Finite baryon density & gauge field dynamics

M. Laine (CERN)

Based on

- D. Bödeker, ML, JHEP 2001 [hep-ph/0108034]
- A. Hart, ML, O. Philipsen, NPB 2000 [hep-ph/0004060] PLB 2001 [hep-lat/0010008]
- ML, C. Manuel, PRD 2002 [hep-ph/0111113]

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Take QCD with N_f flavours, temperature T, quark chemical potential μ, with $\max(\mu,T) \gg \Lambda_{QCD}$.

Integrate out perturbative modes, $P \gtrsim \max(\mu,T)$.
What is effective theory for modes with $P \lesssim \max(gT, g\mu)$?

(If perturbative, eff. theory ≡ quasiparticle picture.
If not, still valid but needs lattice simulations.)

Static observables $(p_0 = 0, |\vec{p}| \lesssim gT) \Rightarrow$
Dimensionally Reduced (DR) effective action.

Non-static observables $(p_0, |\vec{p}| \lesssim \max(gT, g\mu)) \Rightarrow$
Hard Thermal/Dense Loop (HTL/HDL) action.

$$\Rightarrow \delta \mathcal{L}_E = m_D^2 \,\mathrm{Tr}\,(A_0^2 + \ldots),$$

$$m_D^2 = \frac{N_c}{3} g^2 T^2 + \frac{N_f}{2} g^2 \left(\frac{T^2}{3} + \frac{\mu^2}{\pi^2} \right) + \ldots$$

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Effective theories have often extra symmetries, only broken by higher-dimensional operators.

E.g., strangeness violations in QCD induced by

$$\delta \mathcal{L}_E = \frac{g_w^2}{8 M_w^2} \sin 2\theta_C \left[(\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu s_L) + \ldots \right].$$

In analogy, inclusion of μ,

$$\mathcal{L}_E = \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu} + \bar{\Psi} [\gamma_\mu D_\mu - \gamma_0 \mu] \Psi,$$

breaks charge conjugation invariance, C. Does this lead to new operators in HTL/HDL/DR theories?

Since C can be compensated for by $\mu \to -\mu$, coefficients must be odd in μ $\Rightarrow$

$$\delta \mathcal{L}_E = i \mu N_f \frac{g^3}{3\pi^2} \mathrm{Tr}\, (A_0^3 + \ldots).$$

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Applications:

- $\langle \mathrm{Tr}\, F_{12}^2(\bar{x},t)\ \mathrm{Tr}\, F_{03} F_{12}^2(0,0) \rangle \neq 0$
  $\langle \mathrm{Tr}\, [P + P^\dagger](\bar{x},0)\ \mathrm{Tr}\, [P - P^\dagger](0,0) \rangle \neq 0$ Arnold, Yaffe
- $\langle n_B(\bar{x},t)\ e(0,0) \rangle \neq 0$
- $M = \sum_{\pi^\pm} \frac{|\bar{p}_+|^2 - |\bar{p}_-|^2}{|\bar{p}_+|^2 + |\bar{p}_-|^2} \neq 0$ Bronoff, Korthals Altes, Kharzeev, Pisarski, Tytgat
- Off-diagonal quark number susceptibility
  $\langle \int_{\bar{x}} \bar{u} \gamma_0 u(\bar{x},0)\ \bar{d} \gamma_0 d(0,0) \rangle$ Blaizot, Iancu, Rebhan
- ✓ The "sign problem" in the static limit.

Environments:

- In cosmology $\frac{\mu}{T} \sim 10^{-8}$.
- ✓ At and above AGS & SPS energies, $\frac{\mu}{T} \lesssim 1.3$.
- ✓ Fodor & Katz $\Rightarrow$ $\left(\frac{\mu}{T}\right)_{\text{tricritical}} \sim 1.5$.
- In neutron stars $\frac{\mu}{T} \sim \infty$.

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Leading order contributions

Assume $Q_i \lesssim \max(gT, g\mu)$. Do first Matsubara sum, expand then in $\frac{Q_i}{|\vec p|}$; phase space integral $\int_{\vec p}(\cdots)$ contains $\frac{1}{e^{\beta(|\vec p|\pm\mu)}+1}$ and is dominated by $|\vec p|\sim\max(T,\mu)$.

$n=1 \Rightarrow \delta\mathcal{L}_E = -ig N_f \frac{\mu}{3}\left(T^2+\frac{\mu^2}{\pi^2}\right) \mathrm{Tr}\, A_0 = 0.$

$n=2 \Rightarrow$ HTL / HDL of Braaten-Pisarski.

$n=3 \Rightarrow \delta\mathcal{L}_M = -\frac{1}{2\pi^2} g^3\mu N_f \int_v \mathrm{Tr}\left[\tilde A_\mu \tilde A^\mu \frac{1}{v\cdot\partial}\partial^3 \tilde A_3\right],$

$$\tilde A_\mu = \left(\delta^\alpha_\mu - \frac{v^\alpha\partial_\mu}{v\cdot\partial}\right)A_\alpha ,$$

$$v^\alpha=(1,v^i)\ ;\ v\cdot v=0 .$$

(5)

This result is not gauge invariant: for "soft" fields ($\partial\sim g\tilde A\sim g^2T$) higher n contribute at same order.

Fortunately, graphs are easily analysed with gauge choice $v\cdot\tilde A=0$ [Frenkel, Taylor; Elmfors et al].
Turns out only $n=4$ could give a contribution $\propto \mu$, but explicit computation shows result vanishes.

Thus, can remove gauge choice by writing previous result in a gauge invariant way:

$$\tilde A_\mu = \frac{1}{v\cdot D} v^\alpha F_{\alpha\mu}, \ldots \quad \Rightarrow$$

$$\delta\mathcal{L}_M = -\frac{1}{2\pi^2}g^3\mu N_f\int_v \mathrm{Tr}\left(\frac{1}{v\cdot D}v^\alpha F_{\alpha\mu}\right)\left(\frac{1}{v\cdot D}v^\beta F_\beta{}^{\mu}\right)\times \left(\frac{1}{v\cdot D}D^3\frac{1}{v\cdot D}v^\gamma F_{\gamma 3}\right) .$$

(6)

Fortunately, result can be written in a simpler, local form, by introducing additional d.o.f.'s, as suggested by classical kinetic theory!

$$f^{(i)}[A_\mu(x);x,p] \equiv N_c \times N_c - \text{matrices}$$

$$f^{(i)}[0;x,p] \equiv \begin{cases} \frac{\mathbb{1}}{N_c}\cdot\frac{2\theta(p_0)(2\pi)\delta(p\cdot p)}{e^{\beta(p_0-\mu)}+1} & ; i=1,\ldots,2N_cN_f \\ (\mu\to-\mu) & ; i=2N_cN_f+1,\ldots,4N_cN_f \end{cases}$$

$$\begin{cases} [p\cdot D, f^{(i)}] + \frac{g_i}{2}\left\{p^\mu F_{\mu\nu}, \frac{\partial f^{(i)}}{\partial p_\nu}\right\} = 0 \\ \frac{\partial[S\mathcal{L}_M]}{\partial A^a_\mu} = \sum_i g_i \int \frac{d^4p}{(2\pi)^4} p_\mu \operatorname{Tr}[T^a f^{(i)}] \end{cases}$$

This system can in principle be put on a 4+3-dimensional lattice and studied numerically, as has been done in the HTL-case.

[G.D. Moore et al; D. Bödeker et al; Rajantie & Hindmarsh]

Let us study in more detail the static case. Then system is local even without $f^{(i)}$!

$$\mathcal{L}_E = \frac{1}{2}\operatorname{Tr}F_{ij}^2 + \operatorname{Tr}[D_i,A_0]^2 + g^2\left[T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right)+\mu^2\frac{N_f}{2\pi^2}\right]\operatorname{Tr}A_0^2 + ig^3\mu\frac{N_f}{3\pi^2}\operatorname{Tr}A_0^3 + g^4\frac{6+N_c-N_f}{24\pi^2}(\operatorname{Tr}A_0^2)^2;$$

$$\operatorname{Tr} e^{-\beta H} O_1(\bar{x}_1,0)\, O_2(\bar{x}_2,0)\ldots \Rightarrow \int \mathcal{D}A_\mu\, e^{-\beta\int_{\bar{x}}\mathcal{L}_E}\, \hat{O}_1(\bar{x}_1,0)\ldots$$

Starting directly from Matsubara formalism, NLO corrections to action have also been computed.

Observables :

$O \sim \operatorname{Tr}F_{03}F_{12}$	$\to$	colour-electric screening
$\operatorname{Tr}F_{12}F_{12}$		colour-magnetic "
$\bar\Psi\gamma_0\Psi$		susceptibilities
$\mathbb{1}$		free energy
$\vdots$		

Eff. theory reliable down to $T\sim 2T_c$.

The C-odd operator causes now a "sign-problem":

$$\int \mathcal{D}A_\mu \, O \, e^{-S_E(\mu)} \sim \int \mathcal{D}A_\mu \, O \, e^{-S_E(0)} \, e^{-i \frac{\mu}{T} g^3 \# \int_x \mathrm{Tr} A_0^3}$$

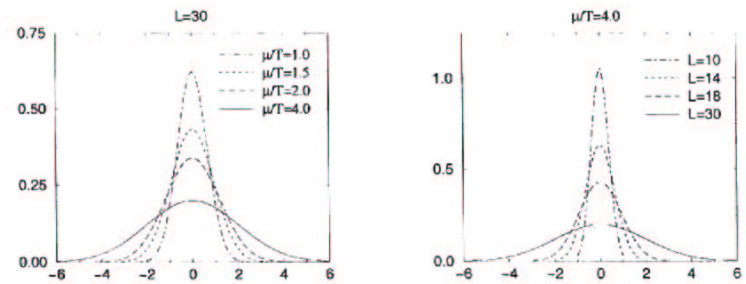
How serious is it ?

- $\left\langle \left[g^3 \# \int_x \mathrm{Tr} A_0^3 \right] \right\rangle_{\mu=0} = 0$
- width $= \left\{ \left\langle [\ldots]^2 \right\rangle \right\}^{1/2} =$ 

$$= \frac{g^3 N_f}{12\pi^3} \left[5 V T^3 \left(\ln \frac{1}{a m_D} + \# \right) \right]^{1/2}$$

Turns out that for $\frac{\mu}{T} \lesssim 4.0$ and realistic V, this is not too serious at all !

Numerical distribution of $\frac{\mu}{T} g^3 \# \int_x \mathrm{Tr} A_0^3$, obtained by doing importance sampling with the real part of the action:



Measurements of correlation lengths with the effective theory can now be used to test various methods for simulating $\mu \neq 0$.

① "Reweighting":

$$\int \mathcal{D}A_\mu \,[O]\, e^{-\mathrm{Re}\,S_E - i\,\mathrm{Im}\,S_E} \to \int \mathcal{D}A_\mu \,[O e^{-i\,\mathrm{Im}\,S_E}]\, e^{-\mathrm{Re}\,S_E}$$

Because of the narrow distribution, reliable infinite V estimates can be obtained up to $\mu \lesssim \pi T$!

An analogous method has been used in 4d simulations by Fodor & Katz, to study the phase diagram. However, generically oscillations become more serious as $\sim V$, so in this case eff. theory may be qualitatively better with $\sim V^{1/2}$!

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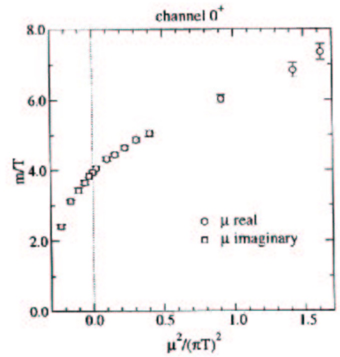
② "Imaginary chemical potential"

Consider C-even observable analytic in μ.

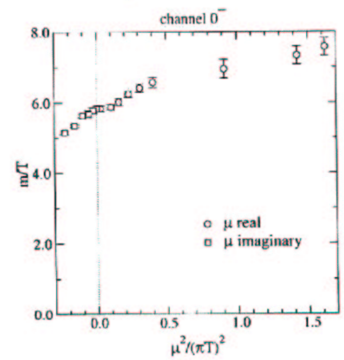
Then $\langle O \rangle = c_0 + c_1 \left(\frac{\mu}{\pi T}\right)^2 + c_2 \left(\frac{\mu}{\pi T}\right)^4 + \ldots$

Measure c_i by using imaginary μ, whereby action is real. Then test against reweighting:

$\mathrm{Tr}\, F_{03}^2 \,;\, (\mathrm{Tr}\, F_{12}^2)$



$\mathrm{Tr}\, F_{03} F_{12}$



ξ^{-1}/T

Success not specific to eff. theory. Recently used in 4d simulations to study phase diagram by de Forcrand & Philipsen.

⑫

③ "Taylor expansions"

The coefficients c_i could be measured from operators at $\mu=0$!
Orders of magnitude:

	c_0	c_1	c_2
O^+	4.0	3.0	-1.0
O^-	5.8	2.0	-1.0

Again, nothing is specific to eff. theory. For $\frac{\mu}{T} \ll \pi$, c_0, c_1 suffice.
Used in 4d simulations to study phase diagram by Allton et al.

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Summary: static case & sign problem

- DR theory allows to put aside real problem of 4d lattice, chiral quarks, and concentrate on $\mathrm{Im}\, S_E$.
- Expansion parameter $\sim \left(\frac{\mu}{\pi T}\right)^2 \Rightarrow$ in the RHIC regime, many methods work and are $\sim$ equivalent.
- For reweighting to work, it is very helpful if oscillations set in as $\sim V^{1/2}$, not as $\sim V$.
- Correlation lengths non-perturbative but known with great precision — however how to connect to Φ?

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Outlook: non-static case

- For $T \gtrsim 2T_c$, eff. theory allows in principle to determine oscillation frequencies, damping rates, etc, non-perturbatively – even at finite μ!

- However a NLO derivation, as well as an exhaustive analysis of UV divergences and renormalisation, are still missing.