Physics of Ultra-Relativistic Heavy-Ion Collisions with the Parton Cascade Model

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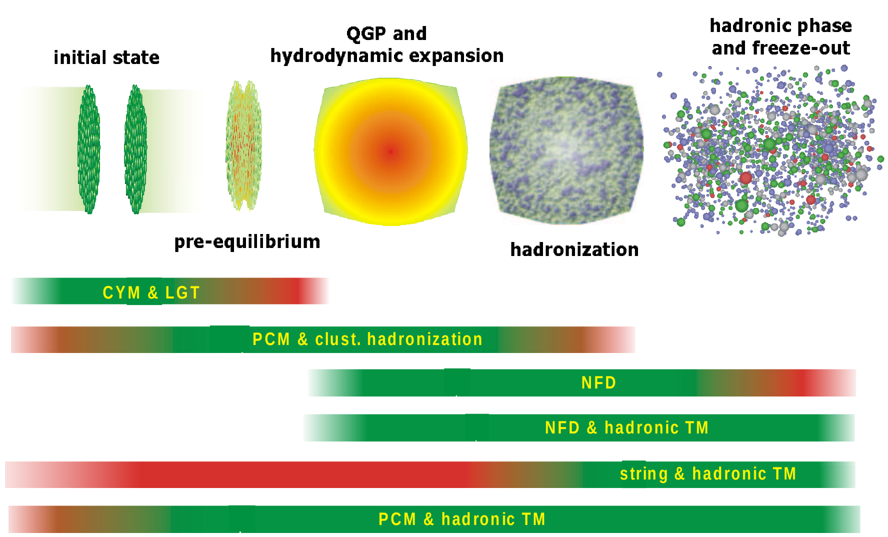
- Motivation
- The PCM: Fundamentals & Implementation
- Tests: comparison to pQCD minijet calculations
- Application: Reaction Dynamics @ RHIC
- Outlook & Plans for the Future

Transport Theory at RHIC



Aims of the Parton Cascade Model

provide a microscopic space-time description of relativistic heavy-ion collisions based on perturbative QCD

- discover novel phenomena associated with the collective behaviour of highly compressed and/or heated QCD matter
- map the route to kinetic and chemical equilibration from a partonic initial state to a Quark-Gluon-Plasma
- identify probes of the partonic phase
- prepare the ground for a study of hadronization and comparison to hadronic observables
- provide initial conditions for other model calculations, e.g. hydrodynamics or hadronic cascades

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Basic Principles of the PCM

- degrees of freedom: quarks and gluons
- classical trajectories in phase space (with relativistic kinematics)
- initial state constructed from experimentally measured nucleon structure functions and elastic form factors
- an interaction takes place if at the time of closest approach d_{min} of two partons

$$d_{\min} \leq \sqrt{\frac{\sigma_{tot}}{\pi}} \quad \text{with} \quad \sigma_{tot} = \sum_{p_3, p_4} \int \frac{d\sigma\left(\sqrt{\hat{s}}; p_1, p_2, p_3, p_4\right)}{d\hat{t}} d\hat{t}$$

- system evolves through a sequence of binary (2→2) elastic and inelastic scatterings of partons and initial and final state radiations within a leading-logarithmic approximation (2→N)
- binary cross sections are calculated in leading order pQCD with either a momentum cut-off or Debye screening to regularize IR behaviour
- guiding scales: initialization scale Q_0, p_T cut-off p_0 / Debye-mass μ_D, intrinsic k_T / saturation momentum Q_S, virtuality > μ_0

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Initial State in the PCM

the initial phase-space distribution can be constructed either from known data on hadrons and nuclei or taken from a model of the initial state of heavy-ion collisions (e.g. a Color-Glass-Condensate)

- for partons of flavour *a* in a nucleus the distribution is given by:

$$F_a(\vec{r},\vec{k}) = \sum_{i=1}^{N_h} P_a^{N_i}(\vec{k},\vec{P},Q_0^2)\times R_a^{N_i}(\vec{r},\vec{R})$$

- with the initial momentum distribution:

$$P_a^{N_i}(\vec{k},\vec{P},Q_0^2) \propto F_a^{N_i}(x,Q_0^2)\times \rho_a^A \times g(\vec{k}_\perp)\times \delta(P_z - P)\times \delta^2(\vec{P}_\perp)$$

(Q_0: initial resolution scale, ρ^A optional shadowing, g: opt. primordial k_T)

- and the initial spatial distribution:

$$R_a^{N_i}(\vec{r},\vec{R}) = \delta(\vec{R}_{AB}^\perp - \vec{b})\times\left[h_a^{N_i}(\vec{r})\times H_{N_i}(\vec{R})\right]_{\text{boosted}}$$

- H_N: distribution of nucleons in nucleus (e.g. Fermi-Distribution)
- h_a: distribution of partons in hadron (based on elastic form factor)

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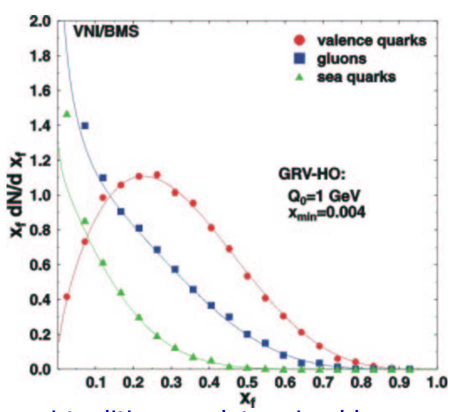
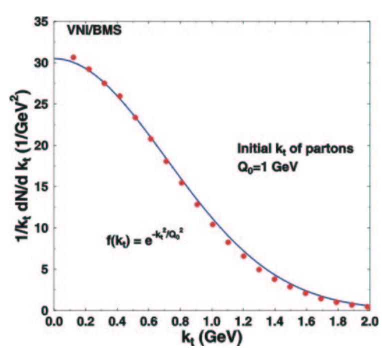
Initial State II: Parton Momenta



- flavour and x are sampled from PDFs at an initial scale Q_0 and low x cut-off x_{min}
- initial k_t is sampled from a Gaussian of width Q_0 in case of no initial state radiation



- virtualities are determined by:

$$\left(\sum E^i\right)^2 - \left(\sum p_x^i\right)^2 - \left(\sum p_y^i\right)^2 - \left(\sum p_z^i\right)^2 = M_N^2$$

with $p_z^i = x^i P_z^N$ and $E^i = \beta_N^{-1} p_z^i$

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Binary Processes in the PCM

- the total cross section for a binary collision is given by:

$$\hat{\sigma}_{ab}\left(\hat{s}\right)=\sum_{c,d}\hat{\sigma}_{ab\to cd}\left(\hat{s}\right)$$

with partial cross sections: $\hat{\sigma}_{ab\to cd}\left(\hat{s}\right)=\int_{\hat{t}_{\min}}^{\hat{t}_{\max}}\left(\frac{d\hat{\sigma}(\hat{s},\hat{t}',\hat{u}')}{d\hat{t}'}\right)_{ab\to cd} d\hat{t}'$

- now the probability of a particular channel is:

$$P_{ab\to cd}\left(\hat{s}\right)=\frac{\hat{\sigma}_{ab\to cd}\left(\hat{s}\right)}{\hat{\sigma}_{ab}\left(\hat{s}\right)}$$

- finally, the momentum transfer & scattering angle are sampled via

$$\Xi\left(\hat{t}\right)=\frac{1}{\hat{\sigma}_{ab\to cd}\left(\hat{s}\right)}\int_{\hat{t}_{\min}}^{\hat{t}}\left(\frac{d\hat{\sigma}(\hat{s},\hat{t}',\hat{u}')}{d\hat{t}'}\right)_{ab\to cd} d\hat{t}'$$

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Parton-Parton Scattering Cross-Sections

g g → g g	$\frac{9}{2}\left(3-\frac{tu}{s^2}-\frac{su}{t^2}-\frac{st}{u^2}\right)$	**q q' → q q'**	$\frac{4}{9}\frac{s^2+u^2}{t^2}$
q g → q g	$-\frac{4}{9}\left(\frac{s}{u}+\frac{u}{s}\right)+\frac{s^2+u^2}{t^2}$	**q qbar → q' qbar'**	$\frac{4}{9}\frac{t^2+u^2}{s^2}$
g g → q qbar	$\frac{1}{6}\left(\frac{t}{u}+\frac{u}{t}\right)-\frac{3}{8}\frac{t^2+u^2}{s^2}$	**q g → q γ**	$-\frac{e_q^2}{3}\left(\frac{u}{s}+\frac{s}{u}\right)$
q q → q q	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}\right)-\frac{8}{27}\frac{s^2}{tu}$	**q qbar → g γ**	$\frac{8}{9}e_q^2\left(\frac{u}{t}+\frac{t}{u}\right)$
q qbar → q qbar	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}\right)-\frac{8}{27}\frac{u^2}{st}$	**q qbar → γ γ**	$\frac{2}{3}e_q^4\left(\frac{u}{t}+\frac{t}{u}\right)$
q qbar → g g	$\frac{32}{27}\left(\frac{t}{u}+\frac{u}{t}\right)-\frac{8}{3}\frac{t^2+u^2}{s^2}$		

- a common factor of $\pi\alpha_s^2(Q^2)/s^2$ etc.
- further decomposition according to color flow

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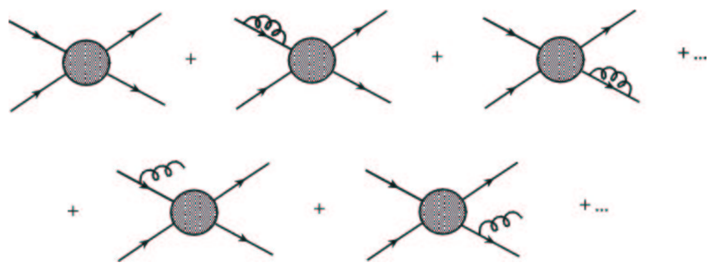
Scale Evolution and Branching

- higher order corrections describing the evolution of the factorization scale and branching processes are treated in the collinear (LLA) approximation
- the differential cross section is modified by a factor of

$$\frac{F_i\left(x_i, Q^2\right)}{F_i\left(x_i, Q_0^2\right)}$$ for each initial state parton



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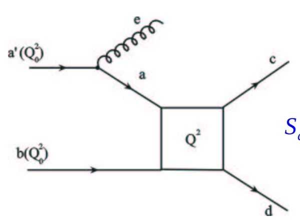
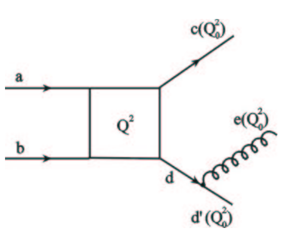
Initial and final state radiation

Probability for a branching is given in terms of the Sudakov form factors:



space-like branchings:

$$S_a\left(x_a, t_{\max}, t\right) = \exp\left\{-\int_t^{t_{\max}} dt' \frac{\alpha_s(t')}{2\pi} \sum_{a'} \int dz\, P_{a'\to ae}(z) \frac{x_{a'} f_{a'}(x_{a'}, t')}{x_a f_a(x_a, t')}\right\}$$



time-like branchings:

$$T_d\left(x_d, t_{\max}, t\right) = \exp\left\{-\int_t^{t_{\max}} dt' \frac{\alpha_s(t')}{2\pi} \sum_{a'} \int dz\, P_{d\to d'e}(z)\right\}$$

- Altarelli-Parisi splitting functions included: $P_{q\to qg}$, $P_{g\to gg}$, $P_{g\to qqbar}$ & $P_{q\to q\gamma}$

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Parton Fusion ($2 \rightarrow 1$) Processes

- $qg \rightarrow q^*$
- $gg \rightarrow g^*$

work in progress

•see e.g. Gunion & Bertsch: **Phys.Rev.D25:746,1982**

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Parton Saturation in VNI/BMS

Parton Saturation and Colour Glass Condensate models postulate that:

- the low p_t physics of relativistic heavy-ion collisions at central rapidities is governed by low-x partons
- the initial state p_t distribution is determined by the saturation scale Q_S

In VNI/BMS we incorporate features of saturation physics by choosing:

- saturation scale as factorization scale: $Q_S=Q_0$
- intrinsic parton transverse momentum $\sim \exp(-k_t^2/Q_S^2)$ instead of k_t generation via initial state radiation
- a low-p_t cut off governed by the space-like virtualities for initial partons
- parton-parton interactions to be screened by a Debye mass $\mu_D= c\, Q_S$ with $c\sim1$ so that there is no need for a low-p_t cut off for secondary scatterings and no artificial division of scatterings with low and high p_t
- the renormalization scale Q^2 to be $\max(Q^2, Q_S^2)$

➢ Q_S is the only scale governing the initial state & evolution of the system

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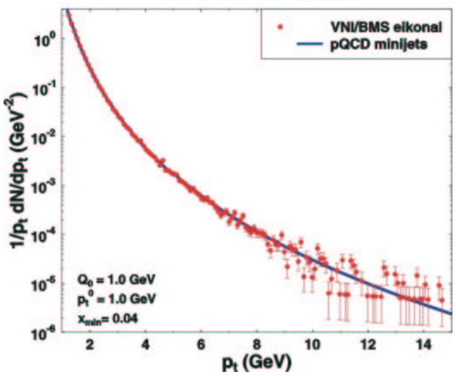
Testing the PCM Kernel: p_t distribution

- the minijet cross section is given by:

$$\frac{d\sigma_{jet}}{dp_t^2 dy_1 dy_2} = \sum_{i,j} x_1 x_2 \left(f_i(x_1,Q^2)\ f_j(x_2,Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}} + (1 \leftrightarrow 2) \right) \left(1 - \frac{\delta_{i,j}}{2} \right)$$



- equivalence to PCM implies:
  - keeping the factorization scale $Q^2 = Q_0^2$ with α_s evaluated at Q^2
  - restricting PCM to eikonal mode, without initial & final state radiation
- results shown are for b=0 fm

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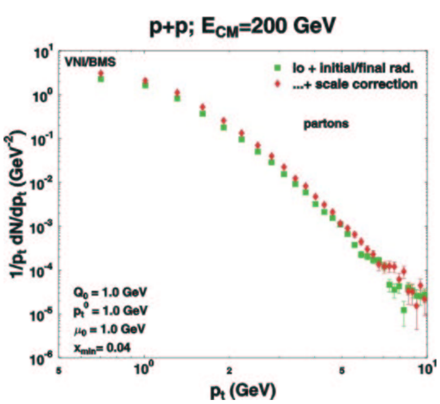
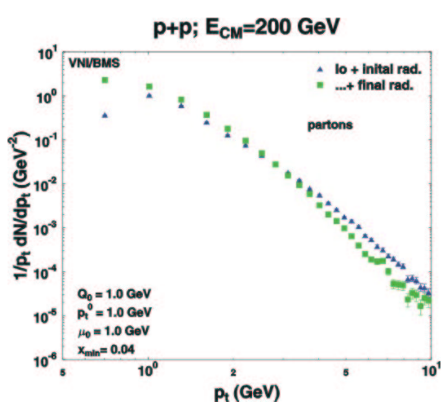
Corrections for Q_0 & Initial/Final Radiation





- dynamic factorization scale correction increases cross section by ~40%

- final state radiation suppresses p_t-distribution by a factor of ~2 at $p_t \approx 5$ GeV
  - jets vs. leading partons

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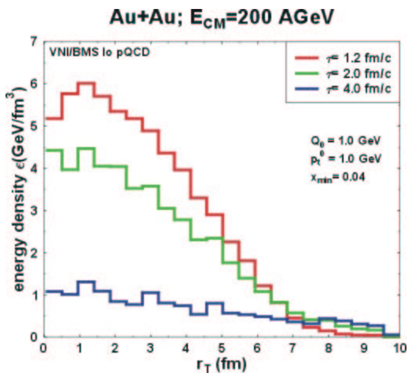
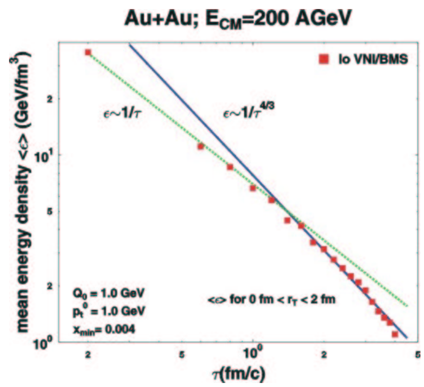
Space-Time Evolution of Energy Density

energy-density at y_{CM} is caculated from: $\varepsilon(r_T) = \frac{dE_T(r_T)}{dy dr_T} \cdot \frac{1}{2\pi\tau\ r_T}$





•hints of transverse expansion?

•conditions for hydrodynamics reached?

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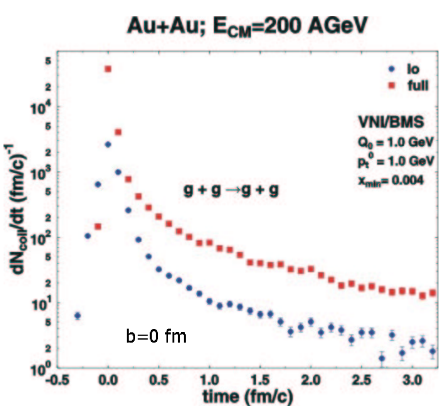
Collision Rates & Numbers



# of collisions	lo	full	sat
q + q	70.6	274	90.6
q + qbar	1.3	38.52	21.0
q + g	428.3	2422.6	747.7
g + g	514.4	4025.6	1265.8

•lifetime of interacting phase: ~ 3 fm/c

•partonic multiplication due to the initial & final state radiation increases the collision rate by a factor of 4-10

➢expect a high degree of thermalization!

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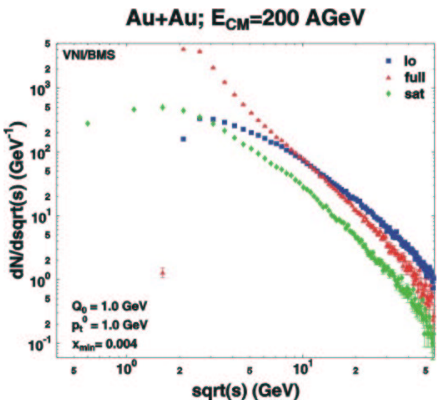
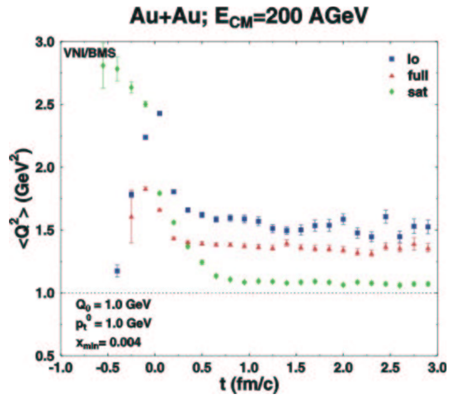
Reaction Dynamics





- radiative multiplication leads to massive rescattering with low s
- saturation: # of collisions suppressed due to virtuality cut off?
- early phase dominated by large momentum transfers
- radiaton/rescattering leads to strong degredation in Q^2

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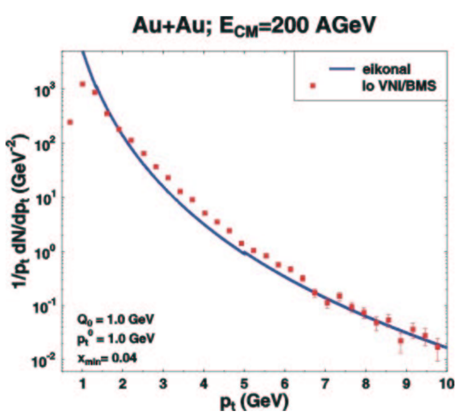
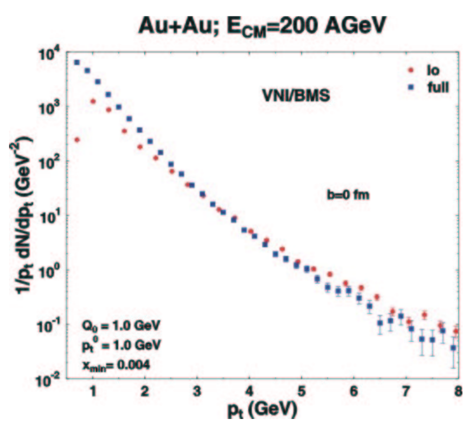
Multiple Scattering and Radiation





- multiple scattering broadens momentum distribution at intermediate p_t
- radiation enhances low p_t domain and leads to suppression at high p_t
  - ➢Jet Quenching at $p_t > 5$ GeV?

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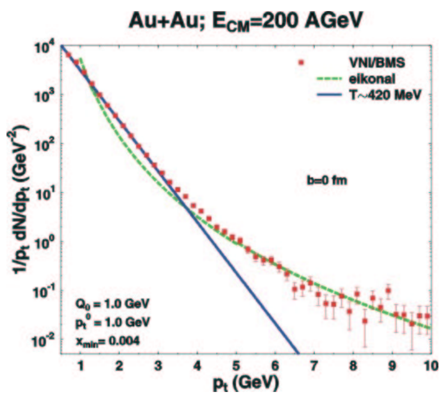
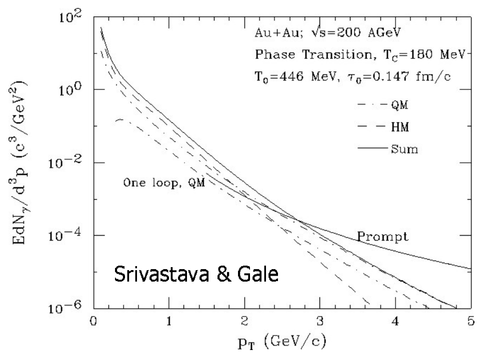
Thermalization?





- spectrum exhibits thermal behaviour for $p_t < 4$ GeV
- thermalization due to radiation and rescattering?
- initial temperature estimated from measured dN/dy and Bjorken's formula: 446 MeV

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Novel Features in VNI/BMS

- initialization in quantitative agreement with PDFs & virtualities
- proper treatment of renormalization scale in transport cross sections
- vastly improved algorithm for sampling t from dσ/dt
- consistent treatment for propagation of space- & time-like partons
- proper treatment of p_t generation in parton showers
- introduction of a fast cascade algorithm
- introduction of factorization scale correction in cross sections
- improved algorithm for the LPM effect
- possibility to simulate eikonal approximation
- incorporation of saturation physics
- output & documentation conforming to OSCAR standards

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Limitations of the PCM Approach

Fundamental Limitations:

- lack of coherence of initial state
- range of validity of the Boltzmann Equation
- parton saturation is input, not result of dynamics
- interference effects are included only schematically
- hadronization has to be modeled in an ad-hoc fashion

Limitations of present implementation (as of May 2002)

- lack of detailed balance: (no N $\rightarrow$ 2 processes)
- no 2 $\rightarrow$ 1 processes involving space-like partons
- lack of selfconsistent medium corrections
- heavy quarks?

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Future Directions …

The VNI/BMS approach provides an ideal framework for:

- study of event by event fluctuations
- investigating the detailed dynamics of jet-quenching
- study of medium modification of QCD processes
- studying the transition of a shattered Colour Glass to a QGP
- study of propagation & recombination of heavy quarks
- investigating models of hadronization
- dovetailing to hydrodynamics & hadronic cascades

- suggestions and collaborative endeavours on these and related issues are most welcome!

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