Can conformal invariance survive KPZ dynamics?

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LPTMS, Université Paris-Saclay, France
joint work with
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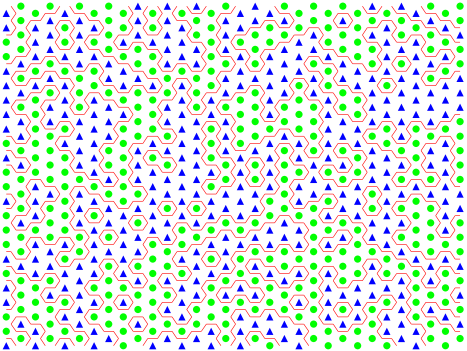
(2d) Conformal invariance

Criticality/scaling $\Rightarrow^{\dagger}$ conformal invariance.



Ex. critical percolation:

- Triangular lattice;
- spin on each site independent

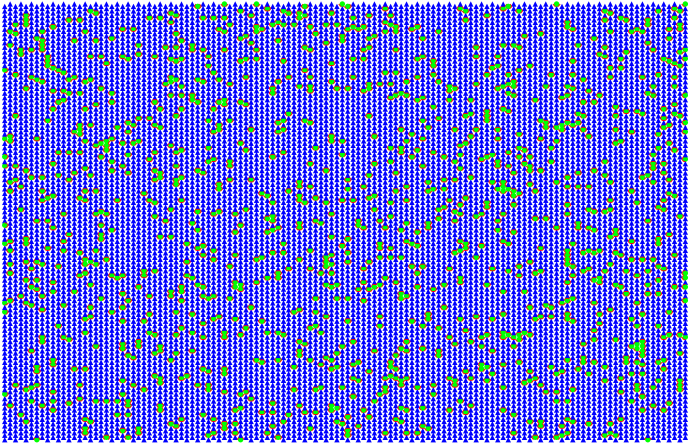
$$\mathbb{P}(\blacktriangle) = p, \ \mathbb{P}(\bullet) = 1 - p$$

- $p \simeq p_c = \frac{1}{2} \Leftrightarrow$ non-trivial frontier loops (& clusters*).

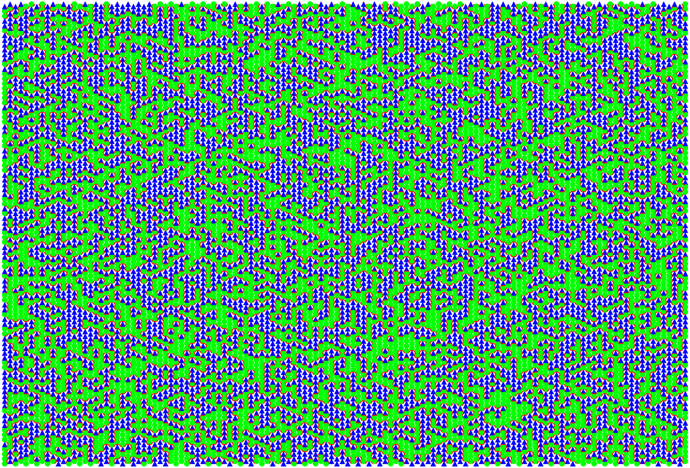
† Under conditions, *e.g.* locality.

* connected components of the surface \ loops.

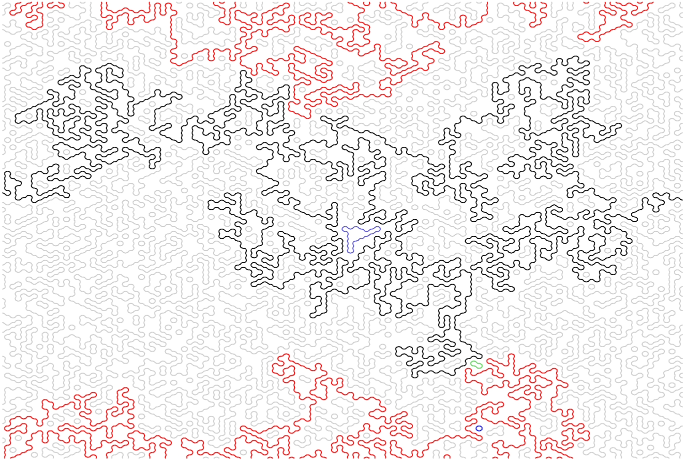
$p \neq p_c$



$p = p_c$



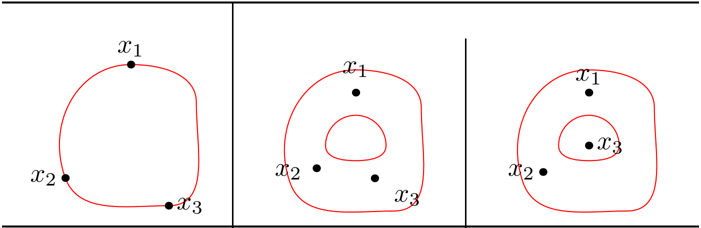
Loop ensemble



Loop/Cluster n–point functions

$$P_l(x_1, \ldots, x_n) = \mathcal{P}[x_1, \ldots, x_n \text{ lie on one loop}].$$

$$P_c(x_1, \ldots, x_n) = \mathcal{P}[x_1, \ldots, x_n \text{ lie in same cluster}].$$



$P_l(x_1, x_2, x_3) = 1$	$P_c(x_1, x_2, x_3) = 1$	$P_c(x_1, x_2, x_3) = 0$

Conformal invariance & three-point constant

Scaling implies

$P_o(x,y) = A|x-y|^{-2\chi_o}$, $o = c, l$.

For critical percolation,

$$\chi_c = \frac{5}{48},\ \chi_l = \frac{1}{4}.$$

[Nienhuis '82]

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[Nienhuis '82]

Conformal invariance implies

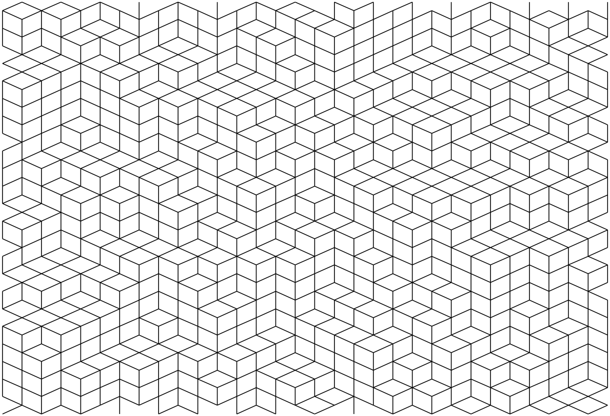
$$C_o := \frac{P_o(x,y,z)}{\sqrt{P_o(x,y)P_o(x,y)P_o(x,y)}}$$

is non-trivial *constant*.
For critical percolation

$$C_l \simeq 0.95\ldots \text{ [num.]},$$
$$C_c = 0.722\ldots.$$

[Delfino-Viti '11, Ribault-Santachiara '15]

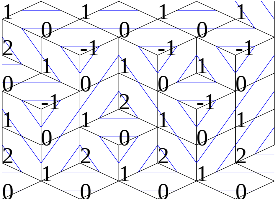
Lozenge tilings



models rough surfaces, has a natural height function

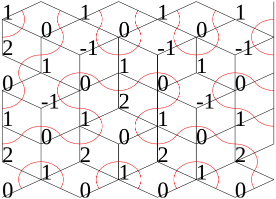
Two loop ensembles on lozenge tilings

Level-lines



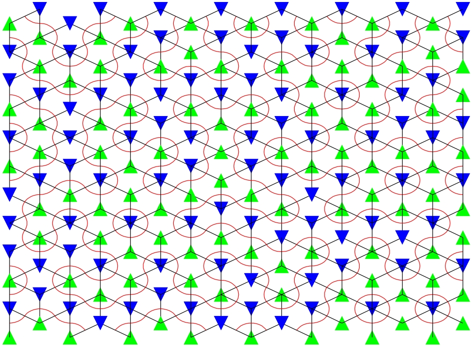
Local construction : 

"Fully-packed loops" (FPL)



Local construction : 

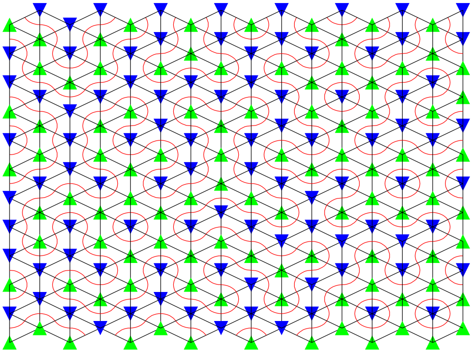
Why FPL? relation with percolation



{lozenge tilings } ⇔ {maximally frustrated spin conf's}

FPL ⇔ percolation frontier loops

Why FPL? relation with percolation



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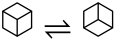
Two dynamics on lozenge tiling

Edwards-Wilkinson



Steady state: equal prob. for any configuration.
Loop ensembles understood.
[Kondev, Jacobsen, Dotsenko *et al*]

Two dynamics on lozenge tiling

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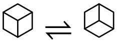

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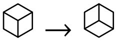
Kardar-Parisi-Zhang ((2 + 1)-d)



Steady state: non-trivial.
[Olejarz *et al* '11, Halpin-Healy '13]

How are the loop ensembles affected ?

Numerical results

	FPL 	level lines 
 $\rightleftharpoons$  EW	$\cong$ Percolation same χ, C	$\cong$ GFF level-lines $\chi_l = \frac{1}{2} \ldots (SLE_4)$

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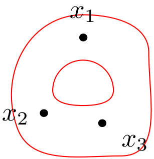
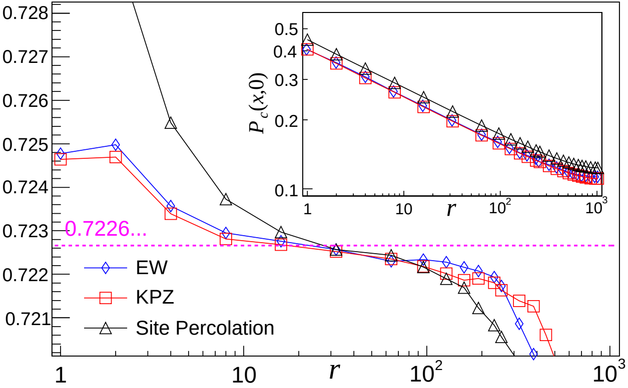
 $\rightleftharpoons$  EW	$\cong$ Percolation same χ, C	$\cong$ GFF level-lines $\chi_l = \frac{1}{2} \ldots (SLE_4)$
 $\rightarrow$  KPZ	$\cong$ Percolation same χ, C	broken conformal inv. $\chi_l \doteq \frac{1}{3}, \nexists C_c, C_l$

FPL loop 3-point constant





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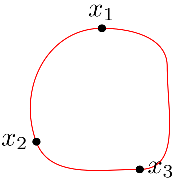
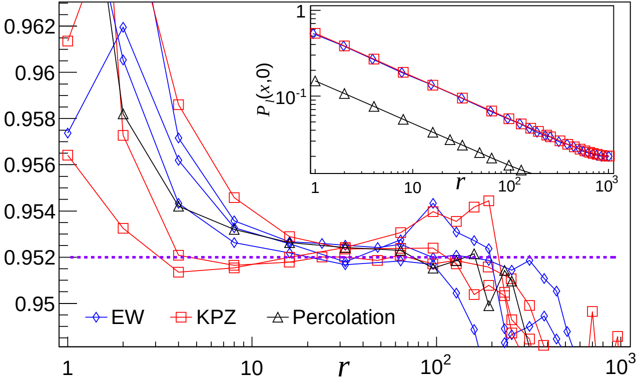
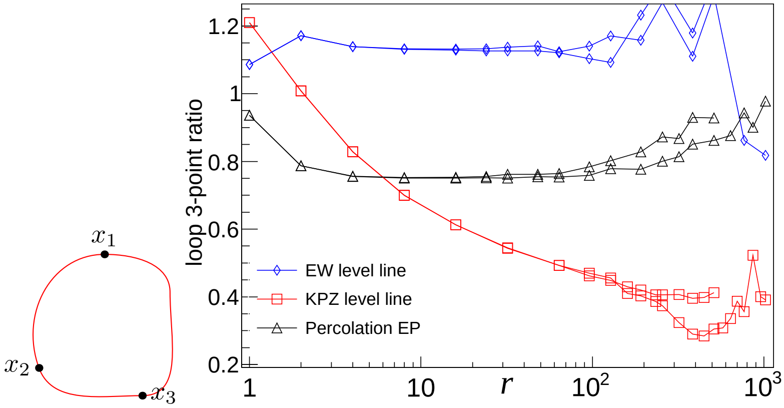




level lines ⟺ loop 3-pt constant



Saberi *et al* conjectured KPZ level lines are the same as percolation external perimeter (EP), having close $\chi_l = \frac{1}{3}$.

Brief explanation

	FPL	level lines
$\rightleftharpoons$ EW	$\cong$ Percolation same χ, C	$\cong$ GFF level-lines $\chi_l = \frac{1}{2} \ldots (SLE_4)$
$\rightarrow$ KPZ	$\cong$ Percolation same χ, C	broken conformal inv. $\chi_l \doteqdot \frac{1}{3}, \nexists C_c, C_l$

- Field theory of EW steady state *decouples* into that of and . [Kondev, Jacobsen, Dotsenko *et al*]
- KPZ affects but leaves intact.

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 $\rightleftharpoons$  EW	$\cong$ Percolation same χ, C	$\cong$ GFF level-lines $\chi_l = \frac{1}{2} \ldots (SLE_4)$
 $\rightarrow$  KPZ	$\cong$ Percolation same χ, C	broken conformal inv. $\chi_l \doteq \frac{1}{3}$, $\nexists C_c, C_l$

- No.
- But one can be fooled into believing so . . .