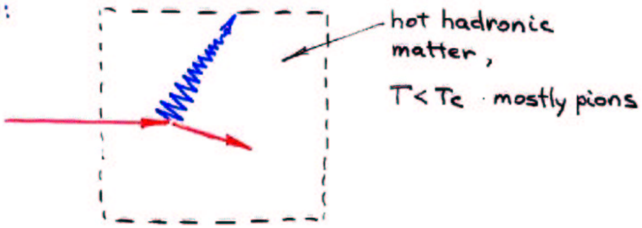
How QCD at finite T responds to time-dependent perturbations?

How do pions propagate in hot QCD?

Why:



Strictly: $T > 0$ - no particles

poles in real-time correlation functions $\to$ quasiparticles

Lattice: $0 < \tau < \frac{1}{T}$; $e^{-\omega\tau} \to e^{i\omega t}$ problematic if $\omega \ll T$ $(t \gg 1/T)$

or $\omega_0 = 0$, $\omega_1 = 2\pi T$, ...

Soft pions are special:

- Real-time propagation from static correlators – – measurable in Euclid (on lattice)
- $T \to T_c$: pion velocity $\to 0$ (phenomenology?)

D. Son, M.S.
PRL 88: 202302, 2002
hep-ph/0204226

DEFINITIONS:

Real time $\langle \pi\pi \rangle$ correlator has poles at $\omega = \omega(\vec{q})$ ← dispersion relation

$m_q = 0$: $\omega^2 = (u\vec{q})^2 + \dots$ u - pion velocity

$m_q \neq 0$: $\omega^2 = u^2(\vec{q}^2 + m^2) + \dots$ m - pion screening mass

$u \cdot m$ - pole mass

$u(T)$, $m(T)$ from static correlators:

m: $\int d\tau\, dV\, e^{-i\vec{q}\vec{x}} \frac{\langle \pi^a(x)\, \pi^b(0) \rangle}{\langle \bar{\psi}\psi \rangle^2} \overset{q, m \to 0}{=} \frac{1}{f^2} \frac{\delta^{ab}}{\vec{q}^2 + m^2}$; $\pi^a = i\bar{\psi}\gamma_5 \tau^a \psi$

"decay const"

u: $u^2 = \frac{f^2}{\chi_{I5}}$

$\int d\tau\, dV \langle A_0^a(x)\, A_0^b(0) \rangle = \delta^{ab} \chi_{I5}$; $A_0^a = \bar{\psi} \gamma^0 \gamma_5 \frac{\tau^a}{2} \psi$

axial isospin

$T = 0$: $f^2 = \chi_{I5} = f_\pi^2$

$f(T)$, $\chi_{I5}(T)$ – from lattice?

Derivation:

Lagrangian approach (conceptually incorrect beyond small q, m – no dissipation)

Microscopic (given) $\longleftrightarrow$ Effective (symmetries, momentum exp. $\to$ few parameters)

$$\mathcal{L}_{micro} = i\bar{\psi}\gamma^\mu D_\mu \psi - (\bar{\psi}_L M \psi_R + h.c.) + \mu_{IS} A_0^3$$

$M = diag(m_u, m_d)$ $\qquad$ $\uparrow$ chem. potl. for A.I.

$\mathcal{L}_{eff}$: d.o.f. pions $\to$ $\Sigma \in SU(2)$;

$$\mathcal{L}_{eff} = \frac{f_t^2}{4} Tr\, \partial_0 \Sigma \partial_0 \Sigma^\dagger - \frac{f_s^2}{4} Tr\, \partial_i \Sigma \partial_i \Sigma^\dagger + \frac{f_m^2}{2} Re\, Tr\, M\Sigma \qquad (\mu_{IS}=0)$$

$$\downarrow$$

$$u^2 = f_s^2 / f_t^2 \quad , \quad m^2 = m_q f_m^2 / f_s^2$$

f_t, f_s, f_m $\to$ by matching to microscopic theory derivatives w.r.t. μ_{IS} and $M(x)$

- How $\mathcal{L}_{eff}$ depends on μ_{IS}?

$$\partial_0 \Sigma \to \partial_0 \Sigma - \frac{i}{2}\mu_{IS} (\tau_3 \Sigma + \Sigma \tau_3)$$

Matching second derivative:

$$\chi_{IS} = \frac{\partial^2 \mathcal{L}_{eff}}{\partial \mu_{IS}^2} = f_t^2$$

- $\mathcal{L}_{eff}(M)$: $\frac{\partial \mathcal{L}}{\partial M}$: $f_m^2 = -\langle \bar{\psi}\psi \rangle$; $\frac{\partial^2 \mathcal{L}}{\partial M(x) \partial M(y)}$: $f_s = f$

$$\downarrow$$

$$f^2 m^2 = -m_q \langle \bar{\psi}\psi \rangle \quad \text{– GOR at finite } T$$

$$\underline{u^2 = f^2 / \chi_{IS}}$$

Operator (hydrodynamic) approach

- Effective description of soft, slow collective modes $\equiv$ HYDRODYNAMICS

D.O.F.? (i) Conserved densities (T^{00}, ρ, etc)

(ii) Goldstone modes (π)

(iii) Near T_c : order parameters (σ)

To linear order: $\phi^a \equiv \frac{\pi^a}{\langle \bar{\psi}\psi \rangle}$ and A_0^a $\qquad$ (parity odd)

Equations (expand in momenta, fields):

- $\partial_0 \phi^a = \frac{1}{\chi} A_0^a + \varkappa \nabla^2 \phi^a + \eta^a$; $\leftarrow$ local noise $\leftarrow$ assumption $\qquad (m_q = 0)$

- $\partial_0 A_0^a = -\partial_i A^{ai}$ and $A_i^a = -f^2 \partial_i \phi^a - D \partial_i A_0^a - \xi_i^a$;

$$\partial_0 A_0^a = f^2 \nabla^2 \phi^a + D\nabla^2 A_0^a + \partial_i \xi^{ai} .$$

Noise:

$$\langle \eta^a(x) \eta^b(0) \rangle = F_\eta \delta^{ab} \delta^4(x)$$

$$\langle \xi_i^a(x) \xi_j^b(0) \rangle = F_\xi \delta^{ab} \delta_{ij} \delta^4(x)$$

Fix parameters:

- canonical commut. $\langle [\phi^a, A_0^b] \rangle = i\delta^{ab}\delta^3(x) \Rightarrow \chi = \chi_{IS}$

- $\langle A_0 A_0 \rangle$, $\langle \phi\phi \rangle$, $\langle A_0 \phi \rangle$, etc can be computed and used to fix parameters:

e.g. $F_\xi, F_\eta \sim T \times D$ – fluctuation-dissipation

Poles: $q_0 = uq - i\Gamma_q/2$, with $u^2 = \frac{f^2}{\chi_{IS}}$

$$\Gamma_q = (D + \varkappa) q^2$$

$u\ (T \to T_c)$

$$u^2 = \frac{f^2}{\chi_{15}}$$

$$\int d\tau\, dV\, e^{-i\vec{q}\vec{x}} \langle \pi^a(x)\pi^b(0)\rangle = \delta^{ab}\frac{\langle\bar{\psi}\psi\rangle^2}{f^2}\frac{1}{q^2} \quad : \quad q \ll m_\sigma$$

Near T_c: $\quad m_\sigma \ll T$

for q: $\quad m_\sigma \ll q \ll T$ (scaling window)

$$\int d\tau\, dV\, e^{-i\vec{q}\vec{x}} \langle \pi^a(x)\pi^b(0)\rangle \sim \frac{1}{q^{2-\eta}}$$

To match at $q \sim m_\sigma$:

- $f^2 = A\, m_\sigma^{-\eta} \langle\bar{\psi}\psi\rangle^2$

  $\Downarrow$

  $f^2 \sim t^{2\beta - \nu\eta} = t^{(d-2)\nu} = t^{\nu}$

η – universal

$m_\sigma \sim t^{\nu}$, $\langle\bar{\psi}\psi\rangle \sim t^{\beta}$ $\qquad t = (T_c - T)/T_c$

$\eta \approx 0.03$, $\nu \approx 0.73$, $\beta \approx 0.38$: $O(4)$, $d=3$

$f^2 \sim m_\sigma$

- χ_{15} – finite at $T = T_c$.

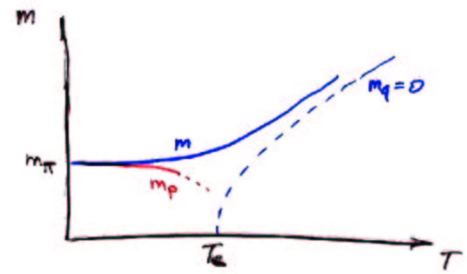
Thus: $\quad u^2 = \frac{f^2}{\chi_{15}} \sim f^2 \sim t^{\nu} \to 0$ at T_c

$m(T) \quad T \to T_c$

$$m^2 = -\frac{m_q\langle\bar{\psi}\psi\rangle}{f^2} \sim m_q t^{\beta-\nu} \qquad \text{- grows}$$

Pole mass:

$$m_p^2 = u^2 m^2 = -\frac{m_q\langle\bar{\psi}\psi\rangle}{\chi_{15}} \sim m_q t^{\beta} \qquad \text{- drops}$$



- Phenomenological consequences (?)
  - Statistical models: $\exp(-\Delta m_p/T)$ enhancement of pion abundance
  - $u < 1$: Cherenkov pions?