From de Sitter hydrodynamics to DFPs

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Based on arXiv: $\underbrace{1603.05344\,,\,2111.04122}_{\text{theory}}$, $\underbrace{2301.09456}_{\text{applications}}$

also: arXiv: 1702.01320, 1809.08484, 1904.09968, 1912.03566, 2207.09887, 2210.17380, 2304.11195$+\cdots$

The Many Faces of Relativistic Fluid Dynamics, KITP, June 20, 2023

$\Longrightarrow$ some time last week:

- Pavel: "······ can you do it without holography?"
- me: "······ no ······"

$\Longrightarrow$ some time last week:

- Pavel: $'' \cdots\cdots$ can you do it without holography?$''$
- me: $'' \cdots\cdots$ no $\cdots\cdots .''$

$\Longrightarrow$ BUT:

- Apart from a simple (physically useful) exercise we will not use holography
- All results are phrased in the language of hydro
- All computations are studies of dynamical horizons of black holes/branes, and following gauge/gravity (gravity/fluid) correspondence interpreted as non-equilibrium (far-from-equilibrium) dynamics of corresponding strongly-coupled QFTs

$\Longrightarrow$ the BENEFITS of holography:

- we can do all-derivative/all-gradient non-equilibrium QFT computations
- results interpreted within hydro gradient truncations teach:
  - ■ what are specific terms that enter at the n-th order of derivative expansion?
  - ■ what are the explicit values of transport coefficients in terms of microscopic parameters

$$\frac{\eta}{s} = \frac{1}{4\pi}\,, \qquad \frac{\zeta}{s}\,, \qquad \cdots$$

  - ■ study transport in the vicinity of critical phenomena — find explicit counter-examples of Onuki classification of bulk viscosity criticality
- study the nonadiabaticity (irreversibility) of off-equilibrium processes in QFT (universality of driven quantum systems)
- Entropy current, thermalization, isotropization of QGP
- $\cdots$

$\Longrightarrow$ a class of problems: **hydrodynamics in cosmology**

- consider FLRW line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 d\boldsymbol{x}^2$$

- the cosmological scale factor $a(t)$ is governed by Einstein's equations coupled to matter:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- under the symmetry assumptions, one typically postulates

$$T_{\mu\nu} \equiv T^{eq}_{\mu\nu} = \mathrm{diag}\{\epsilon, P, P, P\}\,, \qquad \text{with} \qquad \epsilon = \epsilon^{eq}(P)$$

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$\Longrightarrow$ in general, for an interactive QFT,

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{classically}} \qquad = \qquad 8\pi G \quad \underbrace{T_{\mu\nu}}_{\langle T_{\mu\nu}\rangle_{FLRW}}$$

with

$$\langle T_{\mu\nu}\rangle = T^{eq}_{\mu\nu} + \qquad \underbrace{\Pi_{\mu\nu}(\dot{a}, \{\dot{a}^2, \ddot{a}\}, \cdots)}_{\text{derivative corrections to equilibrium } T^{eq}_{\mu\nu}}$$

$\Longrightarrow$ To summarize: we will be interested in:

$$\langle T_{\mu\nu}\rangle\{a(t)\}$$

of strong coupled QFTs

$\Longrightarrow$ A hydro perspective:

- Recall from previous talks: a boost-invariant expansion **is** a QFT dynamics in Milne cosmology from the comoving fluid perspective:

$$ds^2 = -dt^2 + t^2 d\xi^2 + d\boldsymbol{x}_\perp^2\,, \qquad u^\mu = (1,0,0,0) \Longrightarrow \theta \equiv \nabla_\mu u^\mu = \frac{1}{t}$$

- Likewise, a QFT dynamics in FLRW Universe **is** a spatially homogeneous and isotropic flow form the comoving fluid perspective:

$$ds^2 = -dt^2 + a(t)^2\, d\boldsymbol{x}^2\,, \qquad u^\mu = (1,0,0,0) \Longrightarrow \theta \equiv \nabla_\mu u^\mu = 3\frac{\dot{a}}{a}$$

- A particular interesting flow is de Sitter expansion with

$$a(t) = e^{Ht}\,, \qquad H = \text{const}$$

- Note difference in the gradient scales relative to a local temperature scale at late times:
  - boost-invariant —

$$T \propto t^{-1/3} \implies \qquad \lim_{t\to\infty} \frac{\theta}{T} \propto \lim_{t\to\infty} t^{-2/3} = 0$$

  - dS —

$$T \propto e^{-Ht} \implies \qquad \lim_{t\to\infty} \frac{\theta}{T} \propto \lim_{t\to\infty} H \cdot e^{Ht} \to \infty$$

$\implies$ we expect interesting late-time attractor in de Sitter:

<u>Dynamical Fixed Point</u>

Outline:

- FLRW Hydrodynamics
  - first-order hydro
  - resummation and non-hydrodynamic modes
- A trivial DFP: thermal states of $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) in de Sitter
  - gauge theory perspective
  - holographic picture
  - de Sitter vacuum 'entanglement' entropy
- Nontrivial DFP
  - we focus on QFT_{2+1}
  - $\mathcal{N} = 2^*$ and cascading DFP — see the literature
- Applications: holographic gravitational reheating
  - harvesting the entanglement entropy of de Sitter DFPs
- Conclusions and future directions

$\Longrightarrow$ *Thermodynamic equilibrium* is a late-time attractor of dynamical evolution of isolated interacting quantum system:

$$\lim_{t\to\infty} T_{\mu\nu}(t,\boldsymbol{x}) = \mathrm{diag}\left(\mathcal{E}_{eq}, P_{eq}, \cdots P_{eq}\right)$$

▪ $T_{\mu\nu}$ are the component of the stress-energy tensor of the system at time t and the spatial location $\boldsymbol{x}$

$\Longrightarrow$ We also have a theory — **the hydrodynamics** — that describes the approach to that equilibrium (assuming we are not-far from it):

- $\nabla_\mu T^{\mu\nu} = 0$
- $T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots$

•

$$T^{\mu\nu} = \underbrace{\mathcal{E}\ u^\mu u^\nu + P\ (g^{\mu\nu} + u^\mu u^\nu)}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\eta\sigma^{\mu\nu} - \zeta(g^{\mu\nu} + u^\mu u^\nu)\nabla\cdot u \right]}_{\mathcal{O}(\partial^1 u):\ \sigma^{\mu\nu}\sim\partial^\mu u^\nu}$$

$$+ \underbrace{[\qquad \cdots \qquad]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

- u^μ — local fluid velocity
- $g^{\mu\nu}$ — background metric
- η, ζ — shear and bulk viscosities
- expansion parameter of hydro as EFT:

$$\frac{1}{T} \cdot |\partial u| \ll 1$$

where $T = T(t, \boldsymbol{x})$ is the local temperature

$$\mathcal{E} + P = sT\,, \qquad d\mathcal{E} = T ds$$

•

$$\mathcal{S}^\mu = \underbrace{s\ u^\mu}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\frac{1}{T} \ \cdot \ T^{\mu\nu}_{(1)} u_\nu\right]}_{\mathcal{O}(\partial^1 u)} + \underbrace{[\qquad \cdots \qquad]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

- from the conservation of the stress-energy tensor,

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow$$

$$T\ \nabla \cdot \mathcal{S} = \zeta\ \left(\nabla \cdot u\right)^2 + \frac{\eta}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}\ \geq\ 0$$

$\Longrightarrow$ As one approaches the equilibrium,

$$\lim_{t\to\infty} u^\mu = u^\mu_{eq} = (1, \mathbf{0}) \qquad \Longrightarrow \qquad \lim_{t\to\infty} T\ \nabla \cdot \mathcal{S} = 0$$

i.e., in the approach to equilibrium the entropy production rate vanishes

We can now provide a formal definition of a dynamical fixed point (DFP):

A *Dynamical Fixed Point* is an internal state of a quantum field theory with spatially homogeneous and time-independent one-point correlation functions of its stress energy tensor $T^{\mu\nu}$, and (possibly additional) set of gauge-invariant local operators $\{\mathcal{O}_i\}$,

<u>and</u>

strictly positive divergence of the entropy current at late-times:

$$\lim_{t\to\infty}\left(\nabla\cdot\mathcal{S}\right) > 0$$

$\Longrightarrow$ Apart from the requirement of the strictly non-zero entropy production rate at late times, characteristics of a DFP coincide with that of the thermodynamic equilibrium.

Example: $\mathcal{N}=2^*$ QGP — one of the best understood non-conformal top-down holography

- $\mathcal{N}=2^*$: $\mathcal{N}=4$ SYM with $m_b/m_f \neq 0$ for bosonic/fermionic components of a hypermultiplet
- in Minkowski space time:
  - $g^{\mu\nu}=\eta^{\mu\nu}$
  - $$\mathcal{E}_{eq}=\frac{3}{8}\pi^2N^2T^4\left[1+\left\{\frac{\ln\frac{T}{m_b}}{9\pi^4}\left(\frac{m_b}{T}\right)^4+\cdots\right\}+\left\{-\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^4}\left(\frac{m_f}{T}\right)^2+\cdots\right\}\right]$$
  - $$\frac{\eta}{s}=\frac{1}{4\pi}\,,\qquad \frac{\zeta}{\eta}=\beta_f^\Gamma\cdot\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^3}\left(\frac{m_f}{T}\right)^2+\beta_b^\Gamma\cdot\frac{1}{432\pi^2}\left(\frac{m_b}{T}\right)^4+\cdots$$

  where

  $$\beta_f^\Gamma\approx 0.9672\,,\qquad \beta_b^\Gamma\approx 8.0000$$

- in FLRW:
  - $$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \; d\boldsymbol{x}^2$$
  - FLRW cosmology as hydro:

$$\text{local/comoving}: \qquad u^\mu = (1, \mathbf{0})$$

$$\underline{\text{BUT}}: \qquad \nabla \cdot u = 3 \, \frac{\dot{a}}{a} \neq 0$$

  - $$\underbrace{T \; \nabla \cdot \mathcal{S}}_{\frac{T}{a^3} \cdot \frac{d}{dt}[a^3 s]} = \zeta \; \underbrace{(\nabla \cdot u)^2}_{9\left(\frac{\dot{a}}{a}\right)^2} + \frac{\eta}{2} \underbrace{\sigma_{\mu\nu} \sigma^{\mu\nu}}_{=0} + \cdots$$

$\Longrightarrow$ prediction verified in (1603.05344)

$$\underbrace{\frac{d}{dt} \ln[a^3 s]}_{\text{computed from dynamical horizon}} \qquad = \qquad \underbrace{\frac{1}{T} \cdot (\nabla \cdot u)^2 \cdot \frac{\zeta}{s}}_{\text{agreement from Minkowski } \frac{\zeta}{s}} \qquad + \cdots$$

■ We can prove a theorem (from Einstein equations applied to the area growth of dynamical horizon):

$$\frac{d(a^3 s)}{dt} \geq 0$$

■ Contribution to the production rate from operator of dimension Δ in, *e.g*, de-Sitter cosmology (to leading order in $\frac{m}{T}$) reads:

$$\frac{d(a^3 s)}{dt} = N^2 (aT)^2 \; a^{7-2\Delta} \times \Omega_\Delta^2$$

where

$$\Omega_\Delta \equiv \sum_{n=0}^{\infty} c_n(\Delta) \left(\frac{H}{T}\right)^n$$

■ c_0 coefficient describes entropy production due to bulk viscosity

$\Longrightarrow c_n$ for $n \geq 1$ can be computed (semi-)analytically:

$$c_n \;\sim\; n!\,, \qquad n \gg 1$$

$\Longrightarrow$ Thus, "hydrodynamics" of strongly coupled gauge theories in de Sitter

is an asymptotic expansion, with zero radius of convergence

$\Longrightarrow$ To rephrase,

$$T_{\mu\nu} = T^{eq}_{\mu\nu} + \Pi_{\mu\nu}(\dot{a}, \{\dot{a}^2, \ddot{a}\}, \cdots),$$

when organized as a series expansion in derivatives of the scale factor a is a divergent series

$\Longrightarrow$ This asymptotic series can be Borel-resumed; there are **poles** in the Borel transform (resummation)

$\Longrightarrow$ Usually, there is an interesting physics associated with the poles of the Borel transform of asymptotic expansion:

- in QED, it is related to the vacuum instability due to e^+e^- pair production once

$$e^2 \qquad \to \qquad -e^2$$

- in theory of nonlinear elasticity, it is rated to the physics of the material fracture (under stress)

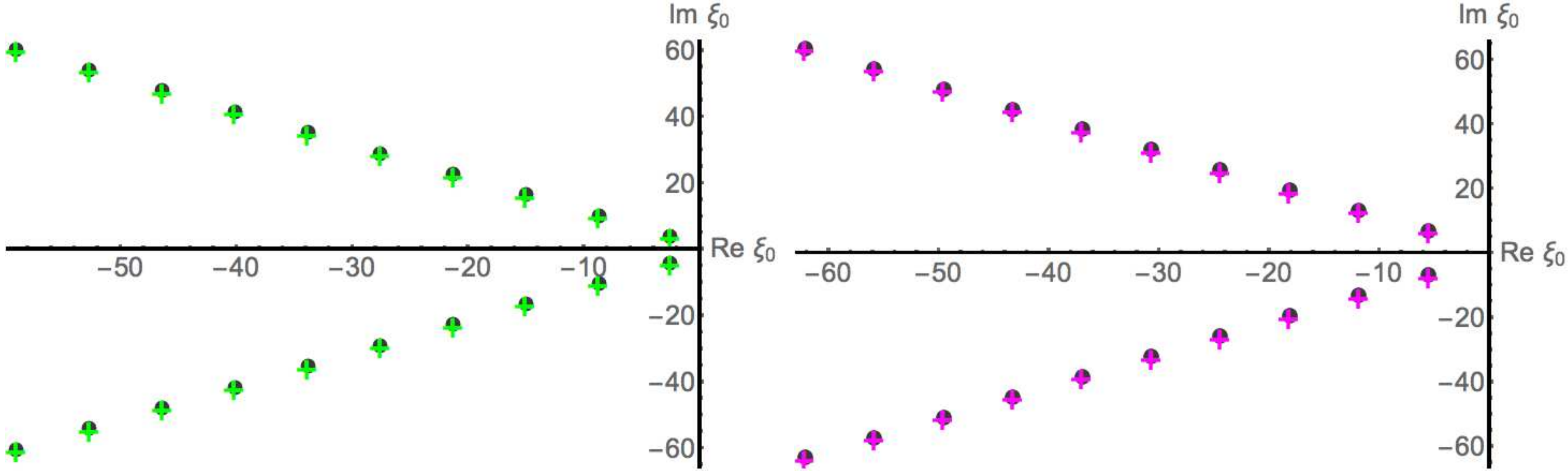
- Here, it is related to the presence of the 'non-hydrodynamic' excitations in strongly coupled gauge theory plasma (black hole quasinormal modes — QNM — in the dual gravitational description).

- We used Pade approximation of

$$\Omega_{\Delta}^{(B)}(\xi) = \sum_{n=0}^{\infty} \frac{c_n}{n!}\xi^n$$

to determine to location of the 10 leading singularities (poles) on the complex Borel plane

- These poles were compared with the BH QNM computations for the $\Delta = \{2, 3\}$ computed by Nunez and Starinets in 2003



Positions on the Borel plane of 10 singularities ξ_0 closest to the origin for $\Omega^{(B)}_{\Delta=2}$ (left) and $\Omega^{(B)}_{\Delta=3}$ (right) are given by solid circles. Crosses correspond to QNM frequencies. One observes a remarkable agreement between the singularities and the QNMs at a fraction of a percent or better.

$\Longrightarrow$ We now move to study FLRW/de Sitter attractors:

$\mathcal{N} = 4$ SYM in FLRW CFT perspective

- FLRW is Weyl equivalent to Minkowski:

$$ds_4^2 = -dt^2 + a^2(t)\, d\boldsymbol{x}^2 = a(t)^2 \left(-\frac{dt^2}{a(t)^2} + d\boldsymbol{x}^2 \right) = a^2 \underbrace{\left(-d\tau^2 + d\boldsymbol{x}^2 \right)}_{ds^2_{Minkowski}}$$

- if $\mathcal{O}_\Delta$ is a primary operator of dimension Δ,

$$\langle \mathcal{O}_\Delta \rangle \Big|_{FLRW} = a^{-\Delta} \, \langle \mathcal{O}_\Delta \rangle \Big|_{Minkowski}$$

- stress-energy tensor is not a primary field:

$$\langle T_{\mu\nu} \rangle \Big|_{FLRW} = a^{-4} \, \langle T_{\mu\nu} \rangle \Big|_{Minkowski} + \text{conformal anomaly}$$

$\Longrightarrow$ for a trace of the stress-energy tensor

$$\langle T^\mu_\mu \rangle \Big|_{FLRW} = a^{-4} \underbrace{\langle T^\mu_\mu \rangle \Big|_{Minkowski}}_{=0} + \frac{c}{24\pi^3} \underbrace{\left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)}_{= -12 \frac{(\dot{a})^2 \ddot{a}}{a^3}}$$

e.g., for $\mathcal{N} = 4$ $SU(N)$ SYM,

$$- \langle T^t_t \rangle \Big|_{FLRW} = \frac{1}{a(t)^4} \mathcal{E} + \frac{3N^2}{32\pi^2} \frac{(\dot{a})^4}{a^4}$$

$$\langle T^{\boldsymbol{x}}_{\boldsymbol{x}} \rangle \Big|_{FLRW} = \frac{1}{a(t)^4} P + \frac{N^2}{8\pi^2} \left\{ \frac{(\dot{a})^4}{4a^4} - \frac{(\dot{a})^2 \ddot{a}}{a^3} \right\}$$

$$\langle T^\mu_\mu \rangle \Big|_{FLRW} = a^{-4} \underbrace{\left(-\mathcal{E} + 3P \right)}_{=0} - \frac{3N^2}{8\pi^2} \frac{(\dot{a})^2 \ddot{a}}{a^3}$$

$\Longrightarrow$ Minkowski space-time thermal equilibrium states of $\mathcal{N} = 4$ SYM (strong coupling) of temperature T_0:

$$\mathcal{E}_0 = \frac{3}{8}\pi^2 N^2 T_0^4 \,, \qquad P_0 = \frac{1}{3}\mathcal{E}_0$$

$\Longrightarrow$ in FLRW cosmology,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4} \,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$

where $T(t)$ is the *effective* temperature

$$T(t) = \frac{T_0}{a(t)}$$

$\Longrightarrow$ Stress-energy tensor in FLRW is covariantly conserved:

$$0 = \langle \nabla^\mu T^\nu_\mu \rangle \qquad \Longleftrightarrow \qquad \frac{d\mathcal{E}(t)}{dt} + 3\frac{\dot{a}}{a}\ \left(\mathcal{E}(t) + P(t)\right) = 0$$

$\Longrightarrow$ entropy density

- In Minkowski space-time:

$$s_0 = \frac{\pi^2}{2} N^2 T_0^3$$

- Assuming the adiabatic expansion in FLRW, the co-moving entropy density, $s_{comoving}$,

$$s_{comoving} \equiv a(t)^3 s(t)$$

  is conserved:

$$\frac{d}{dt} s_{comoving} = 0 \qquad \Longrightarrow \qquad s_{comoving} = s_{comoving}\Big|_{t=0} = s_0$$

  $\Longrightarrow$

$$s(t) = \frac{\pi^2}{2} N^2 T(t)^3$$

- In expanding FLRW, with $a(t) \to \infty$ as $t \to \infty$,

$$\lim_{t\to\infty} s(t) = 0$$

$\Longrightarrow$ Let's rephrase the de Sitter entropy discussion in the language of the entropy current $\mathcal{S}^\mu$:

- A locally static observer has $u^\mu = (1, \mathbf{0})$
- The entropy current (in Landau frame $T^{\mu\nu}_{(1)} u_\nu = 0$) is

$$\mathcal{S}^\mu = s\ u^\mu$$

$$\Longrightarrow$$

$$\nabla \cdot S = \frac{1}{a(t)^3}\ \frac{d}{dt}\left(a(t)^3 s\right) = \frac{1}{a(t)^3}\ \frac{d}{dt} s_{comoving}(t)\ =\ 0$$

That is is why $\mathcal{N} = 4$ SYM (same is true for any conformal theory!) in de Sitter evolved to a trivial DFP

How would a non-trivial DFP arise?

- Imagine that

$$\lim_{t\to\infty} s(t) = s_{ent} \neq 0$$

This limit is natural to call the *vacuum entanglement entropy* density, hence $_{ent}$

- Then,

$$\lim_{t\to\infty} \left(\nabla \cdot \mathcal{S} \right) = 3\ H\ s_{ent}$$

where

$$H = \lim_{t\to\infty} \frac{d}{dt}\ \ln a(t)$$

$\Longrightarrow$ In strongly coupled non-conformal theories with holographic dual

$$s_{ent}\ >\ 0$$

$\Longrightarrow \mathcal{N}=4$ SYM in FLRW holographic perspective

$$S_{\mathcal{N}=4} = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5\xi\sqrt{-g}\left[R+\frac{12}{L^2}\right]$$

$$L^4 = \ell_s^4\ N g_{YM}^2\,, \qquad G_5 = \frac{\pi L^3}{2N^2}\,, \qquad 4\pi g_s = g_{YM}^2$$

$\Longrightarrow$ Consider general spatially homogeneous, time-dependent states:

$$ds_5^2 = 2dt\ (dr - A dt) + \Sigma^2\ d\boldsymbol{x}^2$$

$$A = A(t,r)\,, \qquad \Sigma = \Sigma(t,r)$$

$\Longrightarrow$ We are interested in spatially homogeneous and isotropic states of $\mathcal{N}=4$ SYM in FLRW, so the bulk metric warp approach the AdS boundary $r\to\infty$ as

$$\Sigma=\frac{a(t)r}{L}+\mathcal{O}(r^0)\,,\qquad A=\frac{r^2}{2L^2}+\mathcal{O}(r^1)$$

Indeed, as $r\to\infty$,

$$ds_5^2=\frac{r^2}{L^2}\underbrace{\left(-dt^2+a(t)^2d\boldsymbol{x}^2\right)}_{\text{boundary FLRW}}+\cdots$$

$\Longrightarrow$ Given the metric ansatz, we can derive derive EOMs (without loss of generality we set $L = 2$):

$$
\begin{aligned}
0 &= (d_+\Sigma)' + 2\Sigma' \, d_+ \ln\Sigma - \frac{\Sigma}{2} \\
0 &= A'' - 6(\ln\Sigma)' \, d_+ \ln\Sigma + \frac{1}{2} \\
0 &= \Sigma'' \\
0 &= d_+^2\Sigma - 2A\Sigma' - (4A\Sigma' + A'\Sigma) \, d_+ \ln\Sigma + \Sigma A
\end{aligned}
$$

where

$$
{}' = \frac{\partial}{\partial r}, \qquad \dot{} = \frac{\partial}{\partial t}, \qquad d_+ = \frac{\partial}{\partial t} + A\frac{\partial}{\partial r}
$$

$\Longrightarrow$ These equations can be solve in all generality for arbitrary $a(t)$:

$$A = \frac{(r+\lambda)^2}{8} - (r+\lambda)\,\frac{\dot{a}}{a} - \dot{\lambda} - \frac{r_0^4}{8a^4(r+\lambda)^2}\,,$$

$$\Sigma = \frac{(r+\lambda)a}{2}$$

where

- r_0 is a single constant parameter
- $\lambda(t)$ is an arbitrary function - the leftover diffeomorphism of the 5d

gravitational metric reparametrization $r \to \bar{r} = r - \lambda(t)$:

$$A(t,r) \to \bar{A}(t,\bar{r}) = A(t, r+\lambda(r)) - \dot{\lambda}(t)$$

$$\Sigma(t,r) \to \bar{\Sigma}(t,\bar{r}) = \Sigma(t, r+\lambda(t))$$

$\Longrightarrow$

$$ds_5^2 \;\Longrightarrow\; d\bar{s}_5^2 \qquad = 2dt\,\left(d\bar{r} - \bar{A}dt\right) + \bar{\Sigma}^2\; d\boldsymbol{x}^2$$

$\Longrightarrow$ Identifying

$$\frac{r_0}{2} \equiv T_0$$

$\Longrightarrow$ from holographic computation of the boundary stress energy tensor,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$
$$T(t) = \frac{T_0}{a(t)}$$

Precisely as expected from the Weyl transformation of the thermal state from Minkowski to FLRW!

$\Longrightarrow$ Holography buys us more:

- Chesler-Yaffe pioneered numerical studies of EF metrics:

$$ds_5^2 = 2dt\ (dr - Adt) + \Sigma^2\ d\boldsymbol{x}^2$$

- such metrics has an **apparent horizon** (AH) at r_{AH}

$$d_+\Sigma\Big|_{r=r_{AH}} = 0 \qquad \Longrightarrow \qquad r_{AH} = \frac{r_0}{a(t)} - \lambda(t)$$

- causal dependence **must** include

$$r \in [r_{AH}, +\infty)$$

- region

$$r < r_{AH}$$

  is causally disconnected from the holographic dynamics and **must be** excised

- AH is a dynamical horizon
- $$\underbrace{\left.\frac{\Sigma^3}{4G_5}\right|_{r=r_{AH}}}_{\text{comoving Bekenstein entropy of the AH}} = \frac{N^2 r_0^3}{128\pi}$$

$$= \underbrace{s_{comoving}}_{\text{SYM comoving entropy density in FLRW}} = a(t)^3 s(t) = \frac{\pi^2}{2} N^2 T_0^3$$

Precisely as expected from the CFT arguments!

$\Longrightarrow$ Nontrivial DFP

- The model:

$$S_4 = \frac{1}{2\kappa^2} \int_{\mathcal{M}_4} dx^4 \sqrt{-\gamma} \left[R + 6 - \frac{1}{2} (\nabla\phi)^2 + \phi^2 \right]$$

- ϕ is dual to $\mathcal{O}_\phi$,

$$L^2 m_\phi^2 = -2 \qquad \Longrightarrow \qquad \dim(\mathcal{O}_\phi) = 2$$

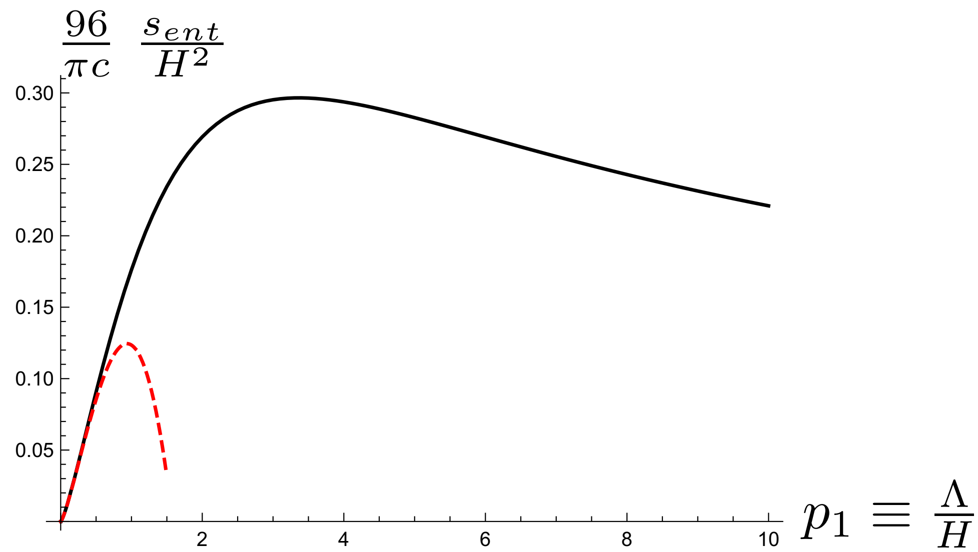
- source terms for the gravitational evolution:
  - the boundary metric is dS_3,

$$ds_3^2 = -dt^2 + e^{2Ht} d\boldsymbol{x}^2$$

  - mass scale Λ of the boundary QFT_3,

$$\phi = \frac{\Lambda}{r} + \mathcal{O}(r^{-2})$$

Recall: $s_{ent} = \lim_{t\to\infty} s(t)$

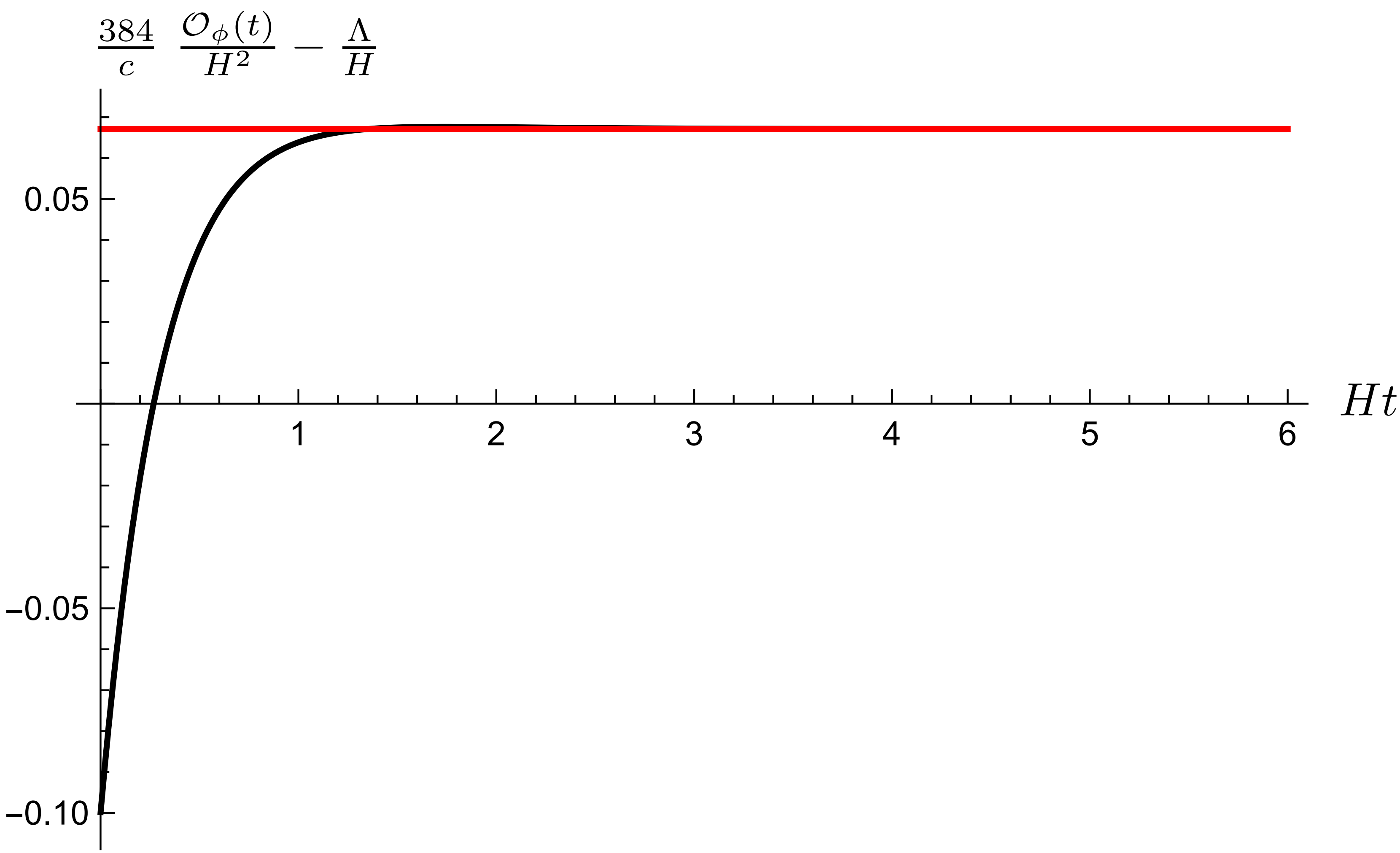


$$\frac{\kappa^2}{2\pi}\ \frac{s_{ent}}{H^2} = \frac{1}{6} 6^{1/3}\ p_1^{4/3} - \frac{1}{12}\ p_1^2 - \frac{5}{216} 6^{2/3}\ p_1^{8/3} - \frac{3359}{311040} 6^{1/3}\ p_1^{10/3} + \mathcal{O}\left(p_1^4\right)$$

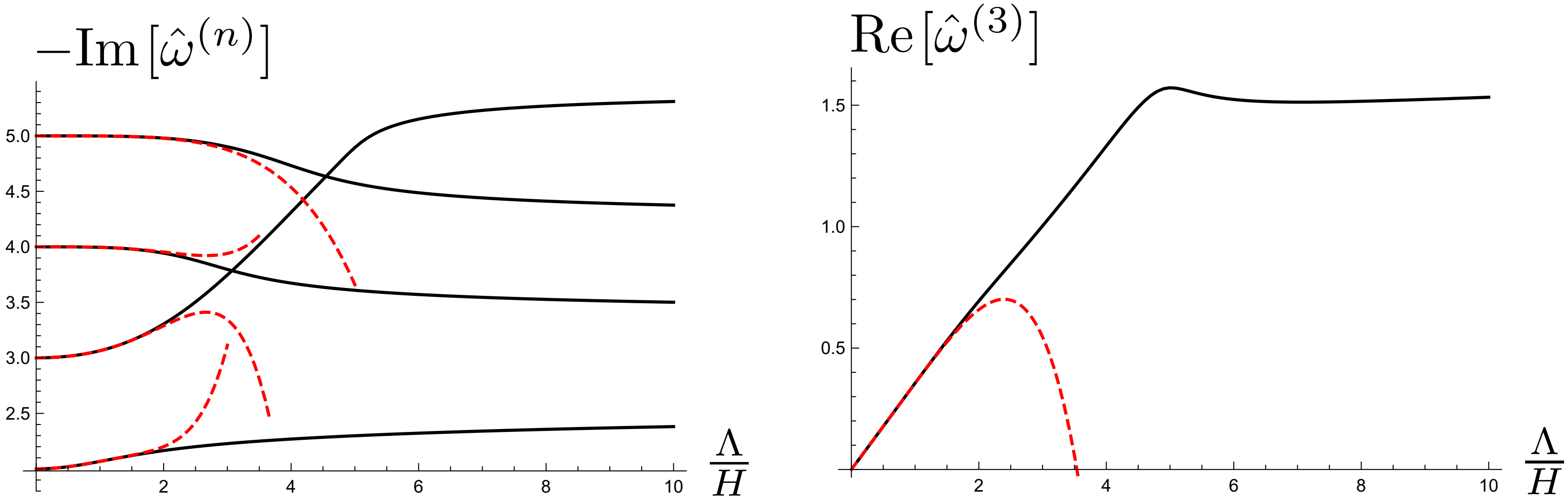
Important:

$$\frac{d(a^2 s)}{dt} = \frac{2\pi}{\kappa^2}\ (\Sigma^2)'\ \frac{(d_+\phi)^2}{\phi^2+6}\bigg|_{r=r_{AH}} \ \geq\ 0$$

$\Longrightarrow$ DFP as a late-time attractor:

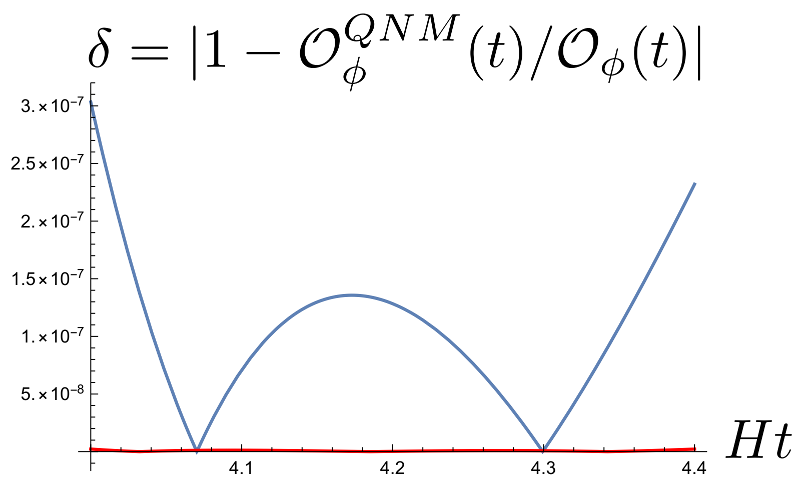


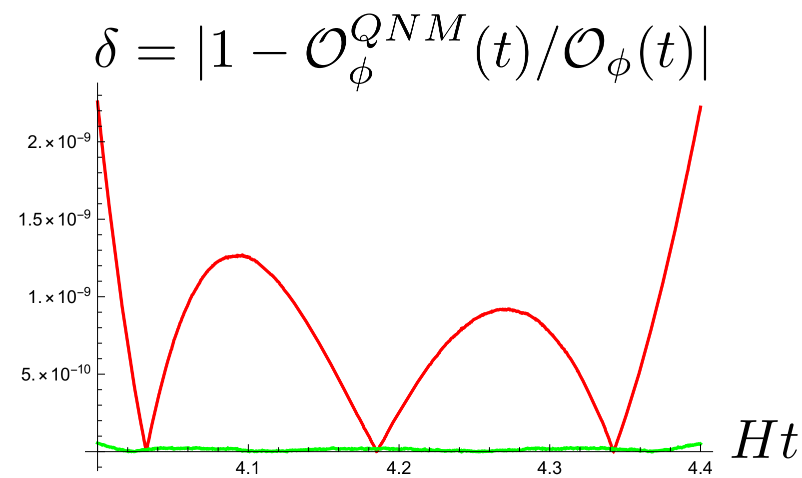
$\Longrightarrow$ Spectrum of DFP fluctuations (*aka* QNMs):



$$\hat{\omega} \equiv \frac{\omega}{H}$$

$\Longrightarrow$ Approach to DFP via 'QNMs':





$$\mathcal{O}_\phi^{QNM}(t) = \mathcal{O}_\phi^{DFP} + \sum_{\text{QNM spectrum}} \mathcal{A} e^{-i(\hat{\omega} H t + \text{phase})}$$

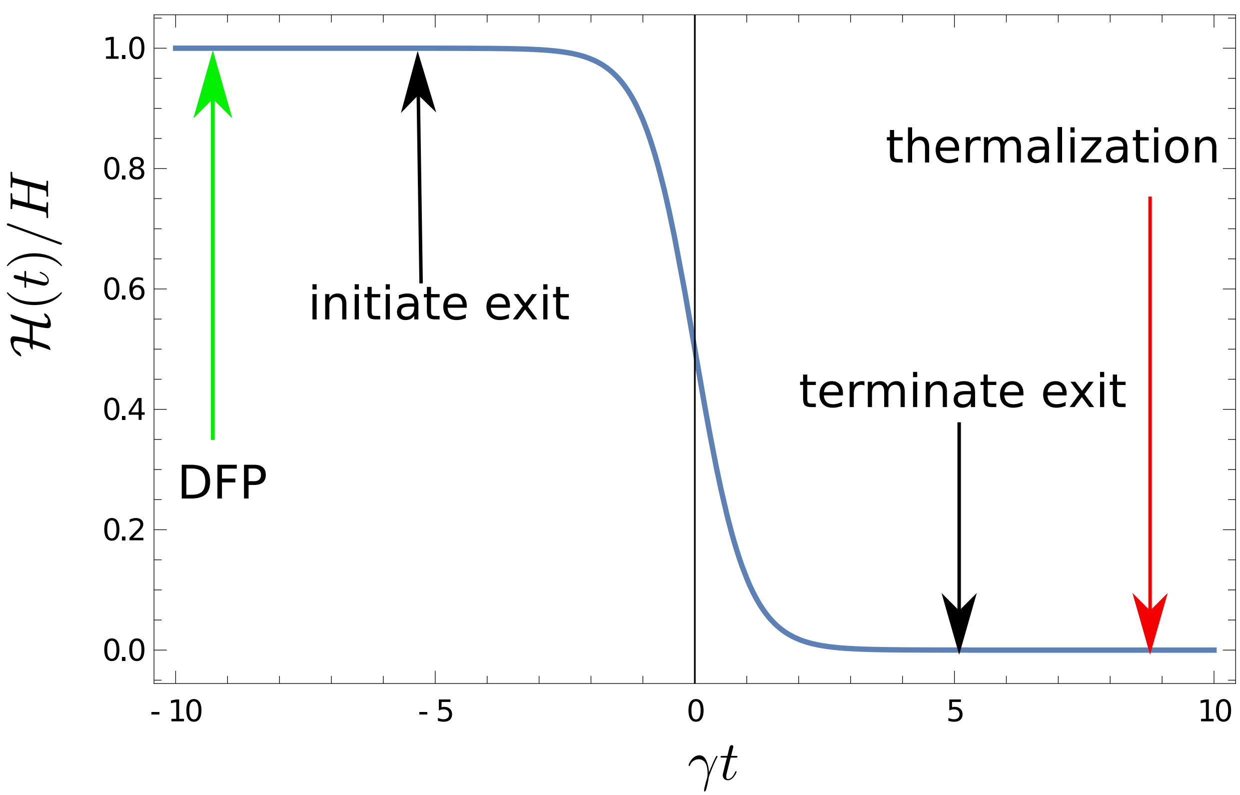
- blue: $n = 2$ QNM only
- red: $n = 2, 3$ QNMs
- green: $n = 2, 3, 4$ QNMs

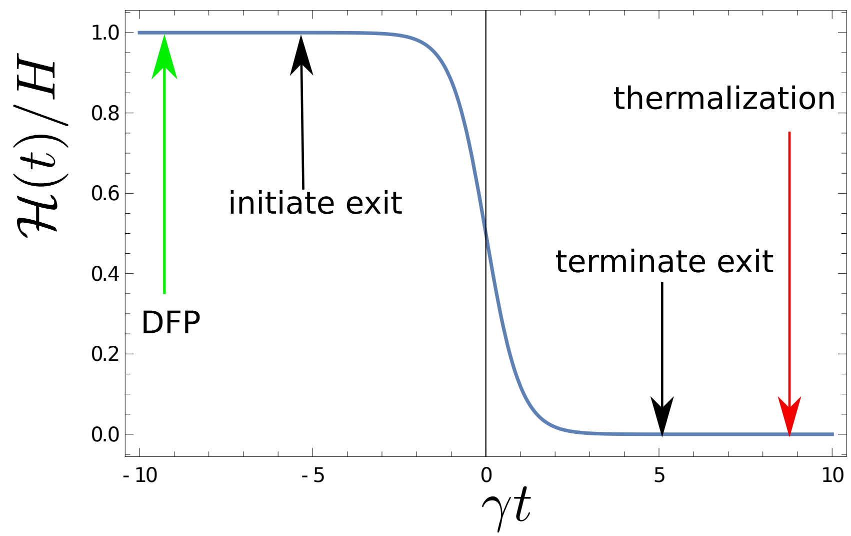
$\Longrightarrow$ Holographic gravitational reheating:

- $$\frac{d(a^2 s)}{dt} = \frac{2\pi}{\kappa^2} \, (\Sigma^2)' \, \frac{(d_+\phi)^2}{\phi^2+6}\bigg|_{r=r_{AH}} \geq 0$$

- consider a scale factor $a(t)$ with a Hubble parameter:

$$\mathcal{H}(t) \equiv \frac{\dot{a}}{a} = \frac{H}{1+\exp(2\gamma t)}$$



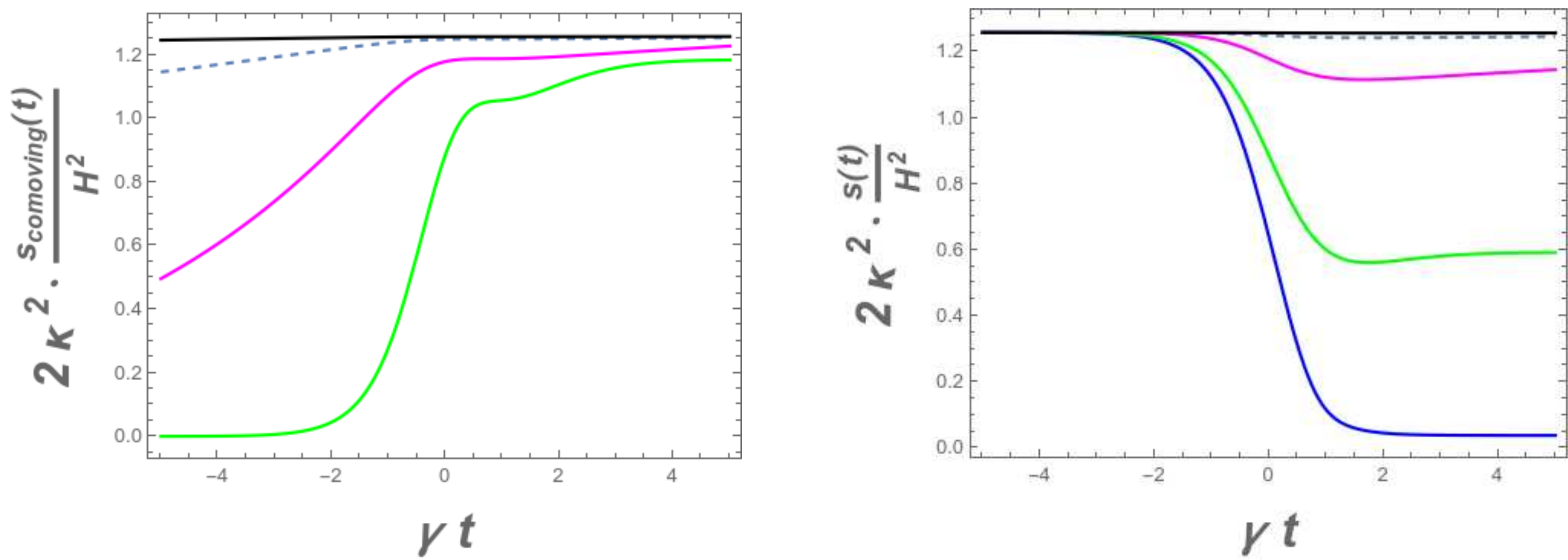


- in the *fast* inflationary exit

$$\ln \frac{a_t}{a_i} = \int_{-5/\gamma}^{5/\gamma} dt \; \mathcal{H}(t) = \frac{5H}{\gamma} \to 0 \qquad \text{as} \qquad \frac{H}{\gamma} \to 0$$

- $$\frac{d(a^2 s)}{dt} \geq 0 \implies s_t a_t^2 \geq s_i a_i^2 \implies s_t \geq s_i \left(\frac{a_i}{a_t}\right)^2 \underbrace{\approx}_{H/\gamma \to 0} s_{ent}^{DFP}$$

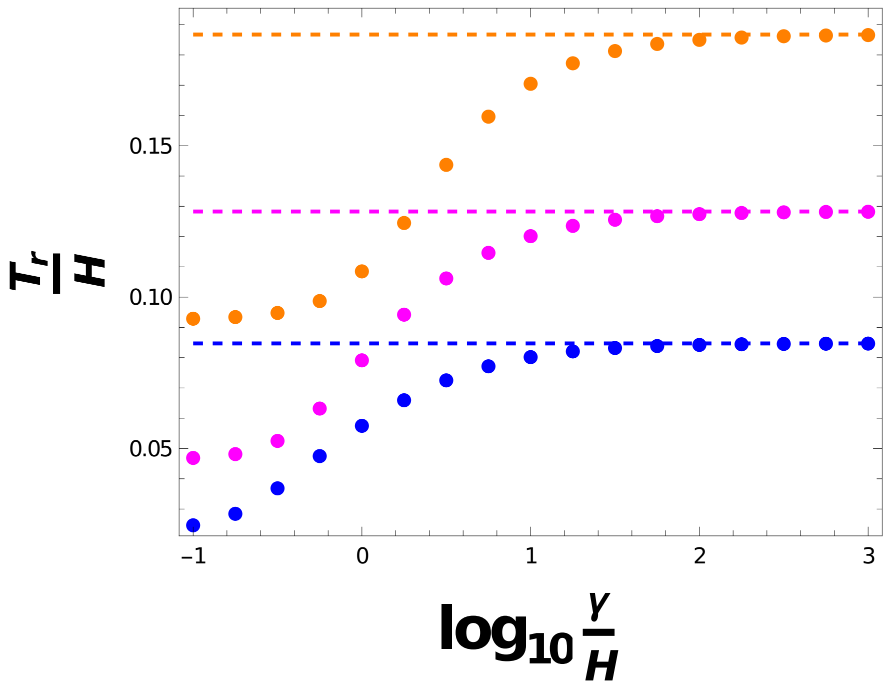
- $$s_{ent}^{DFP} \underbrace{\longrightarrow}_{\text{with further thermalization}} s(t)\Big|_{t\to+\infty} = s_{thermal} \geq s_{ent}^{DFP}$$

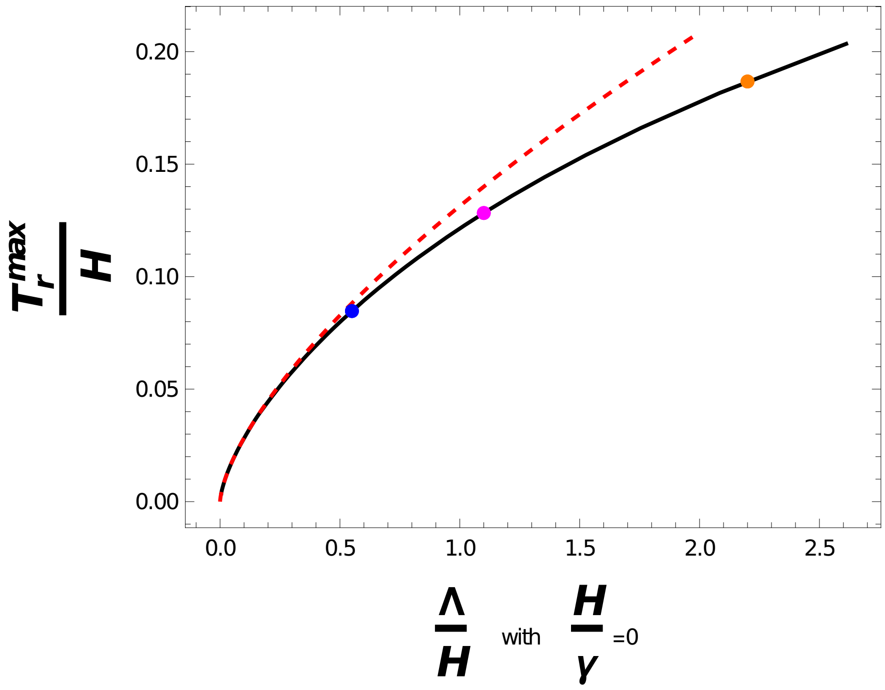


$$\log_{10}\frac{\gamma}{H} = \left\{ \underbrace{-1}_{\text{blue}}, \underbrace{0}_{\text{green}}, \underbrace{1}_{\text{magenta}}, \underbrace{2}_{\text{grey dashed}}, \underbrace{3}_{\text{black}} \right\}$$

$\Longrightarrow$ evolve until the inflationary exit state thermalizes at T_r: $tT_r \sim 1$

$$\frac{\Lambda}{H} = \Big\{ \underbrace{0.55}_{\text{orange}} , \underbrace{1.1}_{\text{magenta}} , \underbrace{2.2}_{\text{blue}} \Big\}$$





$$\frac{T_r^{max}}{H} \approx \frac{3^{2/3}}{2^{7/3}\pi}\left(\frac{\Lambda}{H}\right)^{2/3}, \qquad \text{as} \qquad \frac{\Lambda}{H} \to 0$$

Conclusions:

- A new concept of DFP
- Massive QFT in de Sitter has finite physical entropy density s_{ent}
- In the exit from inflation s_{ent} can be harvested — this solves the problem of the initial Hot Big Bang entropy without the inflaton reheating

$\Longrightarrow$ To do:

- understanding of weakly coupled DFP is missing
- finite coupling, finite-N corrections
- formal: relation of s_{ent} to "simple entropy" of Engelhardt-Wall
- other examples (not dS flows) of DFPs