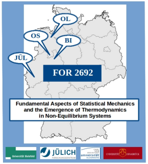
Stability of Quantum Expectation Value Dynamics and the Arrow of Time

Lars Knipschild and Jochen Gemmer

University of Osnabrück,

KITP, Santa Barbara, June 18th, 2018

Funding by the **DFG** through research unit 2692



Outline

- What I will not talk about
- Confession of my ignorance
- The puzzle in more detail
- Numerical experiment (5 slides)
- Heuristic model
- Towards possibly solving the puzzle...

What I will not talk about

- **Validity of the Jarzynski equation for non-thermal initial states (quantum)**
  Phys. Rev. E, 94(1), 012125 , (2016), arXiv: cond-mat/1710.10871
- **Why typicality is not good enough and why we actually need eigenstate thermalization**
  Europhys. Lett., 118, 10006 , (2017)
- **The validity of linear response theory in the non-linear, strong perturbation regime**
  arXiv: cond-mat/1805.11625

If you are interested in these subjects, just urge the organizers to let me give another talk....

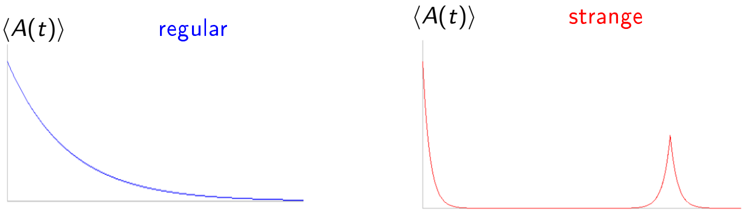
I (still) do not understand where the second law in its most elementary form, namely:

some types of dynamics occur, others do not,

actually comes from.

(Any help welcome !!!)

The Puzzle in More Detail



- Both are possible in closed system QM : $\dot{\rho} = i[\rho, H]$
- Both are possible for "non-fine tuned" initial states $\rho(0)$
- Both are in accord with typicality (Reimann, Goldstein, Short, etc.)
- Both are in accord with the eigenstate thermalization hypothesis (Srednicki, Deutsch, Rigol, etc.)
- Both are in accord with the "equilibration principle": $\langle A(t)\rangle \approx \frac{1}{T}\int_0^T \langle A(\tau)\rangle d\tau$ for most t (Reimann, etc.)
- strange is not necessarily a Poincare recurrence
- Both are "slow" dynamics
- regular occurs routinely, strange does not. Maybe due to regular being more stable to perturbations? $\Rightarrow$

Initial State $\rho(0)$:

We consider "non-fine tuned" initial states of the form

$$\rho(0) \propto 1 + \epsilon A$$

This may be thought of as a high temperature + small perturbation case of $\rho(0) \propto \exp\{-\beta(H + \frac{\epsilon}{\beta} A)\}$.

$\Rightarrow$ expectation value dynamics: $\langle A(t) \rangle \propto Tr\{A(t)A\}$.

This also holds for a much larger class of initial states, given the ETH applies (Srednicki, Rigol, Richter et al.)

Thus we are mow asking why certain forms of the auto-correlation function $Tr\{A(t)A\}$ are more common than others

Hamiltonian H:

We draw 50 000 eigenvalues E_n at random from a uniform distribution from the interval $[-30, 30]$. It turns out that level spacing statistics (Poisson, Wigner-Dyson, Brody, etc.) does not matter here.

Observable A:

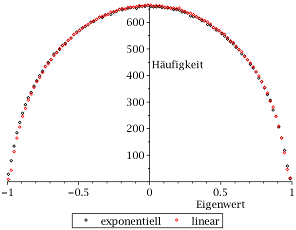
$$A_{nm} = f(|E_n - E_m|) r_{nm}$$

r_{nm}: independent Gaussian random numbers.

This is in full accord with the eigenstate thermalization ansatz. By choosing $f(\omega)$ the dynamics of $\langle A(t)\rangle \propto Tr\{A(t)A\}$ may be chosen at will:

$$Tr\{A(t)A\} \propto \int f^2(\omega)\cos(\omega t)d\omega$$

We investigate four different "paradigmatic" $\langle A(t)\rangle$ (pictures see later). The corresponding observables A have neverthelss almost equal semi-circular spectra (Wigner semi-circle law):



Since all spectra are equal, we say that we address the same observable in the four different examples. The examples than only differ in the orientation of the eigenstates of the Hamiltonian H w.r.t. the eigenstates of the observable A.

Perturbation V:

$$V_{nm} \propto \Theta(\mu - |A_n - A_m|) r_{nm}$$

A_n: eigenvalues of the observable A, Θ: Heaviside-function, r_{nm}: independent Gaussian random numbers.

The perturbation is a random matrix which is banded in the eigenbasis of the observable with bandwith μ. Thus $\mu = 0 \rightarrow [V, A] = 0$. Generally : μ controls how much A and V commute.

We consider the control of the (approximate) commutativity as a physically relevant feature

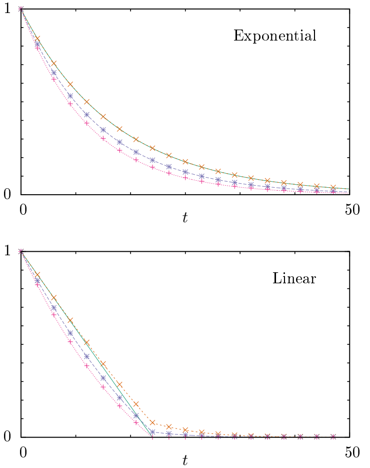
We keep the overall strength of the perturbation V fixed, irrespective of the bandedness μ: $Tr\{V^2\} = \epsilon Tr\{H^2\}$, $\epsilon = 0.03$

Scheme of the numerical experiment:

Dynamics of $\langle A(t) \rangle$ under H: engineered at will.

Dynamics of $\langle A(t) \rangle$ under $H + V(\mu, \epsilon)$: **???**

Stay tuned for the answer on the next slide...



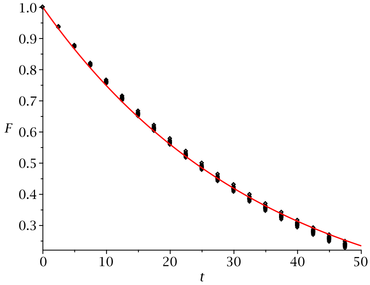
—: engineered, no perturbation, ×: narrowly banded perturbation, ∗: widely banded perturbation , +: unbanded perturbation

- bandedness makes a difference !
- only fully stable setting: exponential decay + narrowly banded perturbation
- exponentials and exponetilally damped oscillations are mapped onto themselves by all perturbations

Do the perturbations affect the dynamics of the quantum state $\rho(t)$ itself differently strongly? Consider as measure the fidelity $F(t)$

$$F = |\langle \tilde{U}(-t)U(t)\rangle|^2$$

$U(t)$ propagator under H, $\tilde{U}(t)$ propagator under $H + V$



The effect of the perturbation on the quantum state is the same for all settings, the macroscopically stable case is not different.

Possible recurrences are not Poincare.

Heuristic Model

Consider the dynamics $\langle A(t)\rangle := a(t)$ as generated by some memory kernel a la Nakajima-Zwanzig:

$$\frac{\mathrm{d}}{\mathrm{d}t}a(t) = -\int_0^t K(t-t')a(t')\mathrm{d}t' = -K * a(t).$$

Then the following appears to hold

$$\tilde{K}(\tau) = K(\tau)\exp(-\epsilon\tau) - \epsilon\beta(\mu)\delta(\tau).$$

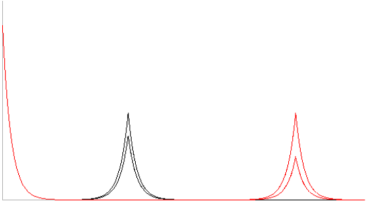
where $\tilde{K}$ generates the perturbed dynamics $\tilde{a}(t)$. We numerically find $\beta(\mu) \to 0$ for $\mu \to 0$. Generally this model implies:

- narrowly banded perturbations ($[V, A] \approx 0$) yield damped memory kernels
- unbanded perturbations yield damped dynamics $\tilde{a}(t) = \exp(-\epsilon t)a(t)$
- exponentials and exponetially damped oscillations are mapped onto themselves by all perturbations

So far the model falls from the sky, but it fits very well !

What about the strange recurrence dynamics?

$a(t), \tilde{a}(t)$



Here the model predicts $\tilde{a}(t) = \exp(-\epsilon t)a(t)$, regardless of the bandedness of the perturbation V: Recurrences are always exponentially more damped the later they occur.

Take home message

- Late recurrences are exponentially instable to all sorts of perturbations
- exponential relaxations are stable to banded perturbations

Thank you for your attention !