Protected qubits using Josephson junctions.

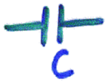
Goal : all-electric protected qubits

Qubits are not elementary devices (classical bits are not elementary either : an implementation consists of several transistors.)

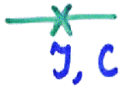
Sub-goal : find a set of basic elements for quantum electric circuits

"Simple" elements:

Capacitor :



Josephson junction :

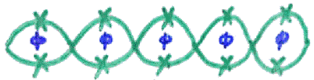


Inductor :

$J \gg \frac{e^2}{C}$

$\left(L_{eff} = N\left(\frac{\hbar}{2e}\right)^2 J^{-1}\right)$

Switch :

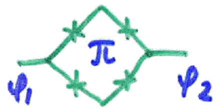


$\phi \approx \pi$ $\Longrightarrow$ transition to an insulating state

(In dimensional units, $\phi \approx \frac{\Phi_0}{2}$)

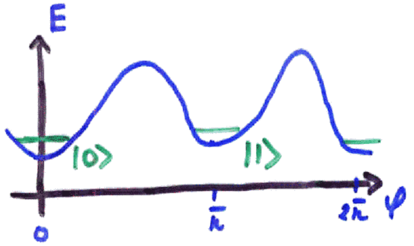
More interesting elements:

1) π-contact (Douçot, Vidal, Ioffe, Feigelman)



$$E=-J_1\cos 2\varphi-\underbrace{J_2\cos\varphi}_{\text{error term}}$$

$$J_2\ll J_1$$



Protection (stabilization)

$$J_{1,\mathrm{eff}}\sim\frac{1}{N}$$

$$J_{2,\mathrm{eff}}\sim\left(\frac{J_2}{t}\right)^{N-1}$$

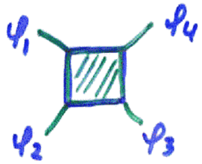
tunneling amplitude

(Douçot, Vidal)

2) Main idea of this talk:

Quantum transformer

(superconducting current mirror)



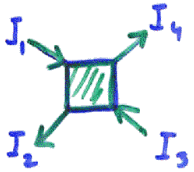
$$\boxed{E=f(\varphi_1-\varphi_2+\varphi_3-\varphi_4)}$$

$$+\underbrace{\alpha\cos(\varphi_1-\varphi_4)+\ldots}_{\text{error terms}}$$

(exponentially small)

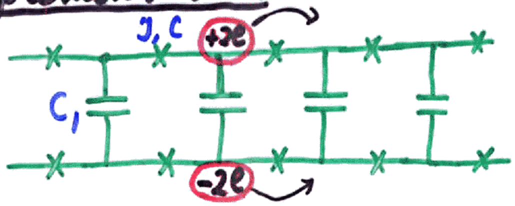
Why transformer?

$$I=\frac{2e}{\hbar}\frac{\partial E}{\partial\varphi}\quad\Rightarrow\quad I_1=-I_2=I_3=-I_4$$



(like )

Implementation



$C_1\gg\ldots\ldots$

(discussed later)

The current mirror idea is not new:

Normal conductors: Cotunneling of electron-hole pairs

(Averin, Korotkov, Nazarov 1991)

Experimental realization in the resistive regime (using Josephson junctions)

(Shimada, Delsing 2000)

Theory of superconducting Josephson ladders:

Bose-condensation of excitons,

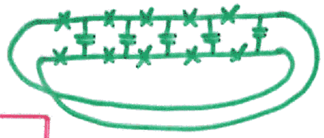
(Choi, Choi, Lee 1998)

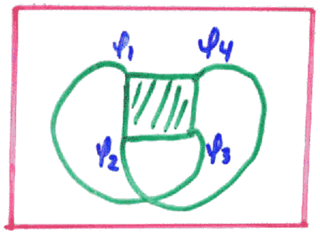
Applications (new)

Related idea

Cristofano, Marotta, Naddeo (2005

1) Qubit



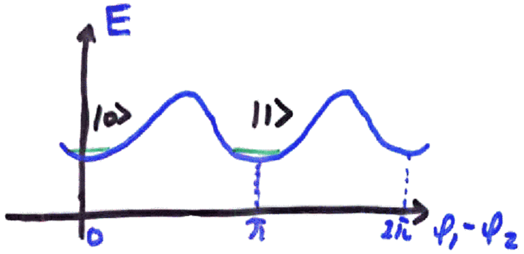


$\varphi_3 = \varphi_1$

$\varphi_4 = \varphi_2$

$E \approx f(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4) = f(2(\varphi_1 - \varphi_2))$

If f has a minimum at 0, then



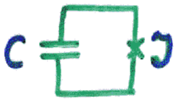
2) Simulation of electromagnetic modes on an arbitrary 3-manifold

(joint work with G. Moore and K.Walke

Discussion of parameters

Start with basics:

Cooper box



$$\boxed{H = \frac{1}{2C}(n - n_0)^2 - J\cos\varphi}$$

Units:
$\hbar = 1 \quad 2e = 1$

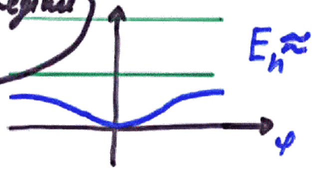
$n = \frac{\partial}{i\,\partial\varphi}$
number of Cooper pairs

offset charge

n_0 is defined modulo 1

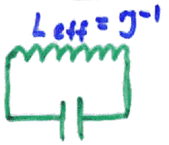
$\boxed{\mu = JC}$

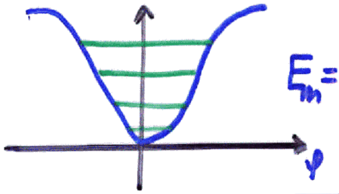
Coulomb regime
$\mu \to 0$



$E_n \approx \frac{(n-\alpha)^2}{2C}$

Harmonic oscillator
$\mu \to \infty$

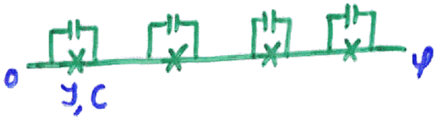




$E_m = \omega_p(m + \frac{1}{2})$

"Plasma frequency" $\omega_p = \sqrt{\frac{J}{C}}$

Josephson junction chains.

1) 

$\mu \lesssim 1$ - insulator

$\mu \gg 1 \qquad E \approx \frac{J}{N}(\varphi + 2\pi k)^2$

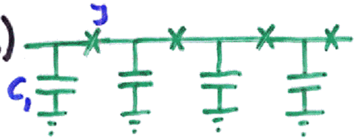
# of trapped elementary fluxes

But there are also phase slips:

E falls off exponentially if

$N > N_c \sim e^{8\sqrt{\mu}}$

Still insulating in the $N \to \infty$ limit

2) 

$$\mathcal{L} = \frac{C_1}{2}\sum_j \dot{\varphi}_j^2 + i\cancel{\sum_j n_{0j}\dot{\varphi}_j} - J\sum_j \cos(\varphi_{j+1} - \varphi_j)$$

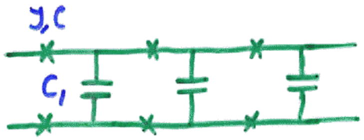
(Using imaginary time)

$JC_1 > \mu_c$ - superconductor

$JC_1 < \mu_c$ - insulator (Kosterlitz-Thouless)

$\mu_c \approx 1$

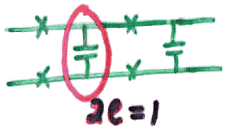
Ladder



Case 1: $JC \ll 1$, $C_1 \gg C$

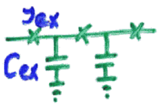
Effective parameters:

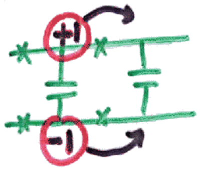
Single charges:



$J_s \approx 2J$
$C_s \approx 2C$ } will be insulating for $N \to \infty$

Excitons:





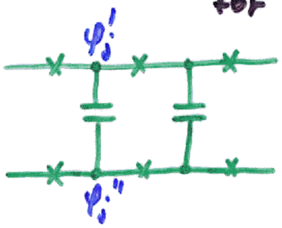
$J_{ex} \approx 4J^2C$
$C_{ex} \approx C_1$

$J_{ex}C_{ex} > \mu_c$

Preliminary results from numerics:
A. Feiguin, S. Trebst

$J = C \sim 0.2$ $C_1 \sim 50$
$N = 3 \div 8$

Case 2: $JC \gg 1$
(a long chain is insulating for single charges anyway)



$$\mathcal{L} = \frac{C}{2}\sum_j \left[(\dot\varphi_{j+1}' - \dot\varphi_j')^2 + (\dot\varphi_{j+1}'' - \dot\varphi_j'')^2 \right] + \frac{C_1}{2}\sum_j (\dot\varphi_j' - \dot\varphi_j'')^2 - - J\sum_j \left[\cos(\varphi_{j+1}' - \varphi_j') + \cos(\varphi_{j+1}'' - \varphi_j'') \right]$$

Continuous limit:
$\varphi'(x)$, $\varphi''(x)$

$\varphi^+ = \frac{\varphi' + \varphi''}{2}$, $\varphi^- = \varphi' - \varphi''$

$$\mathcal{L} = \int \left[C(\varphi^+_{xt})^2 + J(\varphi^+_x)^2 \right] dx + \int \left[\frac{C}{4}(\varphi^-_{xt})^2 + \frac{C_1}{2}(\varphi^-_t)^2 + \frac{J}{4}(\varphi^-_x)^2 \right] dx$$

$M_{eff} = \frac{JC_1}{2} > \frac{4}{\pi^2}$, i.e. $JC_1 \gtrsim 1$

Seems better but in this limit N must be exponentially large

Josephson ladder: conclusion

1) Need to investigate the crossover region $JC \sim 1$ for moderately long chains

2) The important energy parameter

$$\nu = \frac{E}{\omega_p} \approx \frac{J_{ex}}{N}$$

max $f(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4)$

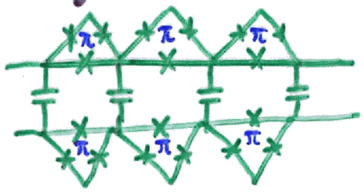
is likely to be small in all interesting cases

$$\nu \lesssim 10^{-2} \div 10^{-3}$$

Need very low temperatures

3) Possible improvements

Magnetic frustration:



Quantum gates

Universal set:

1) Measurement in the $|0\rangle, |1\rangle$ basis

2) Measurement in the $|+\rangle, |-\rangle$ basis $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $|-\rangle = \frac{|0\rangle - |1\rangle}{2}$

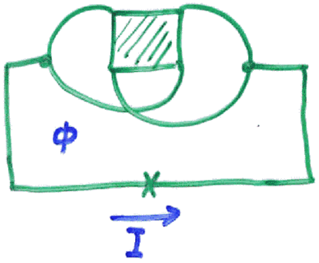
3) $\exp\left(i\frac{\pi}{4}\sigma^z\right)$, $\exp\left(i\frac{\pi}{4}\sigma_1^z\sigma_2^z\right)$ with high precision (protected)

4) $\exp\left(i\frac{\pi}{8}\sigma^z\right)$ with low (~30%) precision (unprotected)

Implementation of the measurements.

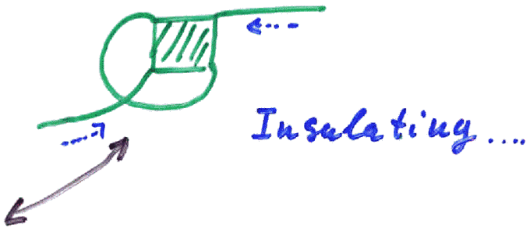
$\underline{|0\rangle, |1\rangle}$

Measuring the phase difference (0 or π)



$\underline{|+\rangle \quad |-\rangle}$ – more interesting

Consider this setup:



$n = \frac{\partial}{i\partial\varphi}$

$$H = \frac{1}{2C_{eff}}(n - n_0 - \hat{\alpha})^2$$

$$\hat{\alpha} = \frac{1+\sigma^x}{4}$$

$|+\rangle \Rightarrow \underline{\alpha = 0}$

$|-\rangle \Rightarrow \underline{\alpha = \frac{1}{2}}$

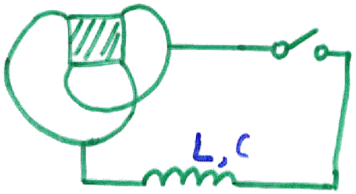
Unprotected $\exp(i\frac{\pi}{8}\sigma^z)$:

Connect for time interval Δt

$$|\psi\rangle \mapsto \exp(i J \Delta t\, \sigma^z)\,|\psi\rangle$$

Protected $\exp(-i\frac{\pi}{4}\sigma^z)$

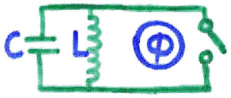
Cf. Gottesma
Kitaev, Preskil
2000



Connect for $\Delta t = \frac{L}{\pi}$

$|\psi\rangle \mapsto \exp(-i\frac{\varphi^2}{2L}\Delta t)|\psi$

Effective circuit:



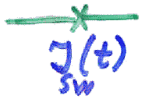
$\underline{R = \sqrt{\frac{L}{C}} \gg 1}$

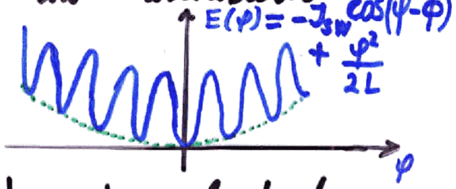
$\varphi = 0$ ($|0\rangle$) $\quad |\psi\rangle \mapsto |\psi\rangle$

or

$\varphi = \pi$ ($|1\rangle$) $\quad |\psi\rangle \mapsto -i|\psi\rangle$

Actually need an adiabatic switch:

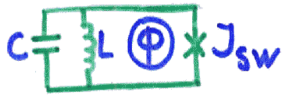




$J_{sw} \lesssim \frac{1}{C}$ – phase is unlocked

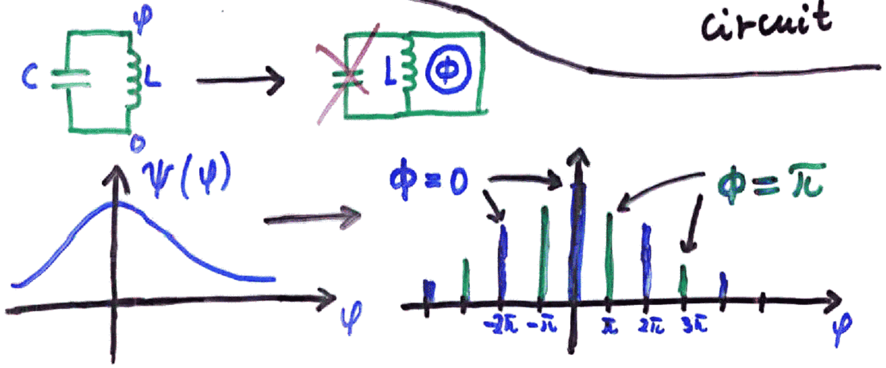
$J_{sw} \gg \frac{1}{C}$ – phase is locked

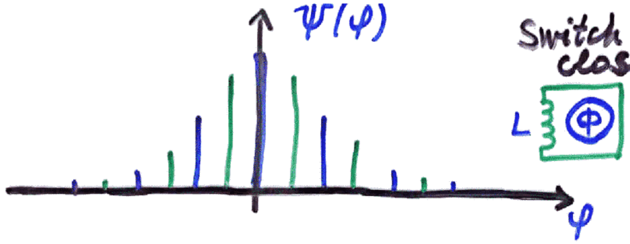
Transition time: $C \ll \tau \ll 2\pi\sqrt{LC}$



not to create oscillations in the J_{sw}-C circuit

to freeze the zero oscillations in the L-C circuit





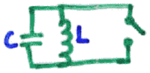
Switch is closed

$\varphi = \Phi + 2\pi k$

$H = \frac{\varphi^2}{2L}$

Each blue peak is multiplied by $\exp\left(-i\underbrace{\frac{(2\pi)^2}{L}\Delta t}_{2\pi}\; k^2\right) = 1$ ↑ peak #

Each green peak is multiplied by $\exp\left(-i\underbrace{\frac{(2\pi)^2}{L}\Delta t}_{2\pi}\left(k+\frac{1}{2}\right)^2\right) = -i$

If Δt is not exact, then excitations will be created in the L-C circuit,  but the qubit will remain (almost) undisturbed.

Conclusions

1) An all-electric QC is possible theoretically

2) A lot of fun for theorists; experimental prospects are not clear yet

3) In short term (mid term), it would be great to implement a quantum transformer as an analog device.