Disentanglement by a Point Contact

Interesting ingredients:

Topological quantum computation, entanglement entropy, quantum Hall effect, chiral conformal field theories, non-abelian statistics, and some very elegant physics...

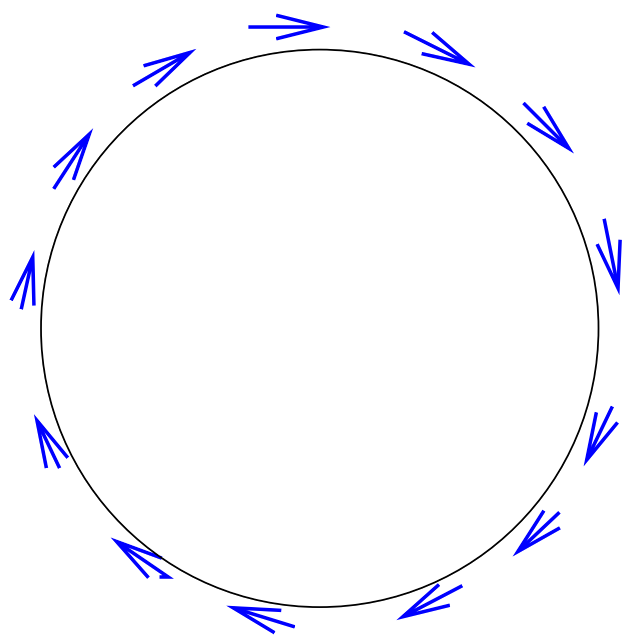
Outline:

1. Entropy on the edge
2. A point contact
3. Tunneling operator
4. Disentanglement

work with Matthew Fisher and Chetan Nayak

The setting:

Quantum states in two dimensions with gapless chiral edge modes.



Simple examples are abelian quantum Hall states, where the edge modes are described by free chiral bosons in 1+1 dimensions.

For topological quantum computation, we need more complicated edge theories, where the edge excitations have non-abelian statistics.

The two key examples for this talk are:

- a $p_x + ip_y$ superconductor

  Read and Green; Ivanov; Stern et al

  See M. Stone's talk this afternoon.

- the Moore-Read state(s)

  See many talks this week!

The edge excitations for $p + ip$ are the chiral Ising field theory.

For the Moore-Read quantum Hall state, the edge theory is a chiral Ising field theory and a free chiral boson.

The Ising model is not free! Correlators of the spin field are extremely non-trivial: see McCoy and Wu's book.

The excitations of the chiral Ising model are labeled

- I (the edge electron in the MR state)
- ψ (edge neutral fermion)
- σ (the interesting one!)

To get a few things out of the way:

edge excitation = excitation= quasiparticle = σ quasiparticle= particle = hole = quasihole

and

$$\sigma \cdot \sigma = I \; + \psi$$

$$\sigma \cdot \sigma = I + \psi$$

Precise meaning in field theory: the operator product expansion of two σ fields contains both the identity field I and the fermion ψ.

Precise meaning in conformal field theory: the “fusion coefficients” for the primary fields I, ψ, and σ are $N_{\sigma\sigma}^{I} = N_{\sigma\sigma}^{\psi} = 1$.

Precise meaning for this talk: There are two possible quantum states for two σ particles, no matter how far apart they are.

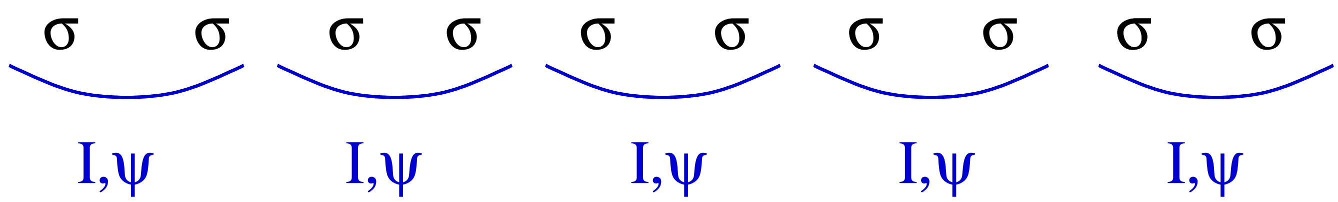
Entanglement!

The full fusion algebra is

$$\begin{aligned}\sigma \cdot \sigma &= I + \psi \\ \sigma \cdot \psi &= \sigma \\ \psi \cdot \psi &= I\end{aligned}$$

The only time there is more than one thing on the right-hand-side is for $\sigma \cdot \sigma$.

The fusion algebra allows us to count the number of quantum states for $2N$ quasiparticles:



There are thus 2^N states for $2N$ quasiparticles.

The entropy per particle is

$$\frac{\ln(\text{\# of states})}{2N} = \ln(\sqrt{2})$$

The quantum dimension of σ is

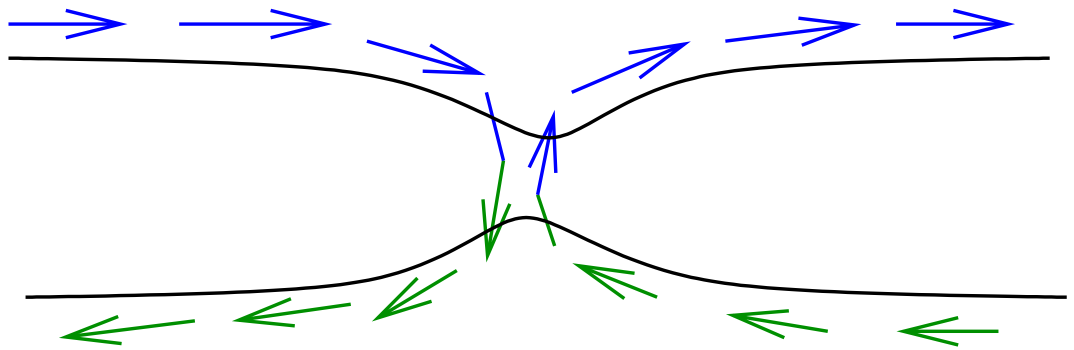
$$d_\sigma = \sqrt{2}$$

The quantum dimensions of the other fields are $d_I = 1$, $d_\psi = 1$.

Non-abelian statistics is possible only when $d > 1$.

Reliving our youths

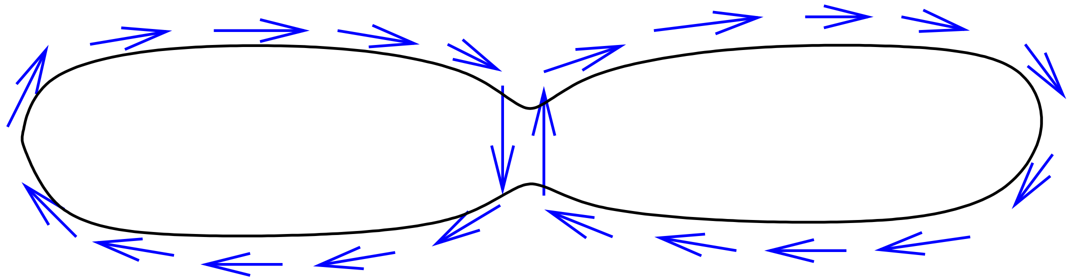
A point contact allows particles to tunnel from one edge to the other. For an abelian quantum Hall state,



Back in those good old days, we could treat the two edges independently.

A tunneling event corresponded to annihilating a left mover on one edge and creating a right mover on the other, or vice versa.

Now in these non-abelian times:



The “left movers” and the “right movers” are indistinghiable particles: they are edge modes for the same bulk. Any pair of σ quasiparticles forms a two-state quantum system no matter where they are.

Let $\mathcal{T}_\sigma(\tau)$ be the operator which tunnels a quasiparticle from one edge to the other at time τ.

Because of the entanglement, we can't just write

$$\mathcal{T}_\sigma = \sigma_L^\dagger \sigma_R \; + \; \sigma_R^\dagger \sigma_L$$

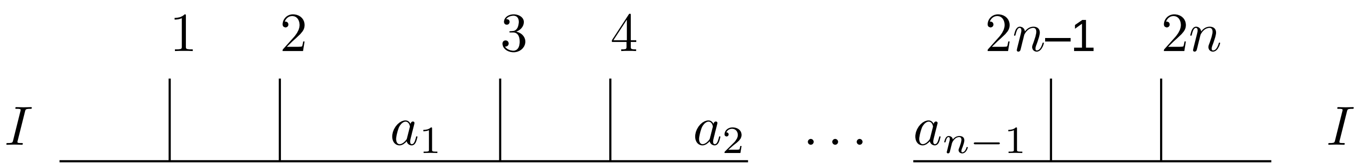
and treat σ_L and σ_R as separate fields.

In fact, correlators of the chiral sigma field

$$\langle \sigma(z_1)\sigma(z_2)\ldots\sigma(z_n)\rangle$$

need more information that just the locations z_1, z_2, $\ldots z_n$ to be defined!

One must also specify the fusion channels. A correlator of $2n$ chiral σ fields (a.k.a a conformal block) is pictorially represented as



where $a_j = I, \psi$ represents the fusion channel for the first $2j$.

To uniquely define the tunneling operator $\mathcal{T}_\sigma$, we must specify what the a_i are.

The simplest kind of point contact will not change the quantum state of the particle being tunneled.

This means the creation and annhilation operators are in the identity channel:

$$\mathcal{T}_\sigma(\tau_1) =$$

where 1 and $1'$ represent the chiral σ fields on the two edges at time τ_1.

This defines the tunneling operator $\mathcal{T}_\sigma$ unambiguously.

To convert this into a more familiar form, we use the braiding and fusing rules developed by Moore and Seiberg.

By “folding” the disc at the point contact, we can can rewrite correlators in the two Ising models in terms of a single free chiral boson.

This is closely related to studying the Ising model with a defect line.

Oshikawa and Affleck

We find a surprising result...

The $p + ip$ superconductor with a point contact is equivalent to the (anisotropic) Kondo problem.

In terms of this new chiral boson φ, we have

$$\mathcal{T}_\sigma = S^+ e^{-i\varphi/2} \; + \; S^- e^{i\varphi/2}$$

where $(S^+)^2 = (S^-)^2 = 0$.

In the Kondo problem, S is the quantum spin of an isolated impurity in a sea of electrons.

This tells us a great deal about how the entropy behaves in the presence of the point contact.

The UV limit corresponds to having no point contact. In the Kondo problem this corresponds to the spin isolated from its environment (i.e. $S^{\pm}$ do not couple to φ).

$\mathcal{T}_{\sigma}$ is a relevant perturbation. Adding it to the Hamiltonian causes a flow to a new fixed point.

The physics of this is well understood in the Kondo problem.

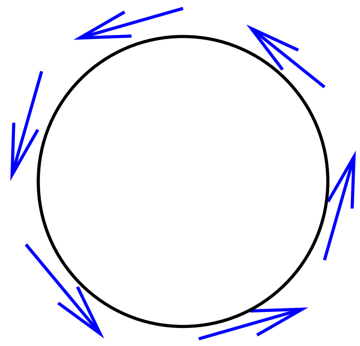
Allowing tunneling across the point contact corresponds to having the Kondo impurity spin interact with its environment antiferromagnetically.

Since the interaction is relevant, the the Kondo impurity spin is screened in the IR limit. This means that it forms a spin singlet with the sea of electrons.

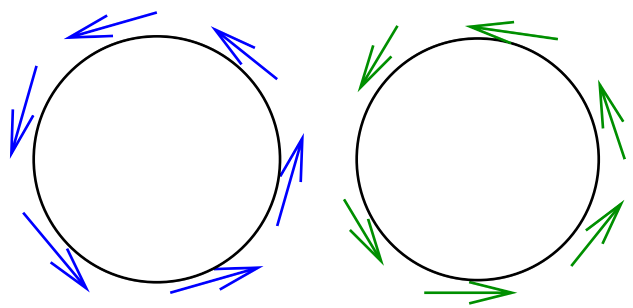
There is thus a change in entropy

$$S_{UV} - S_{IR} = \ln(2) - \ln(0) = \ln(2)$$

In the absence of the point contact, we have



while for the IR, the system effectively splits into two.



So where is this entropy loss?

Entanglement entropy!

The (universal part of the) topological entanglement entropy of a disc is $-\ln\mathcal{D}$, where $\mathcal{D}$ is the total quantum dimension

$$\mathcal{D} = \sqrt{\sum (d_a)^2}$$

Kitaev and Preskill, Levin and Wen

For Ising:

$$\begin{aligned} \mathcal{D} &= \sqrt{(d_I)^2 + (d_\psi)^2 + (d_\sigma)^2} \\ &= \sqrt{1^2 + 1^2 + (\sqrt{2})^2} \\ &= 2 \end{aligned}$$

.

Thus $S_{UV} = -\ln \mathcal{D} = -\ln(2)$.

In the IR, the system has split into two discs, so $S_{IR} = -2\ln(2)$.

Thus we indeed have an entropy loss of

$$S_{UV} - S_{IR} = -\ln 2$$

The two separated halves in the IR are two distinct Hall droplets, which have their own distinct sets of quasiparticles. In the UV, a quasiparticle on the left half can be entangled with one on the right half. In the IR, the two halves are not entangled.

A point contact causes disentanglement!

Another check on these results comes from studying boundary conformal field theory. One has

$$\mathcal{D} = 1/S_0^0$$

where S is the modular S matrix (not to be confused with the Kondo spin S or the entropy S).

Cardy

The fusion coefficients are the integers which encode the fusion rules, e.g. $N_{\sigma\sigma}^{I} = N_{\sigma\sigma}^{\psi} = 1$. The Verlinde formula relates them to S for any rational conformal field theory

$$N_{ab}^{c} = \sum_j \frac{S_a^j S_b^j S_j^c}{S_0^j}$$

Define the matrix Q_a, whose elements are

$$(Q_a)^b_c \equiv N^c_{ab}$$

Think of Q as the matrix which fuses one quasiparticle a with a particle c to get quasiparticles b.

The quantum dimension d_a is the largest eigenvalue of Q_a. Using the Verlinde formula gives

$$\sum_b (Q_a)^c_b \frac{S^b_k}{S^k_0} = \frac{S^k_a}{S^k_0} \frac{S^c_k}{S^c_0}$$

The eigenvalues of Q_a are therefore given by S^k_a / S^k_0. The largest eigenvalue is the one with $k = 0$, so we have

$$d_a = \frac{S^0_a}{S^0_0}$$

An interesting intepretation is in terms of 1+1d boundary entropy.

Affleck and Ludwig

The point contact is the boundary of a boundary!

One can also include bulk σ quasiparticles (i.e. qubits). These change the boundary conditions on the edge modes, and thus change the quantum dimensions. For example, for a single bulk σ quasiparticle, one finds for the entropy

$$S = \ln\left(\frac{d_\sigma}{\mathcal{D}}\right) = \ln(\sqrt{2})$$

For the Moore-Read state, we must include the chiral charge boson φ_c. There are now six quasiparticles:

- I (identity/electron)
- $\sigma e^{\pm\varphi/(2\sqrt{2})}$ (charge $\pm 1/4$)
- ψ (neutral fermion)
- $e^{\pm\varphi/\sqrt{2}}$

These are the six primary fields of the (Neveu-Schwarz sector of) the $\mathcal{N} = 2$ superconformal algebra with $c = 3/2$. Milovanovic and Read

These have quantum dimensions

$$1, \sqrt{2}, \sqrt{2}, 1, 1, 1$$

giving

$$\mathcal{D}_{MR} = 2\sqrt{2}$$

We of course can find this directly from the tunneling operator. The extra boson turns tunneling of charge $\pm 1/4$ particles through a point contact in the Moore-Read state into a variant of the two-channel Kondo problem.

Changing the radius of the charge boson gives different Moore-Read states. This of course changes the total quantum dimension. For the $SU(2)$-invariant bosonic Moore-Read state,

$$\mathcal{D}_{SU(2)_2} = 2$$

Lots of details (e.g. critical exponents) in our paper and soon-to-appear long paper (papers?). But there are important extensions needing to be understood:

- Computing transport quantities (current, noise)
- Allowing the tunneling to change the state (i.e. $\mathcal{T}_\sigma$ not in the identity channel)
- The tunneling operators in more complicated geometries (as in Shtengel's talk to follow)
- The tunneling operator for Read-Rezayi states