

Entanglement on the dot

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Abstract

An overview of quant-ph/0512048 describing how to entangle light from biexciton decay. This is a step towards entangled photons on-demand.

Credits

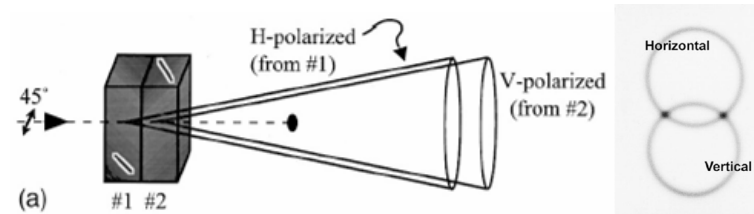
Experiments: N. Akopian and D. Gershoni
Theory: Netanel Lindner and Yosi
Modelling: Netanel, Yoav Berlatzy, Elon Poem
Samples: P. Petrof and B. Gerardot.

Outline

- Down-conversion and on demand
- DQ: Exciton vs positronium, Biexciton cascade
- The incriminating witness
- Witness elimination
- Quantum tomography
- Peres partial transpose
- Misentangled by Nature
- Entangled by witness elimination
- Separable states, negative probabilities

1 On-Demand

1.1 Downconversion



$$\begin{aligned}
 |\Psi\rangle &= |0\rangle \otimes |0\rangle \\
 &+ \varepsilon \left(|0\rangle \otimes |H\rangle + \text{other junk} \right) \\
 &+ \varepsilon^2 \underbrace{(|H\rangle \otimes |V\rangle + |V\rangle \otimes |H\rangle)}_{\text{Bell}} \\
 &+ \varepsilon^2 \left(|HV\rangle \otimes |0\rangle + \text{other junk} \right)
 \end{aligned}$$

Need to post select

Entangled state not available for further use

Good enough for criptography

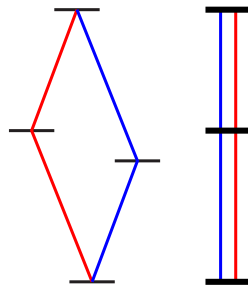
Not good enough for Shor

Need: Event ready

2 Quantum dots, Excitons, biexcitons

Postironum vs exciton

Neutral vs charged



Integer spin: non degenerate

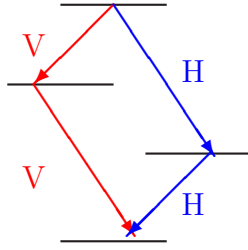
Spin $1/2$: Kramer degenerate

Benson: Use which path ambiguity to entangle

Which one?

3 The witnesses

$$|\psi\rangle = \alpha |HH\rangle \otimes \underbrace{|p_H\rangle}_{\text{photon wave packet}} \otimes |d_H\rangle + \beta |VV\rangle \otimes \underbrace{|p_V\rangle \otimes |d_V\rangle}_{\text{final state of dot}}$$



$$\rho_{polarization} = Tr_{p,d} |\psi\rangle \langle \psi| = \begin{pmatrix} |\alpha|^2 & 0 & 0 & \gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{\gamma} & 0 & 0 & |\beta|^2 \end{pmatrix}$$

$$\gamma = \alpha \bar{\beta} \langle p_H | p_V \rangle \langle d_H | d_V \rangle$$

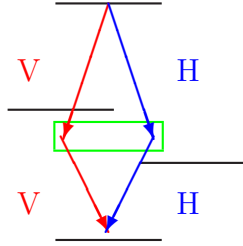
$\gamma = 0$ the photon/dot are faithful witness to decay path

Non-degenerate: Color is a witness

Kramer degenerate: Spin is a witness

4 Witness elimination

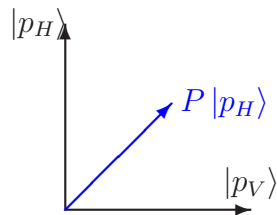
Suppose $\langle p_H | p_V \rangle = 0$ can one nevertheless entangle?



Suppose $|d_H\rangle = |d_V\rangle$. Choose

$$\alpha P |p_H\rangle = \beta P |p_V\rangle$$

$$|\psi\rangle \rightarrow P |\psi\rangle = \underbrace{\left(|HH\rangle + |VV\rangle \right)}_{\text{Bell state}} \otimes P |p_H\rangle \otimes |d\rangle$$



5 Quantum tomography

2 qubits

$$\rho = \sum_{ij=0}^4 a_{ij} \sigma_i \otimes \sigma_j$$

$$\sigma = \{1, \sigma_x, \sigma_y, \sigma_z\}$$

$$\rho = \rho^* \implies a_{ij} \in \mathbb{R}$$

$$\rho = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}$$

$$A = (a_{0j} + a_{3j})\sigma_j, \quad B = (a_{1j} + ia_{2j})\sigma_j, \quad C = (a_{0j} - a_{3j})\sigma_j$$

ρ can be determined by 16 measurements

$$a_{ij} = \frac{1}{4} \text{Tr}(\rho \sigma_i \otimes \sigma_j)$$

Kwiat

6 Partial (=Peres) Transpose

$$\rho^P = \begin{pmatrix} A & B^* \\ B & C \end{pmatrix}$$

P: Preserves reality, does not preserve positivity Ex:

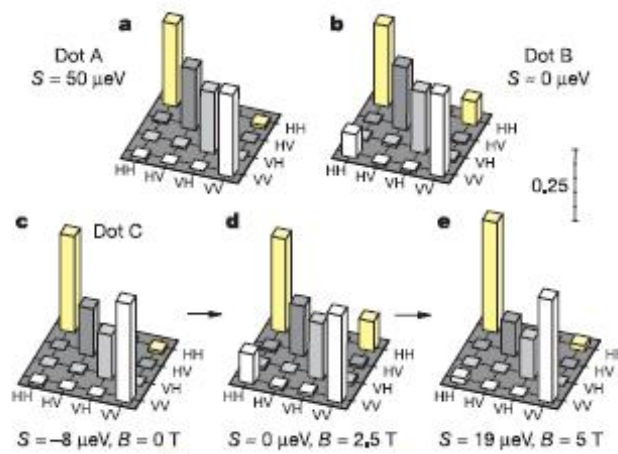
$$\rho = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \geq 0, \quad \rho^P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \text{indefinite},$$

Peres criterion: If $\rho \geq 0$ and ρ^P is indefinite then ρ is entangled.

Horodecki: For 2 qubits: Iff.

7 Misentangled by Nature

Exercise: The Toshiba group in Cambridge, England, led by Andrew Shields, carried the quantum tomography of biexciton decay for a collection of quantum dots. Their data, taken from their Nature article, are reproduced below. For some of the dots the authors claim there is evidence for entanglement. Can you find which one?

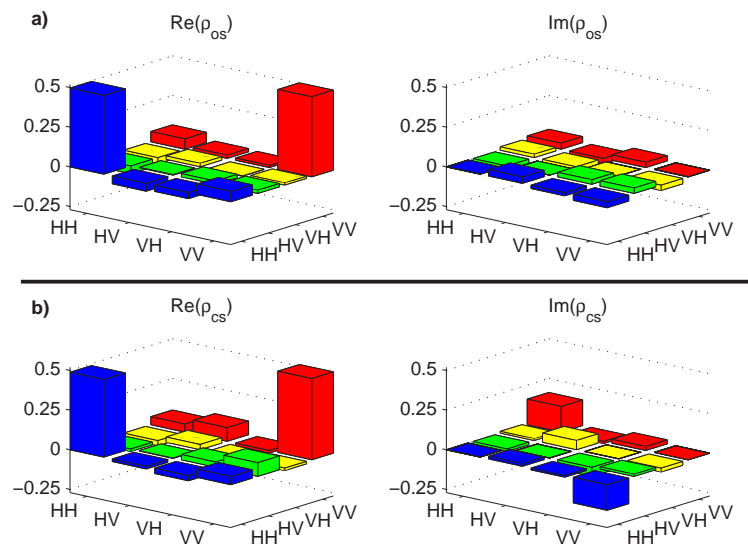


Solution: None.

8 Entangled by witness elimination

An Exercise:

The group of Dudi Gershoni at the Technion carried out the quantum tomography on the biexciton decay of a quantum dot (Berakha). Their data, taken from their upcoming PRL, are reproduced below. Find the data that shows entanglement.



Solution: b is entangled.

9 Separable states, Negative probabilities

Product state

$$\rho = \rho^a \otimes \rho^b$$

Measurement by Ali and Baba independent

$$Prob(i, j) = Tr(E_i \otimes E_j \rho) = Tr(E_i \rho^a) Tr(E_j \rho^b) = Prob(i) Prob(j)$$

Exercise Bell is defined by

$$Bell = A \otimes (A' + B') + B \otimes (A' - B'), \quad A = \sigma \cdot a$$

with a, b unit vectors. Show that a product state satisfies the Bell inequality

$$-2 \leq Tr(\rho Bell) \leq 2$$

Hint: $a - b$ and $a + b$ make a right triangle whose hypotenuse is 2.

Separable states: Convex combinations of product states

$$\rho = \sum p_j \rho_j^a \otimes \rho_j^b, \quad p_j \geq 0, \quad \sum p_j = 1$$

Exercise (Peres): If ρ is separable then $\rho^P \geq 0$.

10 Negative probabilities:

Entangled states can be interpreted as separable with negative probabilities, e.g. the Bell state

$$\frac{1}{4} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \underbrace{\rho_x + \rho_y}_{\text{too much correlations}} - \rho_z$$

where

$$\rho_x = \frac{1}{2} (P_x \otimes P_x + P_{-x} \otimes P_{-x})$$

and

$$P_n = \frac{1 + n \cdot \sigma}{2}$$

Exercise: What would you get if you permute the minus?