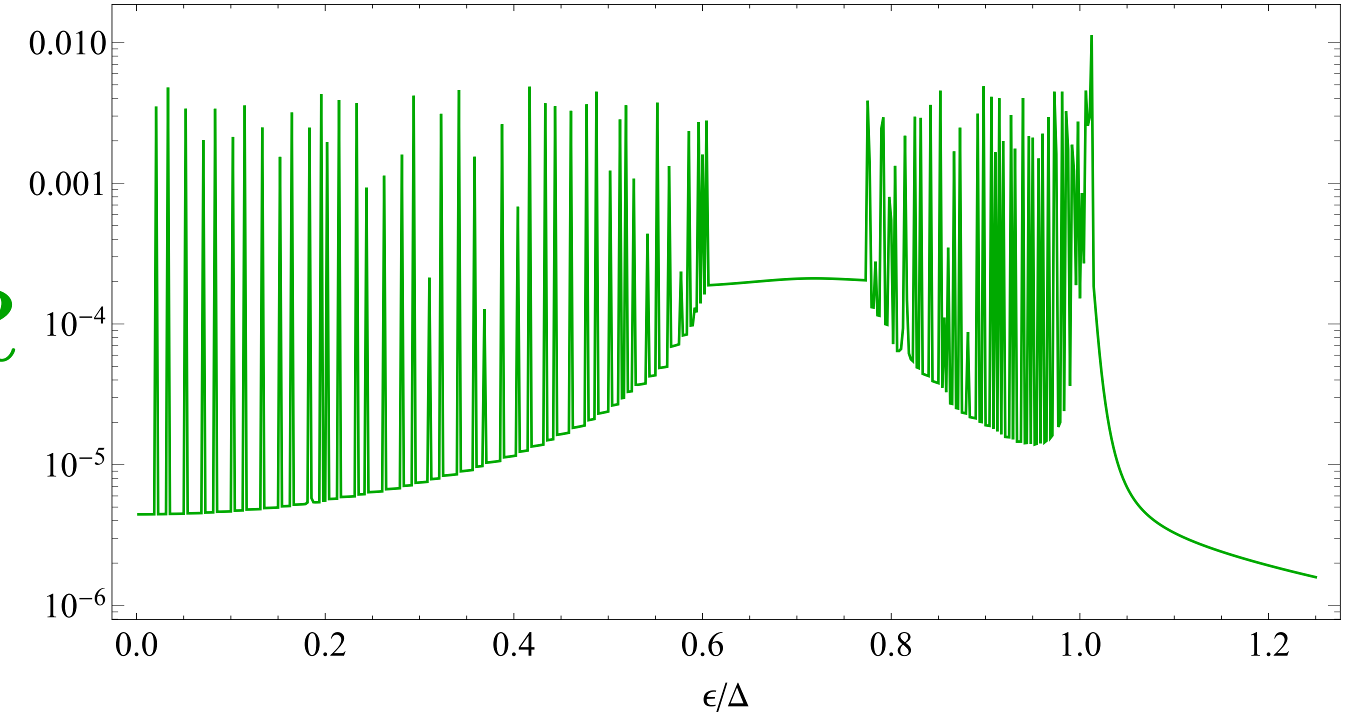
WORK STATISTICS

Averaging over the position

$$\frac{1}{\delta\epsilon}\langle W\rangle$$

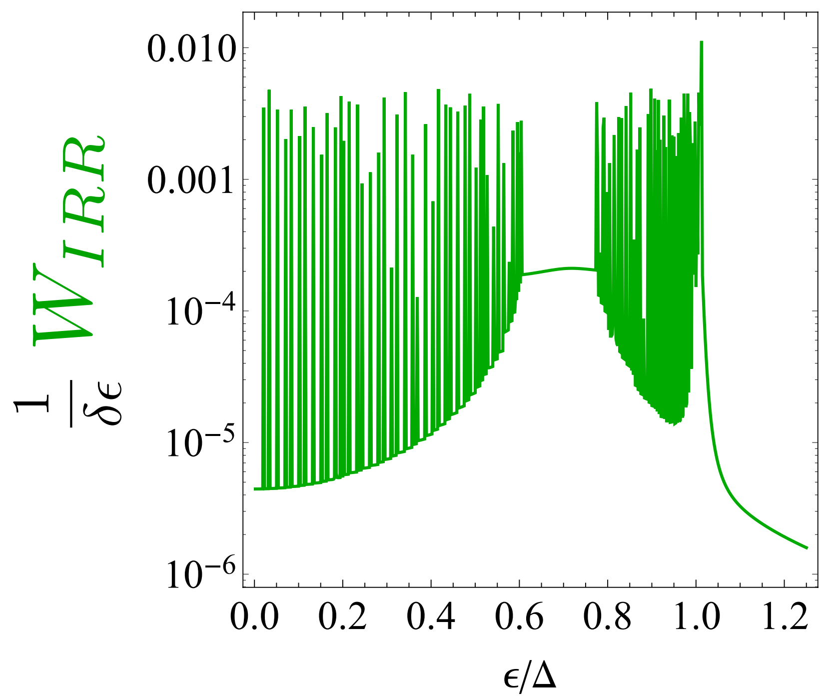


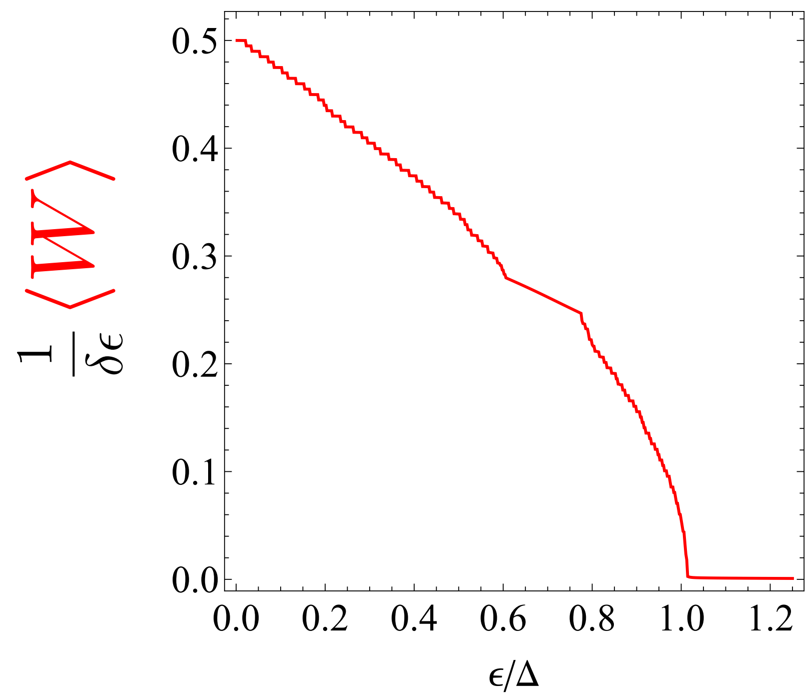
$$\frac{1}{\delta\epsilon}W_{IRR}$$

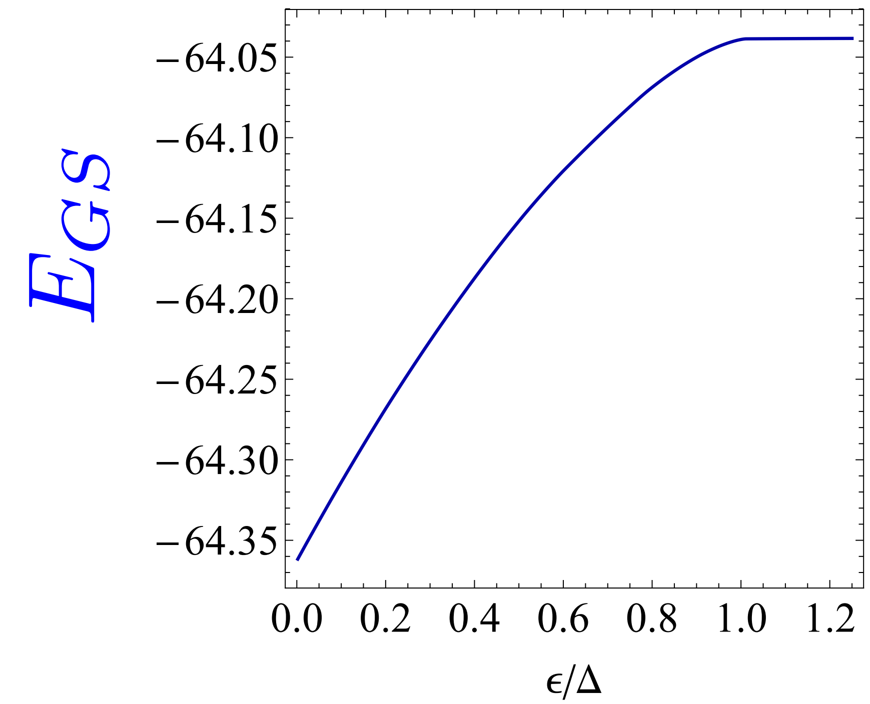


WORK STATISTICS

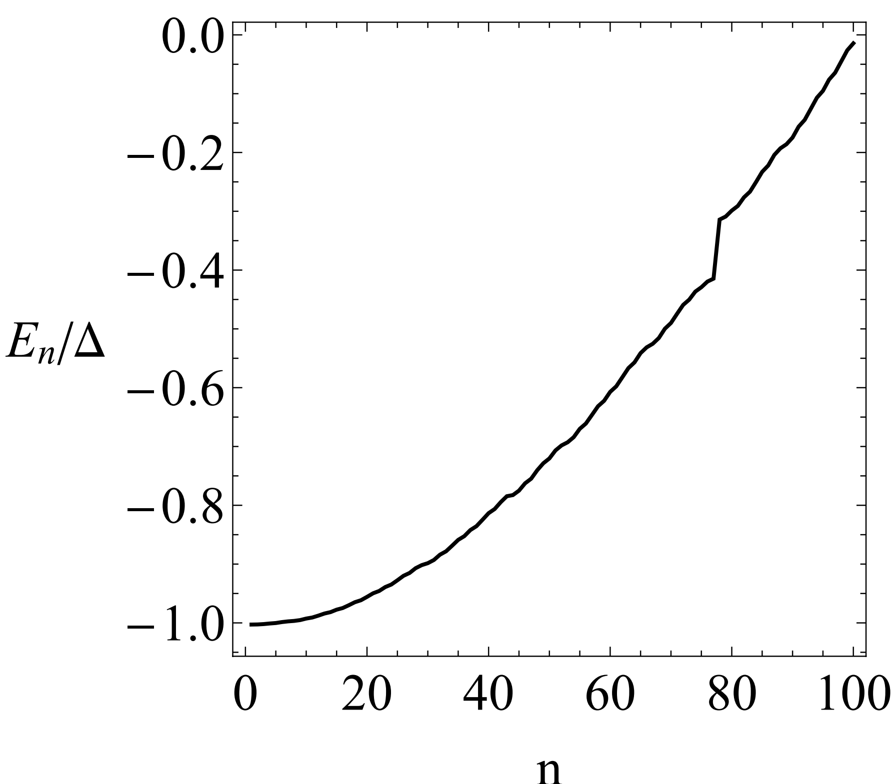
Averaging over the position







Single particle spectrum



ADIABATIC COUPLING

Asymptotic coherences of the probe

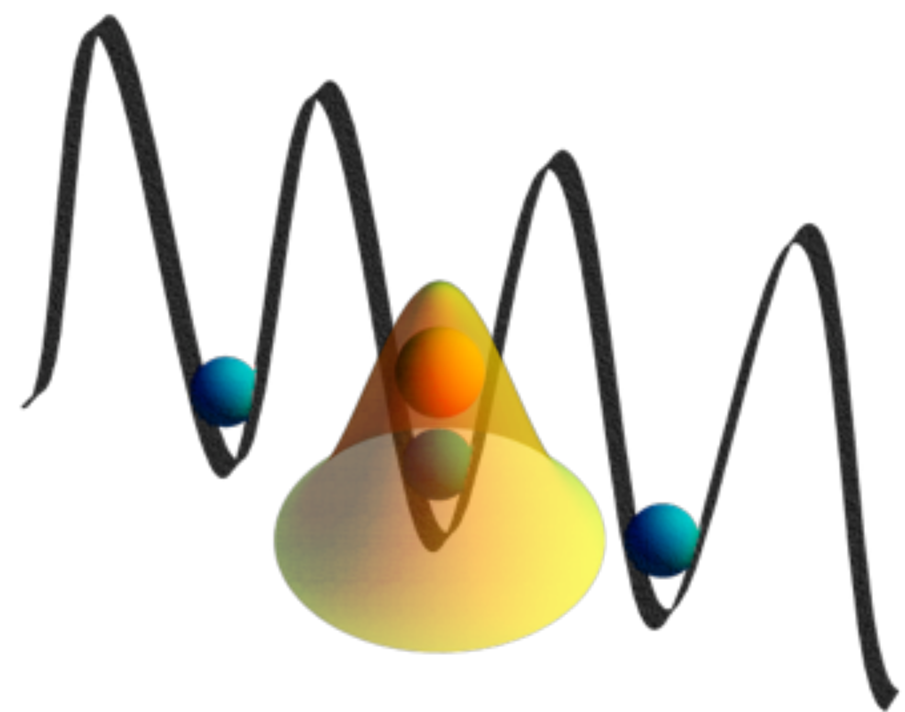
$$\left|\frac{\rho_{eg}(+\infty)}{\rho_{eg}(0)}\right| = |\langle\Psi_0(\epsilon = 0)|U(+\infty)|\Psi_0(\epsilon = 0)\rangle| = |\langle\Psi_0(\epsilon = 0)|\Psi_0(x, \epsilon)\rangle|$$

Ground states fidelity

$$F(x, \epsilon) = |\langle\Psi_0(\epsilon = 0)|\Psi_0(x, \epsilon)\rangle|$$

Statistical Orthogonality catastrophe

$$F_{\text{typ}} \equiv \exp(\overline{\log F}) \sim \exp(-\beta L)$$



V. Khemani, R. Nandkishore, and S. Sondhi, Nature Physics **11** 560–565 (2015)
D. L. Deng, J. Pixley, X. Li, and S. D. Sarma, Physical Review B **92** 220201 (2015)

ADIABATIC COUPLING

Asymptotic coherences of the probe

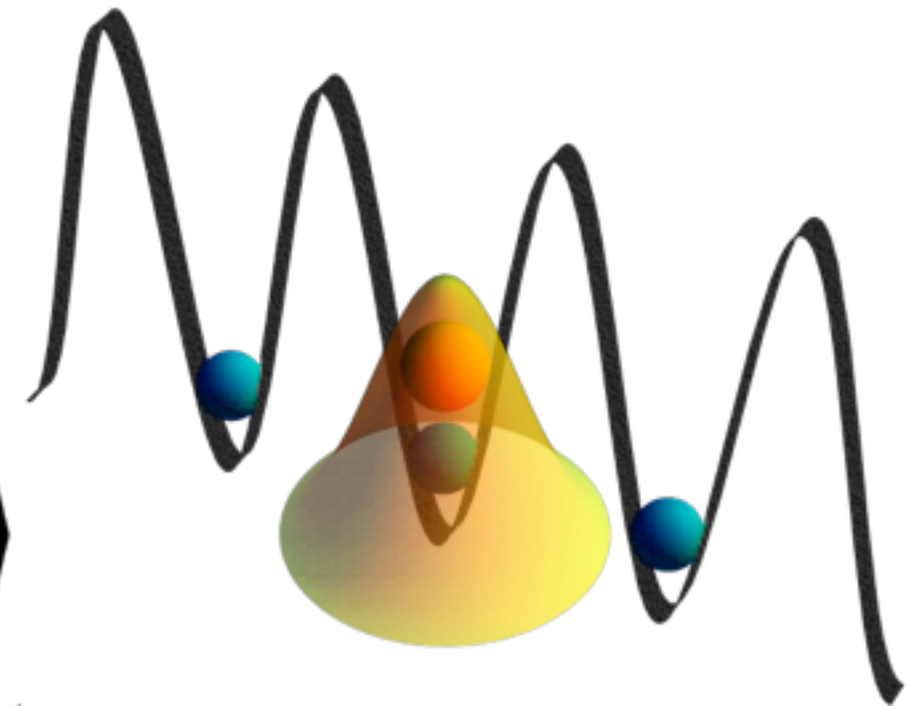
$$\left|\frac{\rho_{eg}(+\infty)}{\rho_{eg}(0)}\right| = |\langle\Psi_0(\epsilon = 0)|U(+\infty)|\Psi_0(\epsilon = 0)\rangle| = |\langle\Psi_0(\epsilon = 0)|\Psi_0(x, \epsilon)\rangle|$$

Ground states fidelity

$$F(x, \epsilon) = |\langle\Psi_0(\epsilon = 0)|\Psi_0(x, \epsilon)\rangle|$$

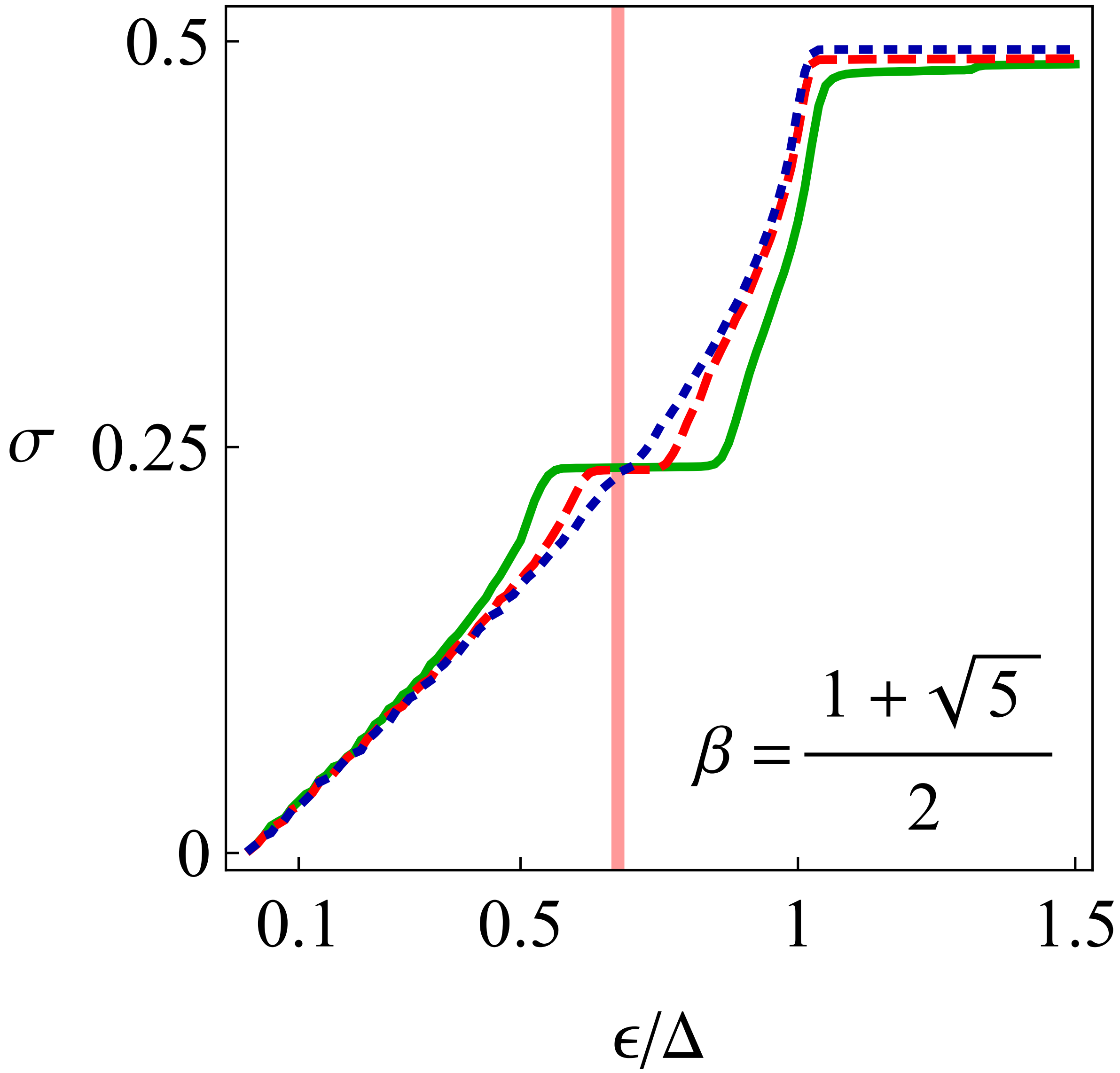
Orthogonality catastrophe "probability"

$$\sigma(\epsilon) = \left\langle \frac{1}{N_s} \sum_{x=1}^{N_s} \theta(\delta - F(x, \epsilon)) \right\rangle$$

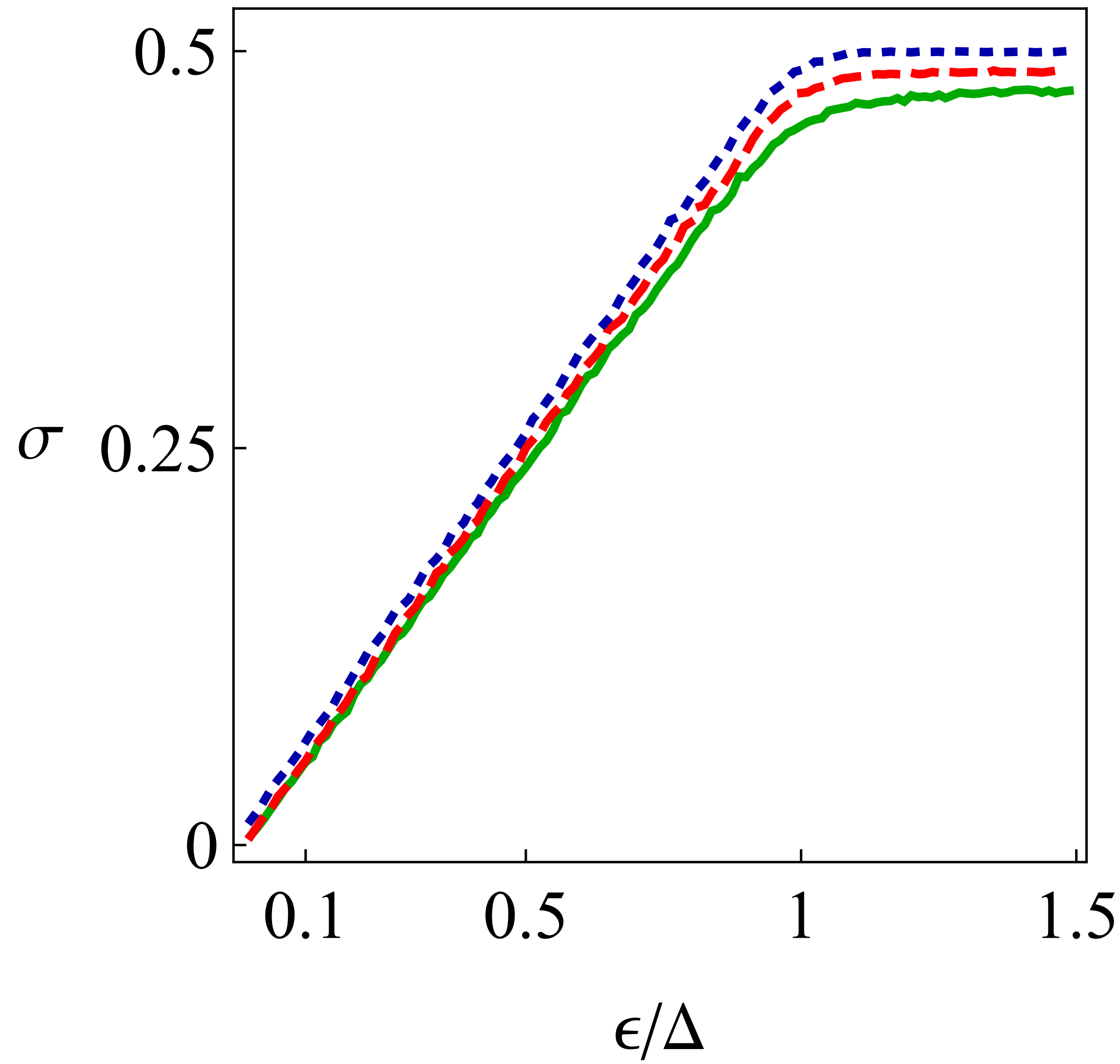


GS FIDELITY

Aubry-André



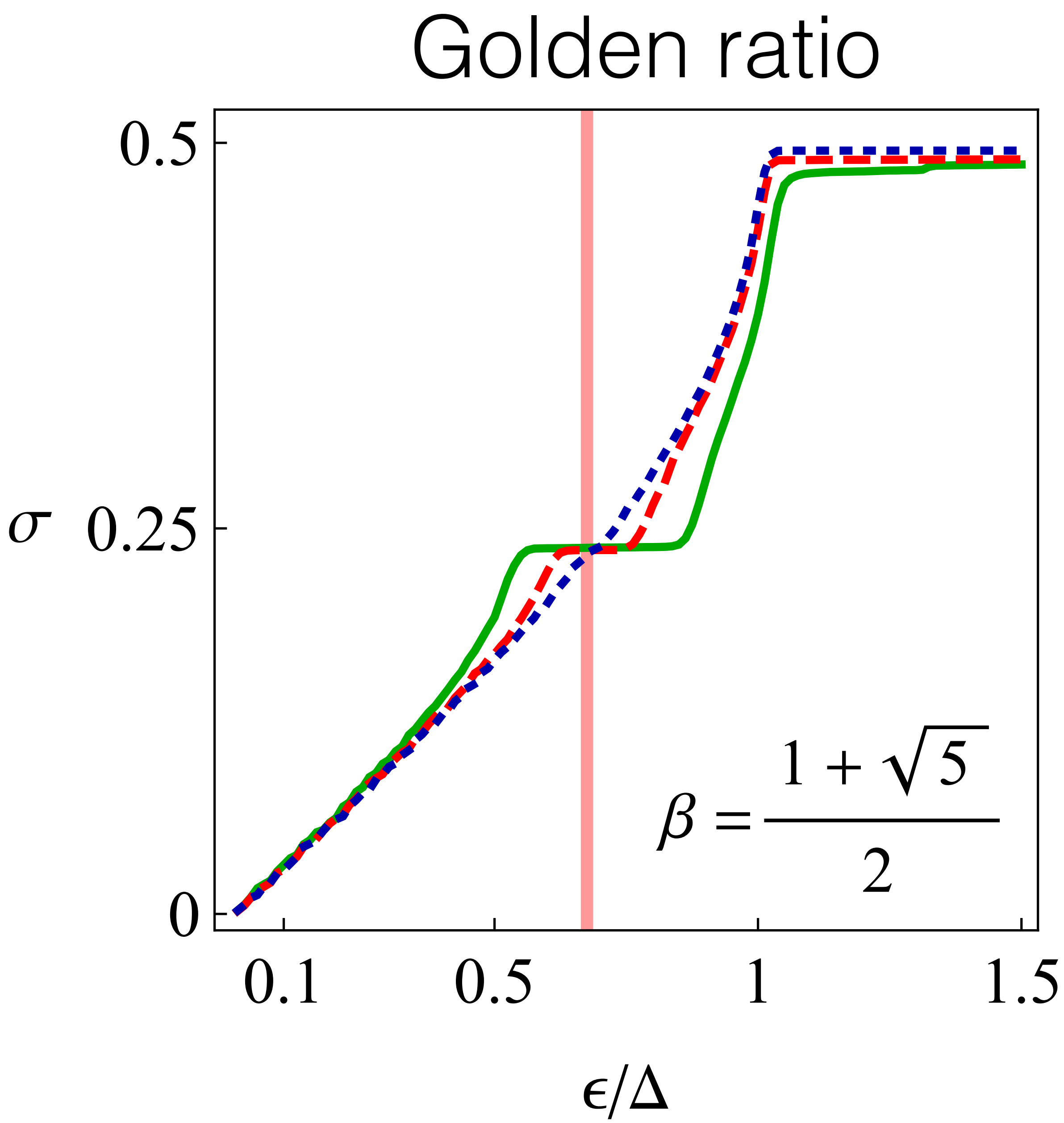
Anderson Insulator

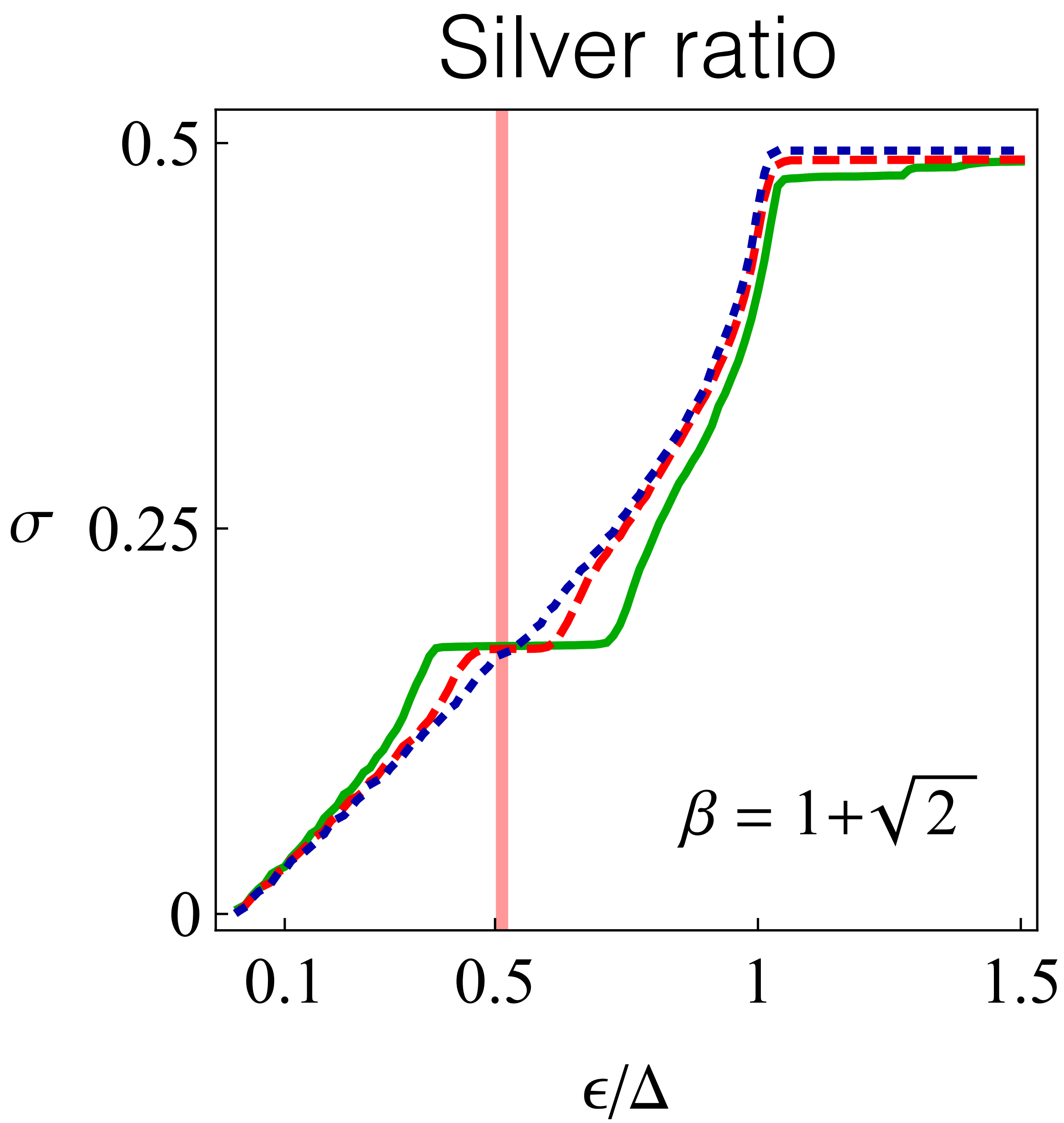


J/Δ=0.1,0.05,0.01

Half filling

GS FIDELITY

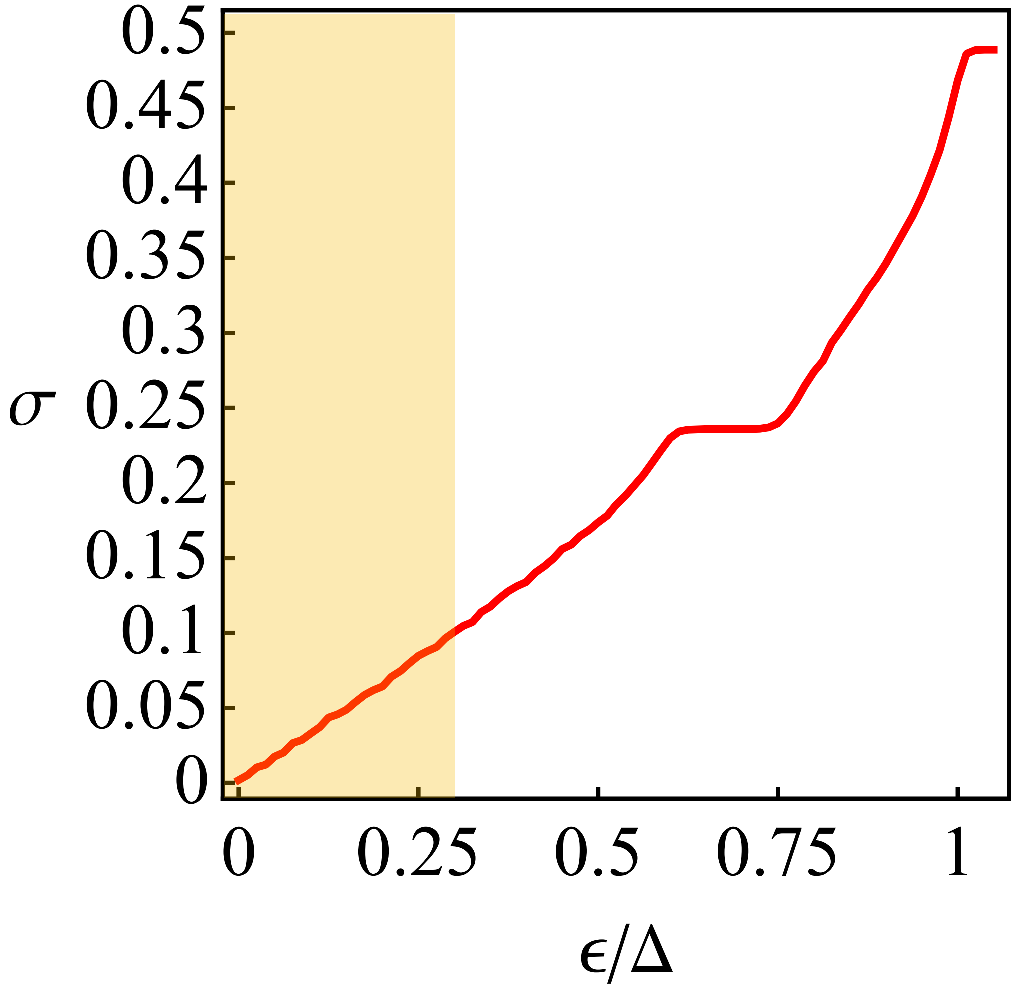




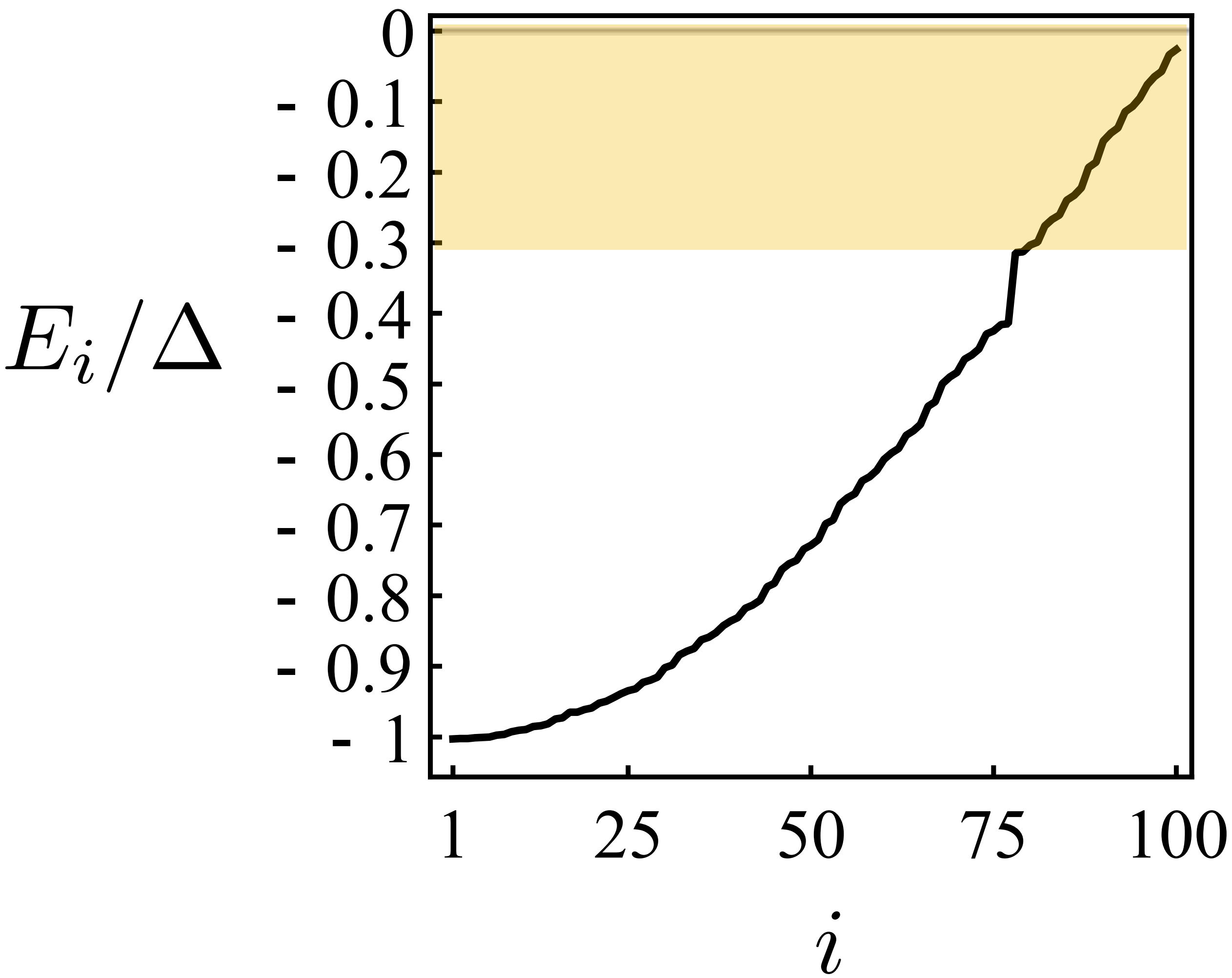
J/Δ=0.1,0.05,0.01

Half filling

PLATEAU-GAP



Occupied spectrum



GAP OPENING

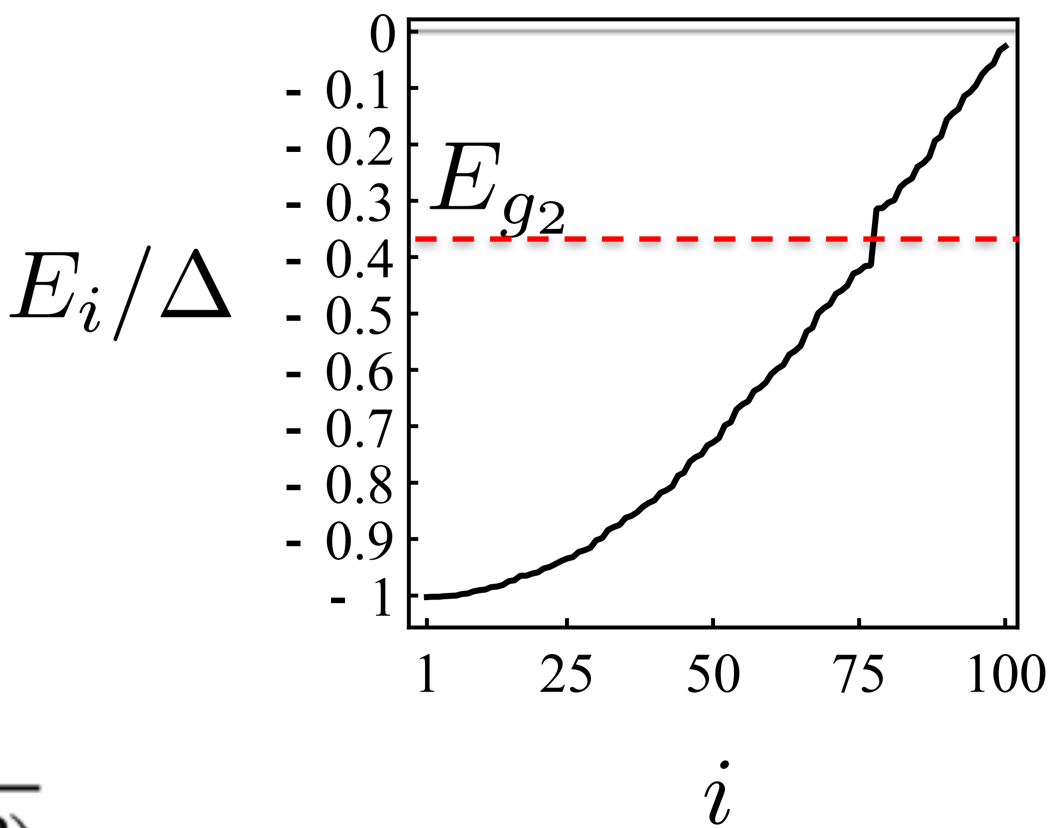
Strongly localised phase

$$E_i \approx \Delta \cos(2\pi\beta i + \phi)$$

Resonance

$$|V_i - V_{i+1}| \lesssim J$$

$$|\sin(2\pi\beta(i + 1/2) + \phi)| \lesssim \frac{J}{2\Delta \sin(\pi\beta)}$$



Gaps opening at $E_{g_2} \simeq \pm\Delta \cos(\pi\beta)$

D. J. Thouless and Q. Niu 1983 J. Phys. A: Math. Gen. 16 1911

GAP OPENING

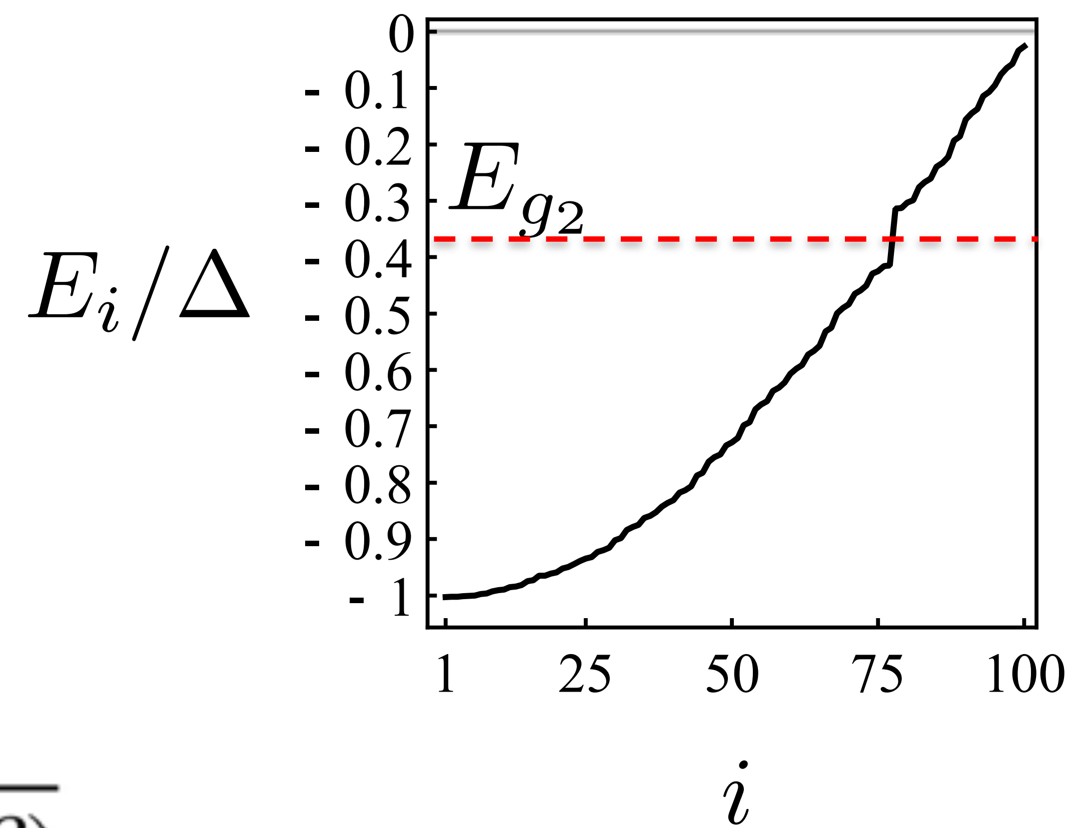
Strongly localised phase

$$E_i \approx \Delta \cos(2\pi\beta i + \phi)$$

Resonance

$$|V_i - V_{i+1}| \lesssim J$$

$$|\sin(2\pi\beta(i + 1/2) + \phi)| \lesssim \frac{J}{2\Delta \sin(\pi\beta)}$$



Gaps opening at $E_{g_2} \simeq \pm\Delta \cos(\pi\beta)$

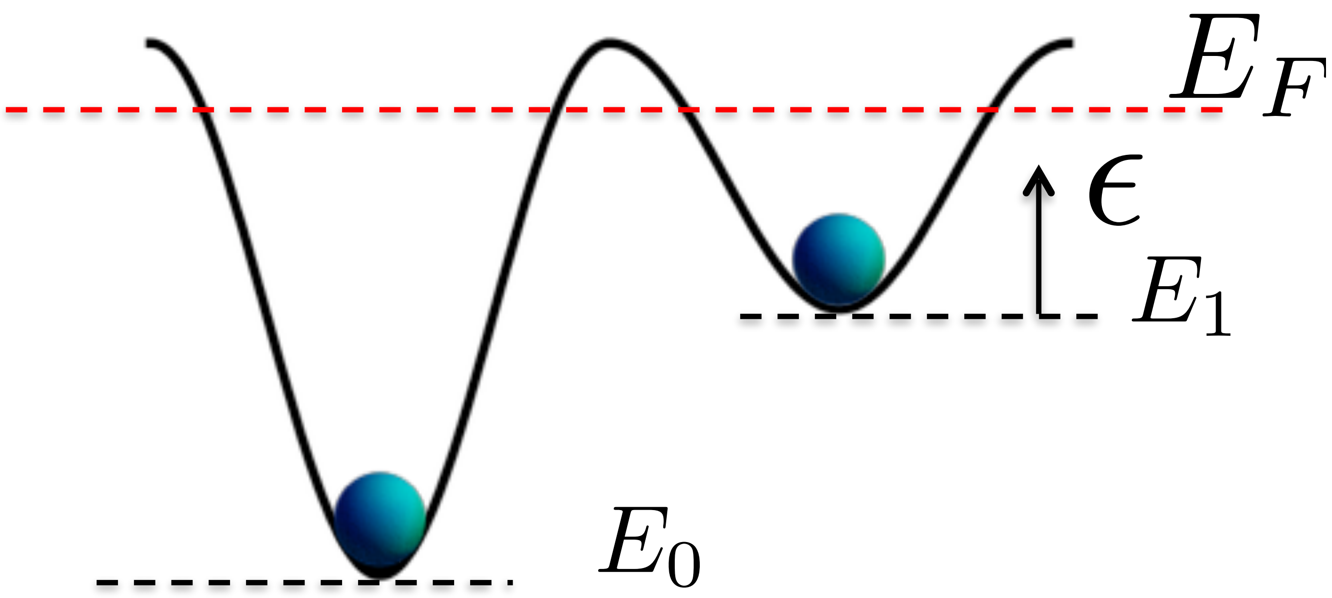
With states on opposite sides of the gap **close** in space

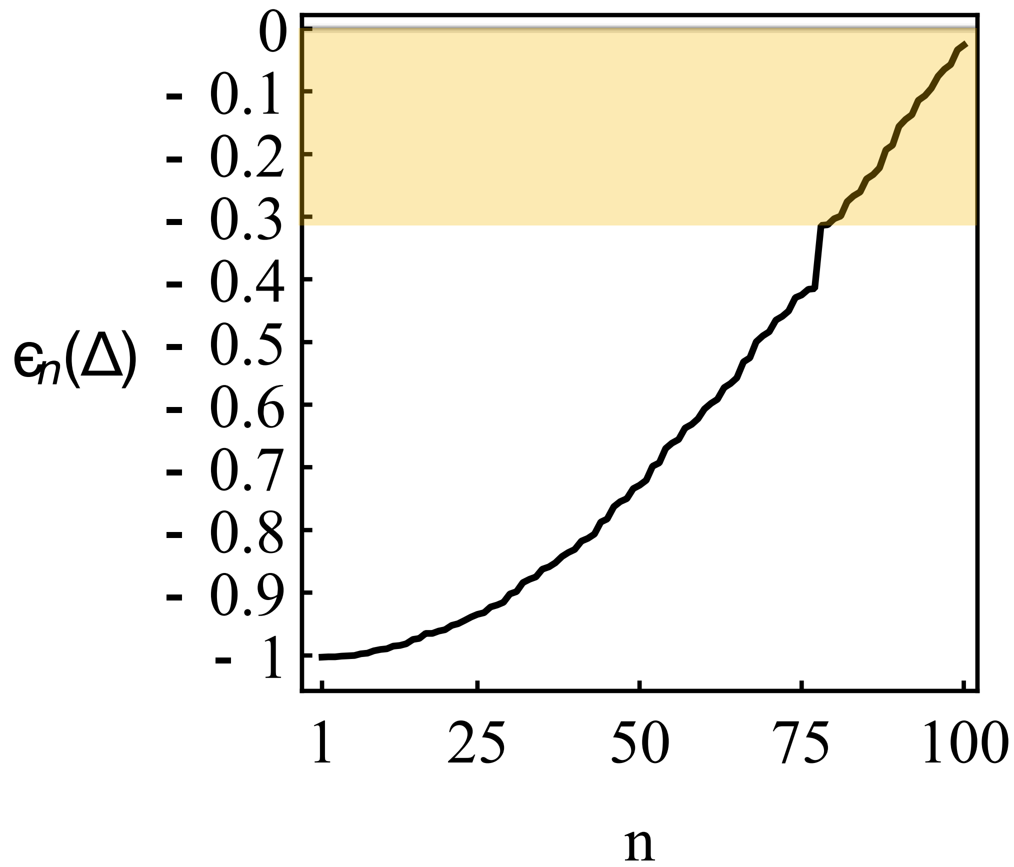
D. J. Thouless and Q. Niu 1983 J. Phys. A: Math. Gen. 16 1911

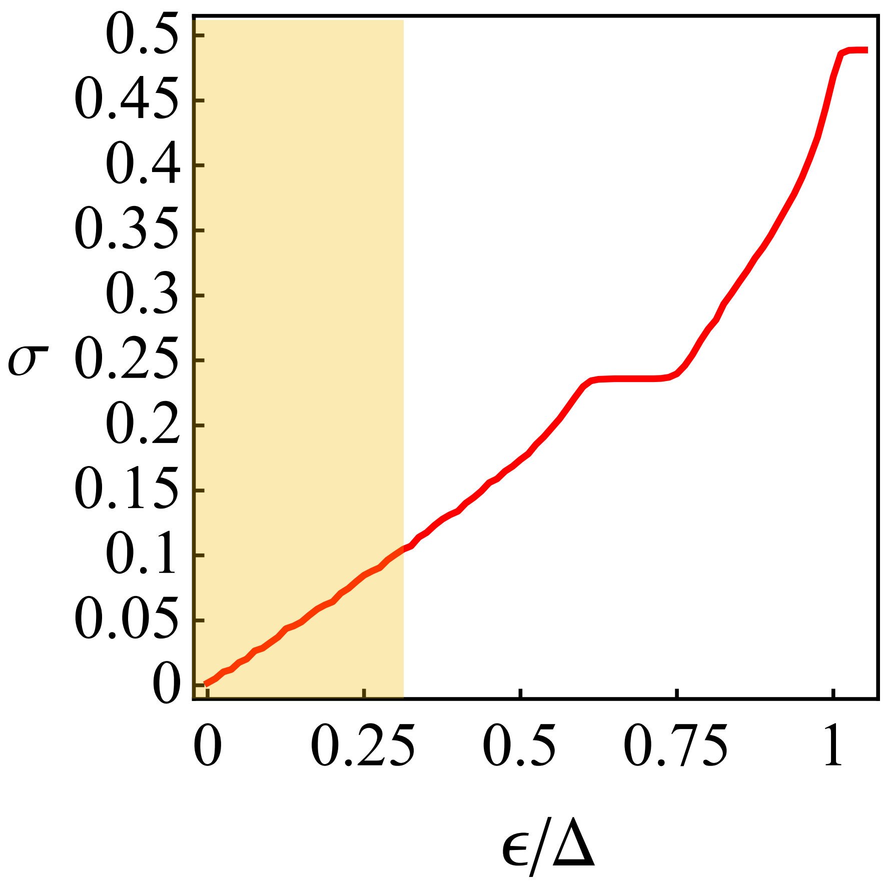
TWO-LEVEL MODEL

Mechanism **weak** impurity

$$H_2 = \begin{bmatrix} \Delta_2 + \epsilon & -J \\ -J & 0 \end{bmatrix}$$



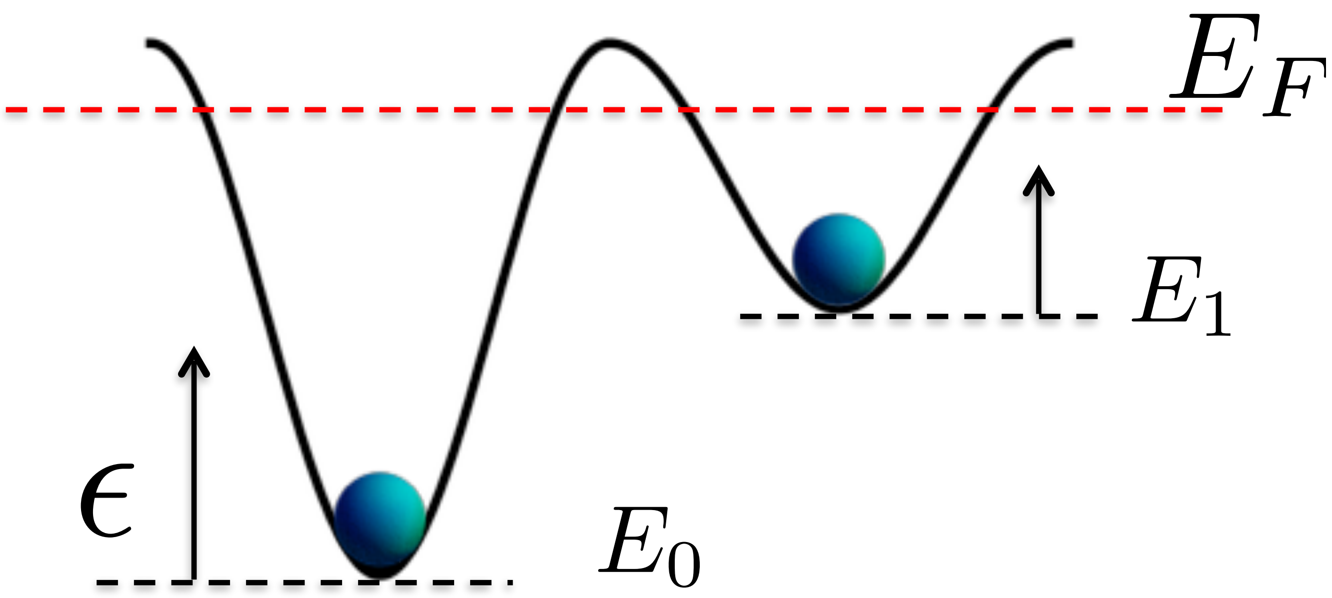


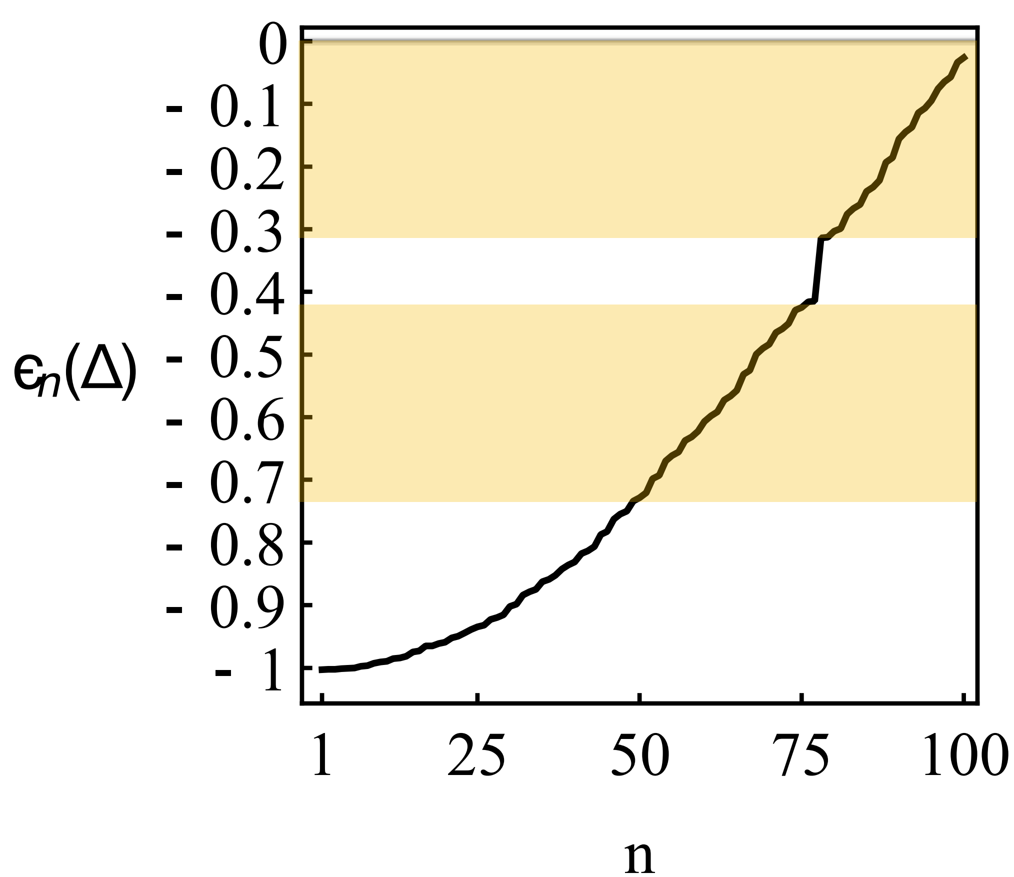


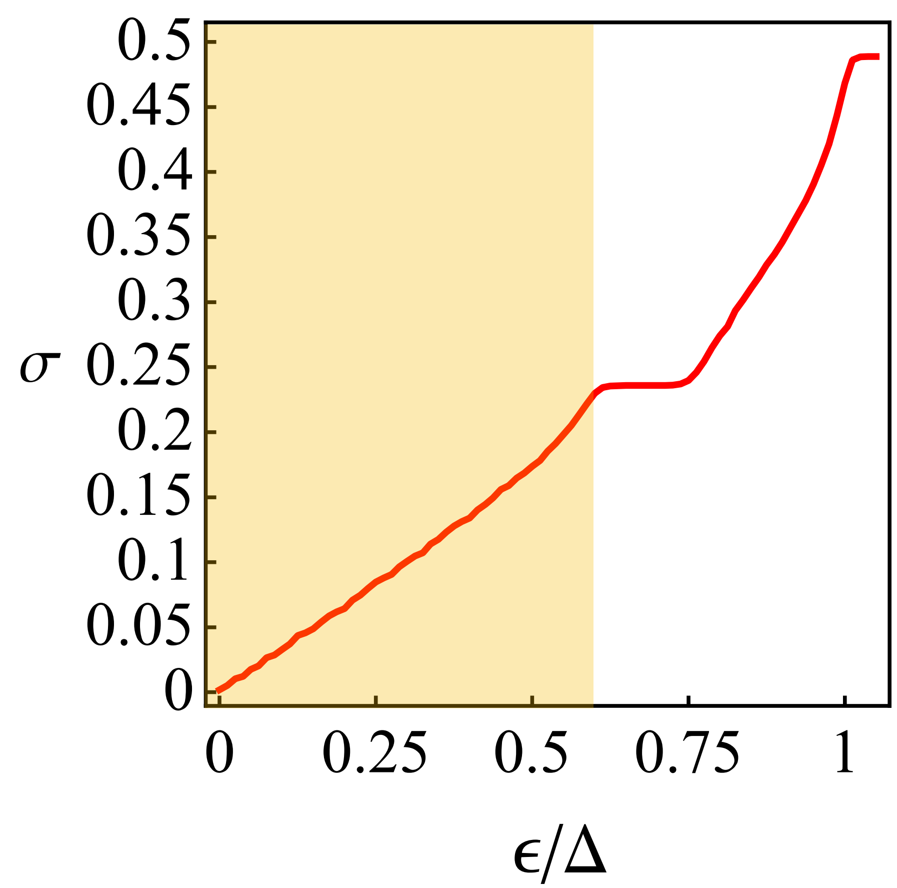
TWO-LEVEL MODEL

Impurity affects neighbouring site

$$H_2 = \begin{bmatrix} \Delta_2 & -J \\ -J & \epsilon \end{bmatrix}$$



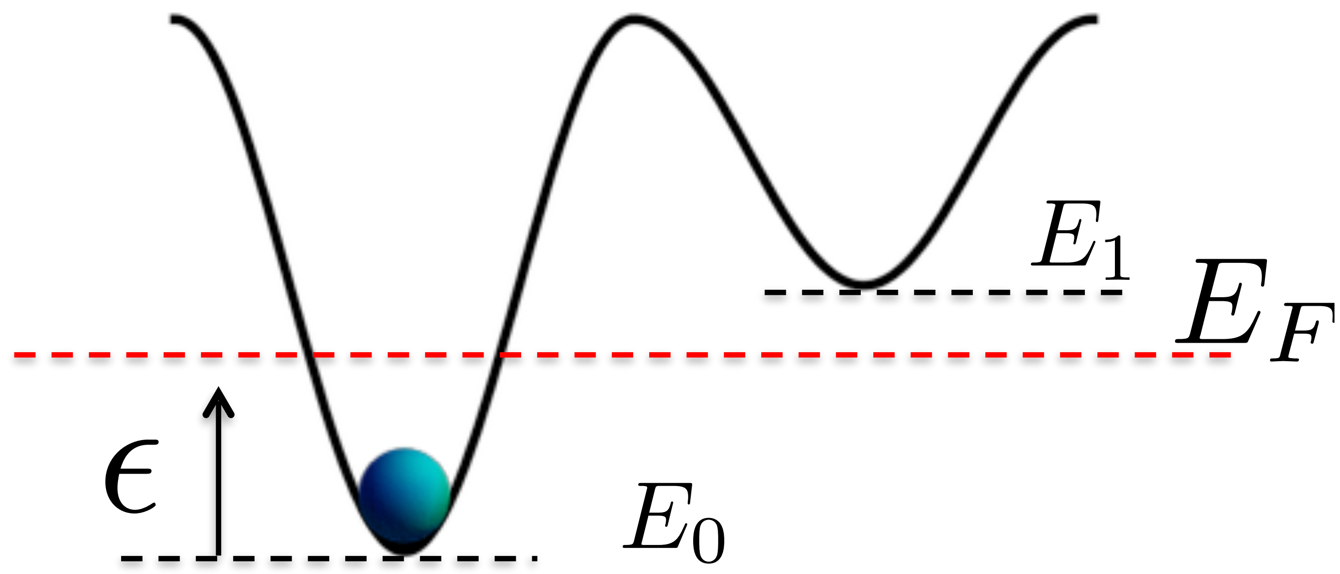


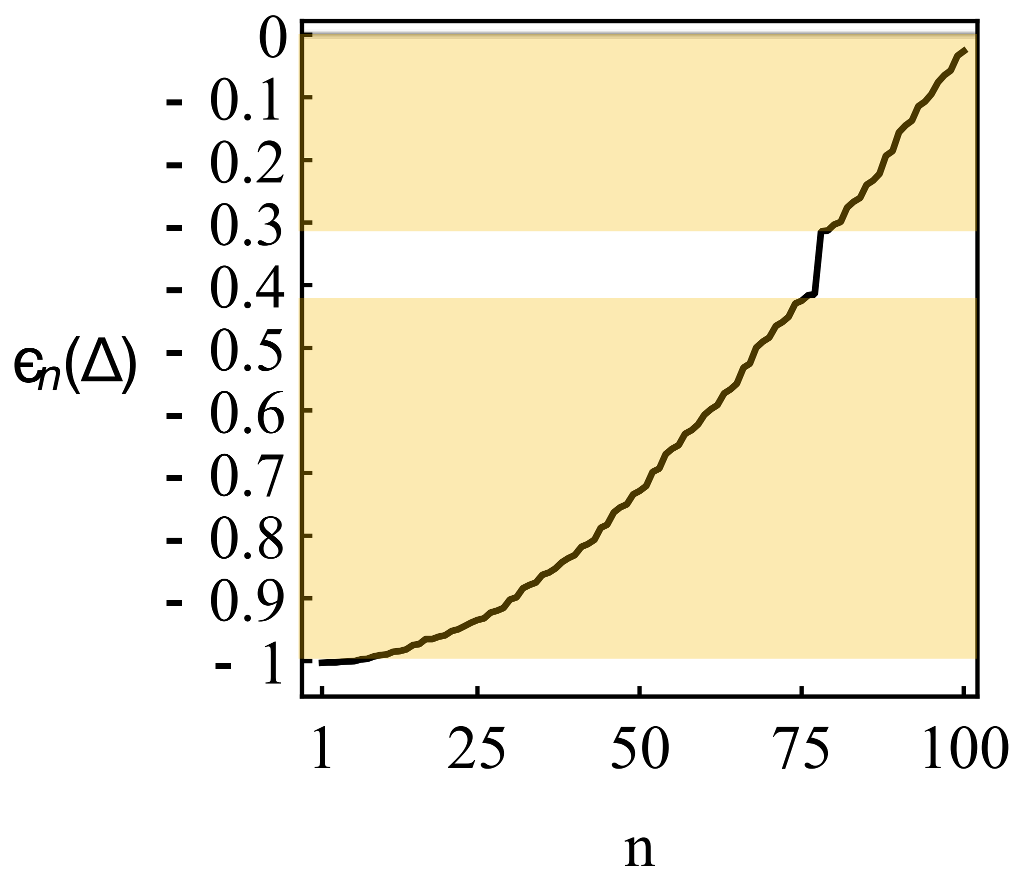


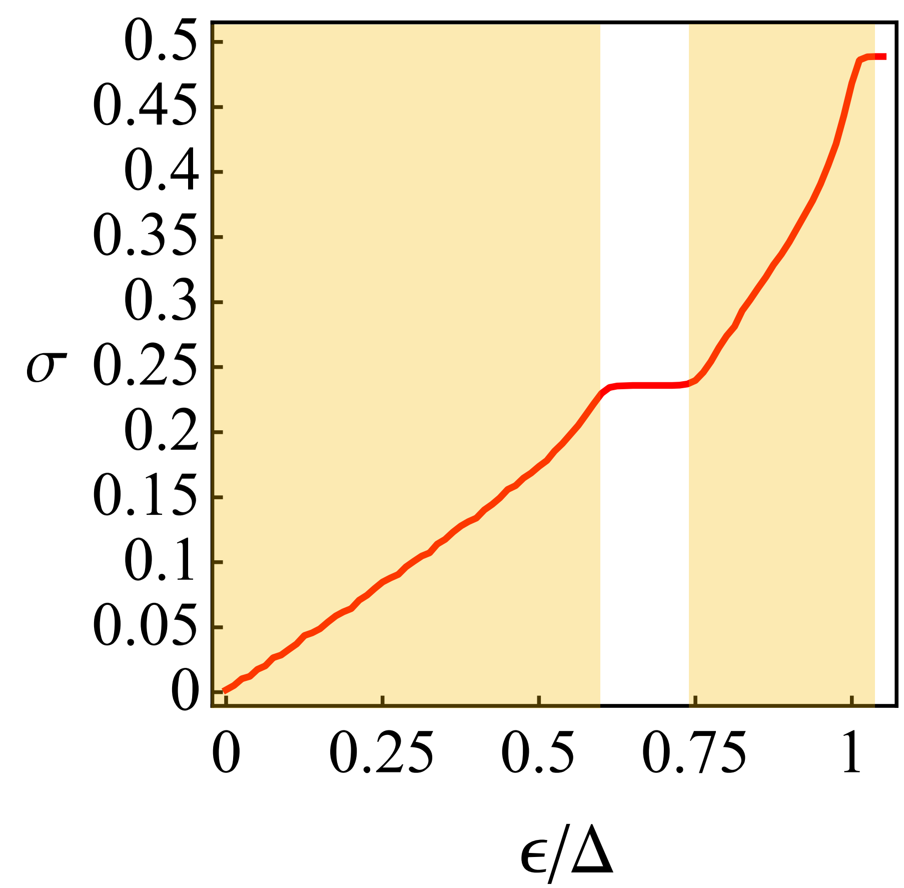
TWO-LEVEL MODEL

No occupied neighbour

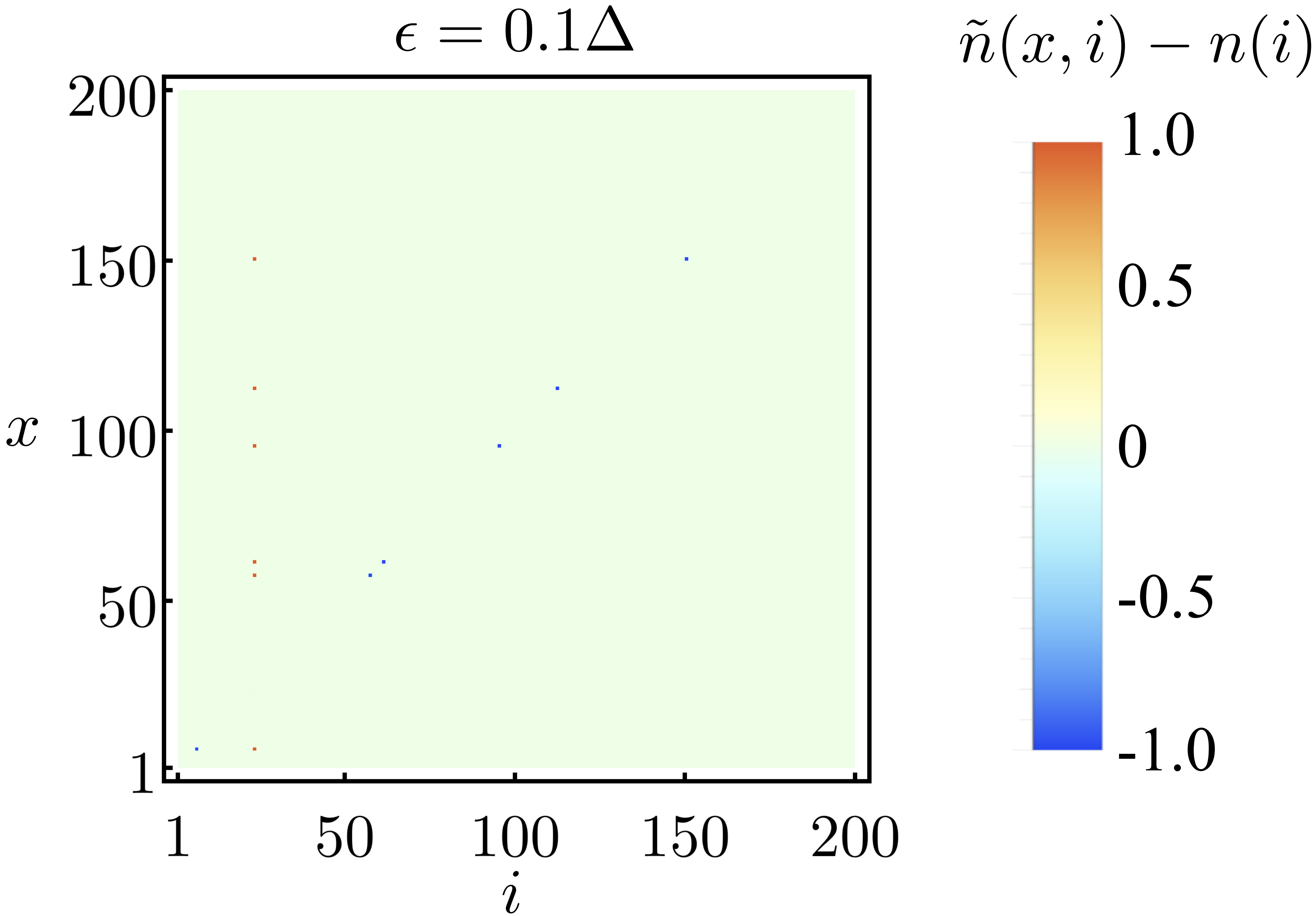
$$H_2 = \begin{bmatrix} \Delta_2 & -J \\ -J & \epsilon \end{bmatrix}$$







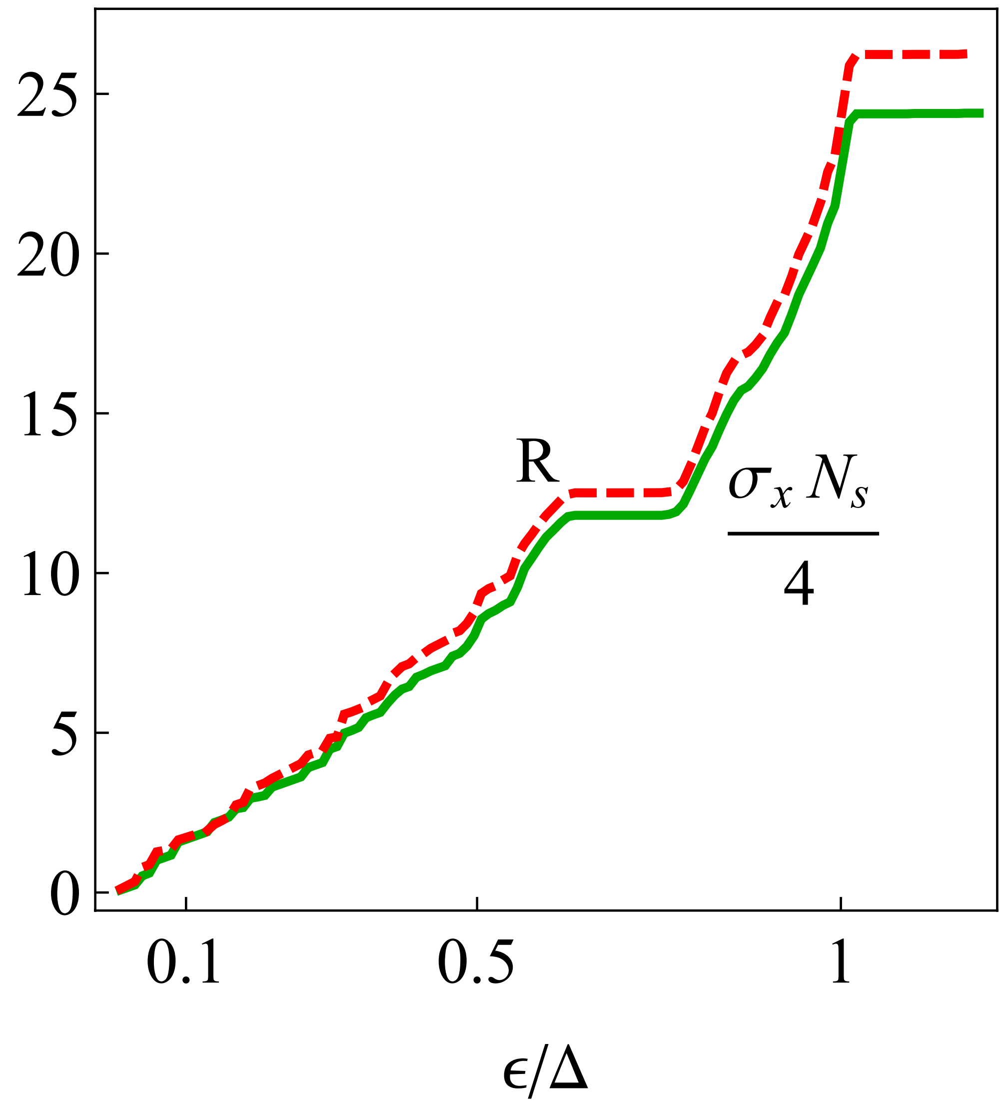
ADIABATIC TRANSPORT



D. L. Deng, J. Pixley, X. Li, and S. D. Sarma, Physical Review B **92** 220201 (2015)

RADIUS OF DISTURBANCE

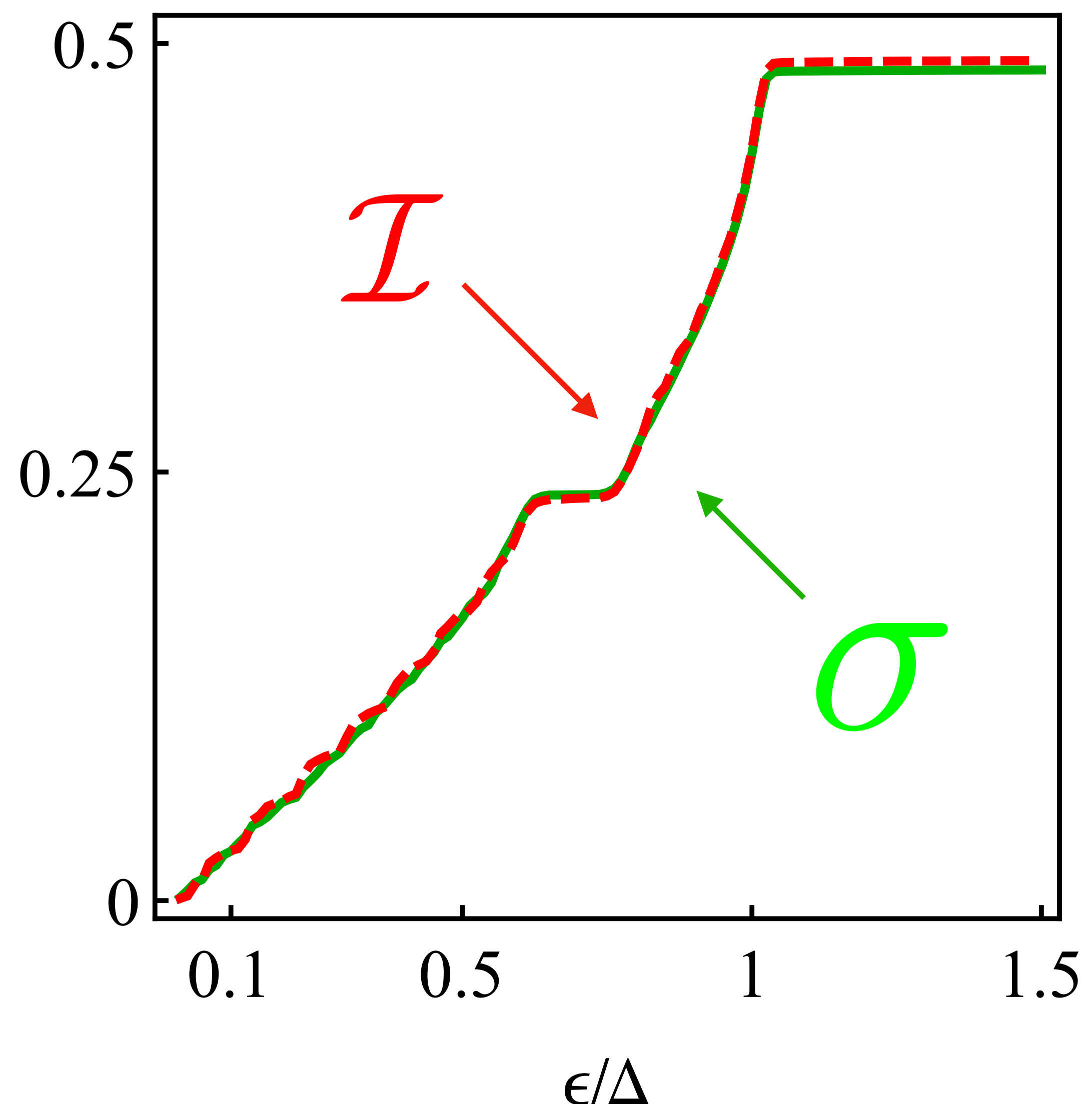
$$\mathrm{R}_x(\epsilon) = \left\langle \sum_{j=1}^{N_s} \left| (j - x) \left[n(j) - \tilde{n}(j, \epsilon, x) \right] \right| \right\rangle$$



IMBALANCE

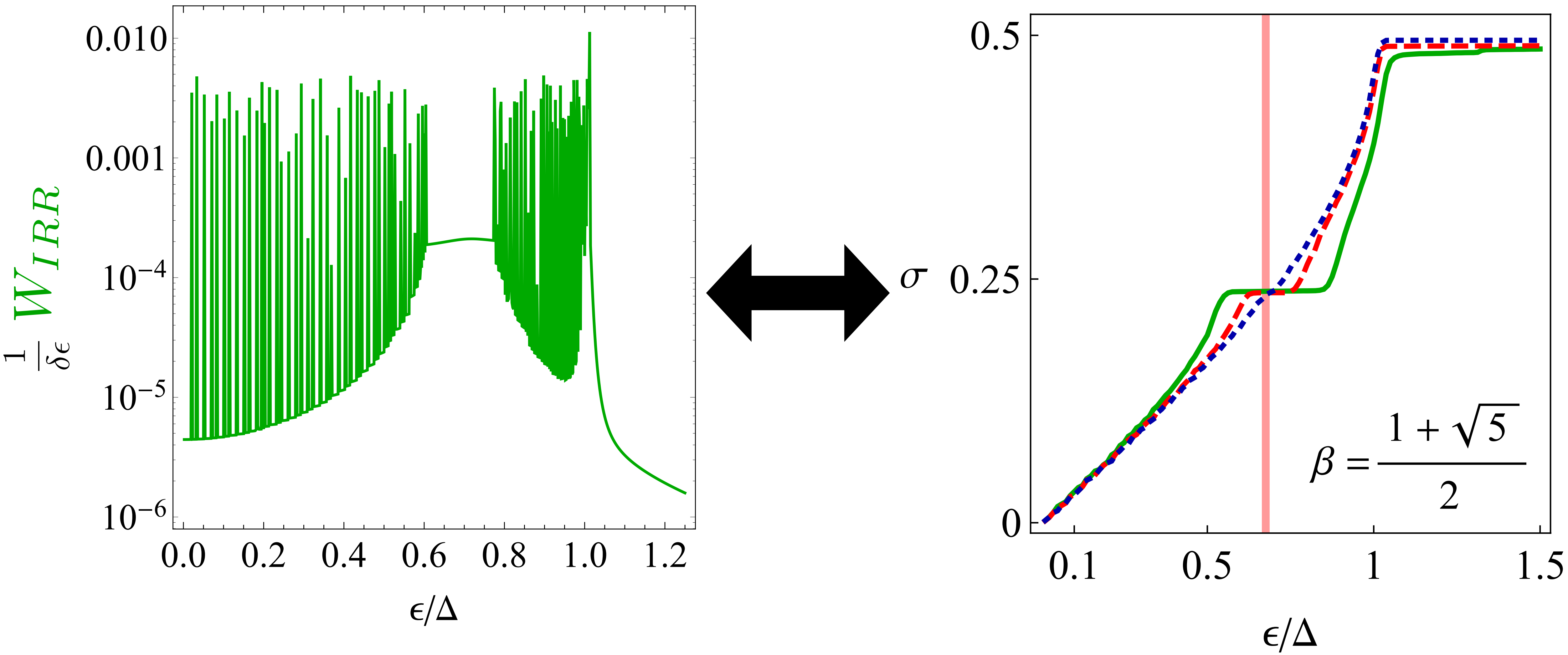
Density imbalance $$\mathcal{I}(\epsilon) = \langle \frac{1}{N_s} \sum_x |\delta n_{odd}(x,\epsilon) - \delta n_{even}(x,\epsilon)| \rangle$$

$$\delta n_{odd/even}(x,\epsilon) = n_{odd/even}(x,\epsilon) - n_{odd/even}(\epsilon = 0)$$



H. P. Lüschen, P. Bordia, S. Scherg, F. Alet, E. Altman, U. Schneider, and I Bloch, Phys. Rev. Lett. **119**, 260401 (2017)

WS AND STOC



0.010
0.001
10^{-4}
10^{-5}
10^{-6}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
ϵ/Δ
$\frac{1}{\delta\epsilon} W_{IRR}$
σ
0.5
0.25
0
0.1
0.5
1
1.5
ϵ/Δ
$\beta = \frac{1+\sqrt{5}}{2}$

CONCLUSIONS

- Different coupling can be used to highlight different properties of a quasi periodic lattice
- Non-trivial connection between energy-space correlation, orthogonality catastrophe and density fluctuations induced by localised impurities
- Connection between non-Markovianity and localisation in quasi-periodic systems
(out of equilibrium environment)

PEOPLE

Sabrina Maniscalco
Elsi-Mari Laine
Massimo Borrelli

TURKU QUANTUM TECHNOLOGY GROUP

UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

Saverio Pascazio

Antonello Scardicchio

The Abdus Salam International Centre for Theoretical Physics

and funding

European Commission

arXiv:1803.04382

THANKS FOR THE ATTENTION