Time correlation properties of KPZ fluctuations: from experimental perspectives

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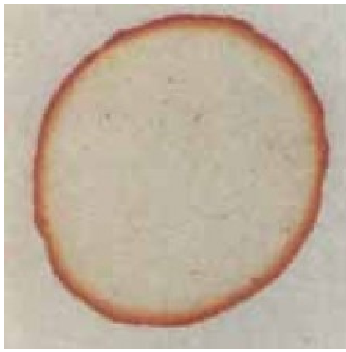
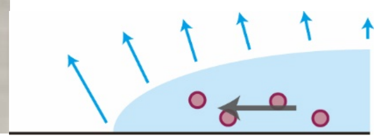
Acknowledgment

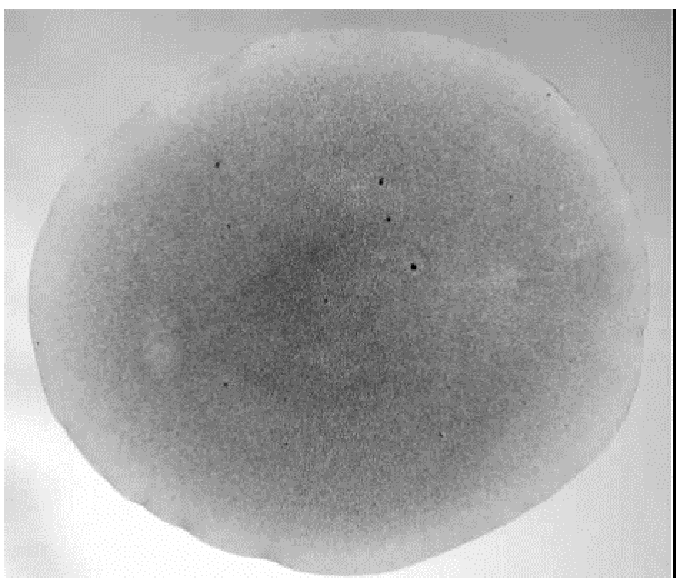
- Masaki Sano (Univ. Tokyo) – experiment
- Takuma Akimoto (Keio Univ.) – sign renewals analysis
- Patrik L. Ferrari & Herbert Spohn – two-time correlation

Growing Interfaces

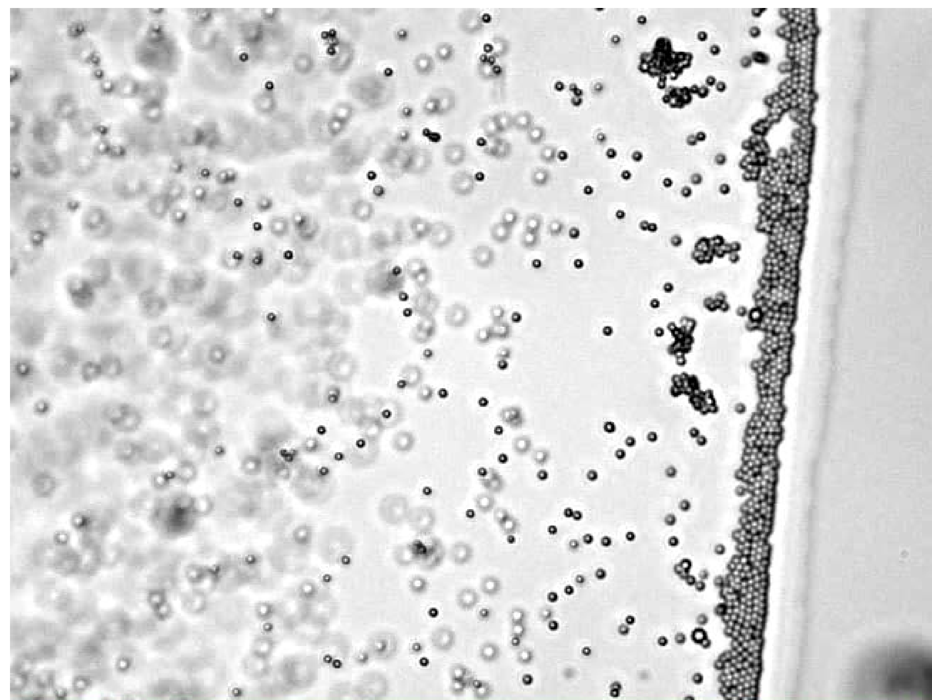
Example 1: particle deposition (a familiar instance: the coffee-ring effect)







Experiment using polystyrene beads

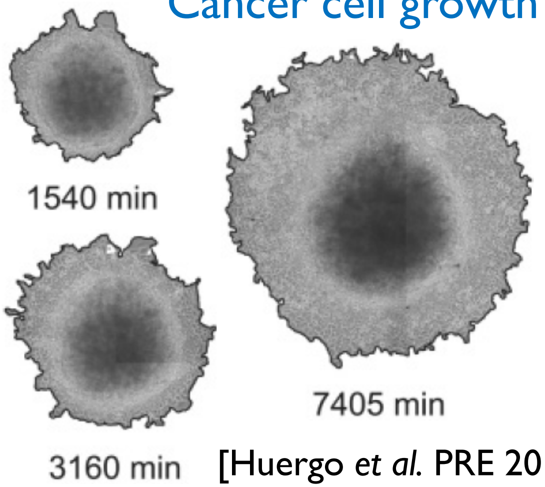


[Yunker *et al.* Nature 2011, PRL 2013]

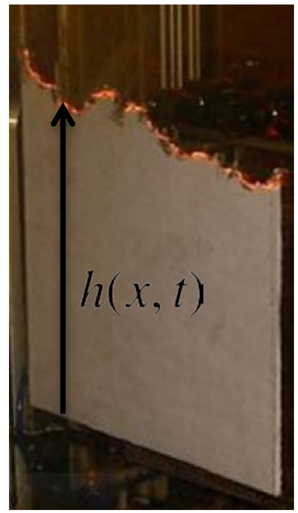
Growing Interfaces

Other examples: (random growth processes with short-range interactions)

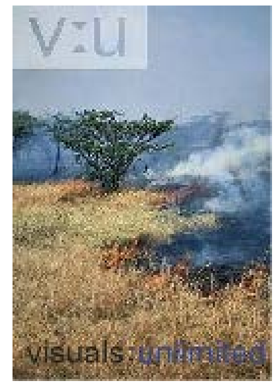
Cancer cell growth



[Huergo *et al.* PRE 2012]



Paper combustion [Maunuksela *et al.* 1997-]
(cf. forest fire)



Random growth processes produce rough interfaces

scale invariant
⇨ universal scaling laws

Kardar-Parisi-Zhang (KPZ) universality class

- KPZ equation: $\frac{\partial}{\partial t}h(x,t) = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \sqrt{D}(\text{noise})$ ⇨ $\delta h \sim t^{\beta}$ or L^{α}, $\alpha = \frac{1}{2}, \beta = \frac{1}{3}$ in 1d
- In 1d, many properties are solved exactly, despite being out of equilibrium. Deep connection to random matrix theory / combinatorics / integrable systems.
- We study experimentally, using liquid-crystal turbulence.

KPZ vs non-KPZ in Real World

Theoretically, KPZ is expected under very generic situations
with short-range interactions, no conservation, no quench disorder…

However, in reality…

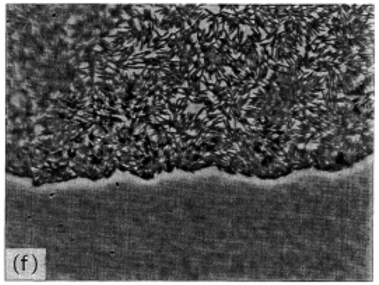
paper combustion

Maunuksela *et al.* 1997-
⇨ KPZ

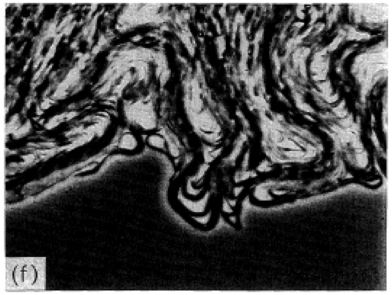
Zhang *et al.* 1992
⇨ not KPZ
$(\alpha \approx 0.70)$

cell colony growth

Wakita *et al.* 1997



mutant bacteria
⇨ KPZ

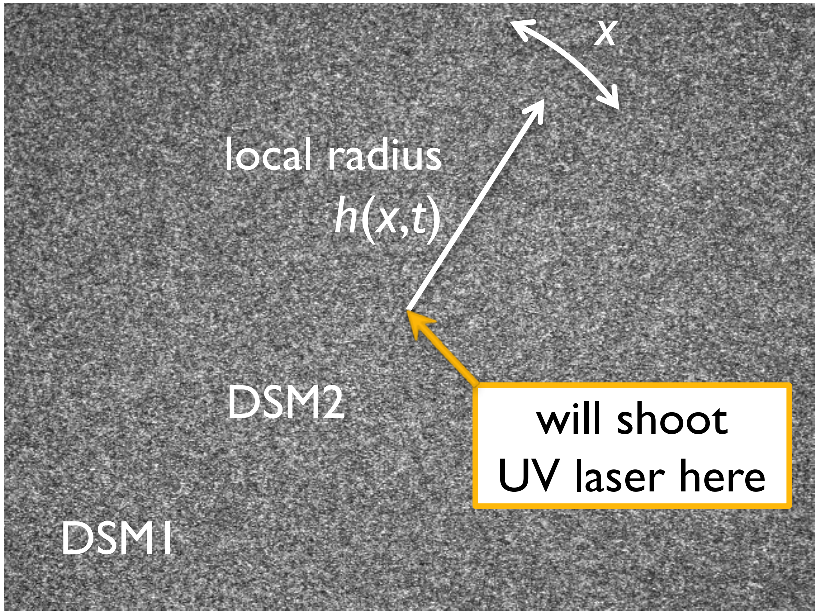


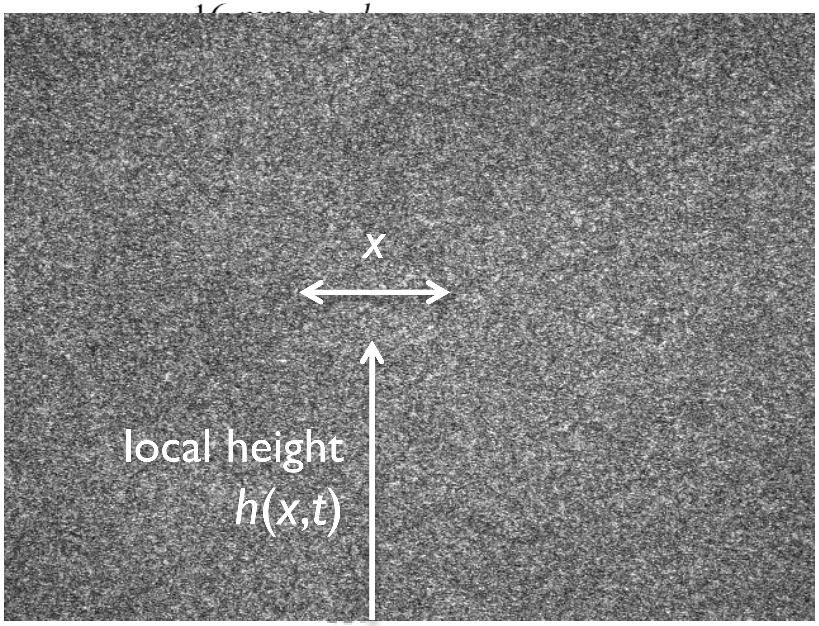
wild-type bacteria
⇨ not KPZ
$(\alpha \approx 0.78)$

There are a great many other examples of non-KPZ interfaces,
but only few experiments showed KPZ (6 examples for 1d KPZ).

Growing Interfaces in Liquid-Crystal Turbulence

- Apply an ac electric field to nematic liquid crystal (here MBBA)
- Convection driven by anisotropic electric properties (Carr-Helfrich instability)
- DSM1 = defect-less turbulence : metastable
  DSM2 = defect-filled turbulence : stable
  at large enough V





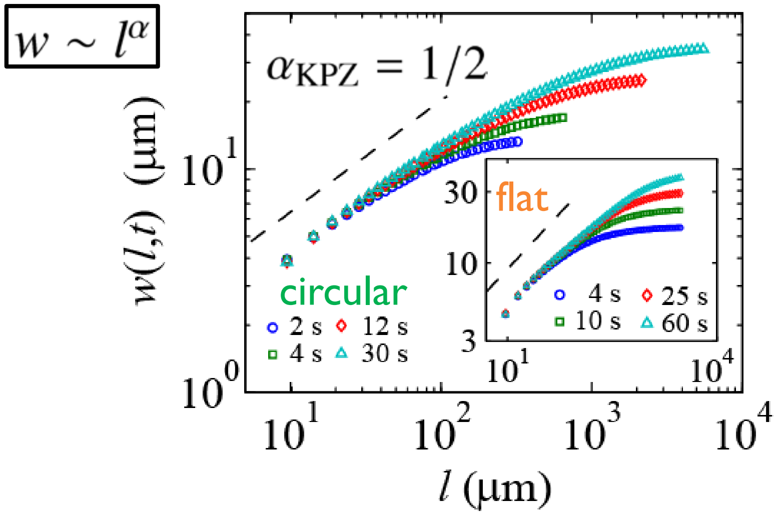
26V, 250Hz Speed x2, — 200μm

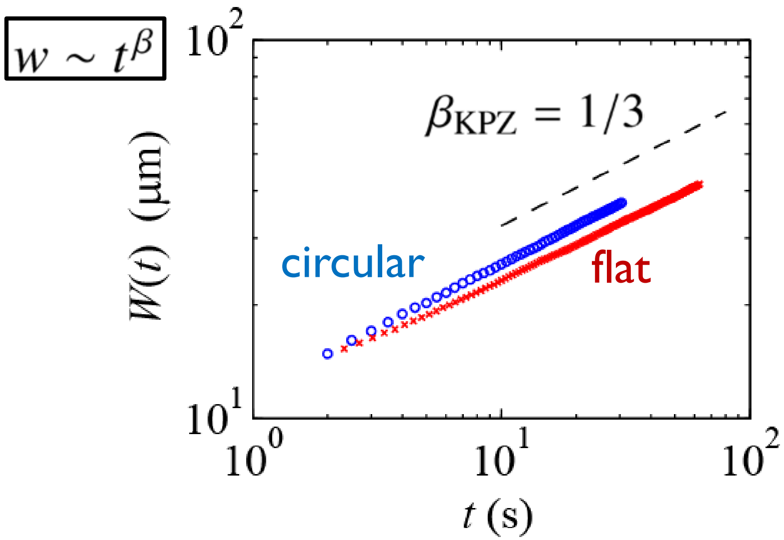
Speed x5

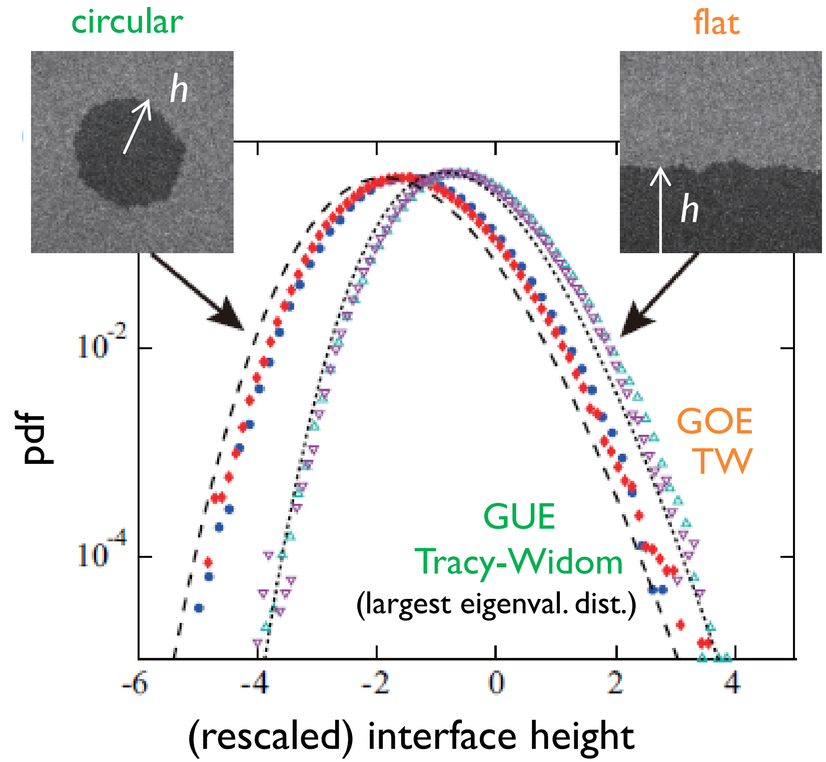
Basic Results

[KaT & Sano, PRL 104, 230601 (2010); J. Stat. Phys. 147, 853 (2012)]

Scaling exponents







circular : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\rm GUE}$

flat : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\rm GOE}$

- Both cases show the KPZ exponents.
- Yet, circular and flat interfaces show different distributions, both developed in random matrix theory.

Spatial Correlation

[KaT & Sano, J. Stat. Phys. 147, 853 (2012)]

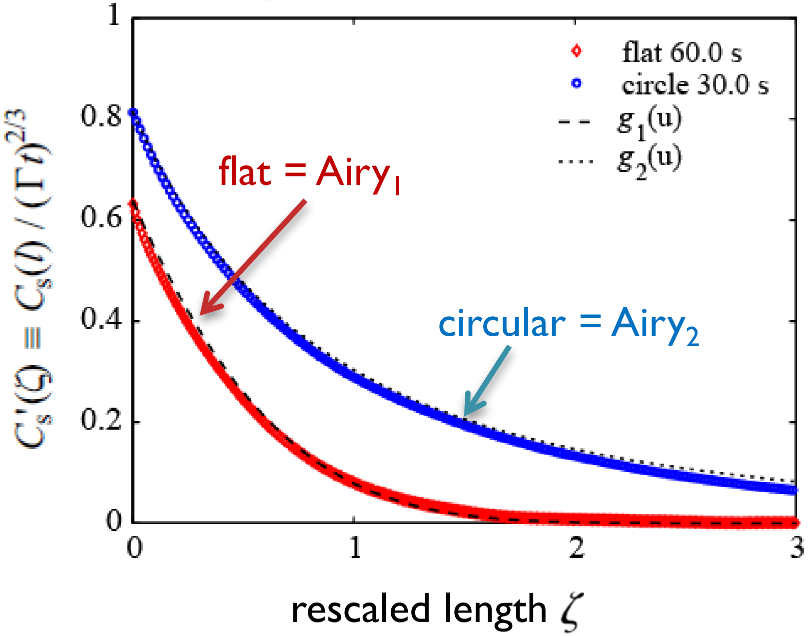
KPZ spatial profile is described by certain stochastic processes (Airy process)

- flat ⇨ Airy_1 process (1pt dist = GOE Tracy-Widom)
- circular ⇨ Airy_2 process (1pt dist = GUE Tracy-Widom)

(cf. Airy_2 = largest-eigenvalue dynamics of Dyson's Brownian motion for GUE, but no such relation between Airy_1 and GOE)

Two-point correlation function

$$C_s(l,t) \equiv \langle h(x+l,t)h(x,t)\rangle - \langle h\rangle^2$$



color symbols = experiments
dashed/dotted lines = Airy correlations
in good agreement!

Qualitatively different decay

- Airy_2 corr. (circular) $\sim l^{-2}$
- Airy_1 corr. (flat) decays faster than exp.

"KPZ Universality Subclasses"

[See review by I. Corwin, Rand. Mat. Theor. Appl. 1, 1130001]

Distribution and spatial correlation had been derived exactly for each subclass (2000~)

Circular (curved) interfaces

- Init. cond. : point / curved / wedge line
- Asymptotics : GUE Tracy-Widom distribution, Airy_2 correlation in space
- Shown for : TASEP [Johansson CMP 2000], PNG, PASEP [Tracy & Widom CMP 2009], KPZ eq. [Sasamoto & Spohn 2010, Amir *et al.* 2011, etc.] … (list is not complete)

Flat interfaces

- Init. cond. : straight line
- Asymptotics : GOE Tracy-Widom distribution, Airy_1 correlation in space
- Shown for : PNG [Prähofer-Spohn PRL 2000], TASEP, KPZ eq. [Calabrese-Le Doussal PRL 2011] ..

Stationary interfaces

- Init. cond. : stationary interface (1d-Brownian motion)
- Asymptotics : Baik-Rains F_0 distribution, $\text{Airy}_{\text{stat}}$ correlation in space
- Shown for : PNG [Baik-Rains JSP 2000], TASEP, KPZ eq. [Imamura-Sasamoto 2012, Borodin *et al.* 2014] ..

Note1) Other subclasses exist.

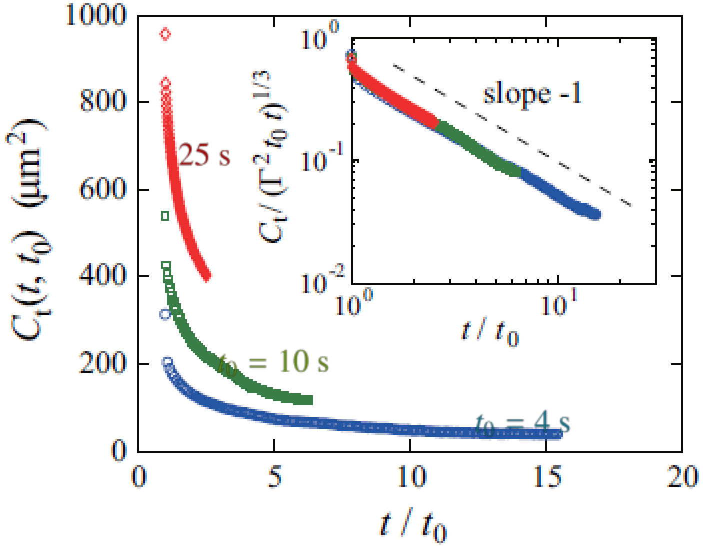
Note2) far less is known about time correlation!

[KaT & Sano, J. Stat. Phys. 147, 853 (2012)]

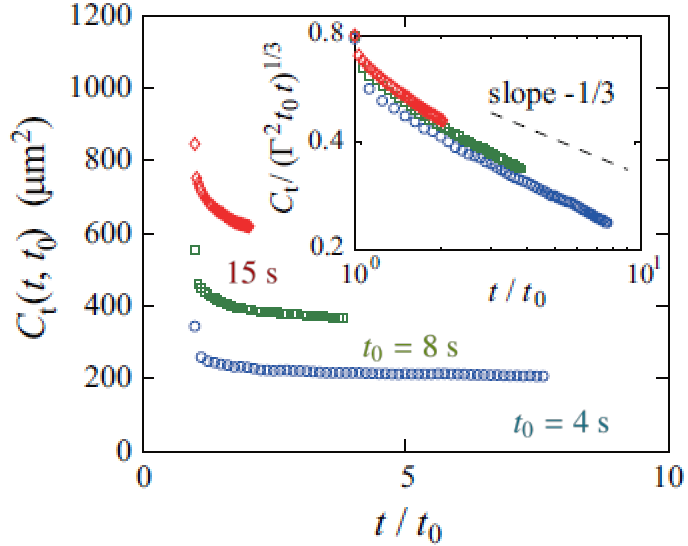
Two-Time Correlation Function

$$C_t(t,t_0) \equiv \mathrm{Cov}[h(x,t),h(x,t_0)] \equiv \langle h(x,t)h(x,t_0)\rangle - \langle h(x,t)\rangle\langle h(x,t_0)\rangle$$

flat



circular



- Natural scaling ansatz works

$$C_t(t,t_0) \simeq (\Gamma^2 t_0 t)^{1/3} F_t(t/t_0)$$

(recall: $h \simeq v_\infty t + (\Gamma t)^{1/3}\chi$)

- Long-time asymptotics

$$F_t(t/t_0) \sim (t/t_0)^{-\bar{\lambda}} \quad \text{with} \quad \bar{\lambda} = 1$$

$$C_t(t,t_0) \sim t^{1/3} F_t(t/t_0) \to 0 \quad (t \to \infty)$$

- Stronger finite-time correction to the scaling ansatz.

- $F_t(t/t_0) \sim (t/t_0)^{-\bar{\lambda}}$ with $\bar{\lambda} = 1/3$

then, $C_t(t,t_0) \sim t^{1/3} F_t > 0 \quad (t \to \infty)$ (!)

Correlation remains! Local ergodicity breaking?

Ferrari & Spohn, arXiv:1602.00486

- Ferrari & Spohn considered two-time correlation function for TASEP

$$C(\alpha) \equiv \mathrm{Cov}[\mathcal{X}(\alpha), \mathcal{X}(1)] = \alpha^{1/3} F_{\mathrm{t}}(1/\alpha)$$

with $\mathcal{X}(\alpha) \equiv \lim_{t\to\infty} -t^{-1/3}\left[h(0,\alpha t) - \frac{1}{4}\alpha t\right]$

where $\alpha = (t/t_0)^{-1}$ in our notation. $(0 \leq \alpha \leq 1)$

- For the stationary subclass, $C(\alpha)$ becomes
  the covariance of the fractional Brownian motion with Hurst exponent 1/3.

$$C(\alpha) = \langle \chi_0^2 \rangle_{\mathrm{c}} \frac{1}{2}\left[1 + \alpha^{2/3} - (1-\alpha)^{2/3}\right]$$

- For the flat and circular subclasses, they obtained asymptotics for $\alpha \to 0, 1$:

(1) circular subclass

$C(\alpha) = \mathcal{O}(\alpha^{2/3})$ for $\alpha \to 0$ ⇨ $F_{\mathrm{t}}(\tau) \sim \tau^{-1/3}$, $\therefore C_{\mathrm{t}}(t, t_0) > 0 \;\; (t \to \infty)$

$C(\alpha) = \langle \chi_{\mathrm{GUE}}^2 \rangle_{\mathrm{c}} - \mathcal{O}((1-\alpha)^{2/3})$ for $\alpha \to 1$

in agreement with experiment

(2) flat subclass

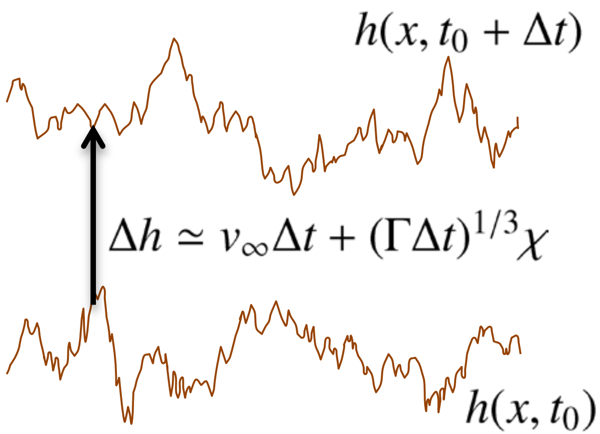
$C(\alpha) = \mathcal{O}(\alpha^{4/3})$ for $\alpha \to 0$ ⇨ $F_{\mathrm{t}}(\tau) \sim \tau^{-1}$

$C(\alpha) = \langle \chi_{\mathrm{GOE}}^2 \rangle_{\mathrm{c}} - \mathcal{O}((1-\alpha)^{2/3})$ for $\alpha \to 1$

Flat-Stationary Crossover

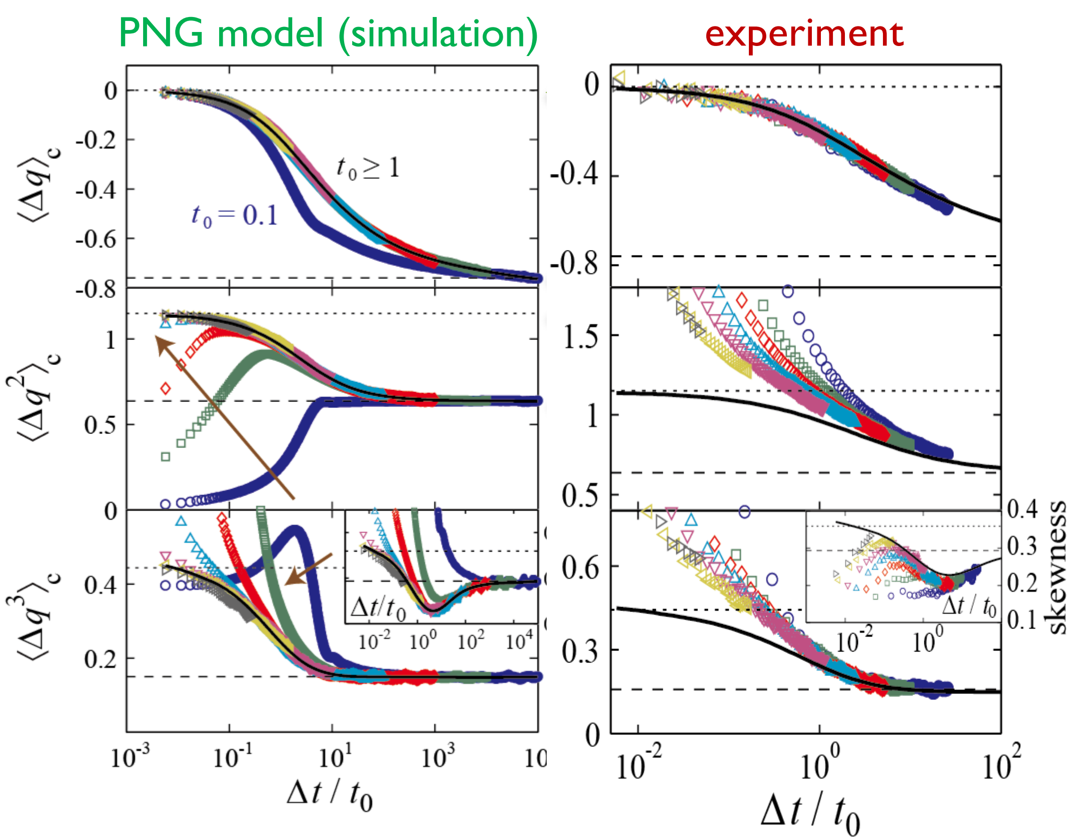
[KaT, PRL 110, 210604 (2013)]

Stationary subclass is theoretically established, but it is never reached in practice.
However, crossover to the stationary subclass can be studied.



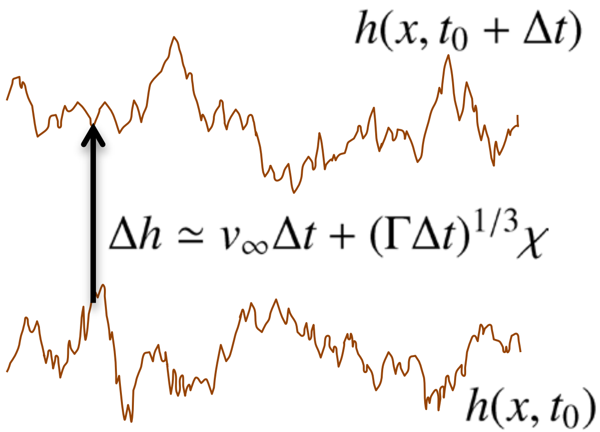
rescaled height difference

$$\Delta q \equiv \frac{\Delta h - v_\infty \Delta t}{(\Gamma \Delta t)^{1/3}} \simeq \chi$$



- Scaling functions $\langle \Delta q^n \rangle_c \simeq \Delta Q_n(\Delta t / t_0)$ describing flat-stationary crossover are found.
- Experiment seems to indicate the same scaling functions, so they are universal.

Relation to Two-Time Correlation



rescaled height difference

$$\Delta q \equiv \frac{\Delta h - v_\infty \Delta t}{(\Gamma \Delta t)^{1/3}} \simeq \frac{\mathcal{X}(1) - \alpha^{1/3}\mathcal{X}(\alpha)}{(1-\alpha)^{1/3}}$$

$\Delta Q_n(\tau) \equiv \langle \Delta q^n \rangle_c$ with $\tau \equiv \Delta t / t_0 = \alpha^{-1} - 1$

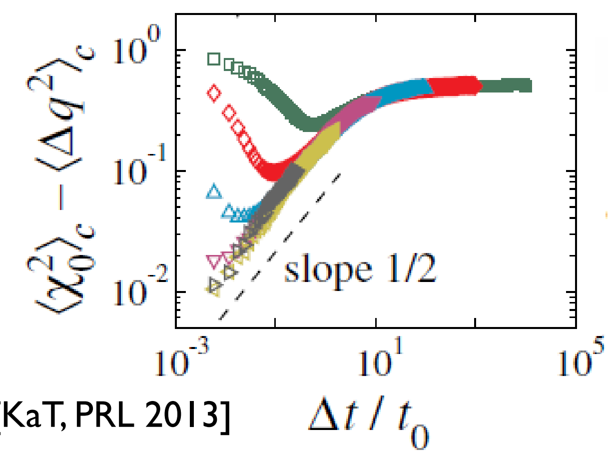
in particular,

$$\Delta Q_2(\tau) = \frac{(1+\alpha^{2/3})\langle \chi_{\mathrm{GOE}}^2 \rangle_c - 2\alpha^{1/3} C(\alpha)}{(1-\alpha)^{2/3}}$$

For the stationary limit ($t_0 \to \infty$, hence $\tau \to 0, \ \alpha \to 1$)

Numerics showed

$$\Delta Q_2(\tau) \to \langle \chi_0^2 \rangle_c - \mathcal{O}(\tau^{1/2})$$



[KaT, PRL 2013]

Theoretically,

$$\Delta Q_2(\tau) \simeq 2\beta^{-2/3} \left[\langle \chi_{\mathrm{GOE}}^2 \rangle_c - C(\alpha) \right] \text{ with } \beta \equiv 1 - \alpha$$

and Ferrari & Spohn tell us

$$C(\alpha) = \langle \chi_{\mathrm{GOE}}^2 \rangle_c - \mathcal{O}(\beta^{2/3})$$

Prediction from numerics is

$$C(\alpha) = \langle \chi_{\mathrm{GOE}}^2 \rangle_c - \frac{1}{2} \langle \chi_0^2 \rangle_c \, \beta^{2/3} + \mathcal{O}(\beta^{7/6})$$

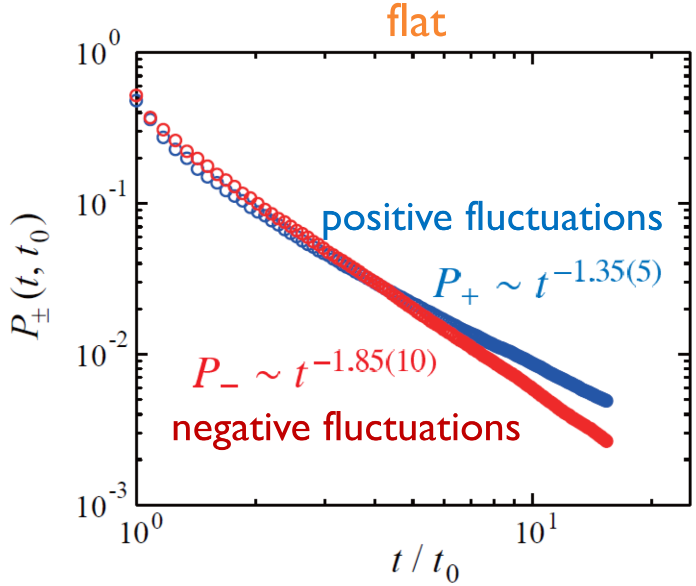
Persistence [KaT & Sano, J. Stat. Phys. 147, 853 (2012)]

typically, $P_{\pm}(t, t_0) \sim t^{-\theta_{\pm}^{(\mathrm{p})}}$

Persistence probability $P_{\pm}(t, t_0)$

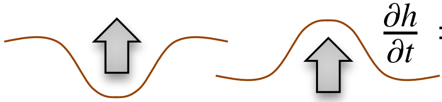
= joint probability that $\delta h \equiv h(x,t) - \langle h \rangle$ at a fixed position x is positive (negative) at time t_0 and keeps the same sign until time t

flat



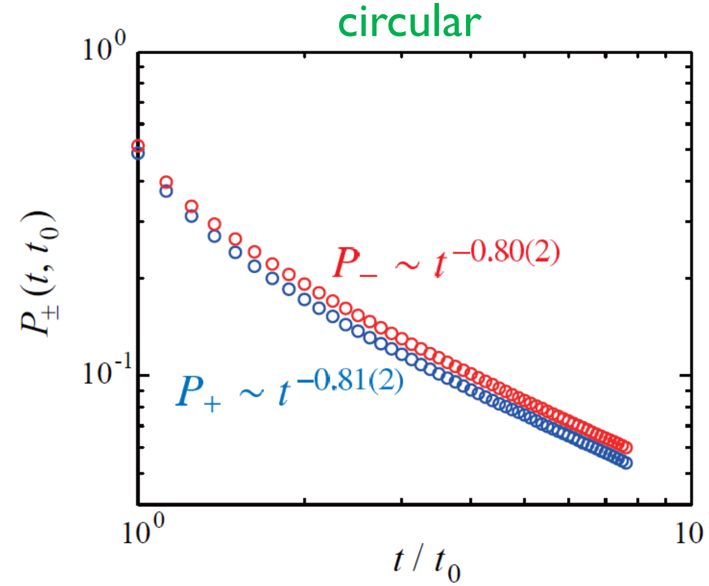
$$\theta_+^{(\mathrm{p})} = 1.35(5), \quad \theta_-^{(\mathrm{p})} = 1.85(10)$$

- $\theta_+^{(\mathrm{p})} < \theta_-^{(\mathrm{p})}$ due to the KPZ nonlinearity



$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

circular



$$\theta_+^{(\mathrm{p})} = 0.80(2), \quad \theta_-^{(\mathrm{p})} = 0.81(2)$$

- Asymmetry cancelled! $\theta_+^{(\mathrm{p})} = \theta_-^{(\mathrm{p})}$
- $\theta_{\pm}^{(\mathrm{p})} < 1$ implies local ergodicity breaking
  ($\because$ mean persistence time $\int P_{\pm} \mathrm{d}t = \infty$)

Ergodicity Breaking?

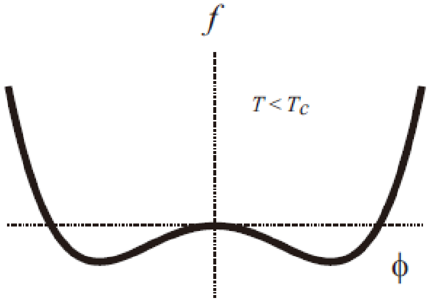
- Ergodicity : time average = ensemble average

- (Usual) ergodicity breaking :

Different equilibrium states (or attractors) coexist, so different results arise.

<u>Examples</u>

- Spontaneous symmetry breaking
- Systems with long-range interactions



- Weak ergodicity breaking : [J. P. Bouchaud 1992]

- Different results in a single state, due to long & random trapping of trajectories
- Time-averaged quantities remain stochastic,
  described by well-defined distribution.

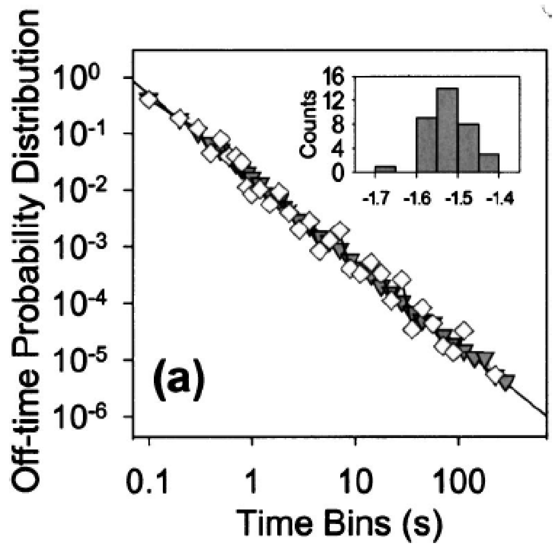
<u>Examples</u>

- experiments: cellular transports, quantum dots, etc.
- models: continuous-time random walk (CTRW), renewal process (RP), etc.

Weak Ergodicity Breaking in Quantum Dots

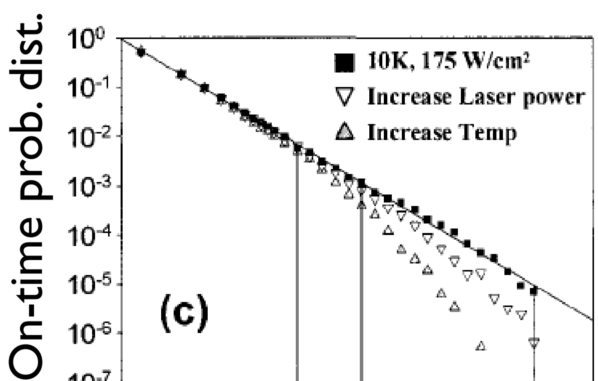
Blinking of fluorescence in CdSe nanocrystal (random on/off switching)

[Brokmann et al., PRL 2003]



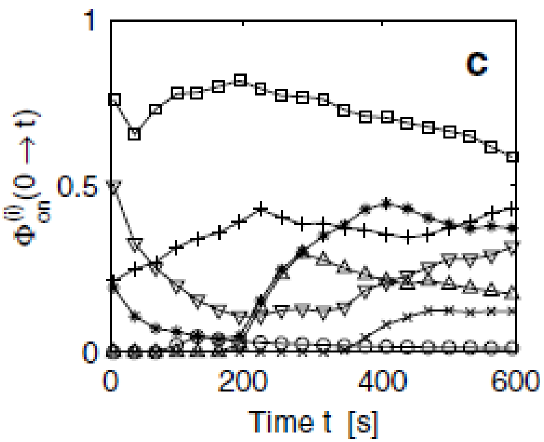
$$\text{Prob[waiting time} > \tau] \sim \begin{cases} \tau^{-0.58} & \text{(on)} \\ \tau^{-0.48} & \text{(off)} \end{cases}$$

⇨ mean waiting time diverges

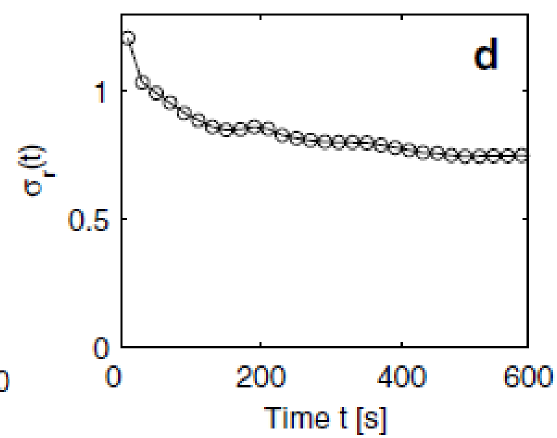


[figures from K.T. Shimizu et al., PRB 2001]

fraction of time of "on" state



standard deviation of Φ_{on}



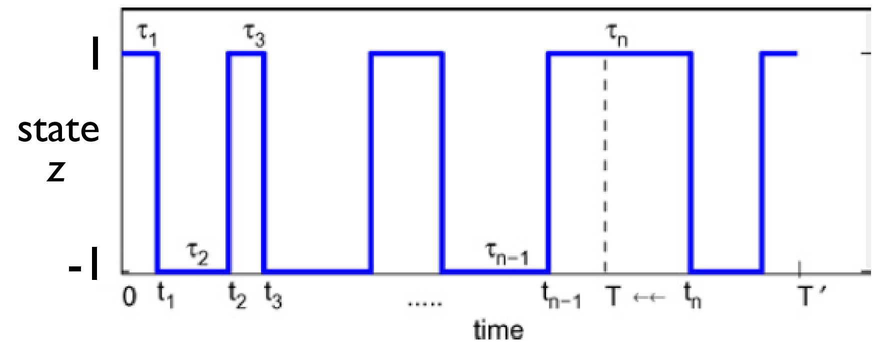
Φ_{on} (time-avgd. quantity) seems to be stochastic even in $t \to \infty$ limit (weak ergodicity breaking)

Simplest Model

Renewal process (RP) [see, e.g., C. Godrèche & J. M. Luck, J. Stat. Phys. 104, 489 (2001)]

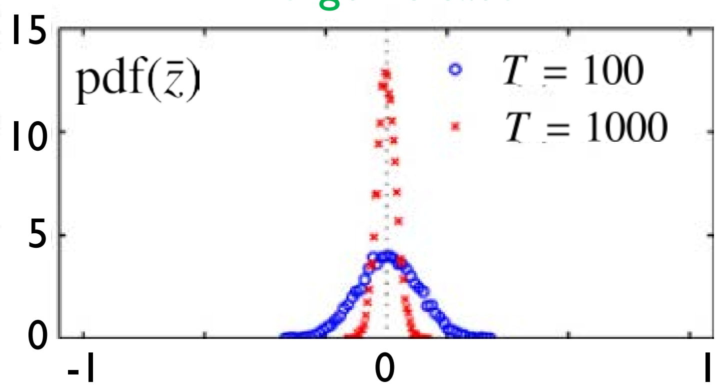
$$\text{Prob}[\text{waiting time} > \tau] \sim \tau^{-\theta}$$

$0 < \theta < 1$: weak ergodicity breaking



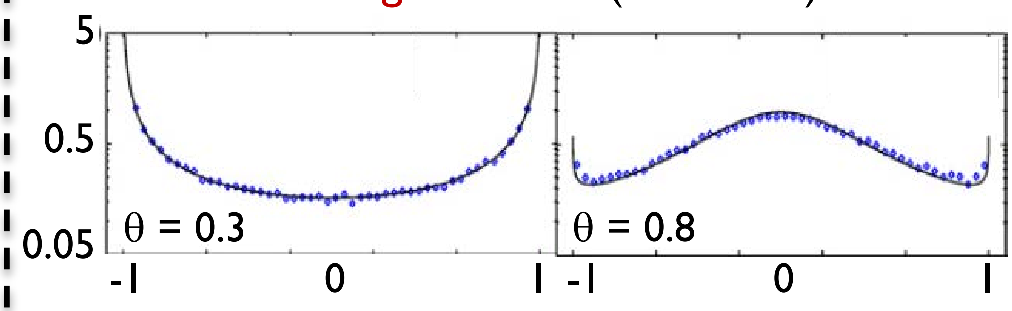
e.g.) time-avg. of z : $\bar{z}(T) = \dfrac{1}{T}\displaystyle\int z(t)\mathrm{d}t$

ergodic case



$$\text{pdf}(\bar{z}) \to \delta(\bar{z} - \langle z \rangle)$$

non-ergodic case (0 < θ < 1)



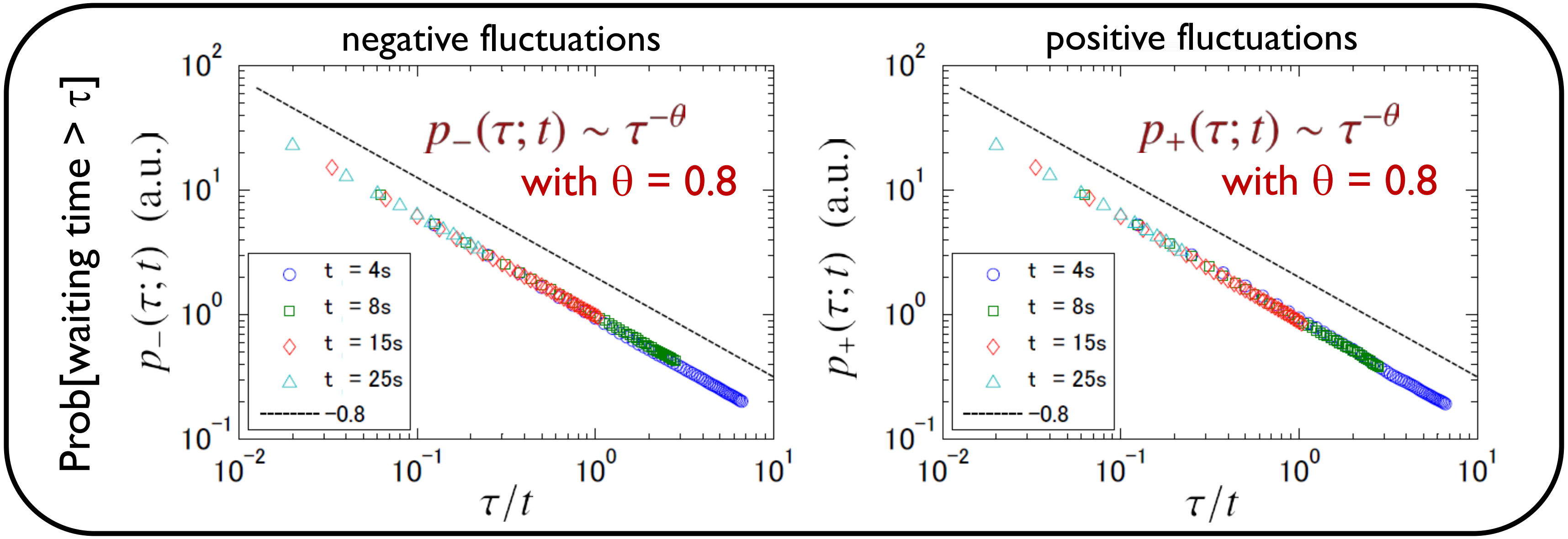
pdf converges to an asymptotic distribution

($\bar{z}$ remains stochastic; Lamperti distribution).

KPZ Interfaces vs Renewal Process (RP)

- Renewal process (RP) is far too simple to describe KPZ.
  two-state model, no space, uncorrelated waiting times…
- But, let's naively compare... one can regard δh's sign as the state z.
  First we measure distribution of waiting time τ of sign renewed at t.

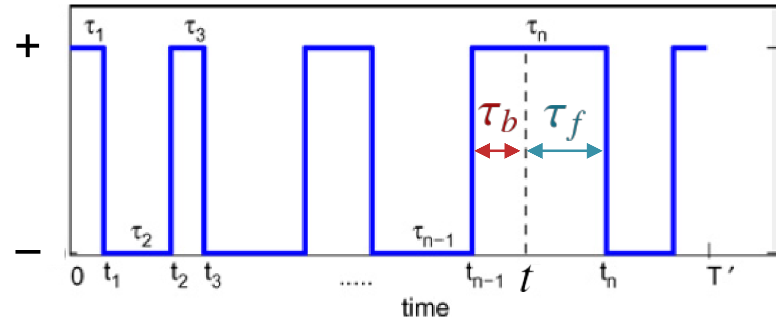
circular interfaces



Circular case

Waiting times show clear power law with $\theta = \theta_{\pm}^{(\mathrm{p})} \approx 0.8$, similarly to RP!

Recurrence Times



Other quantities of interest

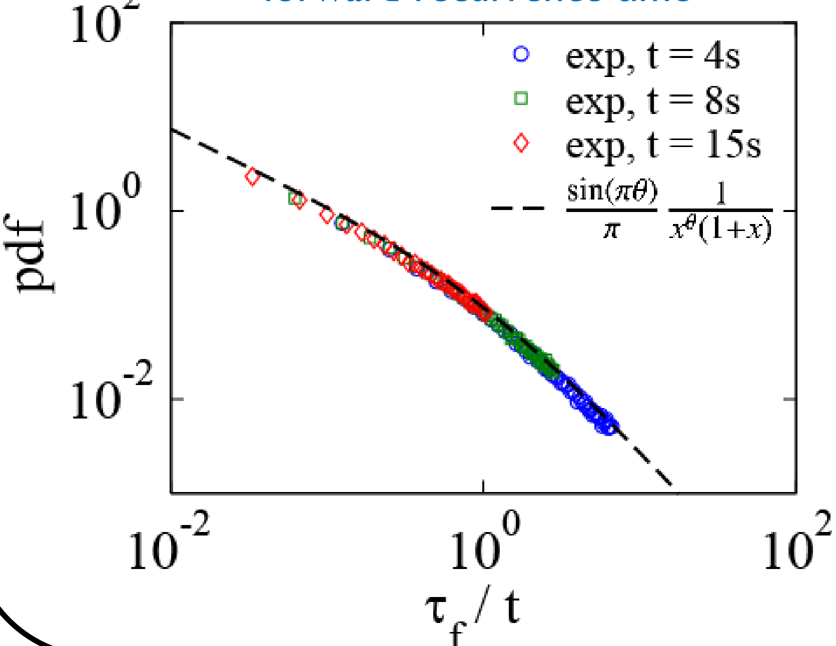
Forward recurrence time $\tau_f(t)$

Backward recurrence time $\tau_b(t)$

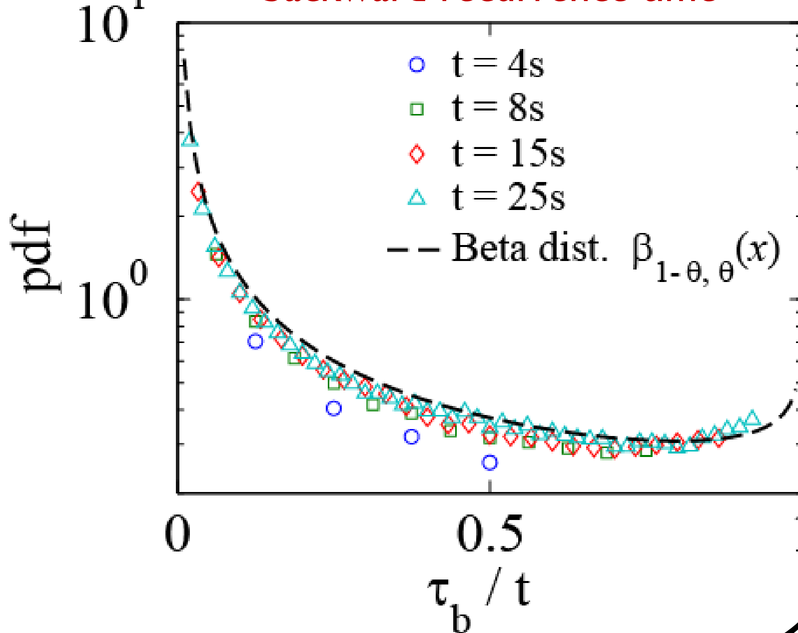
Note) persistence prob. $= \int_\tau^\infty \mathrm{pdf}(\tau_f)\mathrm{d}\tau_f$

circular interfaces

forward recurrence time



backward recurrence time



Circular Remarkable agreement with RP's exact results! (for RP, see Godrèche & Luck 2001)

Flat Case

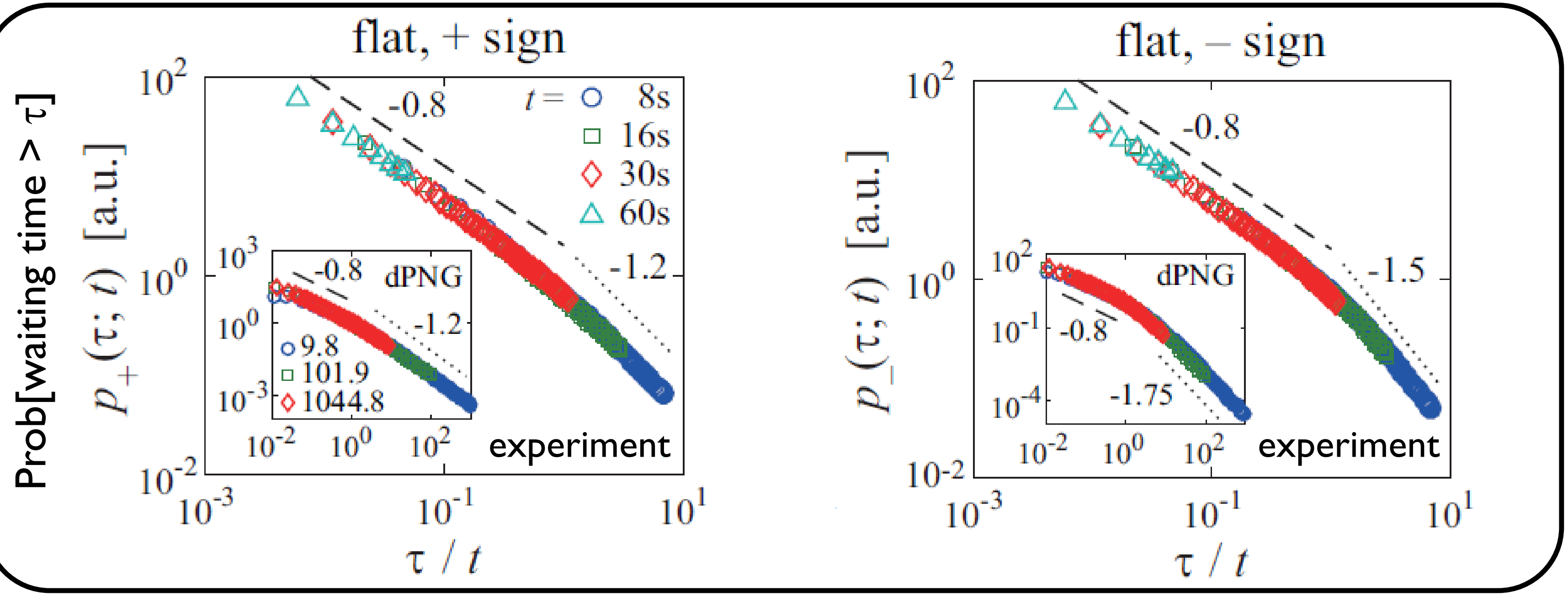
<u>Reminder about flat case:</u>

time correlation of *h* decays : $C_t(t, t_0) \to 0$

persistence exponent $\theta_{\pm}^{(p)} > 1$: $\theta_{+}^{(p)} \approx 1.35, \theta_{-}^{(p)} \approx 1.85$ ⇨ No ergodicity breaking?

However, dynamics is still scale-invariant, unlike RP with $\theta > 1$. Let's have a look!

waiting-time distribution

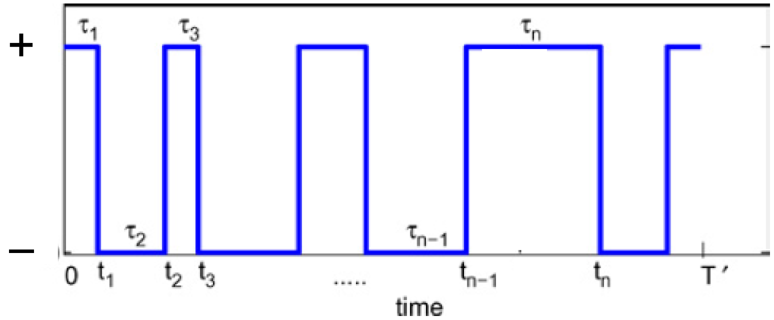


Waiting-time dist. shows crossover: $p_{\pm}(\tau;t) \sim \begin{cases} \tau^{-\theta} & (\tau \ll t) \quad (\theta \approx 0.8) \\ \tau^{-\theta_{\pm}} & (\tau \gg t) \quad (\theta_{\pm} > 1) \end{cases}$

"Renewal Process" for Flat Case

- Standard RP (compared to the circular case)

$$p_{\pm}(\tau) \equiv \mathrm{Prob}[\text{waiting time} > \tau] \sim \tau^{-\theta}$$



- For the flat case, we introduce RP with two-step power-law waiting times:

$$p_{\pm}(\tau;t) \sim \begin{cases} \tau^{-\theta} & (\tau \leq t) \\ \tau^{-\theta'_{\pm}} & (\tau \geq t) \end{cases}$$

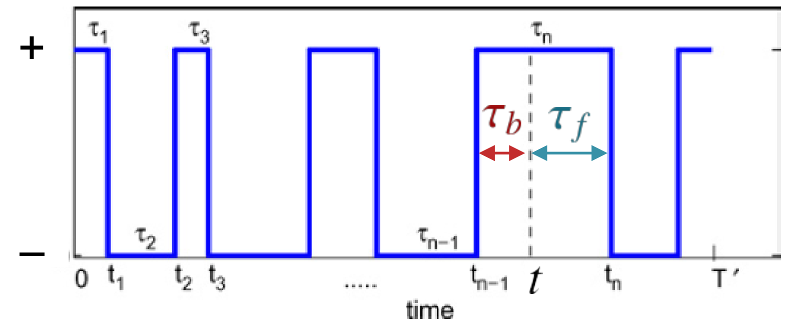
  - Setting threshold $\sim t$ keeps dynamics scale-invariant.
  - Here, we set $\theta = 0.8, \theta'_{+} = 1.2, \theta'_{-} = 1.5$ as observed experimentally.

Recurrence Time Statistics (Flat)

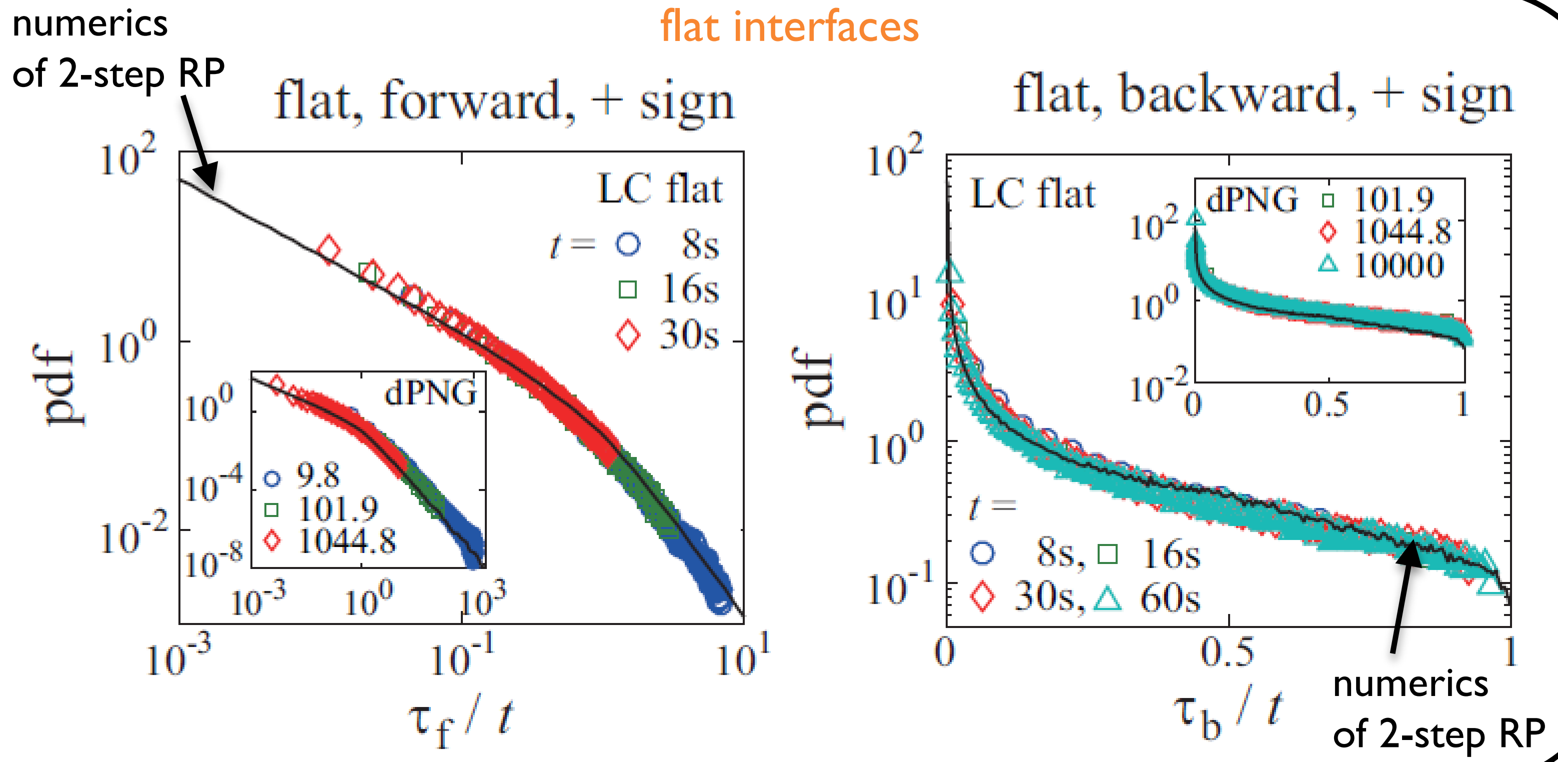
Forward recurrence time $\tau_f(t)$

Backward recurrence time $\tau_b(t)$

Note) persistence prob. $= \int_{\tau}^{\infty} \mathrm{pdf}(\tau_f) d\tau_f$



flat interfaces

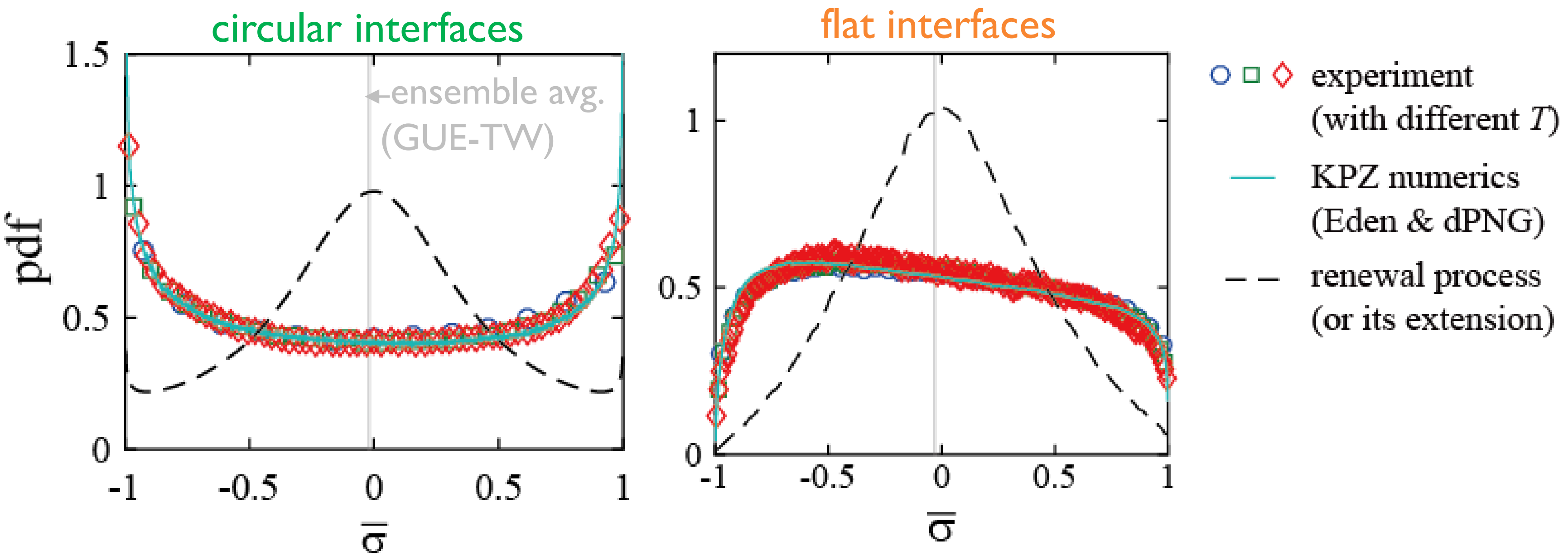


Good agreement with 2-step RP

Distribution of Time-Averaged Sign

$$\bar{\sigma}(x,T) = \frac{1}{T}\int_0^T \mathrm{sign}[h(x,t) - \langle h(x,t)\rangle]\mathrm{d}t$$

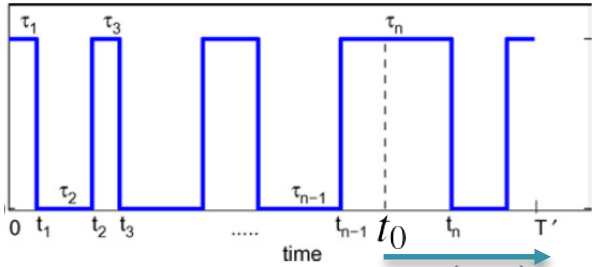
For RP, distribution is exactly known. (Lamperti distribution)



- Pdf converges to a broad distribution. Weak ergodicity breaking.
- Time-avgd. sign distribution is *not* reproduced by RP. Characteristic universal distribution for each KPZ subclass.

Difference from RP is presumably due to correlation between waiting times

Correlation Function of Signs



σ: $\mathrm{sign}[h(x,t) - \langle h(x,t) \rangle]$

$$\boxed{C_{\mathrm{sign}}(t, t_0) \equiv \langle \sigma(x,t)\sigma(x,t_0) \rangle} = \sum_{n=0}^{\infty} (-1)^n p_n(t, t_0)$$

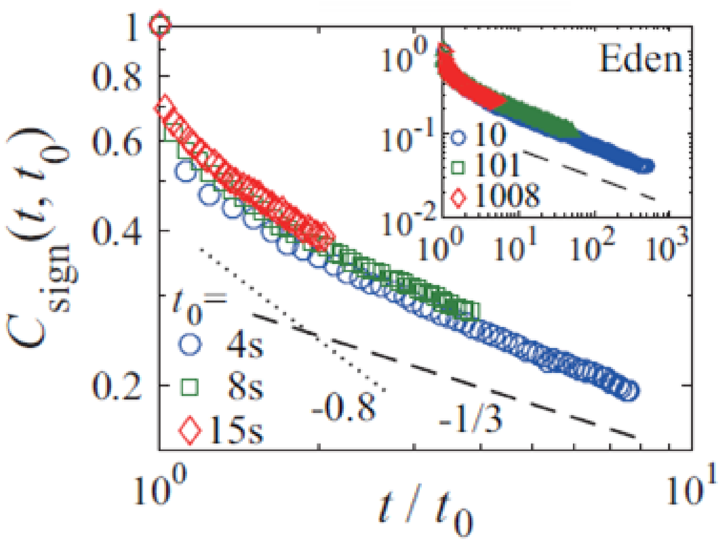
$p_n(t, t_0)$: prob. that sign flips *n* times b/w t_0 and *t*.

(NB; $p_0(t, t_0)$ = persistence probability)

- For RP, one can show that the infinite sum $\sum_{n=1}^{\infty} (-1)^n p_n(t, t_0)$ does not contribute,

$$\boxed{\therefore C_{\mathrm{sign}}(t, t_0) \simeq p_0(t, t_0) \sim t^{-\theta} \text{ (RP)}}$$ [Godrèche & Luck, JSP 2001]

- For KPZ (circular), we find

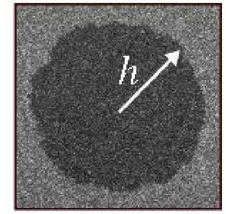


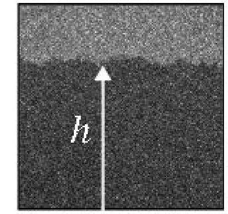
$$\boxed{C_{\mathrm{sign}}(t, t_0) \sim t^{-\bar{\lambda}} \text{ (KPZ)}}$$ *not* $t^{-\theta}$ like RP

$\therefore C_{\mathrm{sign}}$ decays like F_t (KPZ rescaled corr. func. without sign operation)

- KPZ time correlation is encoded in C_{sign}
- $C_{\mathrm{sign}}^{(\mathrm{KPZ})} \neq C_{\mathrm{sign}}^{(\mathrm{RP})}$ because of correlation between waiting times.
- Nevertheless, τ_f & τ_b agree with RP.

Summary





Time correlation properties of KPZ remain largely unexplored, while some experimental results are available.

Two-time correlation function $C_{\mathrm{t}}(t, t_0) \equiv \mathrm{Cov}[h(x,t), h(x,t_0)] \simeq (\Gamma^2 t_0 t)^{1/3} F_{\mathrm{t}}(t/t_0)$

- Experiment: $\begin{cases} F_{\mathrm{t}}(\tau) \sim \tau^{-1} & \text{(flat)} \\ F_{\mathrm{t}}(\tau) \sim \tau^{-1/3} & \text{(circular)} \end{cases}$ ⇦ in particular, $C_{\mathrm{t}}(t, t_0) > 0 \;\; (t \to \infty)$

  [KaT & Sano, J. Stat. Phys. 147, 853 (2012)]

- Ferrari & Spohn's results for TASEP agree with this experiment, but full functional forms of $C_{\mathrm{t}}(t, t_0)$ remain to be solved for the circular & flat cases.

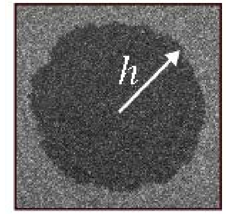
  [Ferrari & Spohn, arXiv:1602.00486]

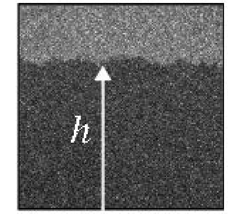
- $C_{\mathrm{t}}(t, t_0)$ also controls the flat-stationary crossover [KaT, PRL 110, 210604 (2013)] : Information on Baik-Rains F_0 distribution is encoded in $C_{\mathrm{t}}(t, t_0)$.

Prediction: (flat case, $\alpha \to 1$)

$$C(\alpha) \equiv \mathrm{Cov}[\mathcal{X}(\alpha), \mathcal{X}(1)] = \langle \chi_{\mathrm{GOE}}^2 \rangle_{\mathrm{c}} - \frac{1}{2} \langle \chi_0^2 \rangle_{\mathrm{c}} (1-\alpha)^{2/3} + \mathcal{O}((1-\alpha)^{7/6})$$

Summary





Time correlation properties of KPZ remain largely unexplored, while some experimental results are available.

Sign renewals of KPZ fluctuations

- Experimental result for persistence probability $P_{\pm}(t, t_0)$

flat: $P_{\pm}(t, t_0) \sim \begin{cases} t^{-1.35} & \text{(+ sign)} \\ t^{-1.85} & \text{(− sign)} \end{cases}$

circular: $P_{\pm}(t, t_0) \sim t^{-0.8}$ (+ & −)

+/- asymmetry disappears for circular case! (no theory yet)

[KaT & Sano, J. Stat. Phys. 147, 853 (2012)]

- Waiting times (until next sign renewal) show clear power-law distribution.
  Circular: single power law, similar to the renewal process (RP)
  Flat: 2 power laws
- Some properties are reproduced by the renewal process (uncorrelated sign renewals)
- Weak ergodicity breaking, characteristic of KPZ.
  (time-avgd. quantity remains stochastic) [KaT & Akimoto, arXiv:1509.03082]