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Cooling of quark stars

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Reference:

- hep-ph/0204132

Symmetry of ground state
$N_f = 3$ (CFL phase)

$$
\begin{aligned}
SU(3)_L \times SU(3)_R \times SU(3)_c &\to SU(3)_{L+R+c} \\
U(1)_B &- \textit{broken} \\
\textit{approx.}\ U(1)_A &- \textit{broken}
\end{aligned}
$$

- Modified electromagnetism $U(1)_{\tilde{em}}$ survives
- Parity is preserved

Low energy dynamics

Chiral limit:

9 NG bosons ($\pi^{\pm}$, π^0, $K^{\pm}$, K^0, $\bar{K}^0$, η, and ϕ)
[Alford,Rajagopal,Wilczek'99]

$\oplus$ η' pseudo-NG boson, $m_{\eta'} \sim \Delta \exp\left(-\frac{\pi}{\alpha}\right)$
[Son,Stephanov,Zhitnitsky'01]

Massive quarks ($m_u, m_d, m_s \ll \mu$):

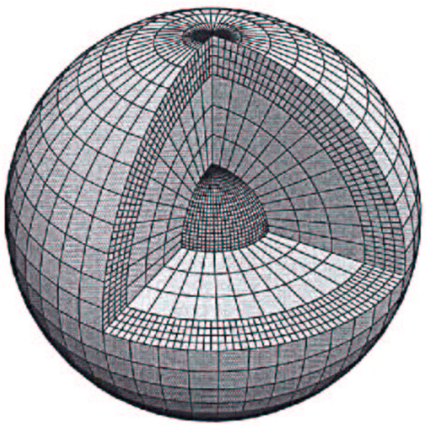
$\pi^{\pm}$, π^0, $K^{\pm}$, K^0, $\bar{K}^0$, and η

get nonzero masses (inverse hierarchy)

Low energy dynamics is dominated by massless ϕ related to breaking of baryon number

A compact star with a quark core

The aftermath of a supernova explosion:



Neutrino trapping $\rightarrow$ deleptonization process by diffusion $\rightarrow$ "initial" stage ($T \lesssim 40$ MeV)

Cooling of NS is dominated by neutrino emission during the first 10^5 years ($T \gtrsim 10$ keV)

Neutrino and photon emissivities of the CFL quark matter are strongly supressed
[Jaikumar,Prakash,Schäfer, astro-ph/0203088]

Would the core remain hot for 10^{24} years?

Thermal conductivity

Thermal conductivity tends to wash away a temperature gradient, i.e.,

$$u_i = -\kappa \partial_i T$$

Kinetic properties are dominated by low-energy degrees of freedom.

Let us estimate the partial contribution of a (pseudo-) NG boson, described by

$$L = \frac{1}{2}\left(\partial_0\varphi\partial_0\varphi - v^2\partial_i\varphi\partial_i\varphi - m^2\varphi^2\right) + \ldots$$

The corresponding heat current is

$$u_i = \frac{\partial L}{\partial(\partial^i\varphi)}\partial_0\varphi = v^2\partial_i\varphi\partial_0\varphi$$

By using the linear response theory, we derive

$$\kappa_{ij} = -\frac{i}{2T}\lim_{\Omega\to 0}\frac{1}{\Omega}\left[\Pi^R_{ij}(\Omega + i\epsilon) - \Pi^A_{ij}(\Omega - i\epsilon)\right]$$

where $\Pi_{ij}(\Omega)$ is the heat current correlator.

Rotational invariance implies that $\kappa_{ij} \equiv \kappa \delta_{ij}$.

At $T \lesssim m$, the direct calculation gives

$$\kappa = \frac{m^5}{24\pi^2 v \Gamma T^2} \int_0^\infty \frac{x^4 dx}{\sinh^2 \frac{m\sqrt{1+x^2}}{2T}}$$
$$\simeq \frac{m^{5/2}\sqrt{T}}{2\sqrt{2}\pi^{3/2} v \Gamma} e^{-m/T},$$

where Γ is the decay width.

Γ is a phenomenological parameter in the propagator of the (pseudo-) NG boson,

$$S(\omega, \vec{k}) = \frac{1}{(\omega + i\Gamma/2)^2 - v^2 \vec{k}^2 - m^2}$$

At $T \lesssim \tilde{T} \ll m_{lpNG}$, dominant contribution to κ comes from the massless NG boson:

$$\kappa_\phi = \frac{4T^3}{3\pi^2 v \Gamma_\phi} \int_0^\infty \frac{x^4 dx}{\sinh^2 x} = \frac{2\pi^2 T^3}{45 v \Gamma_\phi}$$

(In agreement with classical relation $\kappa = \frac{1}{3}\bar{v}\ell c_v$)

Mean free path of the NG boson

Trade width Γ for mean free path: $\ell = \frac{\bar{v}}{\Gamma}$

NG boson is a composite particle, $\phi \to qq$:
[Gusynin,Shovkovy,Nucl.Phys.A700(2002)577]

$$\Gamma_{\phi\to qq}(k) \sim vk \exp\left(-\sqrt{\frac{3}{2}}\frac{\Delta}{T}\right)$$

Then, the mean free path would be

$$\ell_{\phi\to qq} \sim \frac{v}{T} \exp\left(\sqrt{\frac{3}{2}}\frac{\Delta}{T}\right)$$

i.e., $\ell \gtrsim 30$ km for $T \lesssim \Delta/33$ (for $\Delta \simeq 50$ MeV).

NG bosons scattering amplitude $\sim k^4/\mu^4$
[Son,hep-ph/0204199]:

$$\sigma_{\phi\phi} \simeq \frac{T^6}{\mu^8}$$

So, the mean free path scales as

$$\ell_{\phi\phi} \sim \frac{1}{\sigma_{\phi\phi} n_\phi} \sim \frac{\mu^8}{T^9} \approx 8 \times 10^5 \frac{\mu_{500}^8}{T_{MeV}^9} \text{ km}$$

Photon contributions

Photon mean free path $\gg$ star size ($T \lesssim \tilde{T}$)

Photon emission is strongly suppressed

Are there photons inside the quark core?

Nuclear matter is a plasma

The plasma frequency is

$$\Omega_p = \sqrt{\frac{4\pi e^2 Y_e \rho}{m_e m_p}} \simeq 470 \sqrt{\frac{\rho Y_e}{\rho_0}} \text{ MeV},$$

where
$n_e = Y_e \rho / m_p$ is the electron density
ρ is the nuclear matter density
m_p and m_e are the proton and electron masses
$Y_e \simeq 0.1$ is the number of electrons per baryon
$\rho_0 \approx 2.8 \times 10^{14}$ g cm^{-3} is equilibrium nuclear matter density.

There is photon trapping at $T \lesssim \tilde{T}$

Thus, photons also contribute to the thermal conductivity

Total thermal conductivity

Mean free path of photons and NG bosons

$\ell \simeq R_0$, where R_0 is the core size (geometrical restriction)

The expression for the thermal conductivity

$$\kappa_{CFL} = \kappa_\phi + \kappa_\gamma \simeq \frac{2\pi^2}{9} T^3 R_0$$

where $v_\phi = 1/\sqrt{3}$ and $v_\gamma \approx 1$ were used.

Numerically, this yields the value

$$\kappa_{CFL} \simeq 1.2 \times 10^{32} T_{MeV}^3 R_{0,km} \text{ erg cm}^{-1}\text{sec}^{-1}\text{K}^{-1}$$

Compare with

$$\kappa_{NM} \lesssim 10^{24} \text{ erg cm}^{-1}\text{sec}^{-1}\text{K}^{-1}$$

Typical "relaxation time":

$$\frac{R_{0,km}}{v} \simeq 6 \times 10^{-4} \text{ sec}$$

Cooling of a star

Thermal energy vs. emissivity

Thermal energy of the core:

$$E_{CFL}(T) = \frac{4\pi R_0^3}{3}\int_0^T c_v(T')dT' = \frac{6(1+2v^3)T}{5}\left(\frac{\pi T R_0}{3v}\right)^3,$$

or numerically

$$E_{CFL}(T) \simeq 2.1 \times 10^{42} R_{0,km}^3 T_{MeV}^4 \text{ erg}$$

Thermal energy of the nuclear layer:

$$E_{NM}(T) \simeq 8.1 \times 10^{49} \frac{M - M_0}{M_\odot}\left(\frac{\rho_0}{\rho}\right)^{2/3} T_{MeV}^2 \text{ erg}$$

Note that $E_{CFL}(T) \ll E_{NM}(T)$ for $T \lesssim \tilde{T}$

Thus, CFL quark core has a very little slowing effect on the star cooling rate

Other kinetic properties

Shear viscosity: massless NG bosons and photons should also dominate

Electrical conductivity: the lightest *charged* pseudo-NG boson (*i.e.*, the K^+) dominates:

$$\sigma_{el} \sim \frac{m_{K^+}^{5/2}}{\sqrt{T}} \exp\left(-\frac{m_{K^+}}{T}\right)$$

which is suppressed as $T \to 0$ (neutrality)

S2C phase ($N_f = 2$): many kinetic properties are dominated by 2 gapless quarks

Separate issue – bare CFL quark stars:

- Photon free ($\ell \gg R$)
- NG bosons scatter off boundary
- Emissivities are suppressed – very dim
- candidates of dark matter (?)

Conclusion and Outlook

- Studies of dense quark matter are under theoretical control, assuming $\mu \gg \Lambda_{QCD}$: all properties of dense quark matter can be derived from first principles!

- Cooling of CFL quark matter by neutrino and photon emmission is strongly supressed

- Transport properties (conductivities, viscosities, etc.) are dominated by massless NG bosons and photons

- Cooling of compact stars with quark cores does not differ much from that of neutron stars

- Observational signatures of quark stars(?) More work is needed

- Motivation for further studies:
  color superconductivity is likely to exist in the cores of some compact stars