Electrons in a Box: Realization of Szilard Engine and Autonomous Maxwell Demon

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- 1. Heat in circuits: measurement and control
- 2. Single-electron box
- 3. Stochastic thermodynamics and fluctuation relations
- 4. Szilard engine
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- 6. Autonomous Maxwell demon
- 7. Quantum thermodynamics with superconducting qubits and resonators
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Thermodynamics experiments in circuits

Heat engines and refrigerators

Testing and establishing fundamental laws of TD in small systems Curiosity experiments, e.g. Maxwell's Demon





$$W = \Delta U + Q \qquad \langle \Delta S \rangle \ge 0$$

$$P(\Delta S)/P(-\Delta S) = e^{\Delta S/k_B}$$



Dissipation in transport through a barrier



Dissipation generated by a tunneling event in a junction biased at voltage V

$$\Delta Q = (\mu_1 - E) + (E - \mu_2) = \mu_1 - \mu_2 = eV$$

 $\Delta Q = T \Delta S$ is first distributed to the electron system, then typically to the lattice by electron-phonon scattering

For average current *I* through the junction, the total average power dissipated is naturally

$$P = (I e) \Delta Q = IV$$

Indirect and direct measurement of heat



 $E = E_{\rm C}(n - n_{\rm g})^2$ $Q = E_{\rm C}(2n_{\rm g} - 1)$

Indirect measurement of heat (and work) by counting electrons

Direct measurement of heat by thermometry



Fluctuation relations in a circuit

$$\frac{P_{\tau}(\Delta S)}{P_{\tau}(-\Delta S)} = e^{\Delta S/k_{\rm B}}$$

U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012)

$$\langle e^{-\Delta S/k_B} \rangle = 1$$





Experiment on a double quantum dot Y. Utsumi et al. PRB 81, 125331 (2010), B. Kung et al. PRX 2, 011001 (2012).

 $\frac{P_{\tau}(n)}{P_{\tau}(-n)} = e^{neV_{\rm DQD}/k_{\rm B}T}$



 ΔF

Systems **driven** by control parameter(s), starting in equilibrium

$$W_d = W - \Delta F$$
 "dissipated work"
C. Jarzynski 1997 $\langle e^{-\beta W_d} \rangle = 1$ $\langle W \rangle \ge \Delta$
G. Crooks 1999 $p_F(W_d)/p_R(-W_d) = e^{\beta W_d}$

Experiment on a single-electron box

Saira et al., PRL 109, 180601 (2012); Koski et al, Nature Physics 9, 644 (2013)



Information-powered cooling: Szilard engine



Isothermal expansion of the "single-molecule gas" does work against the load

$$W = Q = \int_{V/2}^{V} p dV = \int_{V/2}^{V} \frac{k_B T}{V} dV = k_B T \ln 2$$

Szilard engine for single electrons

J. V. Koski et al., PNAS 111, 13786 (2014), PRL 113, 030601 (2014)

Entropy of the charge states: $S = -k_B \sum_{i=0,1} p(i) \ln[p(i)]$



Extracting heat from the bath in quasi-static drive



Erasure of a bit

Landauer principle: erasing a single bit costs energy of at least $k_B T \ln(2)$ Experiment on a colloidal particle:





A quantum version: L. Yan et al., Phys. Rev. Lett. 120, 210601 (2018)

Realization of the Szilard engine with an electron



Experimental work distributions





Jonne Koski

J. V. Koski et al., PNAS 111, 13786 (2014), PRL 113, 030601 (2014)

Whole cycle with ca. 3000 repetitions: $\langle v \rangle \approx -0.75 k_B T \ln(2)$

Sagawa-Ueda relation

$$\langle e^{-(W-\Delta F)/k_BT-I} \rangle = 1$$

$$I(m,n) = \ln\big(\frac{P(n|m)}{P(n)}\big)$$

T. Sagawa and M. Ueda, PRL 104, 090602 (2010)

For a symmetric two-state system:

$$I(n = m) = \ln(2(1 - \epsilon))$$
$$I(n \neq m) = \ln(2\epsilon)$$

Measurements of *n* at different detector bandwidths





J. V. Koski et al., PRL 113, 030601 (2014)

Entropy minimum in a double dot



S. Singh, E. Roldan et al., arXiv:1712.

Indirect and direct measurement of heat



 $E = E_{\rm C}(n - n_{\rm g})^2$ $Q = E_{\rm C}(2n_{\rm g} - 1)$

Indirect measurement of heat (and work) by counting electrons

Direct measurement of heat by thermometry



Measuring heat currents by thermometry

Measurement of temperature by a fast thermometer



Generic thermal model of an electronic conductor



Separation of time scales: $\tau_{ee} < 10^{-9}$ s, $\tau_{ep} > 10^{-6}$ s

NIS-thermometry

$$I = \frac{1}{2eR_T} \int n_S(E) [f_N(E - eV) - f_N(E + eV)] dE$$

Probes electron temperature of N electrode (and not of S!)





Phys. Rev. Appl. 4, 034001 (2015).

Heat through a single-electron transistor – deviation from Wiedemann-Franz law



Autonomous Maxwell demon

System and Demon: all in one

Realization in a circuit:

$$H(n, N) = E_{\rm s}(n - n_g)^2 + E_{\rm d}(N - N_g)^2 + 2J(n - n_g)(N - N_g)$$





J. V. Koski et al., PRL 115, 260602 (2015). Similar idea: P. Strasberg, ..., M. Esposito, PRL 110, 040601 (2013).

Autonomous Maxwell demon – informationpowered refrigerator



Image of the actual device



N_g = 0: No feedback control ("SET-cooler")

$$H(n,N) = E_{\rm s}(n-n_g)^2 + E_{\rm d}(N-N_g)^2 + 2J(n-n_g)(N-N_g)$$

 $N_{\rm g} = 0$ freezes N = 0

 $E = E_{\rm s}(n-n_{\rm g})^2$

Behaves like in the absence of Demon

JP, J. V. Koski, and D. V. Averin, PRB **89**, 081309 (2014) A. V. Feshchenko, J. V. Koski, and JP, PRB **90**, 201407(R) (2014)



(a)

+V/2

 $R_{T1}, C_1 = R_{T2}, C_2$

(b)

2k_RT

eV

N_g = 0.5: feedback control (Demon)

 $H(n,N) = E_{\rm s}(n-n_g)^2 + E_{\rm d}(N-N_g)^2 + 2J(n-n_g)(N-N_g)$

 $N_{\rm g}$ = 0.5, *N* adjusts to minimize Coulomb repulsion

In particular at $n_{\rm g} = 0.5$

 $E = 2J(n-n_g)(N-N_g)$ $n \to 1, N \to 0$

n -> 0, *N* -> 1

The lower box acts as a Demon on the top System



Both T_L and T_R drop: entropy of the System decreases; T_{det} increases: entropy of the Demon increases

Quantum thermodynamics with superconducting qubits and resonators



Stochastic thermodynamics of a driven qubit Quantum jumps/trajectories

Hekking and JP, PRL 111, 093602 (2013); Horowitz and Parrondo, NJP 15, 085028 (2013)

Classical evolution





Quantum trajectories

Objective: **unravel into single realizations** ("single experiments") instead of averages (the latter ones come naturally from the density matrix)

Construct the Monte Carlo wave function (MCWF) for the system Dalibard, Castin and Mölmer 1992 Plenio and Knight 1998 $|\psi(t)\rangle = a(t)|q\rangle + b(t)|e\rangle$



- At $t = t + \Delta t$, we have three possibilities:
- 1. Relaxation $|\psi(t + \Delta t)\rangle_{\downarrow} = |g\rangle$ with probability $\Delta p_{\downarrow} = \Gamma_{\downarrow}|b(t)|^2 \Delta t$ $Q = +\Delta E$
- 2. Excitation $|\psi(t+\Delta t)\rangle_{\uparrow} = |e\rangle$ with probability $\Delta p_{\uparrow} = \Gamma_{\uparrow}|a(t)|^2 \Delta t$ $Q = -\Delta E$
- 3. Evolution without photon absorption/emission

 $|\psi^{(0)}(t+\Delta(t))\rangle = \mu [1 - \frac{i}{\hbar} \Delta t H] |\psi(t)\rangle, \ \mu = (1 - \Delta p_{\downarrow} - \Delta p_{\uparrow})^{-1/2}$

Here the Hamiltonian is non-hermitian (to preserve the norm)

 $H = H_S - i\hbar\Gamma_{\downarrow}|e\rangle\langle e|/2 - i\hbar\Gamma_{\uparrow}|g\rangle\langle g|/2$

Temperature of a qubit?



Couple the qubit to a true thermal bath



Alternative approach to initialize a qubit to a given "temperature": Y. Masuyama et al., Nature Comm. 9, 1291 (2018).

Quantum Otto refrigerator



Niskanen, Nakamura, Pekola, PRB 76, 174523 (2007); B. Karimi and JP, Phys. Rev. B **94**, 184503 (2016).

System and Hamiltonian



 $H = H_{R_{\rm H}} + H_{R_{\rm C}} + H_{\rm cH} + H_{\rm cC} + H_{\rm Q}$

$$H_{\rm Q} = -E_0(\Delta\sigma_x + q\sigma_z) \qquad \begin{array}{l} \Delta = E_2/(2E_0) \\ q \equiv \delta\Phi/\Phi_0 \end{array}$$

$$\dot{\rho}_{gg} = -\frac{\Delta}{q^2 + \Delta^2} \dot{q} \operatorname{Re}[\rho_{ge} e^{i\phi(t)}] - \Gamma_{\Sigma} \rho_{gg} + \Gamma_{\downarrow}$$



$$\dot{\rho}_{ge} = \frac{\Delta}{q^2 + \Delta^2} \dot{q} (\rho_{gg} - 1/2) e^{-i\phi(t)} - \frac{1}{2} \Gamma_{\Sigma} \rho_{ge}$$

$$E_0^2 M_{\perp}^2 = \Delta^2$$

$$\Gamma_{\downarrow,\uparrow,j} = \frac{E_0 m_j}{\hbar^2 \Phi_0^2} \frac{\Delta}{q^2 + \Delta^2} S_{I,j}(\pm E/\hbar) \qquad P_j = E(t) \left(\rho_{ee} \Gamma_{\downarrow,j} - \rho_{gg} \Gamma_{\uparrow,j}\right)$$

Nearly ideal refrigerator (at intermediate pumping frequencies)



Ideal Otto cycle: brown line

Different coupling to the baths: blue lines

$$P_1 = +\frac{\hbar\omega_1}{2} \left[\tanh(\frac{\beta_1 \hbar\omega_1}{2}) - \tanh(\frac{\beta_2 \hbar\omega_2}{2}) \right] f,$$

$$P_2 = -\frac{\hbar\omega_2}{2} \left[\tanh(\frac{\beta_1 \hbar\omega_1}{2}) - \tanh(\frac{\beta_2 \hbar\omega_2}{2}) \right] f.$$

 $\Pi_j \equiv P_j / (E_0^2 / \hbar)$

Coherent oscillations of heat current at high frequencies



Nearly adiabatic regime (at very low frequencies)



Dimensionless power to reservoir j, $\Pi_j \equiv P_j/(E_0^2/\hbar)$ as a function of dimensionless frequency $\Omega = 2\pi \hbar f/E_0$

$$\Pi_j^{(2)} = \Lambda_j \Omega^2$$

1. Classical rate equation:
$$\dot{\rho}_{gg} = -\Gamma_{\Sigma}\rho_{gg} + \Gamma_{\downarrow}$$

$$\Lambda_{j,\mathrm{CL}} = -\frac{1}{\pi} \int_0^{2\pi} du \sqrt{q^2 + \Delta^2} \left(\frac{\frac{d^2 \rho_{\mathrm{eq,gg}}}{du^2}}{\xi_{\Sigma}^2} - \frac{\left(\frac{d\rho_{\mathrm{eq,gg}}}{du}\right) \left(\frac{d\xi_{\Sigma}}{du}\right)}{\xi_{\Sigma}^3}\right) \xi_{\Sigma,j}$$

2. Full (quantum) master equation:
$$\Lambda_j = \Lambda_{j,\text{CL}} + \delta \Lambda_{j,\text{Q}}$$
$$\delta \Lambda_{j,\text{Q}} = \frac{1}{\pi} \int_0^{2\pi} du \frac{\Delta^2}{(q^2 + \Delta^2)^{3/2}} (\frac{dq}{du})^2 \frac{(\xi_{\downarrow} - \xi_{\uparrow})\xi_{\Sigma,j}}{\xi_{\Sigma}[\xi_{\Sigma}^2 + 16(q^2 + \Delta^2)]} > 0$$

Quantum coherence degrades the performance of the refrigerator

Quantum heat valve A. Ronzani et al., Nature Physics, to be published (2018).



B. Karimi, J. Pekola, M. Campisi, and R. Fazio, Quantum Science and Technology **2**, 044007 (2017).





Experimental realization



THERMOMETERS

Theory vs experiment: low-Q regime



Theory vs experiment: intermediate-Q regime



Quantum Otto refrigerator

a)



Expect about 1 fW cooling power at 1 GHz driving frequency



Calorimetry for measuring mw photons



Typical parameters

Operating temperature T = 0.03...0.1 K $E/k_{\rm B} = 0.3...1$ K, $C = 300...1000k_{\rm B}$

 $\Delta T \sim 1...3$ mK, $\tau \sim 0.01...1$ ms Ideally $\delta E = \sqrt{k_B C} T$

Fast NIS thermometry on electrons



ZBA based thermometry

B. Karimi and JP, in preparation



Heat capacity of copper films



Requirements for single microwave photon detection

Detector noise bounded from below by temperature fluctuations of the absorber coupled to the bath.

$$\langle \delta T^2 \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_T(\omega) = k_B T^2 / C$$

Noise-equivalent temperature, NET

NET
$$\equiv S_T(0)^{1/2} = (2k_B T^2/G_{\rm th})^{1/2}$$

Lines:

Green dashed one: current amplifier limited noise Black: fundamental temperature fluctuations Blue: threshold for detecting a single 1 K microwave photon Red: threshold for detecting a single 2.5 K quantum



Summary

Discussed:

stochastic thermodynamics in single-electron circuits open quantum systems based on superconducting qubits thermometry quantum heat switch based on a superconducting qubit plans for quantum heat engines and single-photon detection





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Main collaborators

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Temperature fluctuations





$$S_T(\omega) = \frac{2k_B T^2}{G_{\rm th}} \frac{1}{1 + \omega^2 C^2 / G_{\rm th}^2}$$

$$\langle \delta T^2 \rangle = k_B T^2 / C$$

$$2\pi f_C = G_{\rm th}/C$$

Heat transported between two resistors



Radiative contribution to net heat flow between electrons of 1 and 2:

$$P_{\nu} = \int_{0}^{\infty} \frac{d\omega}{2\pi} \left[S_{P12}(\omega) - S_{P21}(\omega) \right] = r \frac{\pi k_B^2}{12\hbar} (T_1^2 - T_2^2)$$

Coupling constant:

$$r \equiv \frac{4R_1R_2}{(R_1 + R_2)^2}$$

Linearized expression for small temperature difference $\Delta T = T_1 - T_2$:

$$P_{\nu} = r G_{\rm Q} \Delta T$$

 $G_{\nu} = rG_{\rm Q}$

 $rac{\pi k_{
m B}^2}{--}T$

Photonic heat transport



M. Meschke, W. Guichard and JP, Nature 444, 187 (2006)

"Quantum" of heat transport

$$G_{\rm Q} = \frac{\pi k_{\rm B}^2}{6\hbar} T$$

Schmidt et al., PRL 93, 045901 (2004)

Timofeev et al., PRL 102, 200801 (2009)



M. Partanen et al., Nature Physics 12, 460 (2016).

Shunted λ / 4 resonators, measurement of Q





$$Q = \pi Z_0 / 4R$$





Yu-Cheng Chang et al., in preparation

See also: M. Partanen et al., Nat. Phys. **12**, 160 (2016); arXiv:1712.10256

Spectroscopy to determine circuit parameters



$$\hat{H} = hf_{\rm r} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 + a/2 & g & \tilde{g} \\ 0 & g & r & g \\ 0 & \tilde{g} & g & 1 - a/2 \end{pmatrix}$$
$$r = f_{\rm qubit} / f_{\rm r}$$

$$f_{\rm r} = 5.39 \; {\rm GHz}$$

 $g = 0.020$
 $\widetilde{g} = -0.015$
 $a = 0.008$



Two tone spectroscopy



Since $W = \Delta U + Q$, and $\Delta U = E_f - E_i$, this measurement works only for a closed system

Quantum jump approach for analyzing distribution of dissipation





F. Hekking and JP, PRL 111, 093602 (2013).

Common fluctuation relations (Crooks, Jarzynski) are satisfied

$$p_F(W_d)/p_R(-W_d) = e^{\beta W_d} \qquad \langle e^{-\beta W_d} \rangle = 1$$

Requirements for single microwave photon detection



Standard copper absorber

Efficiency (COP) of the quantum Otto refrigerator



