Heavy-ion phenomenology vs causality, attractors, non-hydro modes et al

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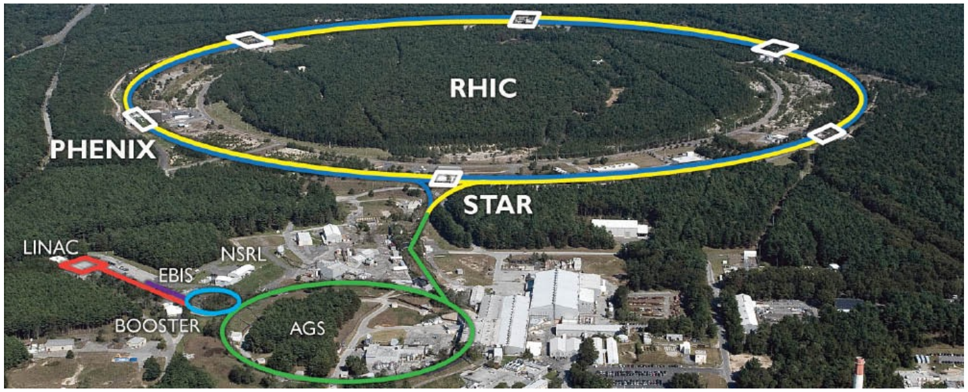
The Many Faces of Relativistic Fluid Dynamics
Kavli Institute for Theoretical Physics
UC Santa Barbara

Relativistic Heavy Ion Collider (RHIC)
[Brookhaven National Lab, Long Island, NY]



Large Hadron Collider (LHC)
[CERN, Geneva, Switzerland/France]

WHAT IS HEAVY-ION PHENOMENOLOGY

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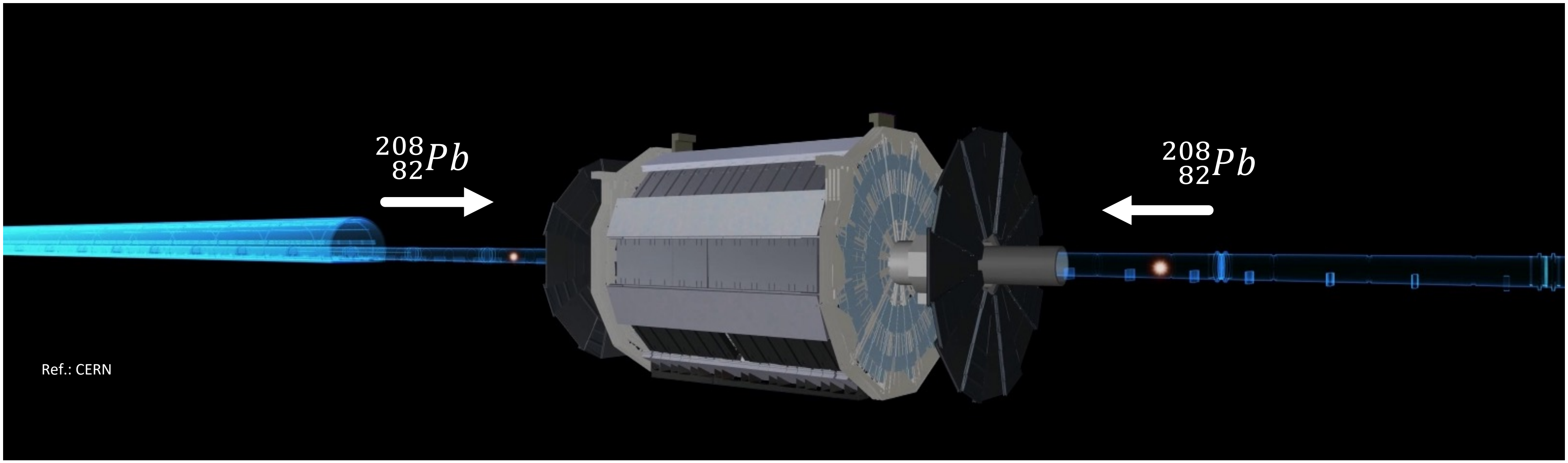

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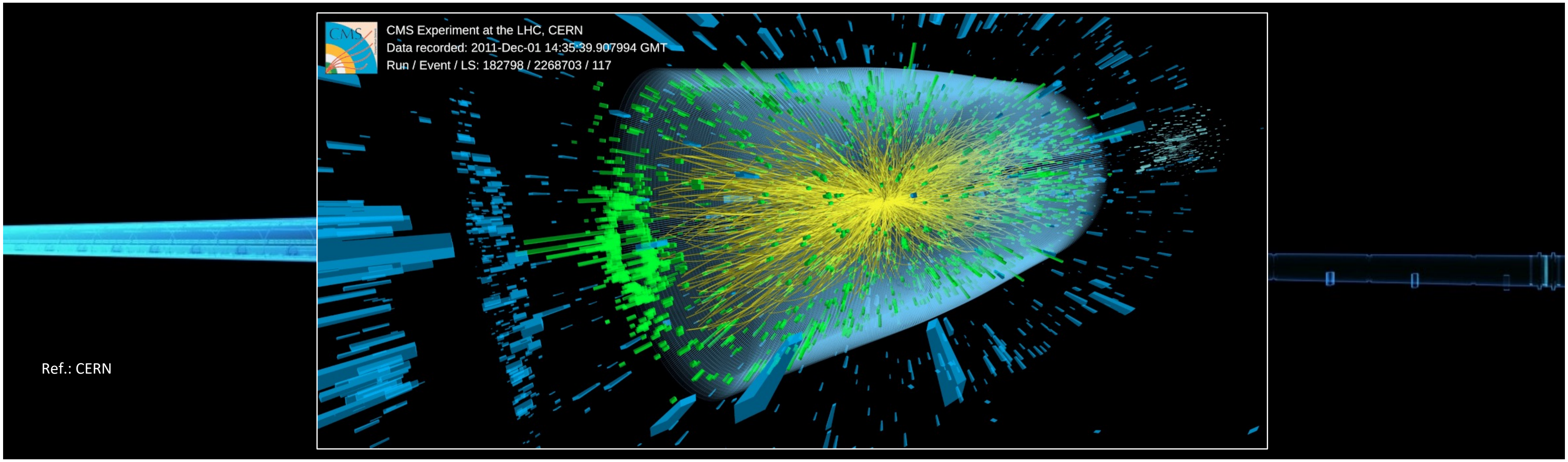
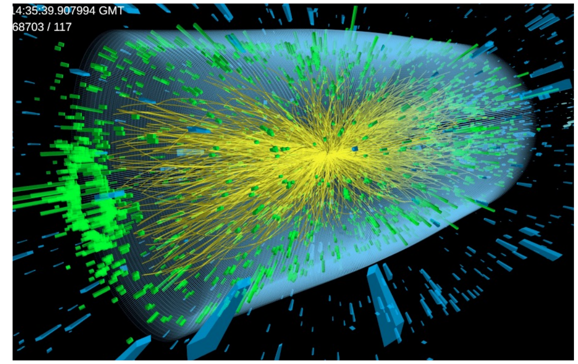
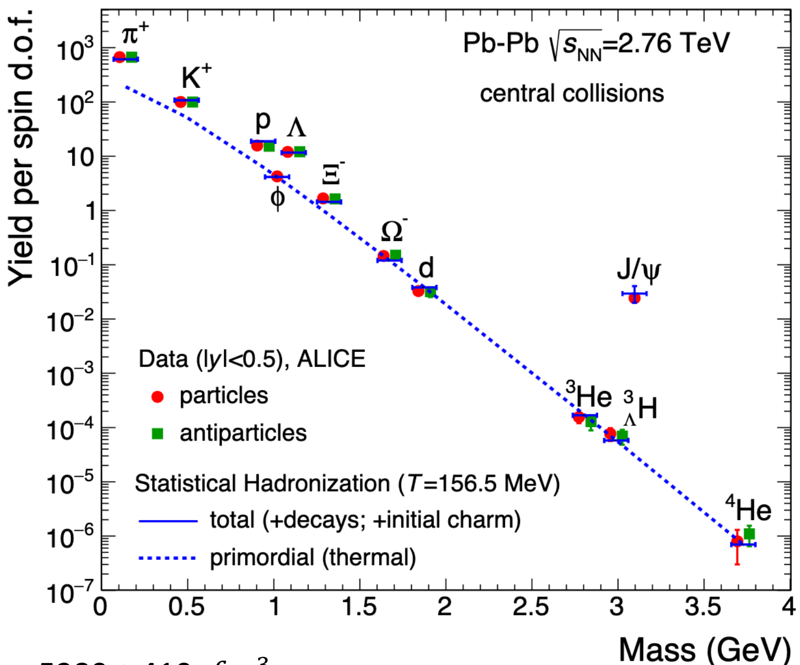


WHAT IS HEAVY-ION PHENOMENOLOGY

Highlights from measurements



- Multiplicity of hadrons



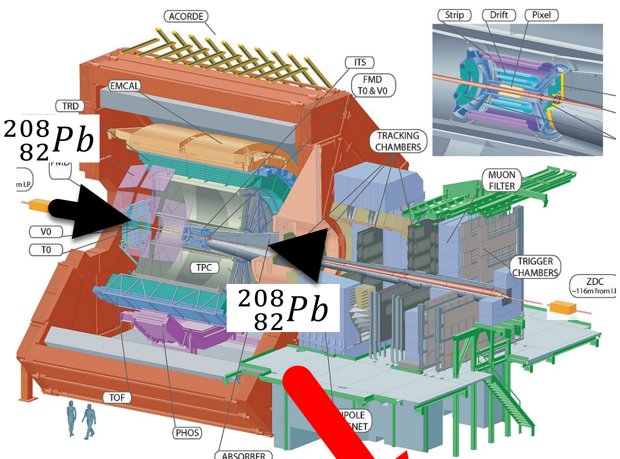
Volume = 5280 ± 410 fm^3

Ref.: Andronic, Braun-Munzinger, Redlich, Stachel (2017) Nature

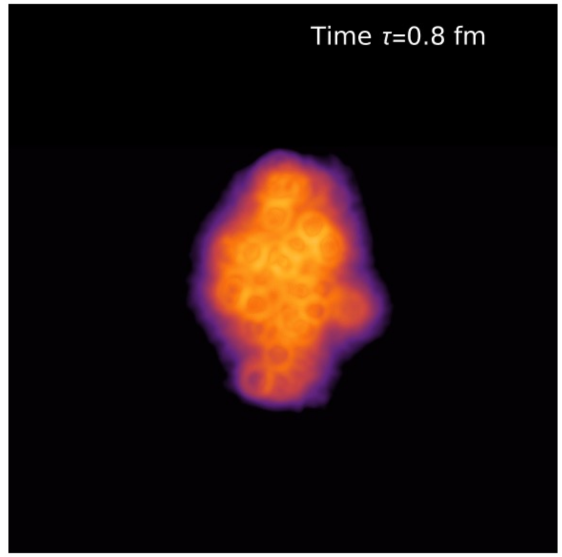
- High transverse momentum photon/Z/$W^{\pm}$/jets

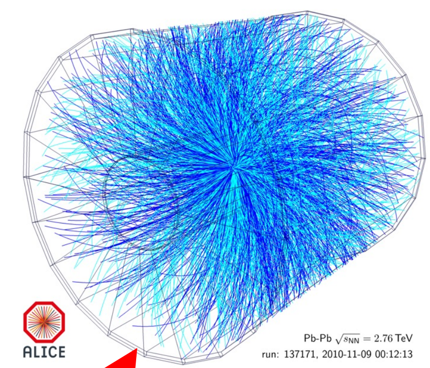
$d\sigma_{AA\to\gamma/Z/W} = N_{bin}\, d\sigma_{pp\to\gamma/Z/W+}$

Heavy-ion collisions

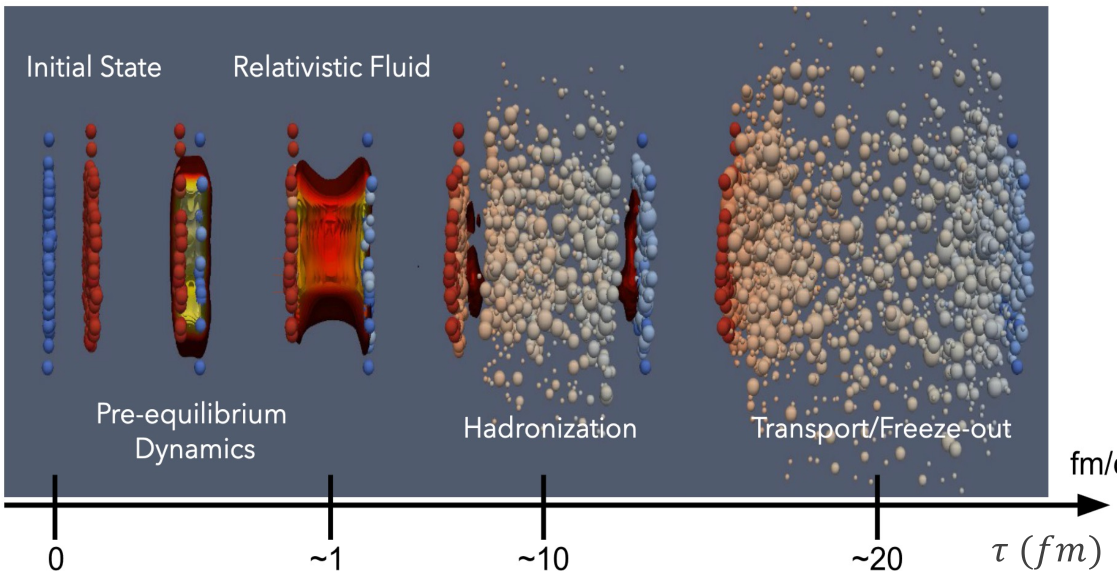


Ref.: ALICE, CERN





Ref: MADAI collaboration, Hannah Elfner and Jonah Bernhard

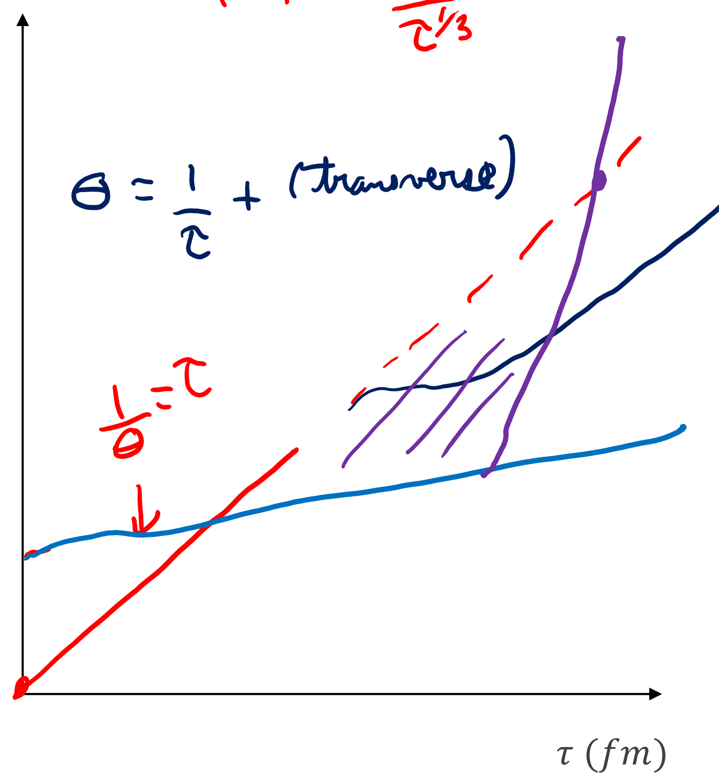


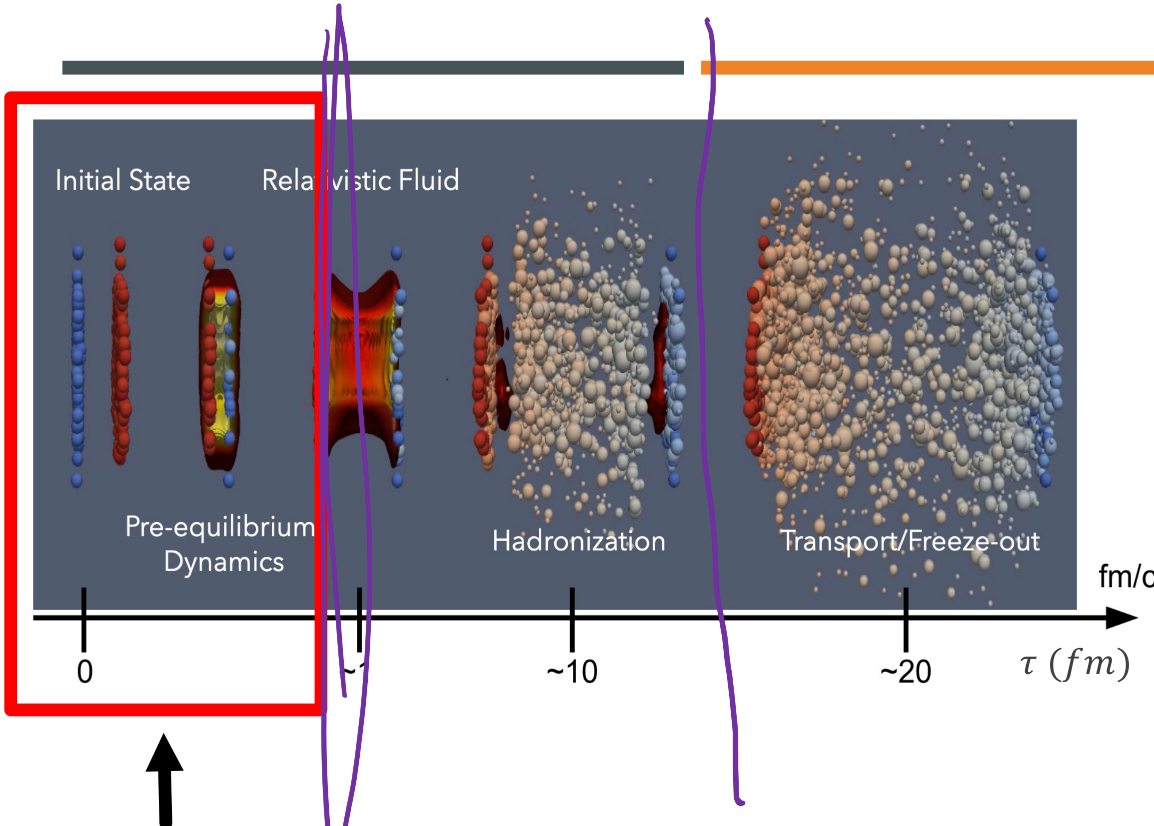
$\tau_\pi = \frac{\eta}{s}\frac{1}{T}\,(3\text{-}5\text{-}10?) \simeq \frac{\eta}{s}\tau^{1/3} \neq$

$\uparrow\ T \propto \frac{1}{\tau^{1/3}}$

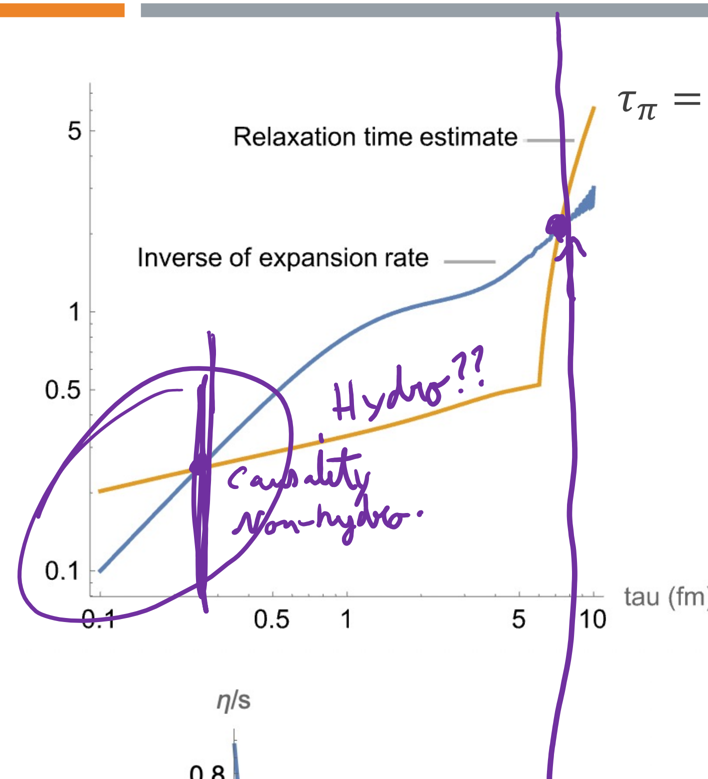
Hydro: $\tau_{relax} \ll \frac{1}{\text{expansion rate}}$

$\theta = \partial_\mu u^\mu = \frac{1}{\tau}\ ;\ \tau = \sqrt{t^2 - z^2}$

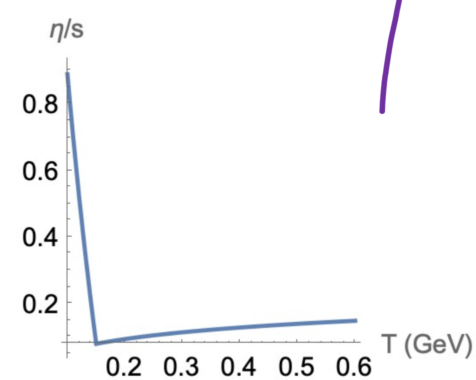




- Causality?
- Attractors?
- Non-hydrodynamic modes & transient hydrodynamics?

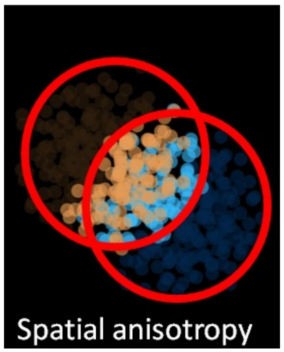


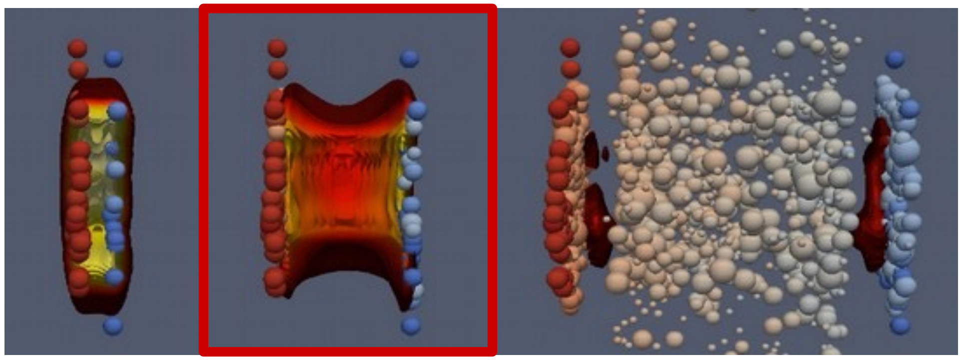
$$\tau_\pi = \frac{\eta}{s}\frac{5}{T}$$

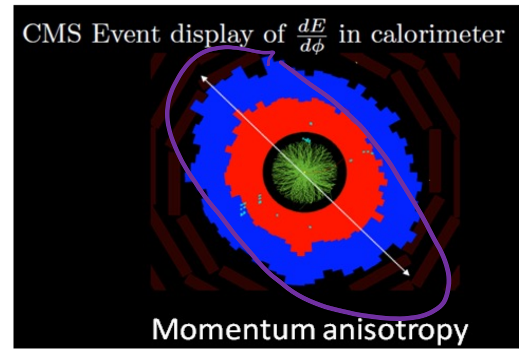


Multistage simulations of heavy ion collisions

Based on figures by Derek Teaney, CMS Coll., MADAI, H. Elfner and J. Bernhard







Energy deposition → **Early dynamics** → **Hydrodynamics** → **Hadronic transport**

- Energy-momentum tensor: $T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} - (P(\epsilon) + \Pi)(g^{\mu\nu} - u^{\mu} u^{\nu}) + \pi^{\mu\nu}$
- Conservation of energy/momentum: $\partial_{\nu} T^{\mu\nu} = 0$
- DNMR transient relativistic viscous hydrodynamics

$$\tau_{\pi} \Delta^{\mu\nu}_{\alpha\beta} \dot{\pi}^{\alpha\beta} + \boxed{\pi^{\mu\nu} = 2\,\eta(T)(\partial^{\mu} u^{\nu} + \cdots)} + (2^{\text{nd}}\ \text{order}); \quad \tau_{\Pi} \dot{\Pi} + \boxed{\Pi = -\zeta(T)\, \partial_{\mu} u^{\mu}} + (2^{\text{nd}}\ \text{order});$$

DNMR

- Energy-momentum tensor:

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} - (P(\epsilon) + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Conservation of energy and momentum:

$$\partial_{\nu} T^{\mu\nu} = 0$$

- Bulk pressure Π equation of motion

$$\tau_{\Pi}\dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

- Shear tensor $\pi^{\mu\nu}$ equation of motion:

$$\tau_{\pi}\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \phi_7 \pi_{\alpha}^{\langle\mu} \pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_{\alpha}^{\langle\mu} \sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

- Definition:

$\theta = \partial_{\mu} u^{\mu}$; $\Delta^{\mu\nu}$=(tensor perpendicular to u^{μ});

$$\Delta_{\alpha\beta}^{\mu\nu} = (symmetric\ traceless\ projection\ operator); \sigma^{\mu\nu} = \Delta_{\alpha\beta}^{\mu\nu}\partial^{\alpha} u^{\beta}$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P(\epsilon) + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu};\ \partial_\nu T^{\mu\nu} = 0$$

- Bulk pressure Π equation of motion

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

- Shear tensor $\sigma^{\mu\nu}$ equation of motion:

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \phi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

$$\phi_7 = \frac{9}{70\mathcal{P}}$$

$$\tau_\Pi = \frac{\zeta}{15\left(\frac{1}{3} - c_s^2\right)^2(\epsilon + \mathcal{P})}$$

$$\delta_{\Pi\Pi} = \frac{2}{3}\tau_\Pi$$

$$\lambda_{\Pi\pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi$$

$$\tau_\pi = \frac{5\eta}{(\epsilon + \mathcal{P})}$$

$$\delta_{\pi\pi} = \frac{4}{3}\tau_\pi$$

$$\tau_{\pi\pi} = \frac{10}{7}\tau_\pi$$

$$\lambda_{\pi\Pi} = \frac{6}{5}$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P(\epsilon) + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu};\ \partial_\nu T^{\mu\nu} = 0$$

- Bulk pressure Π equation of motion

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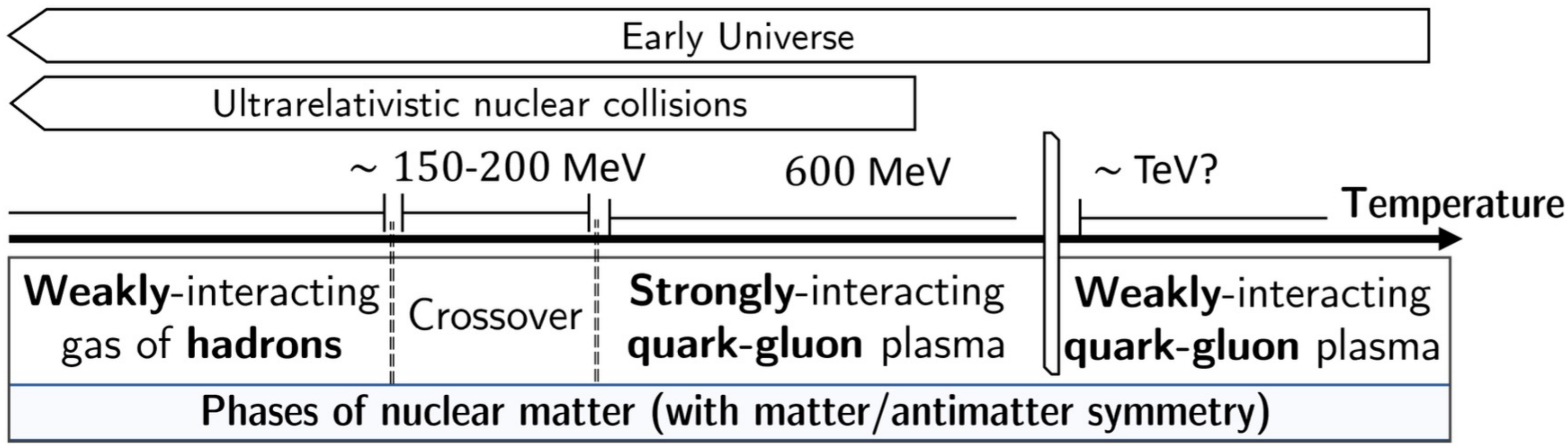
$\tau_\pi = \frac{\eta}{s}\frac{1}{T} \, \#$ $\to 2(2-\ln 2)$

	Gauge/Gravity	Kinetic (BGK)	pQCD
$\epsilon(P)$	3 P	aT	3 P
η	$\frac{\epsilon+P}{4\pi T}$ $\frac{\eta}{s} = \frac{1}{4\pi}$	$\frac{(\epsilon+P)\tau_R}{5}$	$\frac{3.85(\epsilon+P)}{g^4 \ln(2.765 g^{-1})T}$
τ_π	$\frac{2-\ln 2}{2\pi T}$ $\frac{1}{4\pi}\,2$	τ_R	$\frac{5.9\eta}{\epsilon+P}$

Ref.: Romatschke&Romatschke



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τ_π	$\frac{2-\ln 2}{2\pi T}$	τ_R	$\frac{5.9\eta}{\epsilon+P}$

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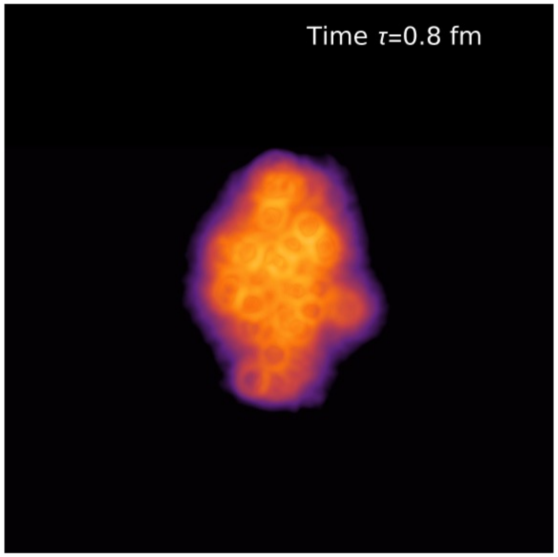
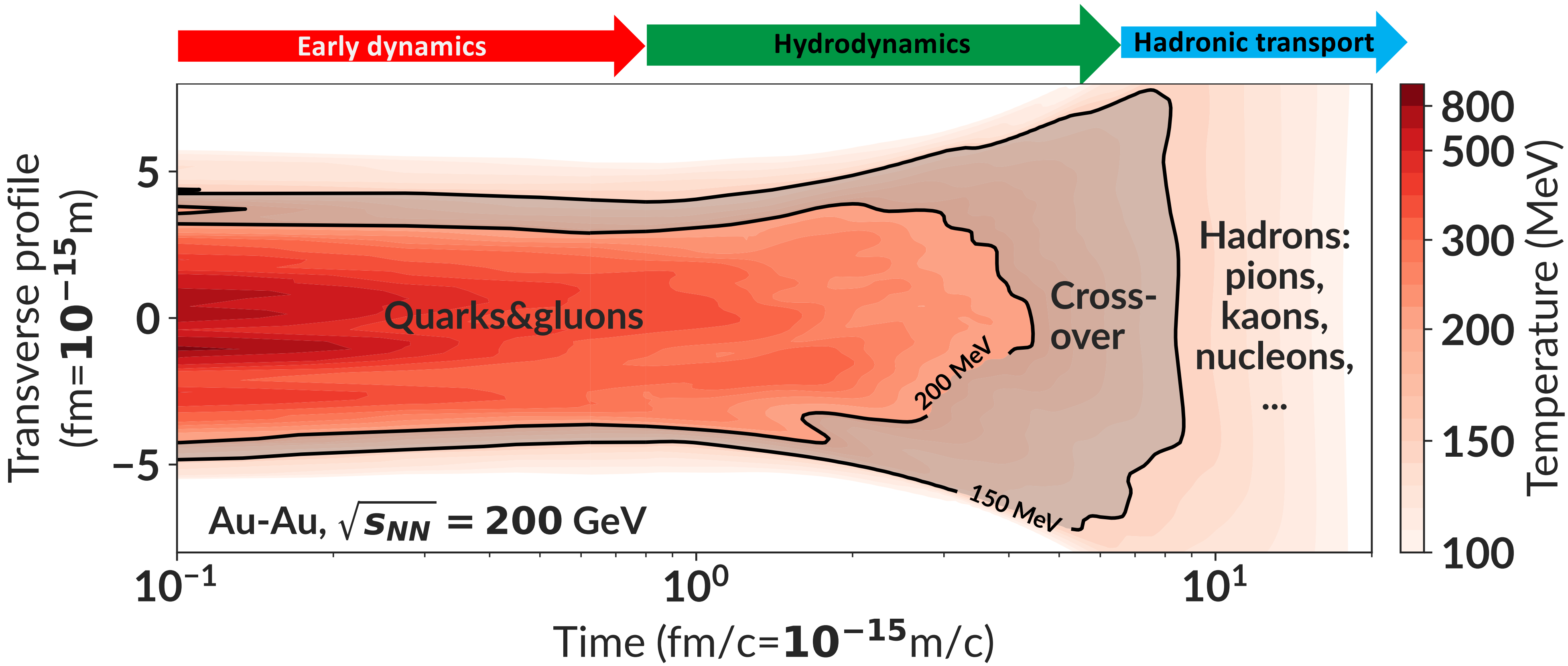
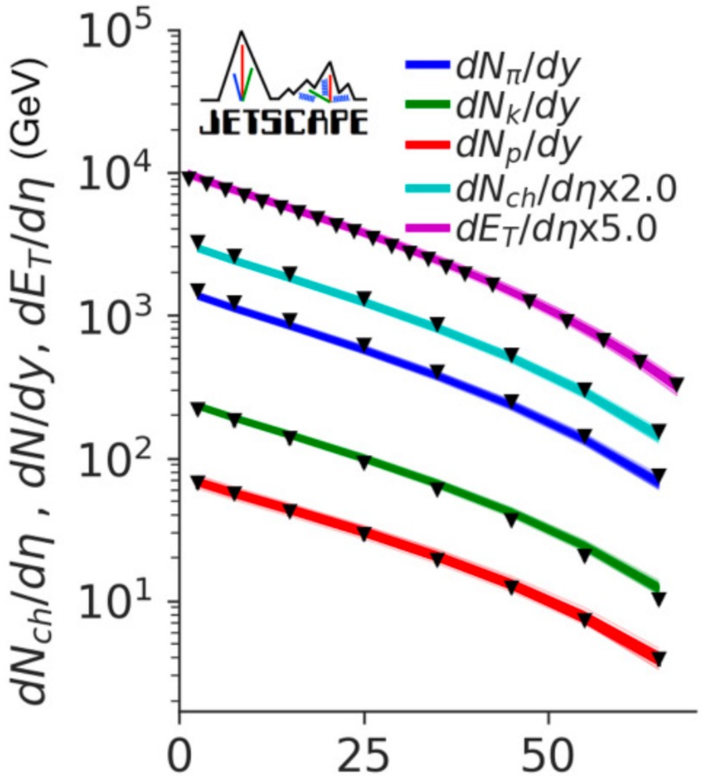
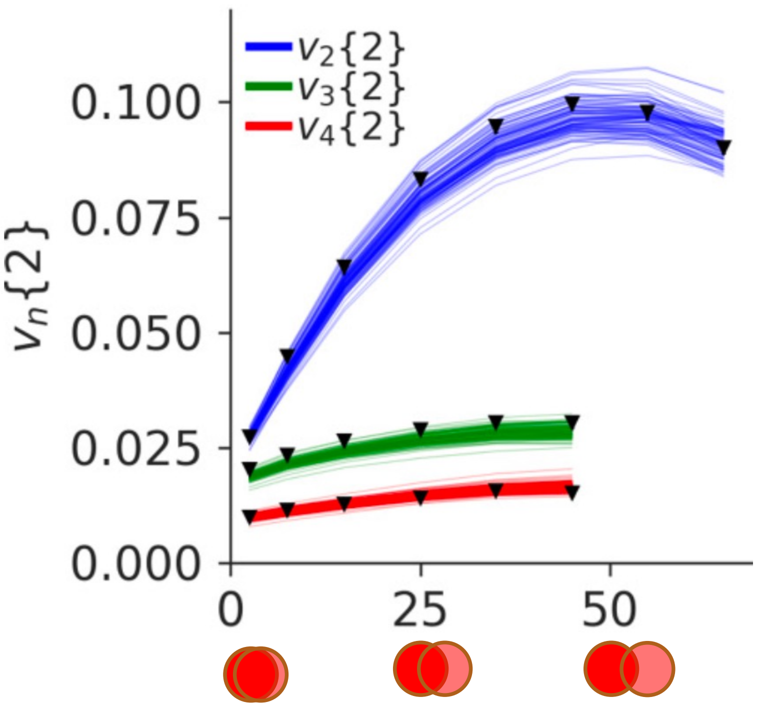

Time τ=0.8 fm

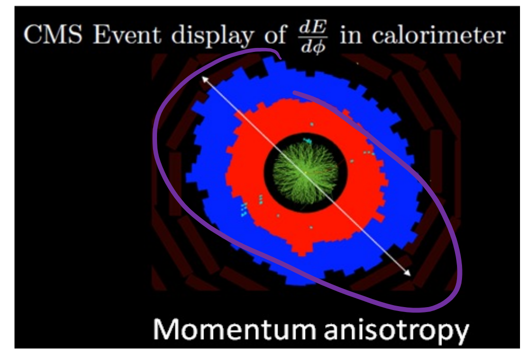


Early dynamics
Hydrodynamics
Hadronic transport
Transverse profile (fm=10⁻¹⁵m)
5
0
−5
Quarks&gluons
200 MeV
150 MeV
Cross-over
Hadrons: pions, kaons, nucleons, ...
Au-Au, √sNN = 200 GeV
10⁻¹
10⁰
10¹
Time (fm/c=10⁻¹⁵m/c)
800
500
300
200
150
100
Temperature (MeV)

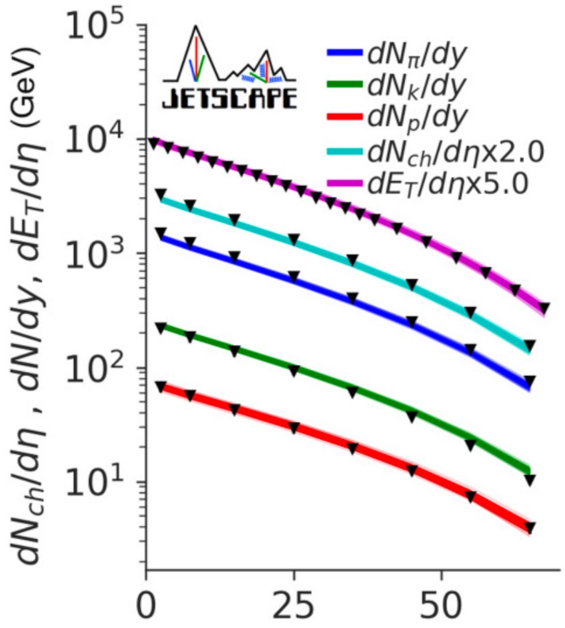
Comparison with data and constraints on viscosity



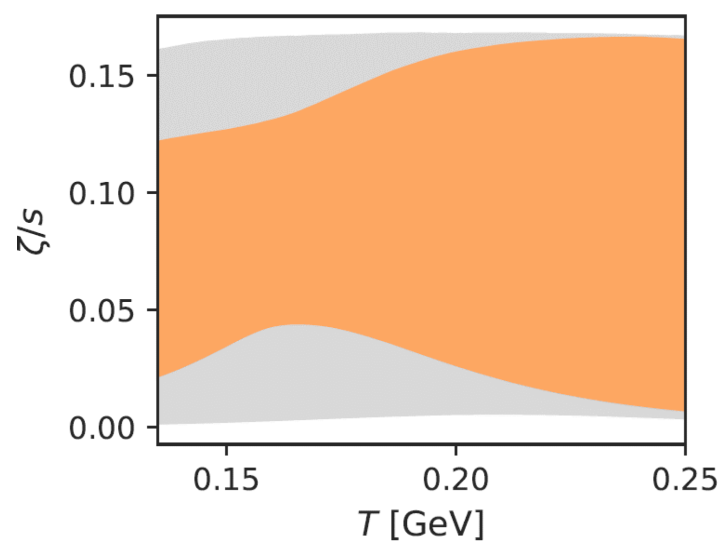




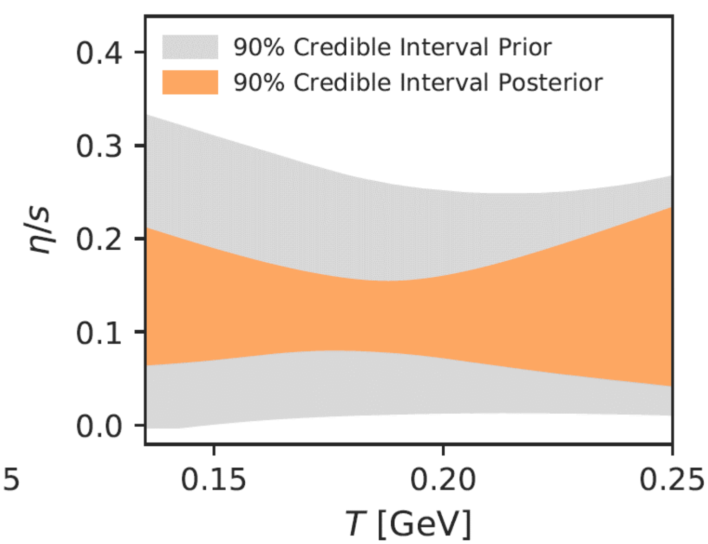
Comparison with data and constraints on viscosity



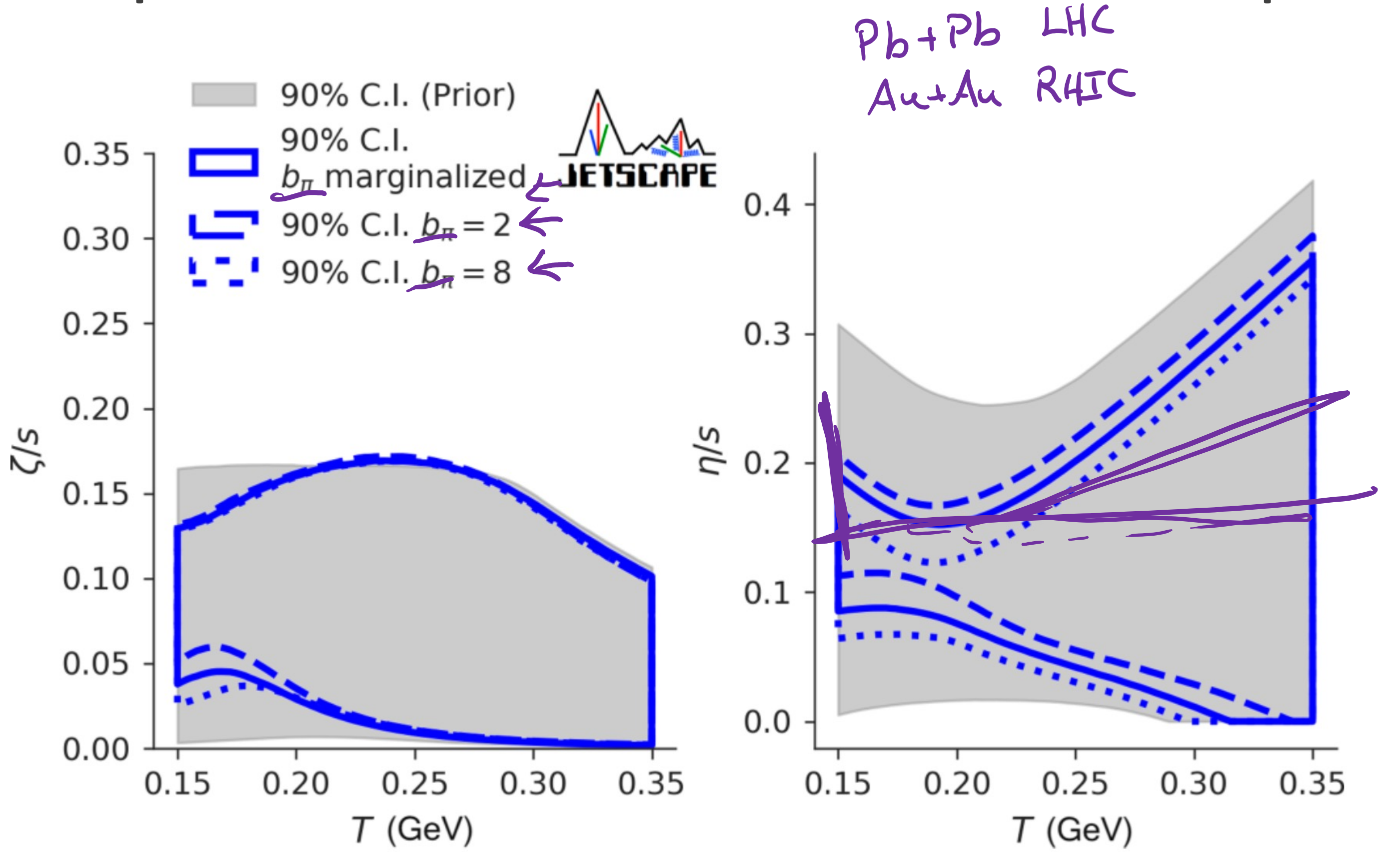
Bulk viscosity



Shear viscosity



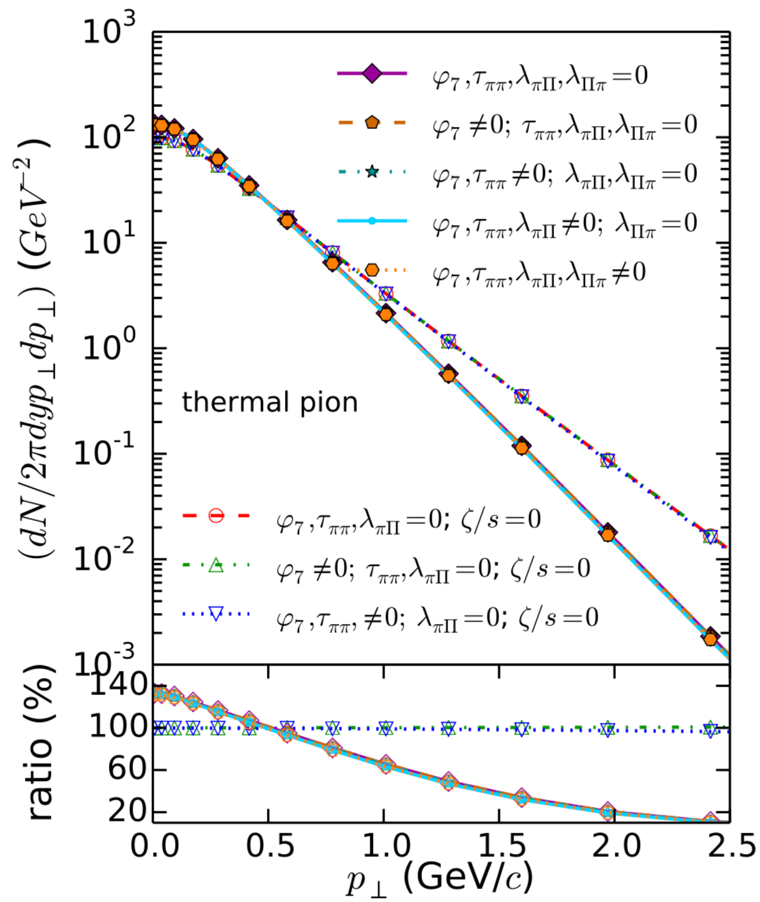
Dependence on second-order transport coefficients



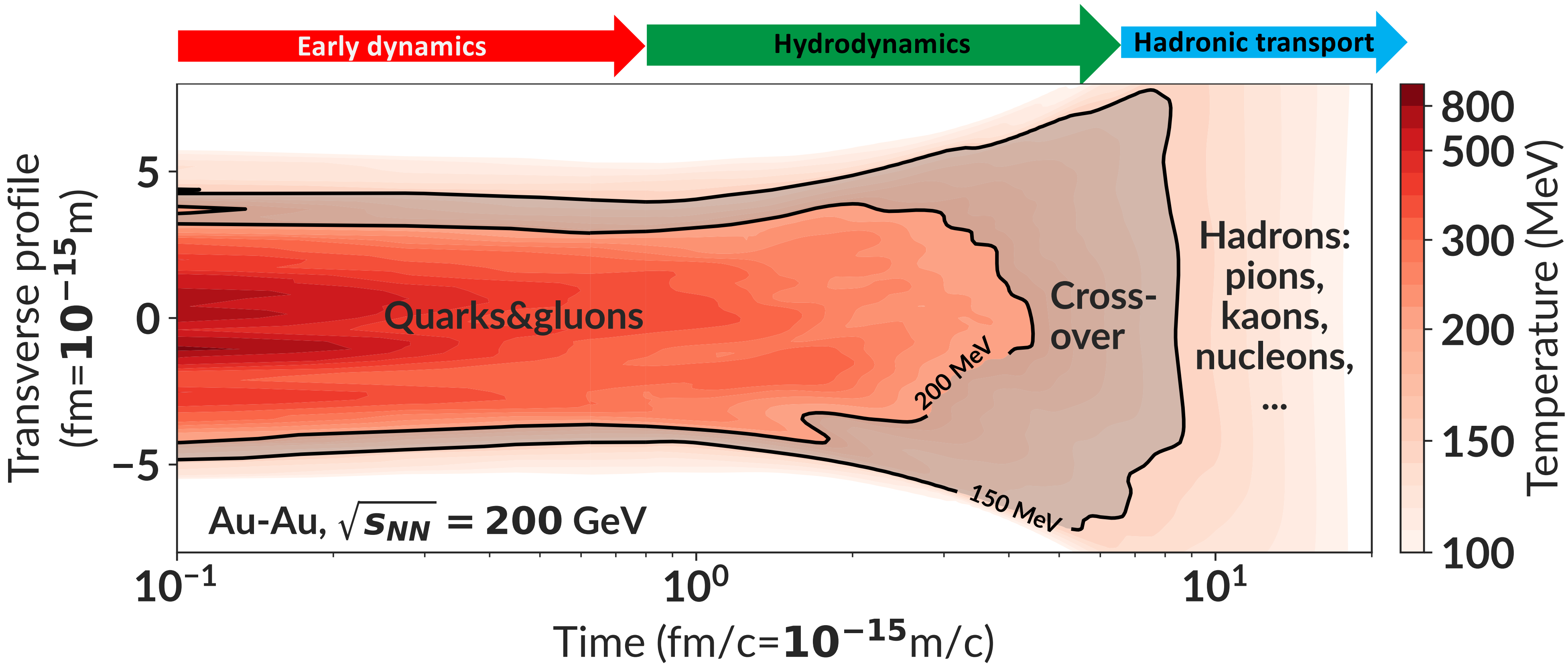
$\tau_\pi = \frac{\eta}{s} \frac{1}{T} b_\pi$

$$
\begin{aligned}
&\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi \\
&= 2\boldsymbol{\eta}\sigma^{\mu\nu} \\
&- \delta_{\pi\pi}\pi^{\mu\nu}\theta \\
&+ \phi_7 \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha} \\
&- \tau_{\pi\pi}\pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} \\
&+ \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}
\end{aligned}
$$

- Jia Liu's PhD thesis



Acausality (c.f. Dekrayat's talk)



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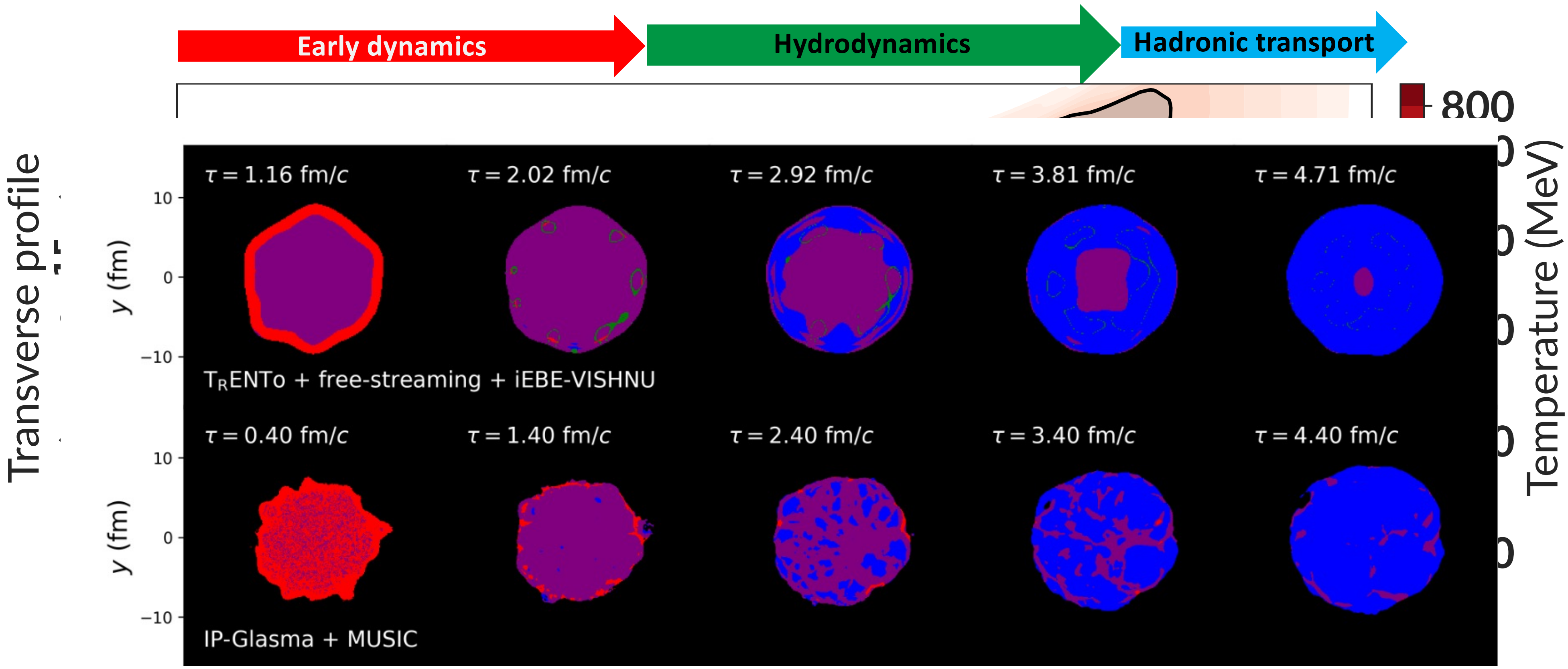
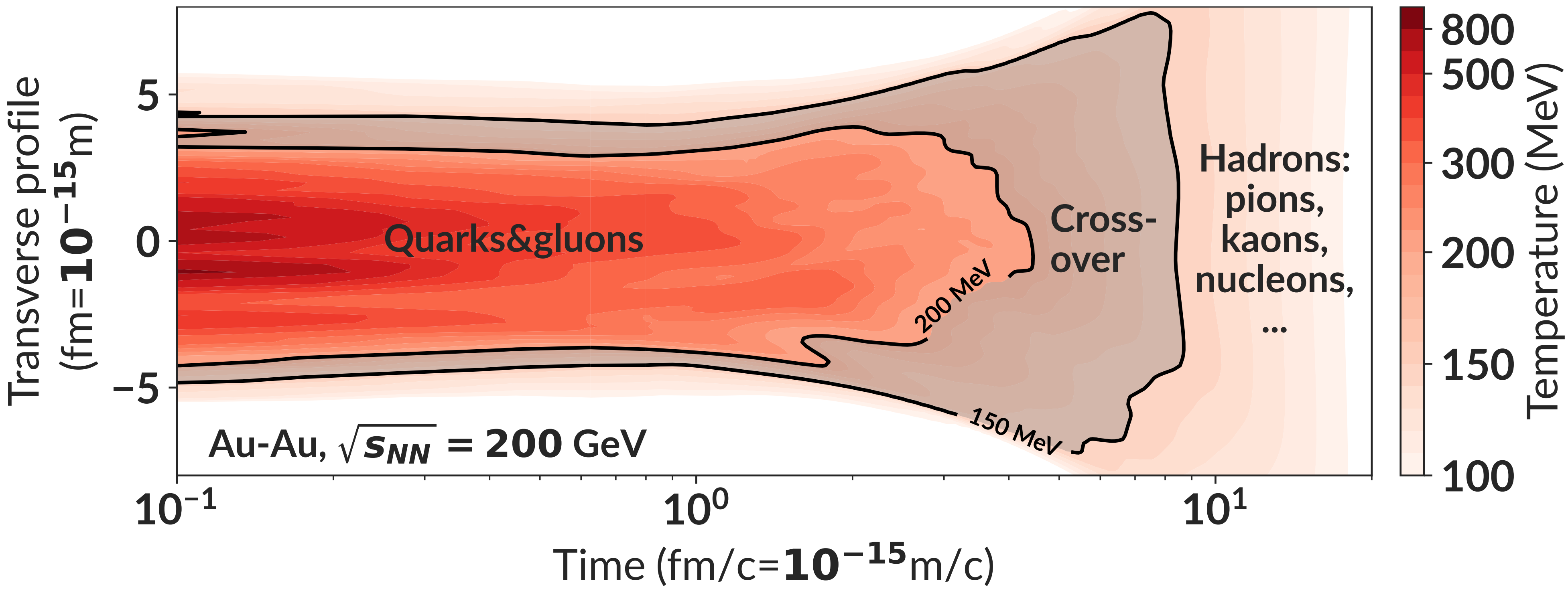
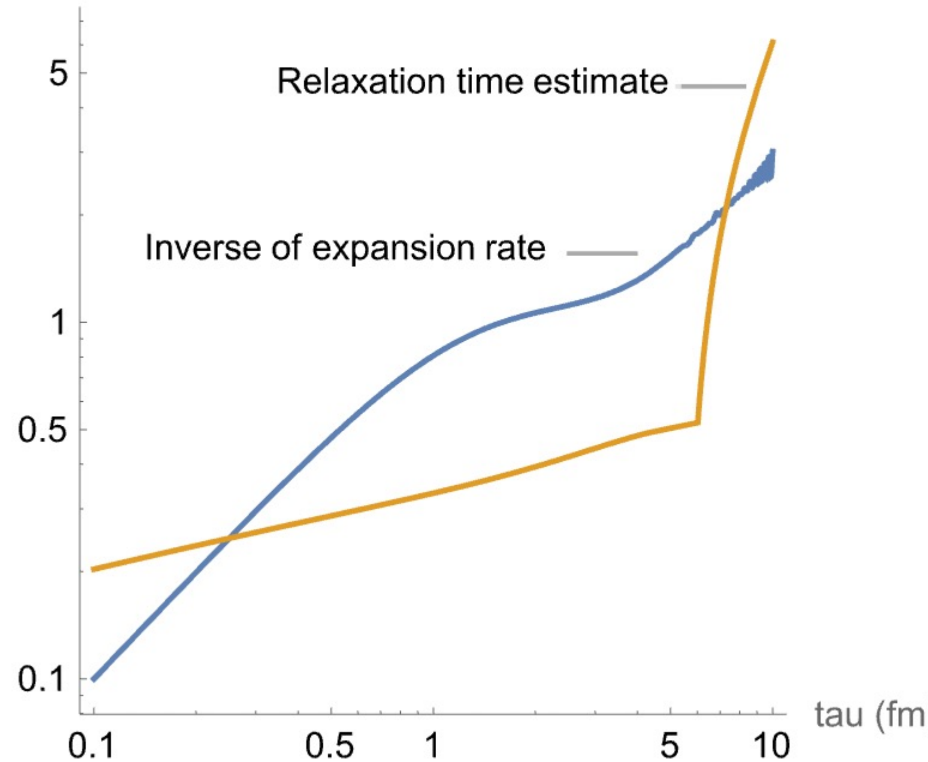


Non-hydro modes + acausality = problem?



Attractors and phenomenology

- Expansion-dominated phase (”free-streaming”)
- Approach to hydrodynamics



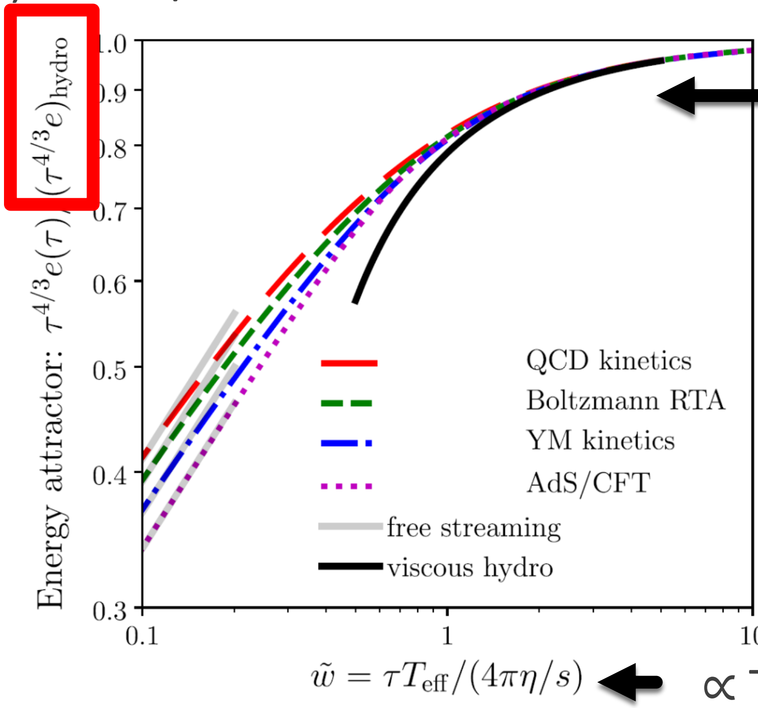
Matching to 0+1D (boost-invariant) hydrodynamics

- In 0+1D hydro, $T^{\mu\nu}$ characterized by single component: energy density
- 0+1D dynamical models with smooth transition to hydrodynamics:
  - Kinetic theory (gluons, QCD, RTA) or AdS/CFT

(note: not all systems have the same final energy density, but this can be rescaled)

Free-streaming: $\propto \tilde{w}^{4/9}$

Navier-Stokes: $1 - \frac{2}{3\pi}\frac{1}{\tilde{w}}$



All systems converge to 0+1D Navier-Stokes

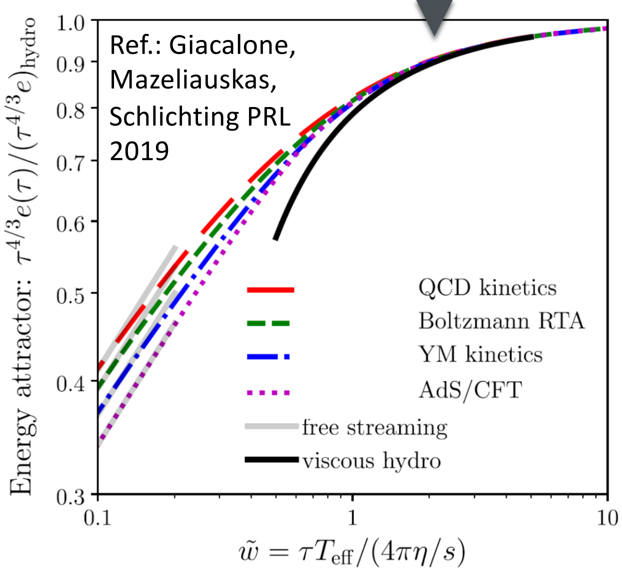
∝ Time/(Relaxation time)

Ref.: Giacalone, Mazeliauskas, Schlichting PRL 2019

Matching to 2+1D hydrodynamics: the "KøMPøST" approach

- Better approximation [KøMPøST]: decompose $T^{\mu\nu}$ in 0+1D background + linear perturbation

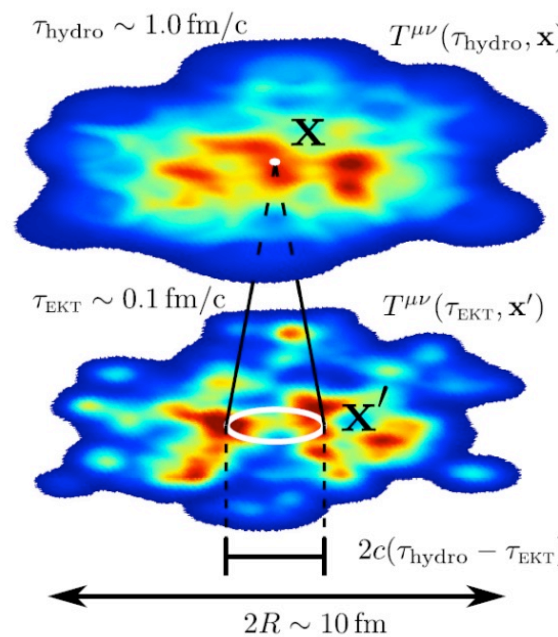
$$T^{\mu\nu}(\tau_{\rm hydro},\mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\rm hydro}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm hydro})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm EKT})} \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}(\mathbf{x},\mathbf{x}',\tau_{\rm hydro},\tau_{\rm EKT})\,\delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\rm EKT},\mathbf{x}')$$



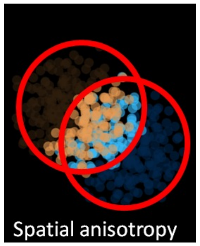
Response functions describing evolution of perturbations

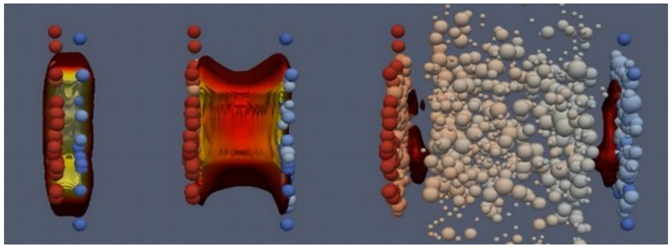
- Also scale with relaxation time

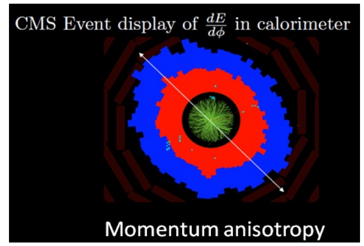
[See also Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD 2020]



Ref.: Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney PRL2019, PRC2019


Spatial anisotropy

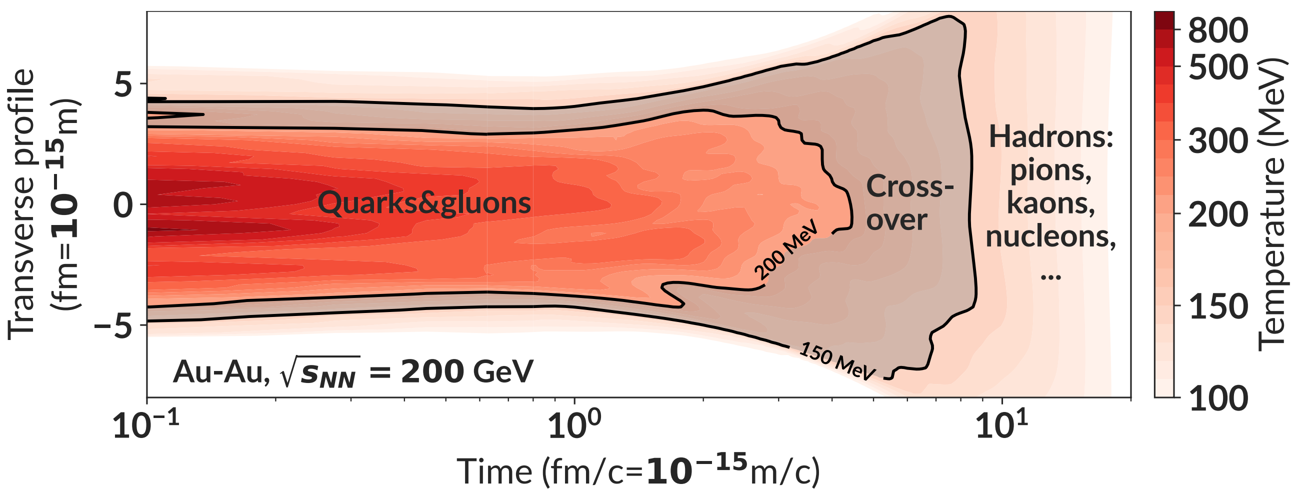



CMS Event display of $\frac{dE}{d\phi}$ in calorimeter
Momentum anisotropy

SUMMARY

Summary

- Need transient hydrodynamics or need early-time model



- Early-time challenges are subtle because we "solve hydrodynamics ~backward"