Simplicity of domain wall velocities at strong coupling

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RJ, M. Järvinen, H. Soltanpanahi, J. Sonnenschein, PRL '22 [2205.06274]
extended hydro: RJ, M. Järvinen, J. Sonnenschein, JHEP '21 [2106.02642]

Outline

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Goal:

Understand bubble wall velocities at strong coupling...

Introduction

- At a 1^{st} order phase transition $T = T_c$, we can have domains of coexisting phases separated by domain walls
- The pressures on both sides are balanced and the domain wall can be static...

Question: What happens when we move away from $T = T_c$?

- This can occur for nucleated bubbles of a stable phase within an supercooled medium
- At an interface between phases away from $T = T_c$
- Or an interface between phases at different temperatures

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- The pressures on both sides are not balanced so one would expect accelerated motion...

- ...but this does not happen — the domain wall ultimately moves with a constant velocity...

- **Common lore:** friction in the second phase balances the net force — challenging to calculate...

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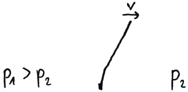

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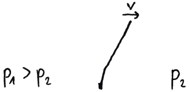

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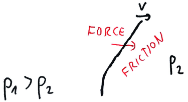

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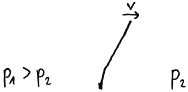

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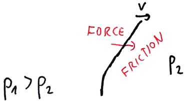

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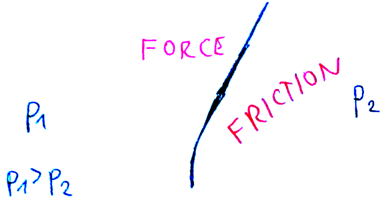

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Perform holographic simulations and read off the pressure profile...

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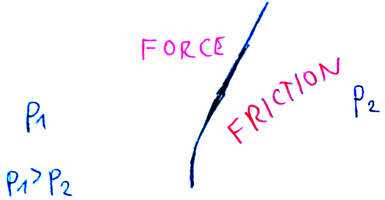

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   full holographic simulation

2. Witten model in 3D – confinement/deconfinement transition

   use simplified model
   (basically extended hydrodynamics)

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- We use a gravity+scalar field system in $D = 3 + 1$ bulk dimensions

$$V(\Phi) = -6\cosh\left(\frac{\Phi}{\sqrt{3}}\right) - 0.2\,\Phi^4$$

- The theory exhibits two **deconfined** phases

studied in RJ, Jankowski, Soltanpanahi 1704.05387, +Belantuono 1906.00061

- Since there is a horizon in both phases it is much easier to setup a numerical relativity computation
- Consequently, we can study directly the gravitational holographic description... includes all non-equilibirum/dissipative effects!

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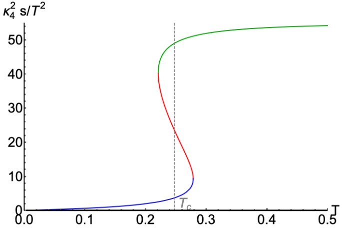

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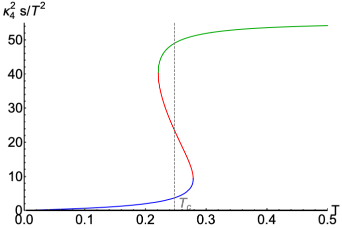

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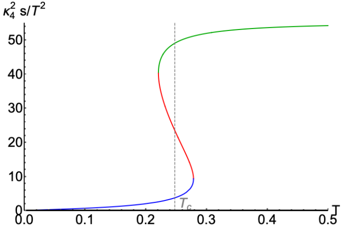

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Confinement/deconfinement transition – the Witten model

- As an example of a holographic theory with a 1^{st} order confinement/deconfinement phase transition we use (a $d = 3$ variant of) the Witten model of '98
- A domain wall solution interpolating between confining and deconf. phase was found numerically Aharony, Minwalla, Weisman '05
- Due to different topologies for the two phases, incorporating time dependence numerically is extremely nontrivial...

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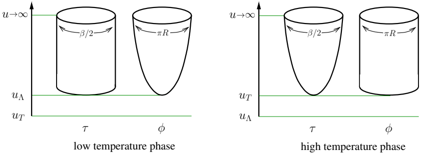

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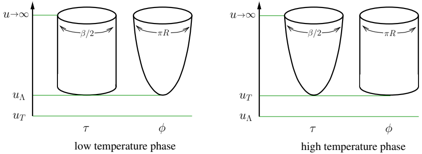

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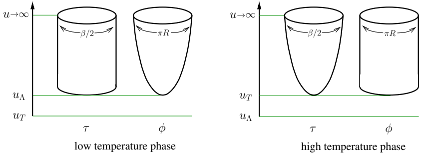

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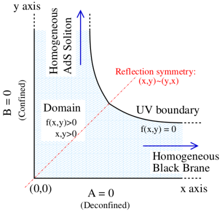

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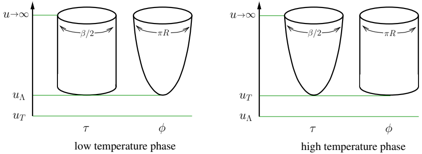

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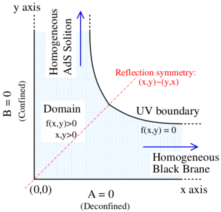

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Interlude: an extension of hydrodynamics

Use instead an effective boundary description:

RJ, M. Järvinen, J. Sonnenschein 2106.02642

1. Should incorporate hydrodynamics for the deconfined phase

$$T_{\mu\nu}^{deconf} = p_{hydro}(T)\,(\eta_{\mu\nu} + 4u_\mu u_\nu)$$

for the $d = 3$ Witten model $p_{hydro}(T) = T^4$

2. Should incorporate confining vacuum

$$T_{\mu\nu}^{conf} = \eta_{\mu\nu}$$

ignoring the compactified ϕ circle...

3. Should reasonably describe the AMW domain wall...

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- $\frac{1}{2}(T_{tt} + T_{\phi\phi})$ from the numerical holographic AMW solution

- Excellent fit by

$$1 - \gamma(x) = \frac{1}{2}\left(1 + \tanh\frac{q_* x}{2}\right)$$

$\gamma = 0$ deconf. phase, $\gamma = 1$ conf. phase

- The field γ looks like a QNM with $\omega = 0$ and imaginary k

(c.f. Sonner, Withers)

- **Add γ to the hydrodynamic degrees of freedom!**
- The $\tanh x$ profile could come from a scalar field Lagrangian with a quartic potential

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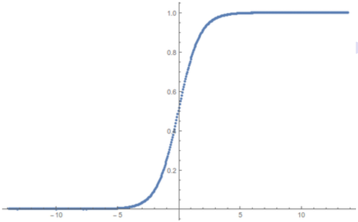

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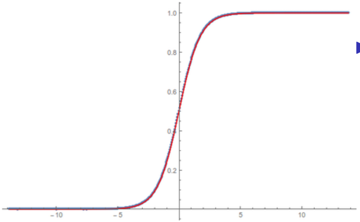

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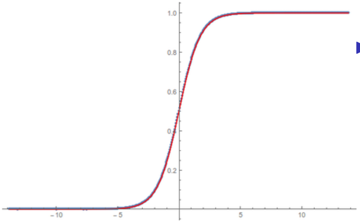

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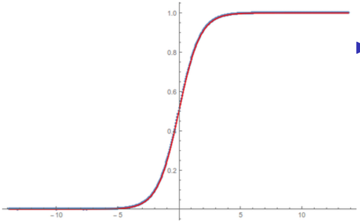

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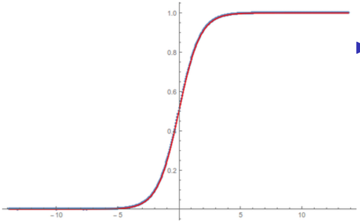

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How to couple γ to hydrodynamics?

- We would like to have

$$T_{\mu\nu} \sim (1-\gamma)\, T_{\mu\nu}^{deconf} + \gamma\, T_{\mu\nu}^{conf} + \underbrace{T_{\mu\nu}^{\Sigma}(\gamma, u^\mu, T)}_{surface\ \ tension}$$

- Use a Lagrangian formulation to get the $T_{\mu\nu}$...
- The first term can be obtained using an action formulation for hydrodynamics of Haehl, Loganayagam, and Rangamani

$$\mathcal{L} = (1-\gamma) p_{hydro}(T) + \gamma + \mathcal{L}(\gamma, T)$$

  which would lead to the above energy-momentum tensor...
- The Lagrangian $\mathcal{L}(\gamma, T)$ should be essentially *scalar field + quartic potential*
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Interlude: an extension of hydrodynamics

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close to $T = T_c = 1$

with

This description (option B) promotes the "order parameter" characterizing the phase to a dynamical field and couples it in a natural way to hydrodynamics...

Equations of motion:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \qquad \text{for } u^\mu,\ T \\ EOM(\mathcal{L}) &= 0 \qquad \text{for } \gamma \end{aligned}$$

This framework seems to be very general...

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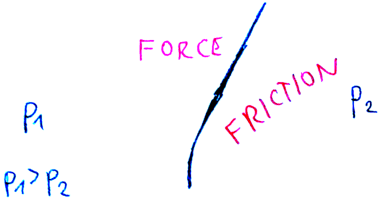

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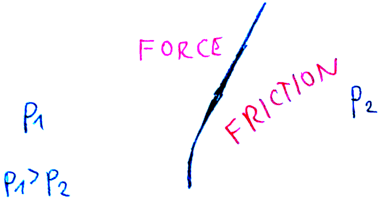

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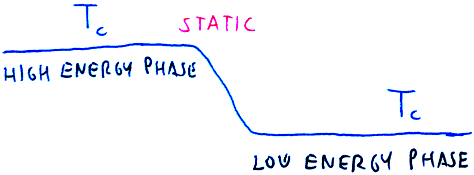
Domain wall velocity – simplest scenario



- We increase the temperature of the high energy phase...
- The initial conditions can be easily constructed as

$$g_{\mu\nu}(x,z,t=0) = \frac{1}{2}(1-\tanh q_* x)\, g_{\mu\nu}^{HIGH}(z) + \frac{1}{2}(1+\tanh q_* x)\, g_{\mu\nu}^{LOW}(z)$$

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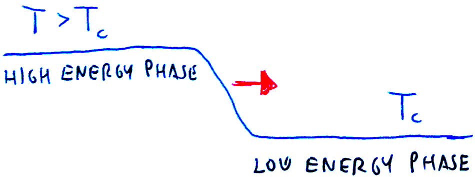

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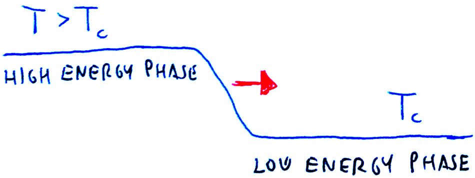

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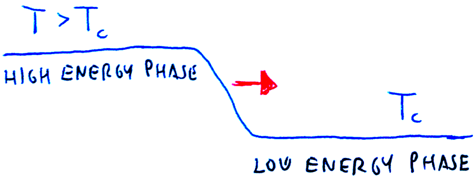

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Spacetime pressure profiles

We obtain a similar picture both in

1. the simplified treatment of the Witten model
2. the full holographic simulations of the nonconformal gravity+scalar model

Key features:

- The large pressure difference appears **away** from the domain wall
- The change in pressures occurs in the high energy density phase
  $\longrightarrow$ **hydrodynamic description**
- The pressure is essentially constant across the domain wall, and very close to $p(T_c)$...

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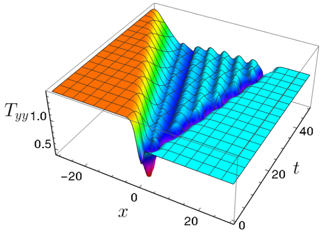

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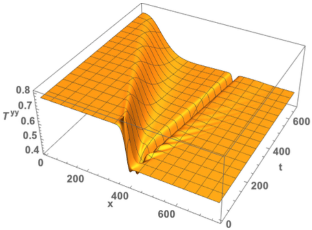

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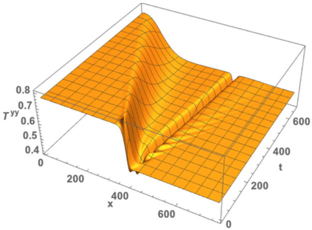

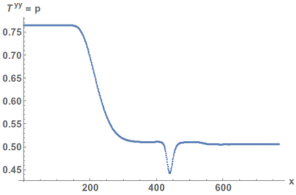

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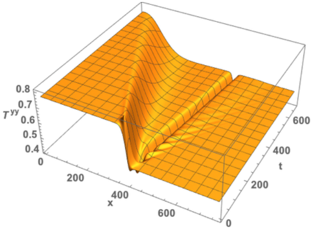

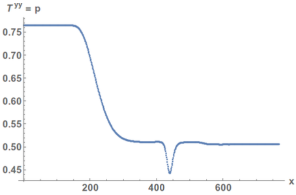

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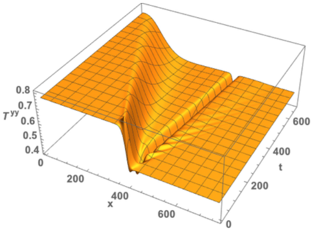

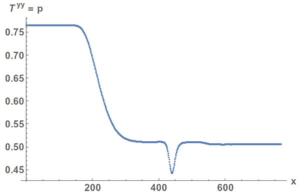

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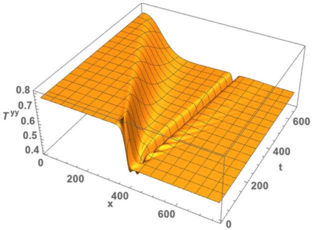

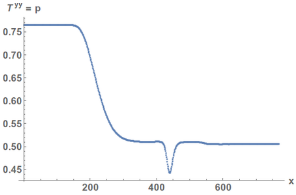

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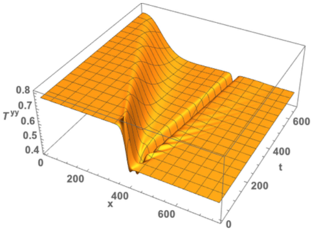

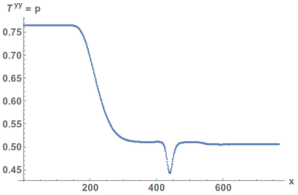

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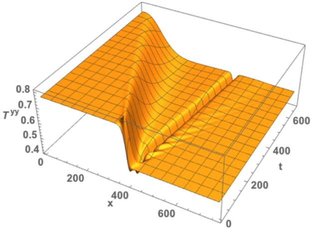

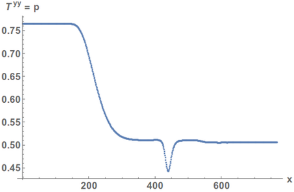

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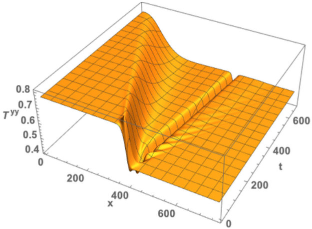

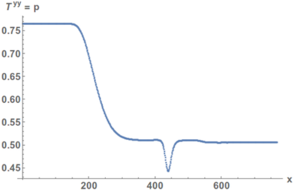

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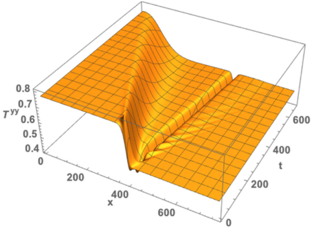

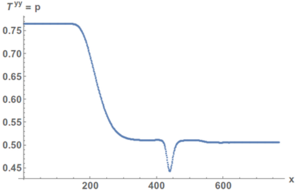

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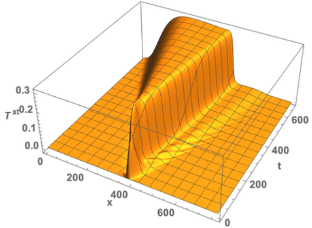

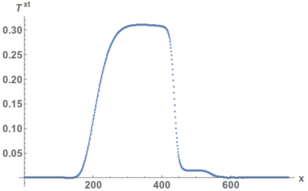

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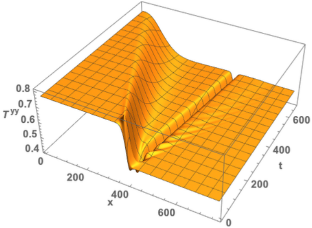

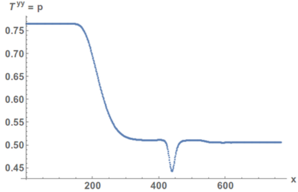

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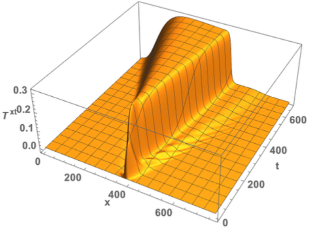

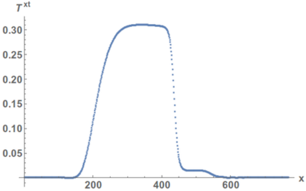

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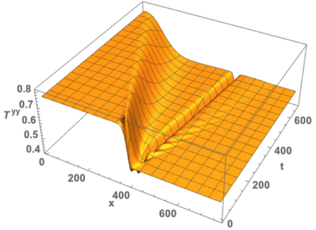

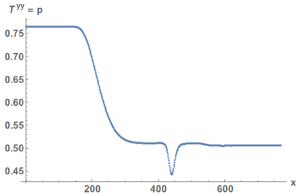

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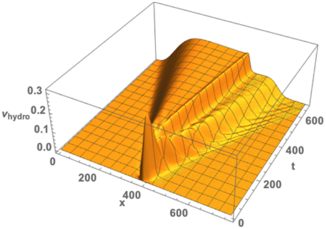

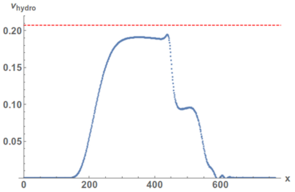

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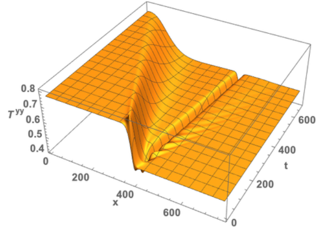

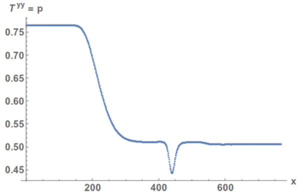

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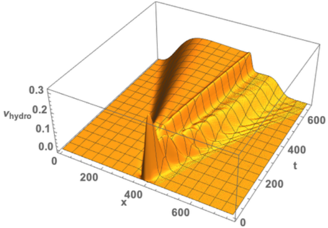

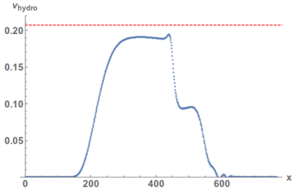

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- The velocity of the plasma close to the domain wall very close to the domain wall velocity...

Why $v_{hydro} \simeq v_{domain\ wall}$?

- Consider the rest frame of the domain wall between high-energy and low-energy phases $v_{hydro} = v_H + v_{domain\ wall}$
- Conservation of energy-momentum links the respective hydrodynamic parameters on both sides of the domain wall

Gyulassy, Kajantie et.al. '84, Espinoza et.al. 1004.4187

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- Since $v_{domain\ wall} \simeq v_{fluid}$, we may access the domain wall velocity through a hydrodynamic computation!
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Linearized approximation

$$p = p_{ref} + \delta p \qquad u^\mu = (\cosh\alpha, \sinh\alpha, 0)$$

then

$$\delta p = f(x + c_s t) \qquad \alpha = -\frac{f(x + c_s t)}{(\varepsilon_{ref} + p_{ref}) c_s} + const$$

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How good is the linearized approximation?

- For moderate Δp we see a dependence on the reference point used for the linearized approximation
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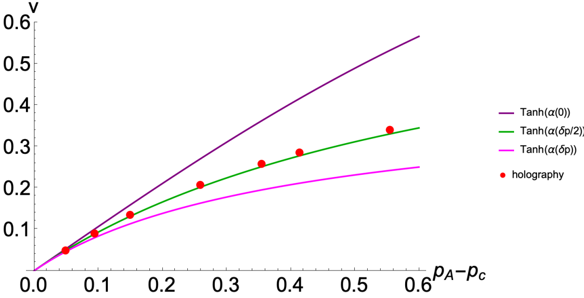

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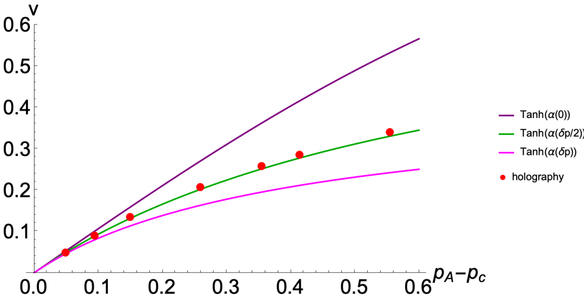

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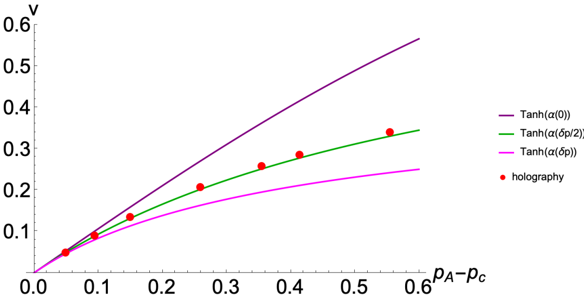

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Nonlinear treatment

- A nonlinear analog of the linearized hydrodynamic wave moving in **one** direction is a so-called **simple wave** (c.f. Landau, *Fluid mechanics*)
- One assumes that all hydrodynamic quantities are functions of a single variable (e.g. pressure)
- Then one gets

$$v_{domain\ wall} = \tanh \int_{p_c}^{p_A} \frac{1}{(\varepsilon + p) c_s} dp = \tanh \int_{T_c}^{T_A} \frac{1}{T c_s} dT$$

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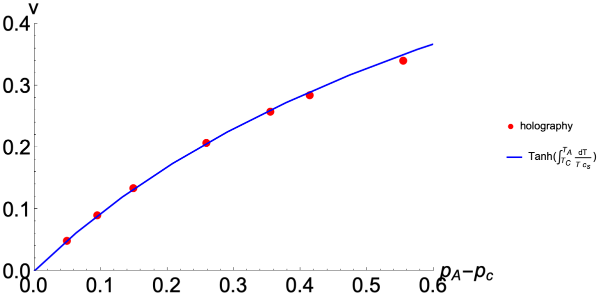

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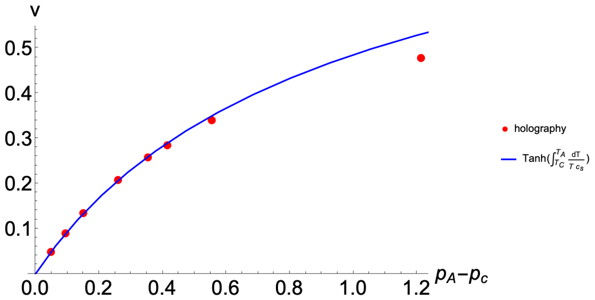
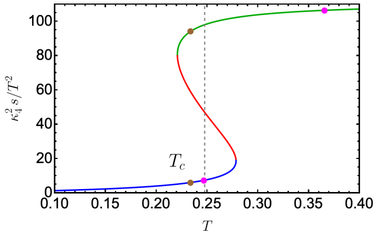


Second scenario: nucleated bubbles in a supercooled phase

velocity profile

- The small bubble of the stable low energy phase is on the left
- The environment is the supercooled high energy phase
- Inside the bubble the fluid is eventually at rest...
- There is a hydrodynamic “wave” travelling in front of the expanding bubble...

Second scenario: nucleated bubbles in a supercooled phase



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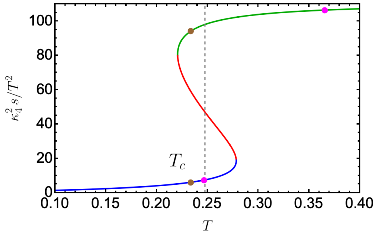

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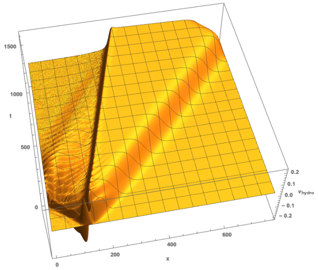

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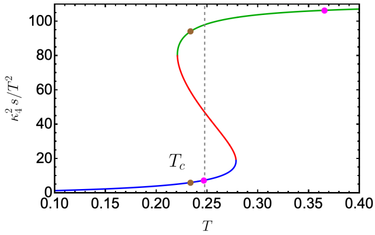

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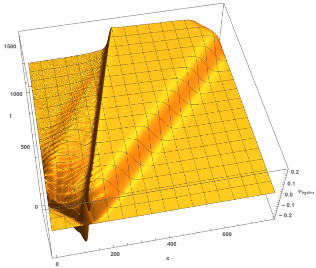

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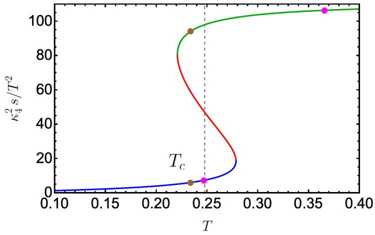

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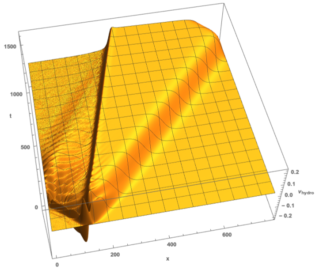

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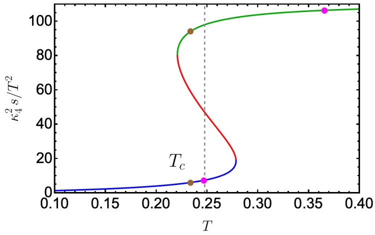

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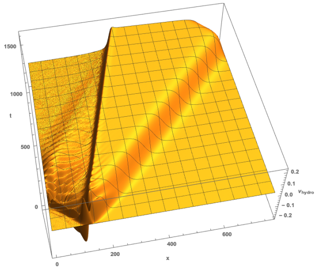

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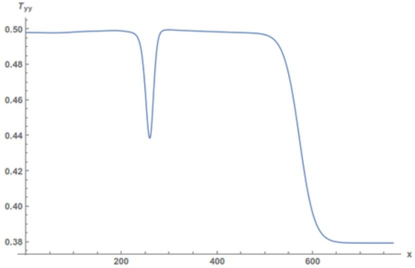

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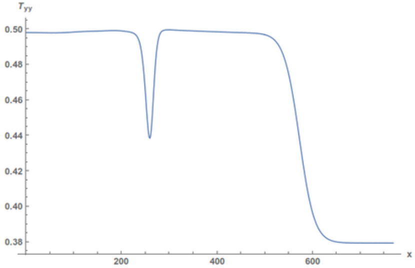

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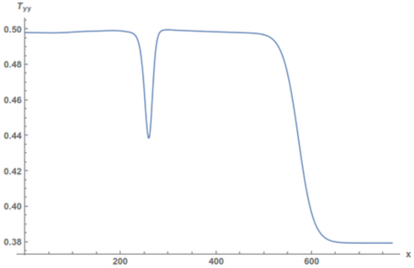

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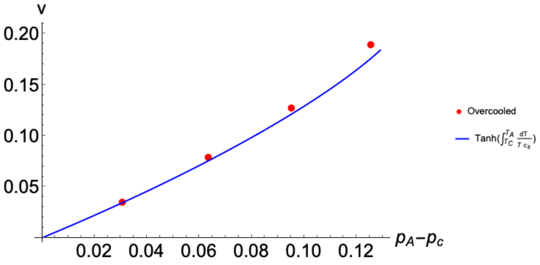

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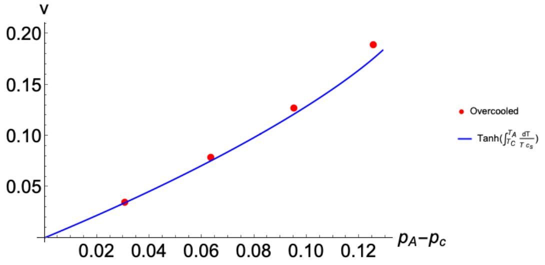

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- Recall from the velocity profile that the fluid in the bubble was practically at rest...
- This means that the velocity of the low energy phase in the domain wall rest frame is $v_L = -v_{domain\ wall}$
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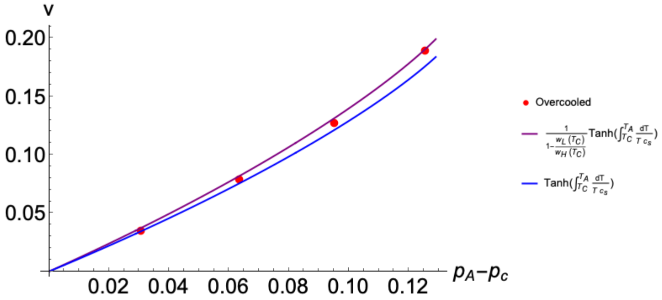
Nucleated bubble wall velocity



We also checked our formula with the numerical simulations of

Bea, Mateos et.al. 2104.05708

– a 5D gravity+scalar system

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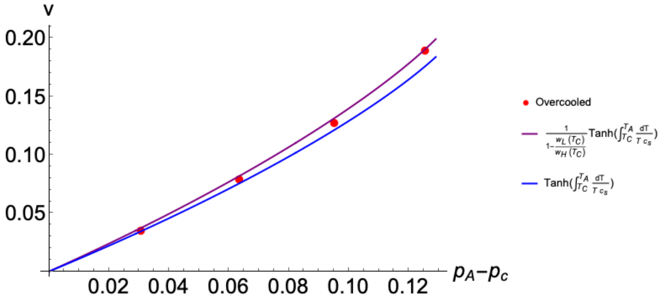

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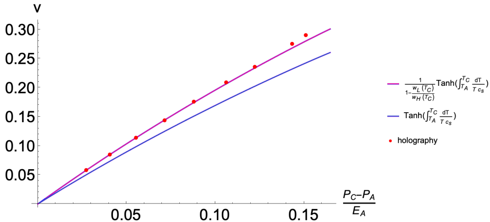

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- ... but **within a hydrodynamic wave** in the high entropy phase
- The fluid velocity in the vicinity of the domain wall is close to the fluid velocity
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