Quasi-holes for non-abelian quantum Hall states

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thanks to …

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outline

1. **quasi-hole heuristics**
2. non-abelian qH states
3. quasi-hole specifics
4. qH-CFT connection
5. quasi-hole counting
6. quasi-hole wavefunctions
7. quasi-hole braiding

quasi-hole heuristics

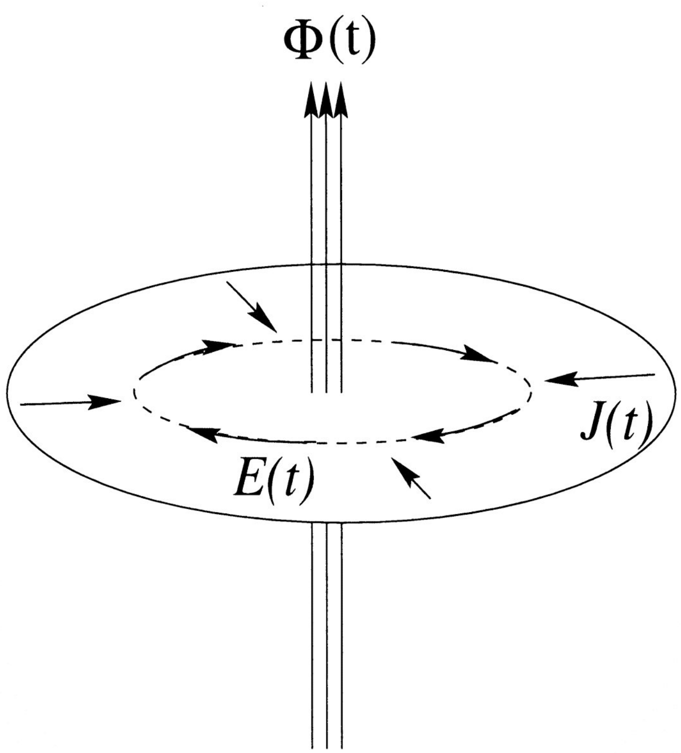
• following Laughlin (1983):
fundamental excitation over $\nu=1/m$ fqH state associated to insertion of a flux quantum Φ_0

• the resulting quasi-hole carries an electric charge equal to

$$q = \sigma_{\mathrm{Hall}}\, \Phi_0 = \nu\, e = e/m$$

• the excitations are anyonic: braiding is represented by a phase factor

$$\exp(i\pi/m)$$



quasi-hole heuristics

- in multi-layer and spin-1/2 state, flux insertion `per layer' or `per spin'
- example --

Halperin *(m+1,m+1,m)* spin-singlet state, filling *ν=2/(2m+1)*,

$$\tilde{\Psi}_{\mathrm{H}}(z_1^{\uparrow},..,z_N^{\uparrow},z_1^{\downarrow},..,z_N^{\downarrow}) = \prod_{i<j}(z_i^{\uparrow}-z_j^{\uparrow})^{m+1}\prod_{i<j}(z_i^{\downarrow}-z_j^{\downarrow})^{m+1}\prod_{i,j}(z_i^{\uparrow}-z_j^{\downarrow})^{m}$$

has quasi-holes with quantum numbers

q = e/(2m+1), S=1/2

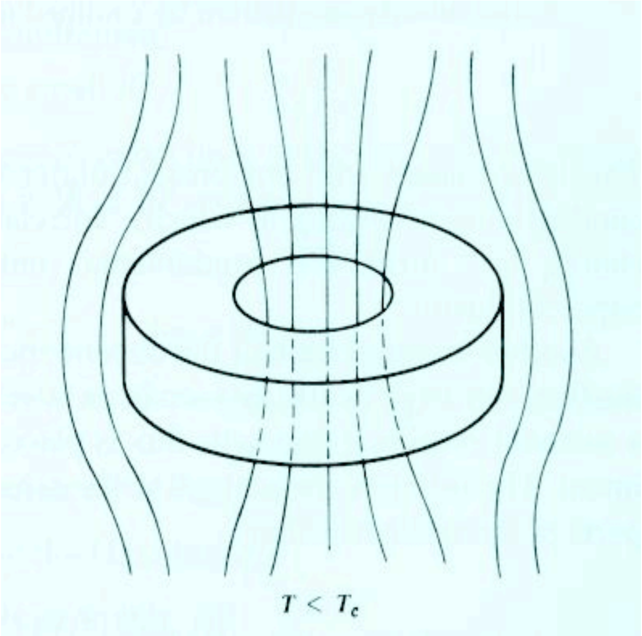
quasi-hole heuristics

• non-abelian quantum Hall states generally characterised by pairing (or order-*k* clustering)

• as in a superconductor, pairing (clustering) of particles leads to a reduction of the minimal allowed flux-insertion, which is now Φ_0/k

• this leads to further fractionalization of quasi-holes

• the resulting quasi-holes are the ones exhibiting non-abelian braid statistics



quasi-hole heuristics

for paired qH states with pfaffian factor the quasi-particle break-up can take different forms:

- $\nu=1/2$ Moore-Read state:

  q=e/2 splits into twice *q=e/4*

  [charge fractionalisation]

- $\nu=2/3$ spin-singlet state [Ardonne et al, 2002]

  [q=e/3, S=1/2] splits into *[q=e/3, S=0], [q=0,S=1/2]*

  [spin-charge separation]

- pfaffian $\nu=1$ state for rotating spin-1 bosons [Reijnders et al, 2002]

  [q=1, S=1] splits into twice *[q=1/2, S=1/2]*

  [spin fractionalisation]

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Read-Rezayi *k*-clustered states

Read-Rezayi, 1999

- qH states for spin-polarized electrons
- filling factor $\nu=k/(kM+2)$
- expected in 2nd LL [$\nu=5/2, 12/5$ (?), ..]
- expected in rapidly rotating BEC [$\nu=1/2,\ 1,\ 3/2$, ..]
- *k*-clustering: *M=0* (bosonic) state obtained as maximal density zero energy eigenstate of hamiltonian

$$H = V \sum_{i_1 < .. < i_{k+1}} \delta^2(z_{i_1} - z_{i_2})\delta^2(z_{i_2} - z_{i_3}) \ldots \delta^2(z_{i_k} - z_{i_{k+1}})$$

- quasi-holes:
  - $q = 1/(kM+2)$
  - **counting, wavefunctions, braiding**

double layer states

Many possibilities for non-abelian states in multi-layer samples.

Example --

double layer pfaffian state

$$\tilde{\Psi}_{\rm pf}(z_1^{\uparrow},..,z_N^{\uparrow},z_1^{\downarrow},..,z_N^{\downarrow}) =$$

$$\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j}(z_i^{\uparrow} - z_j^{\uparrow})^m \prod_{i<j}(z_i^{\downarrow} - z_j^{\downarrow})^m \prod_{i<j}(z_i^{\uparrow} - z_j^{\downarrow})^n$$

Excitations include non-abelian excitons.

For *m=2, n=1* this is a *ν=2/3* spin-singlet state exhibiting spin-charge separation, with non-abelian spinon (spin ½ and zero charge) and holon (charge e/3 and zero spin) excitations.

Ardonne-v. Lankvelt-Ludwig-S, 2002

spin-1/2 spin-singlet states

Ardonne-S 1999

- spin-singlet qH states for spin-1/2 electrons
- filling factor $\nu=2k/(2kM+3)$
- *k*-clustering: *M=0* (bosonic) state obtained as maximal density zero energy eigenstate of hamiltonian

$$H = V \sum_{i_1 < .. < i_{k+1}} \delta^2(z_{i_1} - z_{i_2})\delta^2(z_{i_2} - z_{i_3}) \dots \delta^2(z_{i_k} - z_{i_{k+1}})$$

- quasi-holes:
  - *[q= 1/(2kM+3), S=1/2]* or *[q= 2/(2kM+3), S=0]*
  - **counting, wavefunctions, braiding**

spin-1/2 spin-singlet states: *k=2*

- paired states; filling factor *ν=4/3 [M=0], ν=4/7 [M=1], etc.*
- explicit *[M=0, $N_\uparrow$=2, $N_\downarrow$=2]*

$$\tilde{\Psi}_{\mathrm{AS}}(z_1^\uparrow, z_2^\uparrow, z_1^\downarrow, z_2^\downarrow) = (z_1^\uparrow - z_1^\downarrow)(z_2^\uparrow - z_2^\downarrow) + (z_1^\uparrow - z_2^\downarrow)(z_2^\uparrow - z_1^\downarrow)$$

- the pairing is governed by same hamiltonian as for Moore-Read state; the AS state smoothly connects to the MR state upon varying $N_\uparrow$, $N_\downarrow$ from spin-singlet to fully polarized with (for *M=0*)

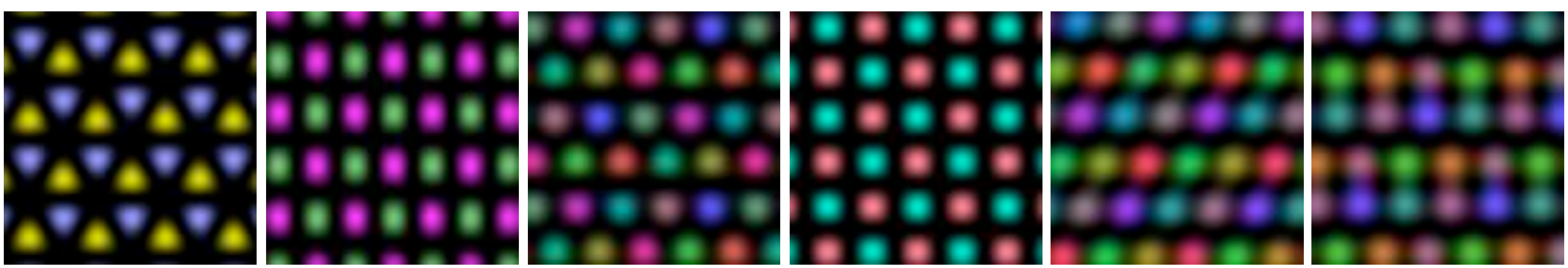
$$\nu = \frac{N_\uparrow^2 + 2N_\uparrow N_\downarrow + N_\downarrow^2}{N_\uparrow^2 + N_\uparrow N_\downarrow + N_\downarrow^2}$$

- the quasi-hole braiding is in the universality class of Fibonacci anyons; hence the prospect of universal QC with a paired qH state!
- experimental access to spin-singlet states: hydrostatic pressure and tilted field [Kang et al. 1997, Cho et al. 1998]

qH states for rotating (spin-1) bosons

Reijnders-v. Lankvelt-Read-S, 2002

- if qH states for rotating bosons can be achieved, then spin-1 bosons are an interesting and viable option
- two independent channels *(S=0,2)* in contact interactions give non-trivial parameter $g=g_2/g_0$
- spin interactions can be ferromagnetic [$g<0$, ^{85}Rb] or antiferromagnetic [$g>0$, ^{23}Na]
- qH states compete with various lattices of spin-textures (below)
- qH states include spin-1 version of RR series [$SU(4)_k$] and a series [$SO(5)_k$] exhibiting spin fractionalisation



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quasi-holes: the issues

Quasi-holes generated by inserting excess flux in qH ground states come with an `internal register' associated to fusion channel assignments. Issues

• counting the dimensionality of `internal register' associated with given number of quasi-holes

• explicit wavefunctions, setting the amplitudes of various fusion channel states as a function of the positions of the quasi-holes

• braiding properties: enabling operations on the internal register by weaving braids of quasi-hole positions

quasi-holes: the strategies

- **direct analysis:** (numerically) determining zero-energy eigenstates for excess flux, and braiding e.g. with a Berry phase approach [Read-Rezayi, Tserkovnyak-Simon, …]

- **`coordinate CFT':** concrete evaluation of quasi-hole wavefunctions, leading to braiding properties and beyond [Nayak-Wilczek, Ardonne-S]

- **`algebraic CFT':** relying on quantum group structure associated to RCFT, for example to match fusion and braiding properties [Slingerland-Bais,…]

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qH wavefunctions from CFT

ground state wave function

$$\Psi_{\rm GS}(z_1,\ldots,z_N) \cong \left\langle \psi_{\rm e}(z_1)\ldots\psi_{\rm e}(z_N)\psi_{\rm background}(z_\infty)\right\rangle_{\rm CFT}$$

electron (boson)
condensate operator

neutralizing
background charge

quasi-hole excitations : fixed by

$$\phi_{\rm qh}(w)\psi_{\rm e}(z_1) = (z-w)^{\rm integer}\left[\phi_2(w) + \ldots\right]$$

excited state wave function:

$$\Psi_{\rm qh}(w_1,w_2,\ldots;z_1,z_2,\ldots) \cong \left\langle \phi_{\rm qh}(w_1)\phi_{\rm qh}(w_2)\ldots\psi_{\rm e}(z_1)\psi_{\rm e}(z_2)\ldots\right\rangle_{\rm CFT}$$

parafermions and vertex operators

Generic form of electron and quasi-hole operators

$$\psi_{\mathrm{e}} = \psi^{\mathrm{PF}} \times \mathrm{V.O.(spin)} \times \mathrm{V.O.(charge)}$$

$$\phi_{\mathrm{qh}} = \sigma^{\mathrm{PF}} \times \mathrm{V.O.(spin)} \times \mathrm{V.O.(charge)}$$

Parafermionic fields taken from CFT of the form

$$\mathrm{PF}(G,k) \cong \hat{G}_k / [U(1)^{\mathrm{rank}(G)}]$$

Gepner, 1987

Vertex Operators for spin and charge have the form

$$\mathrm{V.O.(charge)} \cong \exp(i\alpha_{\mathrm{c}}\varphi_{\mathrm{c}}) \;,\; \mathrm{V.O.(spin)} \cong \exp(i\alpha_{\mathrm{s}}\varphi_{\mathrm{s}})$$

CFT: deformations of (chiral) WZW

The parafermion theory *PF(G,k),* together with the vertex operators for spin and charge (depending on the Laughlin exponent *M*), build up a deformation of the chiral WZW model *WZW(G,k)*

$$\text{CFT} \quad \text{data}: \quad G_{k,M}$$

$G_k=SU(2)_k$ for the RR series and $G_k=SU(3)_k$ for the non-abelian spin-singlet states.

The properties of the $SU(2)_k$ and $SU(3)_k$ parafermions guarantee that the PF correlators give wave functions that satisfy the order-k clustering property.

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quasi-hole counting

Agreement between the `direct' and `parafermion' approaches for the counting problem in spherical geometry

- **direct approach:**
  numerical evaluation of zero energy eigenstates of clustering hamiltonian in presence of excess flux, meaning a fixed number of quasi-holes
- **parafermion approach:**
  analytic counting formulas based on parafermion combinatorics and on a physical picture relating parafermions to broken clusters in the condensate

[Read-Rezayi, Rezayi-Gurarie, Ardonne-Read-Rezayi-S, Ardonne]

direct approach

k=2 **spin-singlet state:**

N=8 **electrons**

n=8 **quasi-holes**

(total # states:
$\#_{N=8,n=8} = 1719$)

	S=0	S=1	S=2	S=3	S=4
L=0	4	1	3	0	1
L=1	1	7	2	1	0
L=2	7	7	6	1	0
L=3	3	9	3	1	0
L=4	6	6	4	0	0
L=5	2	5	1	0	0
L=6	3	2	1	0	0
L=7	0	1	0	0	0
L=8	1	0	0	0	0

parafermion approach

Analytical expression for general $N_\uparrow$, $N_\downarrow$, $n_\uparrow$, $n_\downarrow$:

$$\#_{N,n} = \sum_{\substack{F_1, F_2 \\ N_\uparrow + N_\downarrow = N \\ n_\uparrow + n_\downarrow = n}} \begin{Bmatrix} n_\uparrow & n_\downarrow \\ F_1 & F_2 \end{Bmatrix}^{(k)} \binom{(N_\uparrow - F_1)/k + n_\uparrow}{n_\uparrow} \binom{(N_\downarrow - F_2)/k + n_\downarrow}{n_\downarrow}$$

[with explicit expressions for the symbols $\{\ \}^{(k)}$ …]

Ardonne-Read-Rezayi-S 2000, Ardonne 2001

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quasi-hole wavefunctions

Ardonne-S, in preparation

We determined explicit wavefunctions for the excitations with 4 quasi-holes (of various kinds) over the *k=3* Read-Rezayi state and the *k=2* non-abelian spin-singlet state, generalizing results of Nayak and Wilczek, 1996, for the MR state. From these, braiding properties follow by inspection.

Here we present the case of 4 spin-less quasi-holes ϕ_3 , of charge 2/3, over the paired spin-singlet state. For $M>0$ these are the lowest dimension quasi-holes, and thereby the most natural agents in a QC protocol.

quasi-hole wavefunctions

The qH-CFT correspondence gives

$$\Psi_{3333}(w_1, w_2, \ldots; z_1^{\uparrow}, z_2^{\uparrow}, \ldots, z_{1'}^{\downarrow}, z_{2'}^{\downarrow}, \ldots) =$$

$$\left\langle \sigma_3(w_1)\sigma_3(w_2)\ldots\psi_1(z_1^{\uparrow})\psi_1(z_2^{\uparrow})\ldots\psi_2(z_{1'}^{\downarrow})\psi_2(z_{2'}^{\downarrow})\ldots \right\rangle_{\mathrm{CFT}}$$

$$\times\left[\Psi^{221}\left(\left\{z_i^{\uparrow}, z_{j'}^{\downarrow}\right\}\right)\right]^{1/2}\prod_{i,j}\left(z_i^{\uparrow} - w_j\right)^{1/2}\prod_{i,j}\left(z_{i'}^{\downarrow} - w_j\right)^{1/2}\prod_{i<j}\left(w_i - w_j\right)^{1/3}$$

quasi-hole wavefunctions

Going into the correlator

$$\left\langle \sigma_3(w_1)\sigma_3(w_2)\ldots\psi_1(z_1^{\uparrow})\psi_1(z_2^{\uparrow})\ldots\psi_2(z_{1'}^{\downarrow})\psi_2(z_{2'}^{\downarrow})\ldots \right\rangle_{\mathrm{CFT}}$$

SU(3)₂ parafermion algebra

$$\psi_1(z)\psi_1(w) = (z-w)^{-1} I + \ldots,$$

$$\psi_2(z)\psi_2(w) = (z-w)^{-1} I + \ldots$$

$$\psi_1(z)\psi_2(w) = (z-w)^{-1/2}\psi_{12} + \ldots$$

and the spin-field OPE, with two independent fusion channels

$$\sigma_3(z)\sigma_3(w) = (z-w)^{-1/5} I + (z-w)^{2/5}\rho_3(w) + \ldots$$

quasi-hole wavefunctions

For the MR state, Nayak and Wilczek applied bosonization to obtain the analogous correlator, giving a final expression of the type

$$\Psi^{(0,1)}(w_1,w_2,w_3,w_4;z_1,z_2,\ldots) =$$

$$A^{(0,1)}(\{w_i\})\Psi_{[12,34]}(\{w_i;z_j\}) + B^{(0,1)}(\{w_i\})\Psi_{[13,24]}(\{w_i;z_j\})$$

pre-factors depending on fusion channel *(0,1)* and on quasi-hole locations w_i

basis for two-fold degenerate internal register; polynomial in w_i , z_j

quasi-hole wavefunctions

To obtain a similar expression we proceed in a number of steps

Step1. In absence of quasi-holes, we have the following expression for wavefunction [Cappelli et al, Ardonne et al]

$$\Psi_{\mathrm{GS}} = \frac{1}{N} \sum_{\{S_1, S_2\}} \Psi^{221}_{S_1}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right) \Psi^{221}_{S_2}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

with particles in disjoint subsets S_1, S_2 each forming a Halperin 221 state

$$\tilde{\Psi}^{221}(z_1^{\uparrow}, .., z_N^{\uparrow}, z_{1'}^{\downarrow}, .., z_{N'}^{\downarrow}) = \prod_{i<j}\left(z_i^{\uparrow} - z_j^{\uparrow}\right)^2 \prod_{i<j}\left(z_{i'}^{\downarrow} - z_{j'}^{\downarrow}\right)^2 \prod_{i,j'}\left(z_i^{\uparrow} - z_{j'}^{\downarrow}\right)$$

quasi-hole wavefunctions

Step 2. Basis for 4 quasi-hole state obtained by distributing the quasiholes over de sets S_1, S_2 ; two independent choices for this are $\Psi_{12,34}$ and $\Psi_{13,24}$ with

$$\Psi_{12,34} = \frac{1}{N} \sum_{\{S_1, S_2\}} \left[\prod_{i,j' \in S_1} \left(z_i^{\uparrow} - w_1\right)\left(z_i^{\uparrow} - w_2\right)\left(z_{j'}^{\downarrow} - w_1\right)\left(z_{j'}^{\downarrow} - w_2\right) \right] \Psi_{S_1}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

$$\times \left[\prod_{i,j' \in S_2} \left(z_i^{\uparrow} - w_3\right)\left(z_i^{\uparrow} - w_4\right)\left(z_{j'}^{\downarrow} - w_3\right)\left(z_{j'}^{\downarrow} - w_4\right) \right] \Psi_{S_2}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

$$\Psi_{13,24} = \ldots$$

quasi-hole wavefunctions

Step 3. Decompose wavefunction over $\Psi_{12,34}$ and $\Psi_{13,24}$ and impose consistency upon fusing some of the parafermions $\psi_{1,2}$ with the σ_3 .

This requires detailed knowledge of short distance Operator Products Expansions (OPE), and of 4-point functions in the $SU(3)_2$ WZW model [Knizhnik-Zamolodchikov, 1984]

Building blocks are hypergeometric functions

$$F_1^{(0)} = x^{-8/15}(1-x)^{1/15} F(\frac{1}{5},\frac{-1}{5},\frac{2}{5},x)$$

$$F_2^{(0)} = \frac{1}{2} x^{7/15}(1-x)^{1/15} F(\frac{6}{5},\frac{4}{5},\frac{7}{5},x)$$

$$F_1^{(1)} = x^{1/15}(1-x)^{1/15} F(\frac{2}{5},\frac{4}{5},\frac{8}{5},x)$$

$$F_2^{(1)} = -3x^{1/15}(1-x)^{1/15} F(\frac{2}{5},\frac{4}{5},\frac{3}{5},x)$$

$$x = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_4)(w_3 - w_2)}$$

quasi-hole wavefunctions

Final result.

$$\Psi^{(0,1)}_{3333}(w_1,w_2,w_3,w_4;z^{\uparrow}_1,z^{\uparrow}_2,\ldots,z^{\downarrow}_{1'},z^{\downarrow}_{2'},\ldots) =$$

$$A^{(0,1)}_{3333}(\{w_i\})\Psi_{[12,34]}(\{w_i;z_i,z_{j'}\}) + B^{(0,1)}_{3333}(\{w_i\})\Psi_{[13,24]}(\{w_i;z_i,z_{j'}\})$$

$$A^{(0)}_{3333} = \left[w_{12}w_{34}\right]^{4/5} x^{-2/15}(1-x)^{2/3} F^{(0)}_2(x)$$

$$B^{(0)}_{3333} = \left[w_{12}w_{34}\right]^{4/5} x^{-2/15}(1-x)^{2/3} F^{(0)}_1(x)$$

$$A^{(1)}_{3333} = \left[w_{12}w_{34}\right]^{4/5}(-1)^{8/5} C\, x^{-2/15}(1-x)^{2/3} F^{(1)}_2(x)$$

$$B^{(1)}_{3333} = \left[w_{12}w_{34}\right]^{4/5}(-1)^{8/5} C\, x^{-2/15}(1-x)^{2/3} F^{(1)}_1(x)$$

$$C^2 = \frac{1}{9}\frac{\Gamma^3\left(\frac{2}{5}\right)\Gamma\left(\frac{4}{5}\right)}{\Gamma^3\left(\frac{3}{5}\right)\Gamma\left(\frac{1}{5}\right)}$$

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quasi-hole braiding

The full parafermion theory has eight sectors

$$\{I, \psi_1, \psi_2, \psi_{12}, \sigma_1,\ \sigma_2,\ \sigma_3, \rho\}$$

with Fibonacci type fusion if we set

$$0 \equiv \{I, \psi_1, \psi_2, \psi_{12}\} \qquad 1 \equiv \{\sigma_1,\ \sigma_2,\ \sigma_3\,, \rho\}$$

One thus expects Fibonacci braiding properties, suitable for universal QC.

quasi-hole braiding

Evaluation of braiding by direct inspection of:

$$\Psi^{(0,1)}_{3333}(w_1,w_2,w_3,w_4;z_1^{\uparrow},z_2^{\uparrow},\ldots,z_{1'}^{\downarrow},z_{2'}^{\downarrow},\ldots)$$
$$= A^{(0,1)}_{3333}(\{w_i\})\Psi_{[12,34]}(\{w_i;z_i,z_{j'}\})$$
$$+ B^{(0,1)}_{3333}(\{w_i\})\Psi_{[13,24]}(\{w_i;z_i,z_{j'}\})$$

$$A^{(0)}_{3333} = [w_{12}w_{34}]^{4/5}x^{-2/15}(1-x)^{2/3}F_2^{(0)}(x)$$
$$B^{(0)}_{3333} = [w_{12}w_{34}]^{4/5}x^{-2/15}(1-x)^{2/3}F_1^{(0)}(x)$$
$$A^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5}C\,x^{-2/15}(1-x)^{2/3}F_2^{(1)}(x)$$
$$B^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5}C\,x^{-2/15}(1-x)^{2/3}F_1^{(1)}(x)$$

Example: $w_2 \leftrightarrow w_3$.

this swaps $\Psi_{12,34}$ and $\Psi_{13,24}$; furthermore

$$F_2^{(0)}(1-x) = C_0^0 F_1^{(0)}(x) + C_1^0 F_1^{(1)}(x), \quad \text{etc.}$$
$$C_0^0 = \frac{1}{2}\left(\sqrt{5}-1\right) = \tau\,, \quad C_1^0/C = -\sqrt{\tau}$$

$$\Rightarrow \quad U_{2\leftrightarrow 3} = (-1)^{4/5}\begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

quasi-hole braiding

Full set of braiding relations on the 4 quasi-hole wavefunctions at *M=0*

$$U_{1\leftrightarrow 2} = (-1)^{-2/3} \begin{pmatrix} (-1)^{4/5} & 0 \\ 0 & (-1)^{-3/5} \end{pmatrix}$$

$$U_{2\leftrightarrow 3} = (-1)^{4/5} \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$$U_{1\leftrightarrow 3} = (-1)^{8/15} \begin{pmatrix} \tau & (-1)^{-3/5}\sqrt{\tau} \\ (-1)^{-3/5}\sqrt{\tau} & (-1)^{-1/5}\tau \end{pmatrix}$$

Similar relations are obtained for spin-full quasi-holes over the *k=2* spin-singlet state and for quasi-holes over the *k=3* RR state.

Ardonne-S, in preparation

summary

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