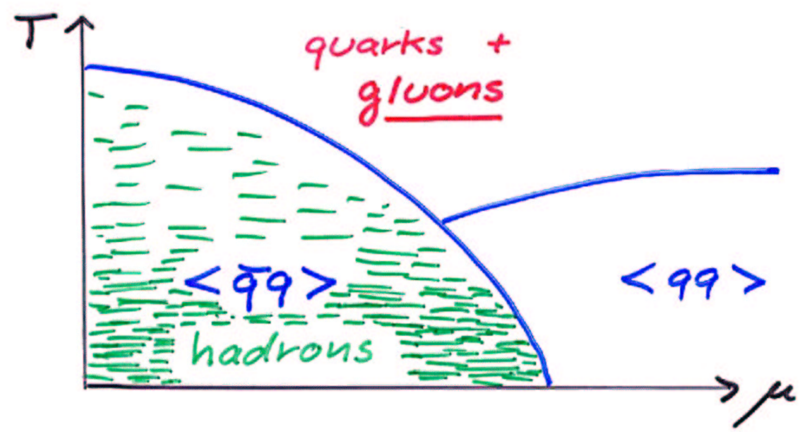
Color octet condensation in the QCD - Phase Transition ?

Analytical understanding of the QCD phase diagram



problem: change in effective degrees of freedom !

a) high T : Gluons + quarks for $T \gg T_c$
Pions for $T \ll T_c$

b) high density : Very different Fermi surfaces for quarks and baryons

Quark descriptions (NJL model)

fail to describe phase transition

a) high T :

chiral aspects could be ok

(quark gas to pion gas)

but glue!

b) different Fermi surface for quarks and baryons ($T=0$)

mean field theory : factor 27 !

confinement very important

„baryon enhancement"

Berges, Jungnickel

Universe cools below 170 MeV :

both gluons and quarks disappear from thermal equilibrium

mass generation ...

chiral symmetry breaking

$\Rightarrow$ fermion masses

gluons ?

Electroweak phase transition :

similar situation

understood by Higgs mechanism

⇒ masses for fermions

and gauge bosons

analogy for QCD :

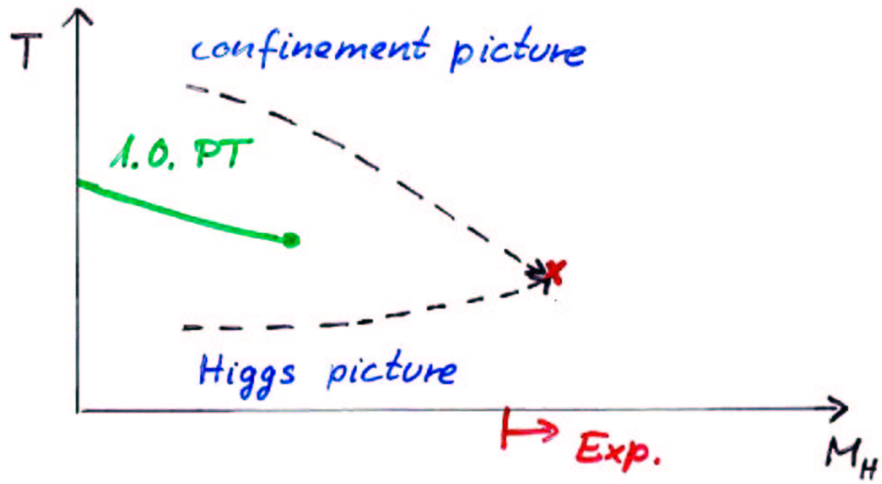
Higgs description of

QCD vacuum

Phase diagram :

Electroweak interactions

at high temperature



Reuter, W.

Philipsen, Buchmüller

Laine, Kajantie, Rummukainen,
Shaposhnikov

Crossover in the standard model !

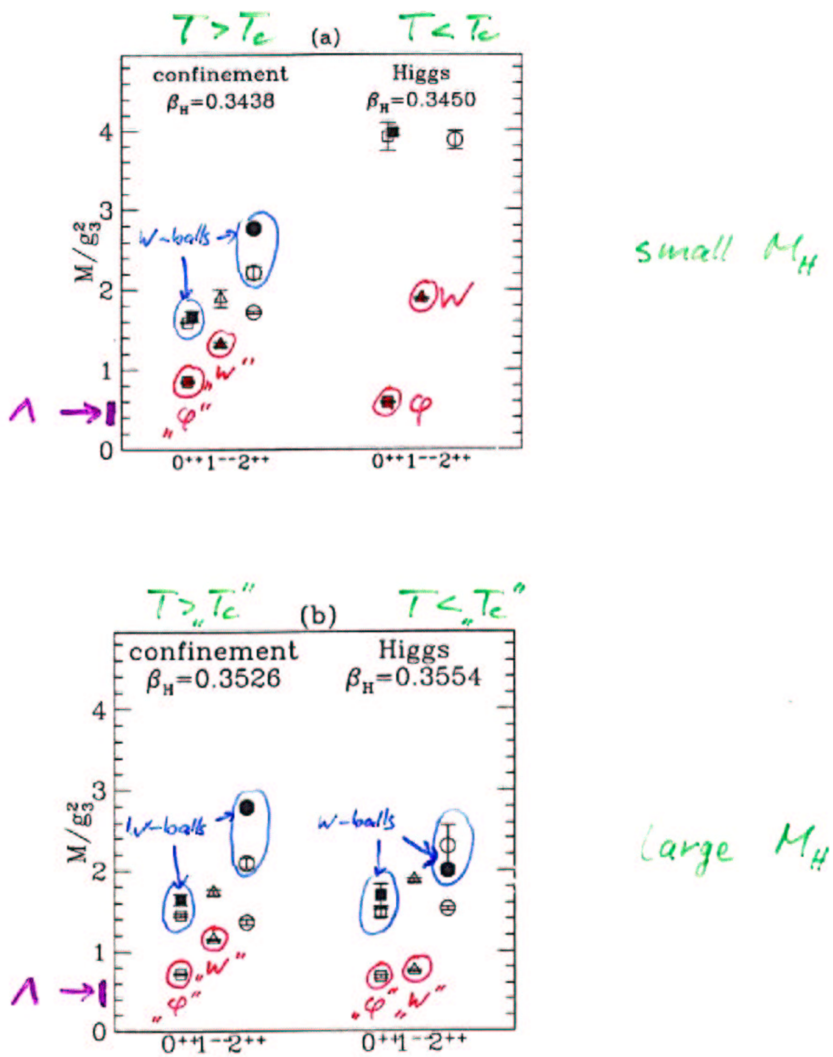


Figure 1: *The lowest states of the spectrum in the confinement (left) and Higgs (right) regions for (a) $\lambda_3/g_3^2 = 0.0239$, (b) $\lambda_3/g_3^2 = 0.274$. Full symbols denote pure gauge states.*
O.Philipsen, M.Teper and H.Wittig, preprint HD-THEP 97-37 (in preparation);

Analogy :

Strong interactions

~

Weak interactions

* gauge symmetry „spontaneously broken"
* Higgs mechanism gives gauge bosons and fermions a mass

$\rho \sim W$

$M_\rho \approx 800\,\mathrm{MeV} \sim M_W \approx 80\,\mathrm{GeV}$

Spontaneous breaking of

color

in the

QCD- vacuum

* equivalence of Higgs -
  and confinement description
  in real QCD in vacuum
  (no phase transition !
  similar to high T electroweak
  interactions)

* no „fundamental" scalars
  (as in usual chiral symmetry
  breaking)
  „symmetry breaking" by $\langle \bar{\psi} \psi \rangle$

Octet condensate in QCD

$\Rightarrow$

spontaneous breaking of color

here: $N_f = 3$

for $N_f = 2$: additional diquark cond.
J. Berges

quarks : $\psi_{L,R\, a\, i}$

a: flavor, i: color

condensate in vacuum :

$$\langle \bar{\psi}_{L\, jb}\, \psi_{R\, ai} \rangle =$$

$$\frac{1}{\sqrt{6}}\, \bar{\xi}_0 \left(\delta_{ia}\, \delta_{jb} - \frac{1}{3}\, \delta_{ij}\, \delta_{ab} \right)$$

color octet

$$+ \frac{1}{\sqrt{3}}\, \bar{\sigma}_0\, \delta_{ij}\, \delta_{ab}$$

color singlet

$\langle \text{octet} \rangle \neq 0$:

spontaneous breaking of color

Higgs mechanism

massive gluons

$$M_\rho^2 = g^2 \hat{Z} \xi_0^2$$

- infrared regulator for QCD
- „simple mechanism for confinement"

$\langle \text{octet} \rangle \neq 0$:

spontaneous breaking of

electromagnetic U(1)-symmetry

(similar to $Q = I_3 + \frac{1}{2} Y$

in electroweak theory)

„combined" U(1) -symmetry survives

(similar to standard model hypercharge, weak isospin $\Rightarrow$ e.m.)

Electric charge

$$Q = \frac{1}{2}\left(\lambda_3^{(L)} + \lambda_3^{(R)} - \underline{\underline{\lambda_3^{(c)}}}\right) + \frac{1}{2\sqrt{3}}\left(\lambda_8^{(L)} + \lambda_8^{(R)} - \underline{\underline{\lambda_8^{(c)}}}\right)$$

	$-\frac{1}{2}\lambda_3^{(c)}$	$-\frac{1}{2\sqrt{3}}\lambda_8^{(c)}$	$\frac{1}{2}\lambda_3^{(v)}$	$\frac{1}{2\sqrt{3}}\lambda_8^{(v)}$	Q	
u_1	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	0	Σ^0, Λ^0, S^0
u_2	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	1	Σ^+
u_3	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	1	p
d_1	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	-1	Σ^-
d_2	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	Σ^0, Λ^0, S^0
d_3	0	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	n
s_1	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	-1	X^-
s_2	$\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	0	X^0
s_3	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	Λ^0, S^0

octet condensate preserves

global SU(3) symmetry

„ diagonal in color and flavor "

„ color - flavor - locking "

cf Alford, Rajagopal, Wilczek; Schaefer, Wilczek

representations with respect to the

„ eightfold way "

quarks : 8 + 1

gluons 8

quarks and gluons carry the observed values of isospin and strangeness spontaneous symmetry breaking

$$\langle \varphi_{ab} \rangle = \underline{\underline{\sigma_0}}\, \delta_{ab}$$

$$\langle \chi_{ij,ab} \rangle = \frac{1}{\sqrt{6}}\, \underline{\underline{\chi_0}} \left(\delta_{ia}\, \delta_{jb} - \frac{1}{3} \delta_{ij}\, \delta_{ab} \right)$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

chiral symmetry breaking

$$SU(3)_c \times SU(3)_V \to SU(3)_P$$

color symmetry breaking
(color-flavor-locking)

$SU(3)_{Ph}$:

quarks : $\underbrace{\bar{3}}_{\text{color}} \times \underbrace{3}_{\text{flavor}} = 8 + 1$

gluons : $\underbrace{8}_{\text{color}} \times \underbrace{1}_{\text{flavor}} = 8$

quarks = baryons

$p, n, \Sigma, \Lambda, \Xi$ $(+5°)$

quark-baryon duality

gluons = vector mesons

ρ, K^*, ω

gluon-meson duality

$\bar{M}_\rho = 850$ MeV

„gluon mass"

gluons carry electric charge and strangeness

K. Bardakci, M. Halpern ; I. Bars (72)

G. t' Hooft (80)

S. Dimopoulos, S. Raby, L. Susskind (80)

T. Matsumoto (80)

M. Yasue (90)

M. Alford, K. Rajagopal, F. Wilczek (99)

T. Banks, E. Rabinovici (79)

E. Fradkin, S. Shenker (79)

R. Mohapatra, J. Pati, A. Salam (76)

A. De Rujula, R. Giles, R. Jaffe (78)

B. Iijima, R. Jaffe (81)

* <u>standard QCD</u>

  (no new theory)

* <u>vacuum</u>

  (not high density QCD)

  (no phase transition physics)

* <u>dynamical mechanism</u>

  in principle, everything should be computable in terms of QCD - parameters

Effective low energy model for QCD

+ composite scalars
  ($\bar{\psi}\psi$ bound states)

+ gauge invariance

+ approximation:
  renormalizable interactions
  for QCD with scalars

comparison with observation ?

Einfacher Ansatz für effektive Wirkung

$$\mathcal{L} = Z_\psi \{ i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \underline{g}\, \bar{\psi}_i \gamma^\mu A_{ij,\mu} \psi_j \}$$

$$+ \tfrac{1}{2} G^{\mu\nu}_{ij} G_{ji,\mu\nu} \qquad \text{„QCD"}$$

$$+ \mathrm{Tr}\{ (D^\mu \chi_{ij})^\dagger (D_\mu \chi_{ij}) \} + \underline{U}(\chi)$$

$$+ Z_\psi \bar{\psi}_i \left[(\underline{h} \varphi \delta_{ij} + \underline{\tilde{h}} \chi_{ij}) \frac{1+\gamma_5}{2} \right.$$

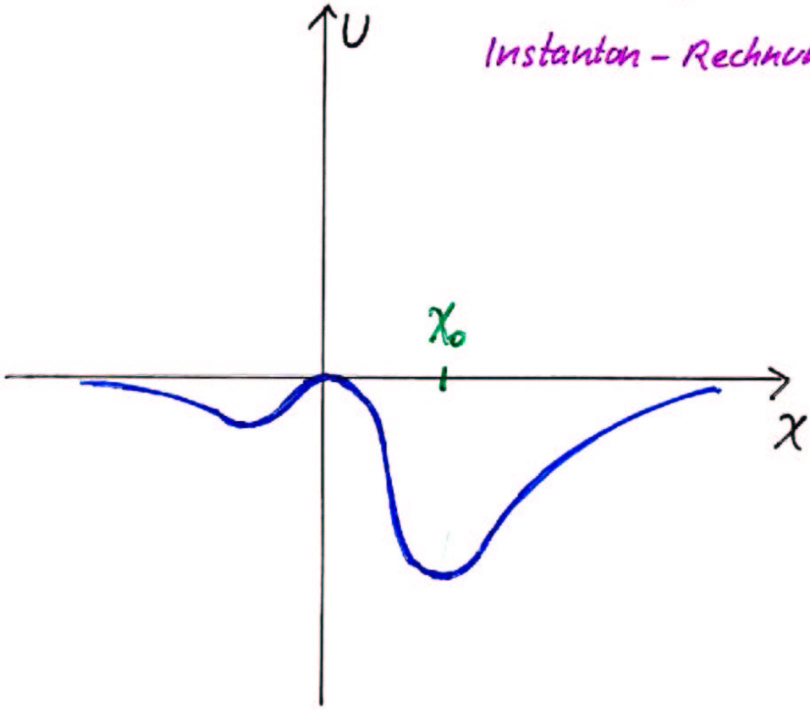
$$\left. - (\underline{h} \varphi^\dagger \delta_{ij} + \underline{\tilde{h}} \chi^\dagger_{ij}) \frac{1-\gamma_5}{2} \right] \psi_j$$

$$A_{ij,\mu} = \tfrac{1}{2} A^z_\mu (\lambda_z)_{ij}$$

Farb + Spinor - Indizes kontrahiert

Effektives Oktet - Potential

Ein-Schleifen-
Instanton-Rechnung



$\chi_0 \approx 150$ MeV

$\bar{M}_\rho \approx 800$ MeV

Anomalie !

Average vector meson mass :

$$\bar{M}_\rho = g\chi_0 =: 850 \text{ MeV}$$

Baryon masses :

$$M_8 = h\sigma_0 - \frac{\tilde{h}}{3\sqrt{6}}\chi_0 = 1.15 \text{ GeV}$$

$$\pm M_1 = h\sigma_0 + \frac{8}{3}\frac{\tilde{h}}{\sqrt{6}}\chi_0 = 1.4 \text{ GeV}$$

$$\frac{\tilde{h}}{g} = 0.24 \quad / \; 2.5$$

$$h\sigma_0 = 1.18 \text{ GeV} \; / \; 0.87 \text{ GeV}$$

$(M_1 > 0)$ $\quad$ $(M_1 < 0)$

$M_1 < 0 \Rightarrow$ singlet has opposite parity of octet

J. Berges, ..

5 undetermined parameters

$\chi_0,\ \sigma_0,\ g,\ h,\ \tilde{h}$

fixed by 5 observable quantities

(for $m_q = 0$, averages over $SU(3)$ multiplets)

$\bar{M}_\rho = 850$ MeV

$\bar{M}_N = 1150$ MeV

$M_1 = 1400$ MeV

$\bar{f} = 110$ MeV $\quad (\bar{f} = \frac{2}{3} f_K + \frac{1}{3} f_\pi)$

$\Gamma(\rho \to \mu^+\mu^-)$, $\Gamma(\rho \to e^+e^-) = 7$ keV

„predictions"

* $\Gamma(\rho \to 2\pi) \approx 150$ MeV
* β-decay of neutrons: $g_A = 1$ (Exp: $g_A = 1.26$)
* vector dominance in electromagnetic interactions of pions, $g_{\gamma\pi\pi}/e = 0.04$

* all predictions of chiral perturbation theory

+

determination of parameters

		„Exp"
L_1	0.87	0.7 ± 0.3
L_2	1.74	1.7 ± 0.7
L_3	-5.2	$-(4.4 \pm 2.5)$

Conclusions (1)

- Spontaneous color breaking plausible in QCD
- Computation of effective action at $\mu_\rho \approx 850$ MeV needed
- Simple effective action can account for mass spectrum of light baryons and mesons as well as their couplings

Gluon – Meson – Duality

Quark – Baryon – Duality

Interesting consequences ?!

High temperature phase transition in QCD :

Melting of octet condensate

Lattice simulations :

deconfinement temperature

= critical temperature for restoration of chiral symmetry

Why ?

simple explanation :

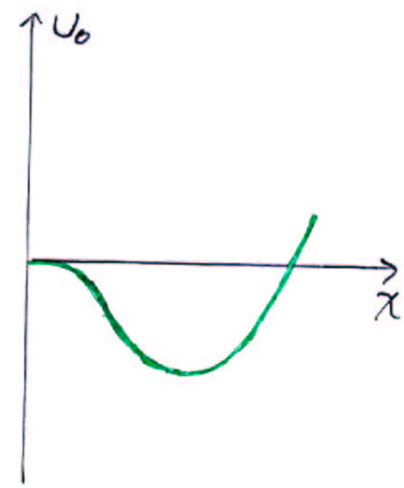
octet condensate → „confinement"
octet condensate → chiral symmetry breaking

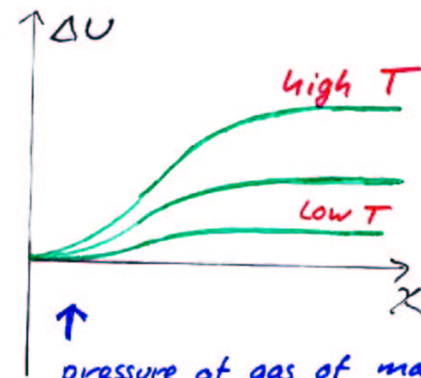
melting of octet condensate → „deconfinement"
melting of octet condensate → chiral symmetry restoration

„ quarks and gluons become massless simultaneously "

temperature dependent effective potential

$$U(\chi) = U_0(\chi) + \Delta U(\chi, T)$$





pressure of gas of massless or light particles

quarks, gluons, pseudoscalar mesons with χ - dependent mass

* $\mu_g^2 = g^2(\mu_g)\,\chi^2$ gluons
* $M_q^2 = h_\chi^2\,\chi^2$ quarks
* $M_G^2 = c_G \dfrac{\partial U}{\partial(\chi^2)}$ pseudoscalars

temperature corrections to effective octet potential

$$\Delta U(\chi, T) = 24\, J_B(\mu_\rho^2) - 12 N_f\, J_F(M_q^2) + (N_f^2-1)\, J_G(M_G^2)$$

$$J_B(M^2, T) = T\int_0^\infty \frac{dq\, q^2}{2\pi^2} \ln\left(1 - \exp\left(-\frac{\sqrt{q^2+M^2}}{T}\right)\right)$$

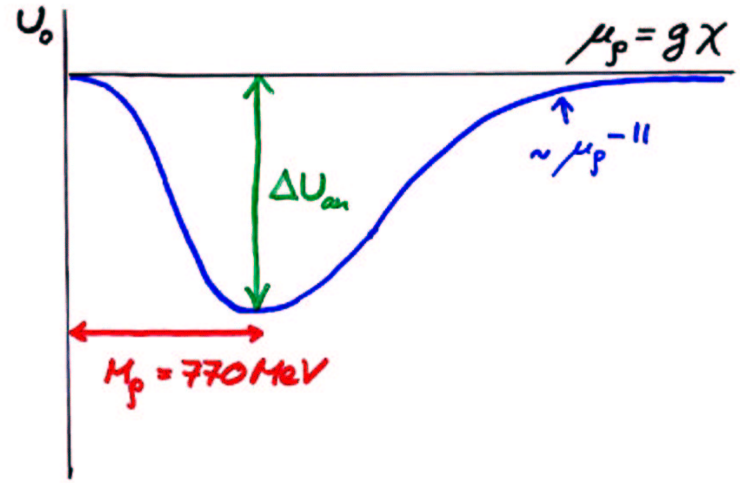
$$J_F(M^2, T) = T\int_0^\infty \frac{dq\, q^2}{2\pi^2} \ln\left(1 + \exp\left(-\frac{\sqrt{q^2+M^2}}{T}\right)\right)$$

$$J_G = J_B \quad \text{for } M^2 > 0$$

„particle" masses M depend on χ !

vacuum effective potential $(T=0)$

instanton dominated



$$\Delta U_{an} = \frac{f_{\eta'}^2 M_{\eta'}^2}{2 N_f^2} \approx 10^{-3}\ \mathrm{GeV}^4$$

	vanishing quark masses	equal $m_u = m_d = m_s \neq 0$ $2M_K^2 + M_\pi^2 = (390\,\mathrm{MeV})^2$
$\bar{M}_\rho$	700 MeV	770 MeV
f	68 MeV	116 MeV
T_c	154 MeV	170 MeV
$\bar{M}_\rho(T_c)$	290 MeV	290 MeV
$\bar{M}_g(T_c)$	580 MeV	600 MeV

screening masses

equation of state : pion gas → QGP

$$\frac{\varepsilon - 3p}{\varepsilon + p} \approx \tau(T_c)\, \frac{T_c^4}{T^4} \qquad (T \gg T_c)$$

$\tau(T_c)$	0.37	0.53

quantitatively :

rough approximation

qualitatively :

analytical computation

interpolates between

pion gas $(T \ll T_c)$ and

quark-gluon plasma $(T \gg T_c)$

correct degrees of freedom !

(without „gluing ad hoc")

interesting relation between T_c

and η' - properties

$$T_c^4 \approx 10^{-2} M_{\eta'}^2 f_{\eta'}^2$$

$(m_s = 0)$

$M_{\eta'} = 960\,\mathrm{MeV}$

$f_{\eta'} \approx 150\,\mathrm{MeV}$

Conclusions (2)

coherent picture for phase diagram of QCD is emerging

gluon - meson duality allows for analytical calculations

quark - baryon duality : direct contact to quantities of nuclear physics

Gauge invariant formulation

Higgs picture only as guide for ideas.

Can be misleading for certain questions!

$U(\varphi, \chi)$ gauge invariant

only assumptions:

a) minimum preserves global SU(3)

b) minimum not for $\chi = 0$

(for appropriate gauge and normalization of χ)

1) color singlet scalar

$$\phi = \sigma_0 U$$

U : 3×3 matrix, $U^\dagger U = 1$

$$U = \exp\left(-\frac{i}{3}\vartheta\right) \exp\left(i \frac{\Pi_z \lambda_z}{f}\right)$$

$z = 1 \ldots 8$

$\Pi_z : (\vec{\pi}, K^0, \bar{K}^0, K^\pm, \eta)$;

$\vartheta \rightarrow \eta'$

(only pseudoscalars are kept here)

2) color octet scalar

$$\chi_{ij,ab} = \frac{1}{\sqrt{6}}\,\chi_0 \left\{ (W_R v)_{ai} (W_L^* v^*)_{bj} - \frac{1}{3} U_{ab}\, \delta_{ij} \right\}$$

$W_{L,R}\,,\ v$: unitary 3x3 matrices

$$U = W_R W_L^\dagger$$

v : color triplet , $B = -\frac{2}{3}$

(v^+ : diquark)

3) How quarks get dressed as baryons

$$\psi_L = Z_\psi^{-1/2}\, W_L\, \underline{N_L}\, v$$

$$\psi_R = Z_\psi^{-1/2}\, W_R\, \underline{N_R}\, v$$

$\psi \equiv \psi_{ai}$: 3x3 matrix , quark field

$\underline{N}$: " , baryon field

N: gauge singlet !

$$N = \underbrace{Z_\psi^{1/2}\, W^\dagger}_{\text{cloud}}\, \underbrace{\psi}_{\text{quark}}\, \underbrace{v^\dagger}_{\text{cloud}}$$

baryons : $B = 1$!

4) gauge bosons

$$A_\mu = -\, v^T \underline{V_\mu}^T v^* - \frac{i}{g}\, \partial_\mu v^T v^*$$

V_μ : vector mesons $(\vec{\rho}, K^*, \omega)$

* express $\mathcal{L}$ in terms of

$W_{L,R}$, V_μ , $N_{L,R}$, v

$\mathcal{L}$ is independent of v

$\Rightarrow$ gauge invariant !

* extract physical propagators and vertices for π, V, N

(they only involve gauge invariant fields !)

π : pseudoscalars

V : vector mesons (ρ, K^*, ω)

...

Non Linear local symmetry

not postulated

* consequence of local color symmetry + „ SSB"

* gauge bosons $\hat{=}$ gluons

predictions correct !

$$\mathcal{L}_V = \frac{1}{2}\,\mathrm{Tr}\,\{V^{\mu\nu}V_{\mu\nu}\} \qquad (\xi^2 = U)$$
$$+ \bar{M}_\rho^2\,\mathrm{Tr}\,\{V^\mu V_\mu\}$$
$$+ \frac{2}{g}\bar{M}_\rho^2\,\mathrm{Tr}\,\{V^\mu \tilde{v}_\mu\} \leftarrow \rho\pi\pi \text{ coupling}$$
$$+ \frac{1}{g^2}\bar{M}_\rho^2\,\mathrm{Tr}\,\{\tilde{v}^\mu \tilde{v}_\mu\}$$
$$- g\,\mathrm{Tr}\,\{\bar{N}_8\gamma^\mu N_8 V_\mu\}$$
$$- \mathrm{Tr}\,\{\bar{N}_8\gamma^\mu\tilde{v}_\mu N_8\} + \dots$$

$$\tilde{v}_\mu = -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\,\partial_\mu\xi^\dagger\right)$$

$$\Rightarrow \quad \tilde{v}_\mu = -\frac{i}{8f^2}\,[\lambda_y, \lambda_z]\,\Pi^y\partial_\mu\Pi^z + \dots$$

* Note: $\mathcal{L}$ does not depend on v!

It involves only color-singlets.

Include electromagetic interactions

by covariant derivatives

e.g. $\hat{v}_\mu = -\frac{i}{2}\left(\xi^\dagger D_\mu\xi + \xi D_\mu\xi^\dagger\right)$

$$= v_\mu - \frac{e}{2}B_\mu\left(\xi^\dagger Q\xi + \xi Q\xi^\dagger - 2Q\right)$$

transforms homogenously

$$\mathcal{L}_V = a f_\pi^2\,\mathrm{Tr}\,\underbrace{\left(\hat{v}_\mu + \frac{1}{2}g\,\vec{\rho}_{V\mu}\vec{\tau} - \frac{1}{2}eB_\mu\tau_3\right)^2}_{}+\dots$$

dictated by local reparametrization symmetry and electromagnetic gauge invariance!

(restriction to ρ-mesons)

$$a = \frac{\chi_0^2}{f_\pi^2} \approx 2.4 \frac{x}{1+x}$$

$$\mathcal{L} = \frac{1}{2} \underline{M_\rho^2} \vec{\rho}_V^{\,\mu} \vec{\rho}_{V\mu}$$

$$- e \underline{g_{\rho\gamma}} \rho_{V3}^\mu B_\mu \qquad (\rho^0 - \gamma\text{-mixing})$$

$$+ \underline{g_{\rho\pi\pi}} \vec{\rho}_V^{\,\mu} (\vec{\pi} \times \partial_\mu \vec{\pi}) \qquad (\rho \to 2\pi)$$

$$+ \underline{g_{\gamma\pi\pi}} B^\mu (\vec{\pi} \times \partial_\mu \vec{\pi})_3 + \dots$$

$$M_\rho^2 = a g^2 f_\pi^2 \quad , \quad g_{\rho\gamma} = a g f_\pi^2$$

$$g_{\rho\pi\pi} = \frac{1}{2} a g \quad ,$$

$$g_{\gamma\pi\pi} = e \left(1 - \frac{2 g_{\rho\pi\pi}^2 f_\pi^2}{M_\rho^2}\right) \approx 0 \quad ✓ \; ⚠$$

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2}{48\pi} \frac{(M_\rho^2 - 4M_\pi^2)^{3/2}}{M_\rho^2} \propto 150\,\text{MeV}$$

$$\Rightarrow g_{\rho\pi\pi} \approx 6$$

$$\Gamma(\rho_0 \to e^+e^-) = 6.62\,\text{keV}$$

$$\Rightarrow g_{\rho\gamma} = 0.12\,\text{GeV}^2$$

prediction :

$$g_{\rho\gamma} = 2 g_{\rho\pi\pi} f_\pi^2 \quad ✓$$

KSFR - relation ⚠

$$M_\rho^2 = \frac{4}{a} g_{\rho\pi\pi}^2 f_\pi^2 \quad \Rightarrow \quad a \approx 2$$

A few common questions:

→ Baryon number ?

Quarks : $B = \frac{1}{3}$

Baryons : $B = 1$

physical states have triality 0

⇒ B integer , Q integer

≙ baryons

N: nucleon field ($(SU(3))_c$ - singlet, integer Q_{em})

$$N = \underbrace{Z_\psi^{1/2} W^+}_{\text{cloud}} \underbrace{\psi \; \upsilon^+}_{\text{quark}}$$

$\upsilon^+ : B = \frac{2}{3}$

→ What about $N_f \neq 3$?

* $N_f = 2$?

diquark condensation in vacuum ?

consistent with conserved B

J. Berges, ...

* heavy quarks ?

integer charged heavy mesons and baryons

* $N_f = 0$?

$$\langle F_{\mu\nu}^{ik} F_{\mu\nu}^{kj} \rangle \neq 0 \quad ?$$

$N_f = 0$: Higgs description of gluodynamics does not work well

→ Dynamical mechanism for octet condensation ?

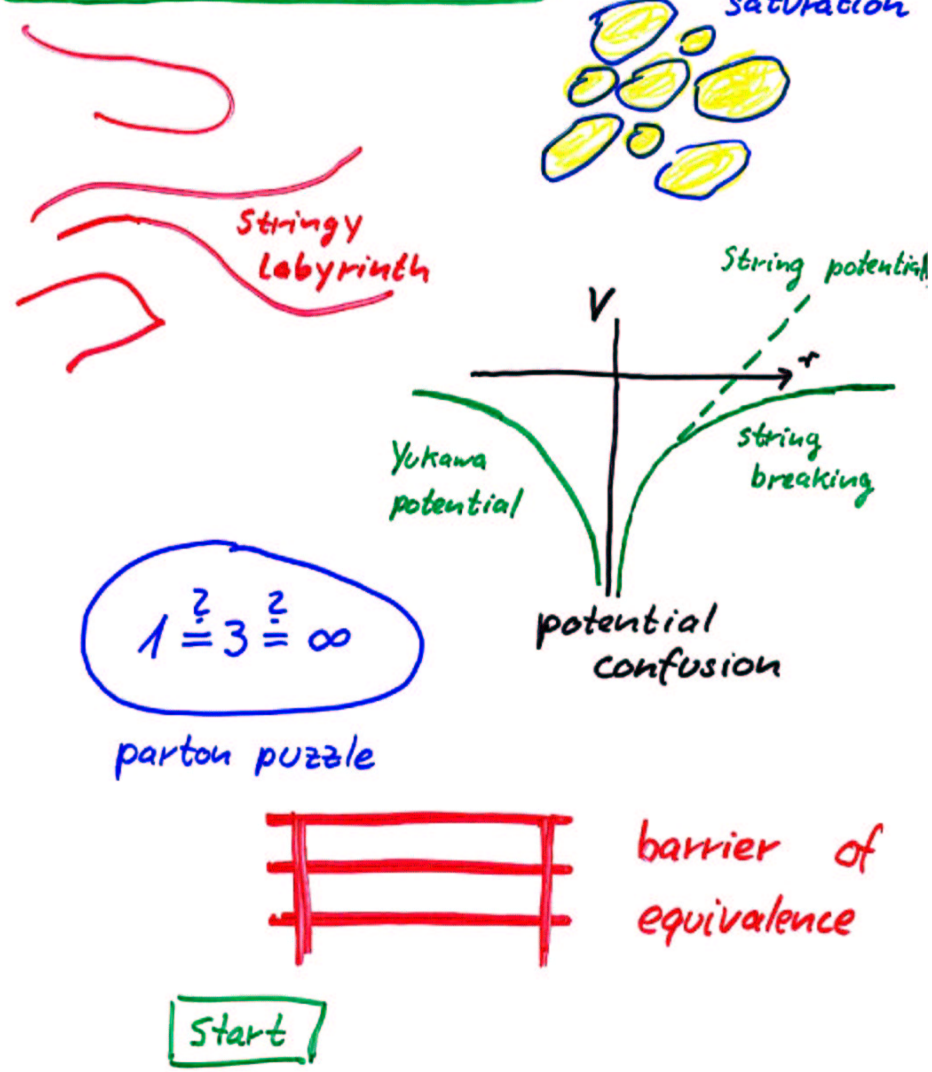
Instantons !

$\bar{q}q$ interactions in octet channel are attractive for nonzero background $\langle \chi \rangle \neq 0$

* octet condensate provides effective IR cutoff for large instantons

* bosonization of effective instanton interactions for $N_F = 3$ can be achieved

If you think you're done :
Get the next pictures..



Lattice tests

a) continuity

- add „fundamental" scalar octets and start in perturbative Higgs phase
- remove scalars continuously by increasing quadratic term in potential

?

phase transition or analytical behaviour

?

contact with parton model

structure function of the proton ?

Large virtuality (large Q^2) $\Rightarrow$

perturbative QCD

electromagnetic vertex

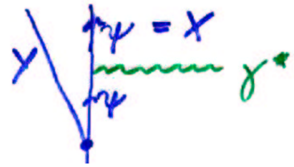
$$\sim e \int_{q,\tilde{p}} \mathrm{Tr}\, \bar{\psi}(\tilde{p}+q)\gamma^\mu \tilde{Q}\, \psi(\tilde{p})\, \tilde{B}_\mu(q)$$

relation between quark and proton field :

$$\psi_{LR}(\tilde{p}) = \int_{p,p'} (Z_\psi^{-1/2} W_{LR})(\tilde{p}-p-p')\, N_L(p)\, v(p')$$

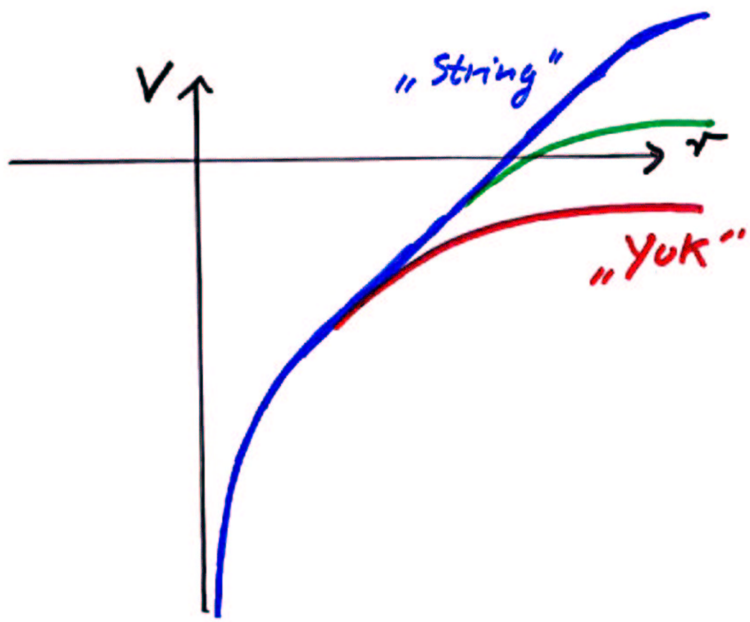
$$Z_\psi = Z_\psi(-D^2(A))$$

$$N + \gamma^* \to X + Y$$



b) heavy quark potential

- start with large m_q : stringy potential
- lower m_q : continuous transition to Yukawa potential?



c) Z_3 -strings

- look for „macroscopic" Z_3 - strings in vacuum
- they should disappear for $T > T_c$

$$\chi_{ab} = \frac{\chi_0(r)}{\sqrt{24}} \lambda^z_{ab} \left(v^T(\varphi)(\lambda^z)^T v^{*}(\varphi) \right)$$

$$v = \exp\left(\frac{i}{\sqrt{3}} \varphi \lambda_8 \right)$$

$$A_\varphi = \frac{1}{\sqrt{3}} \frac{1}{g} \lambda_8$$