Domain Walls and Strings in Dense Quark Matter

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based on papers with
Son, Stephanov,
Forbes, Buckley

Introduction. General remarks.

1. We work in the limit

d (dimensionality) $= 4 = 3+1$

N_c (# of colors) $= 3$

$\mathcal{N}$ (supersymmetry, # of supercharges) $= 0$

N_f (# of light flavors) $= 3$

2. It is well-known that there are no topological defects in the Standard Model (like monopoles, domain walls, strings ....)

$$SU(3)_c \times SU(2) \times U(1) \times (\text{Global}) \to SU(3)_c \times U(1)_{EM}$$

However! In dense matter the situation could be very different

I. $N_f = 3$ CFL (color-flavor-locking) phase. ($m_u = m_d = m_s = 0$)

1. General properties:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{L+R+C}$$

a) color gauge group is completely broken

b) $U(1)_B$ is also spontaneously broken

c) Electromagnetic $U(1)_Q$ is not broken

d) $U(1)_A$ is broken spontaneously and explicitly

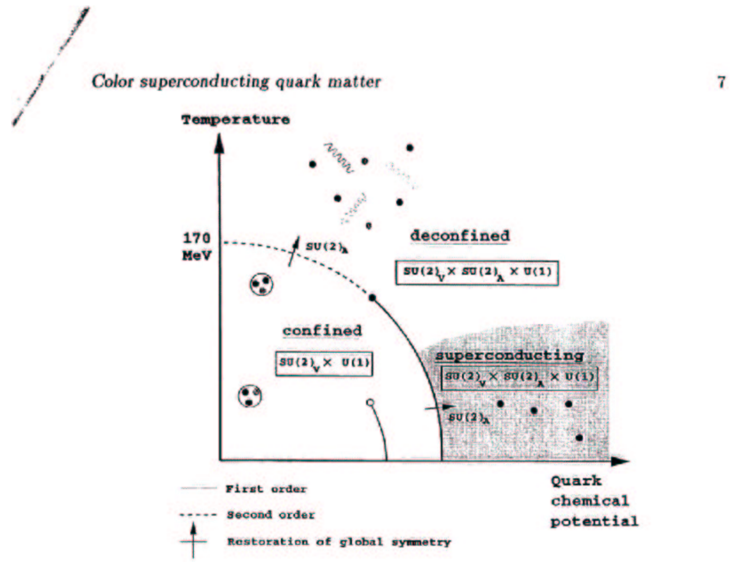
2. Condensates

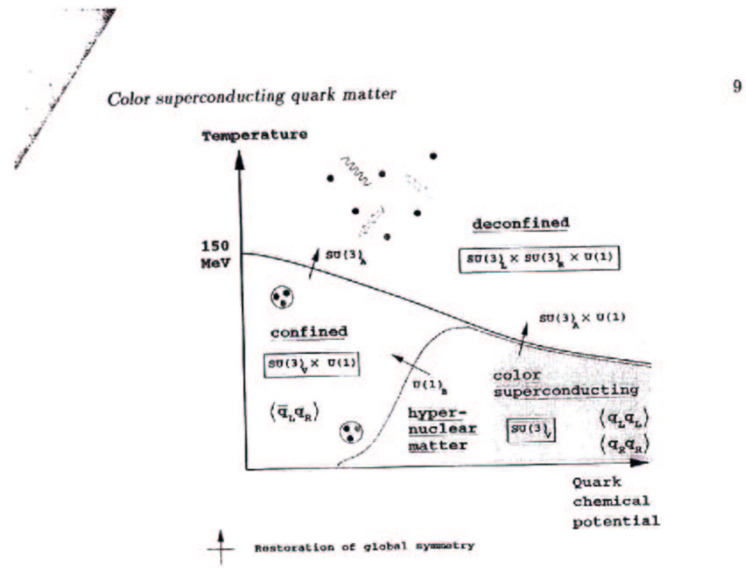
$$\langle \psi^{i\alpha}_{L,a}\, \psi^{j\beta}_{L,b} \rangle^* \sim \epsilon^{ij}\, \epsilon^{\alpha\beta\gamma}\, \epsilon_{abc}\, X^c_\gamma$$

$$\langle \psi^{i\alpha}_{R,a}\, \psi^{j\beta}_{R,b} \rangle^* \sim \epsilon^{ij}\, \epsilon^{\alpha\beta\gamma}\, \epsilon_{abc}\, Y^c_\gamma$$

$$\Sigma^\alpha_\beta = Y^\dagger X = |\Sigma|\, e^{i\frac{\pi^A \lambda^A}{f_\pi} - i\xi'}$$

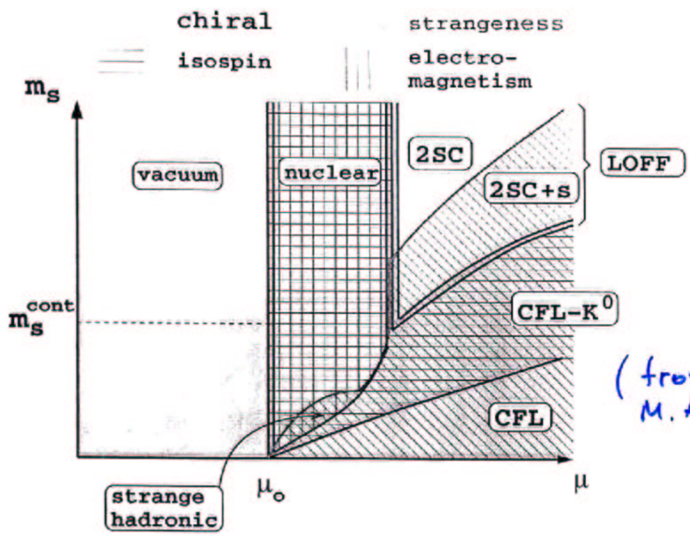
$$\langle \Sigma \rangle = |\Sigma| \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad f_\pi^2 = \frac{21 - 8\ln 2}{18}\, \frac{\mu^2}{2\pi^2}$$





(from M. Alford + ...)

3. CFL + K^0 - phase



(from M. Alford...)

If m_s is large enough, the K^0 condensation occurs

$$\langle \Sigma \rangle = |\Sigma| \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{i\varphi} \\ 0 & -\sin\theta e^{-i\varphi} & \cos\theta \end{pmatrix}$$

$$\cos\theta = \frac{m_0^2}{\mu_{eff}^2} \qquad m_0^2 = \frac{3\Delta^2}{\pi^2 f_\pi^2} m_u (m_d + m_s) \qquad \mu_{eff}^2 = \frac{m_s^2}{2\mu}$$

II Topological Defects

1. $U(1)_B$ is spontaneously broken → there are global vortices (similar to what is observed in He^4)

2. $U(1)_A$ is broken spontaneously (by the condensate $\langle \Sigma \rangle \neq 0$) and explicitly (by the instantons) → There are $U(1)_A$ vortices and domain walls (symmetry $\theta \to \theta + 2\pi n$)

3. CFL + K^0 phase.
$U(1)_Y$ is broken spontaneously (by $\langle K \rangle$ condensate) and explicitly (by weak interactions $\sim G_F$) → There are K-strings and domain walls.

4. CFL + K^0 phase →
$\langle K^+ \rangle$ condensate may occur in the core of K^0-string (similar to what is observed in He^3-B phase)

III. $U(1)_A$ Domain Walls in the chiral limit in QCD ($\mu \gg \Lambda$)

1. General properties:

a) $U(1)_A$ - is a symmetry of QCD at the classical level.

b) $U(1)_A$ is broken spontaneously (by the condensate $\langle \Sigma \rangle$) and explicitly (by instantons)

c) ξ' is the phase of the condensate $\langle \Sigma \rangle$

d) Transformation properties under $U(1)_A$:

$$q \to e^{i\alpha\gamma_5 \frac{1}{2}} q, \quad X \to e^{-i\alpha} X, \quad Y \to e^{+i\alpha} Y$$

$$\Sigma \to e^{-i2\alpha} \Sigma$$

e) Define the Goldstone mode ξ' as:

$$\Sigma = |\Sigma| e^{-i\xi'}, \quad \xi' \to \xi' + 2\alpha$$

f) At low energies:

$$L_{eff} = f^2 \left[(\partial_0 \xi)^2 - u^2 (\partial_i \xi)^2 \right], \quad f^2 = \frac{\mu^2}{8\pi^2}, \quad u^2 = \frac{1}{3}$$

+ small instanton contribution.

g) If we knew dependence on θ-parameter, we would know dependence on dynamical field ξ.

$(\theta - \xi)$ - is the only allowed combination.

(similar to $(\theta - i \log \mathrm{Det}\, U)$ at $\mu = 0$).

2. Instantons at large $\mu \to \infty$.

a) $n(\rho) \sim e^{-N_f \mu^2 \rho^2}$ (Shuryak, 1980)

Therefore, the instantons have small size ($\rho \sim \mu^{-1}$) and the dilute instanton gas approximation becomes reliable.

b) For small instantons ($\mu = \infty$) dependence on θ is: $e^{-i\theta}$

c) $$L_{inst.} = \int d\rho\, n_0(\rho) \left(\frac{4}{3}\pi^2\rho^3\right)^2 \Big\{ \bar{u}_R u_L \bar{d}_R d_L + \frac{3}{32}\Big[\bar{u}_R \lambda^a u_L \bar{d}_R \lambda^a d_L - \frac{3}{4}\bar{u}_R \sigma_{\mu\nu}\lambda^a u_L \bar{d}_R \sigma_{\mu\nu}\lambda^a d_L\Big]\Big\}.$$

['t Hooft, SVZ]

d) $$V_{inst}(\varphi) = \int d\rho\, n_0(\rho) \left(\frac{4}{3}\pi^2\rho^3\right)^2 12 \, |X|^2 \cos(\varphi - \theta)$$

$|X| = \frac{3}{2\sqrt{2}\pi} \frac{\mu^2 \Delta}{g}$, Δ - is the BCS gap

$$n_0(\rho) = C_N \left(\frac{8\pi^2}{g^2}\right)^{2N_c} \rho^{-5} e^{-\frac{8\pi^2}{g^2}} e^{-N_f \rho^2 \mu^2}$$

$$C_N = \frac{0.466\, e^{-1.679 N_c}\, 1.34^{N_f}}{(N_c-1)!\,(N_c-2)!}$$

e) $V_{inst} = -a \mu^2 \Delta^2 \cos(\varphi - \theta)$,

$$a \simeq 5 \cdot 10^4 \left(\frac{\Lambda_{QCD}}{\mu}\right)^{11-\frac{4}{3}} \left(\ln \frac{\mu}{\Lambda_{QCD}}\right)^7 \quad N_f = 2$$

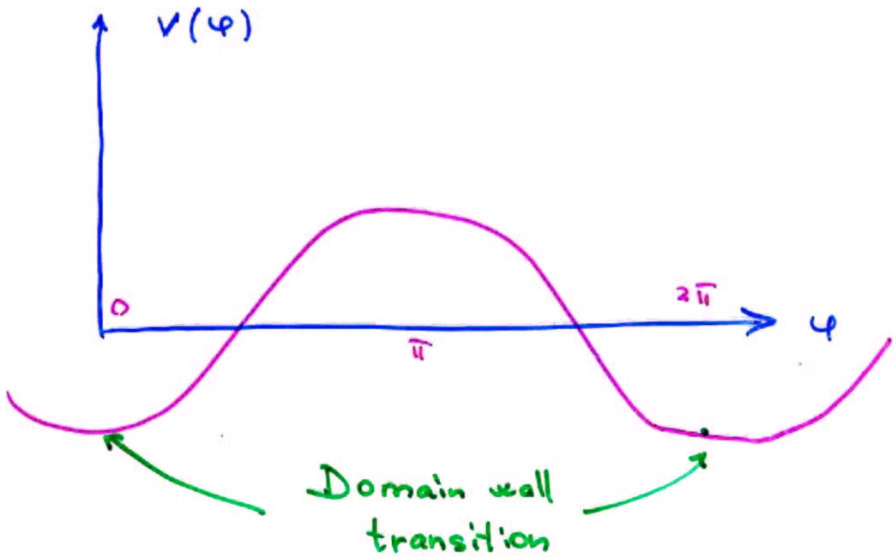
$$a' \simeq 7 \cdot 10^3 \left(\frac{m_s}{\mu}\right) \left(\frac{\Lambda_{QCD}}{\mu}\right)^9 \left(\ln \frac{\mu}{\Lambda_{QCD}}\right)^7 \quad N_f = 3$$

When $\mu \gtrsim 1$ GeV, the theoretical calculations are under control

IV $U(1)_A$ Domain Walls. Decay.

1. $L_{eff} = f^2\left[(\partial_0\varphi)^2 - u^2(\partial_i\varphi)^2\right] + a\mu^2\Delta^2\cos(\varphi-\theta)$, $a \sim \left(\frac{\Lambda_{QCD}}{\mu}\right)^6 \to 0$

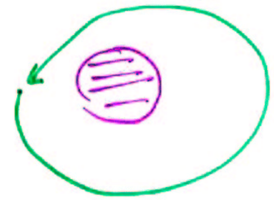
$\varphi = 4\tan^{-1}\exp\left(\frac{m x}{4}\right)$, $m = \sqrt{\frac{a}{2}}\frac{\mu}{f}\Delta$ – η' mass in the chiral limit



$\varphi = 0, 2\pi, 4\pi, \ldots$ – are the same physical points

$\sigma = 8\sqrt{2a}\, u f \mu \Delta$ – domain wall tension

2. If states: $\varphi = 0, 2\pi, 4\pi, \ldots$ were physically different states, this domain wall would be absolutely stable (ferromagnetic d.w.)

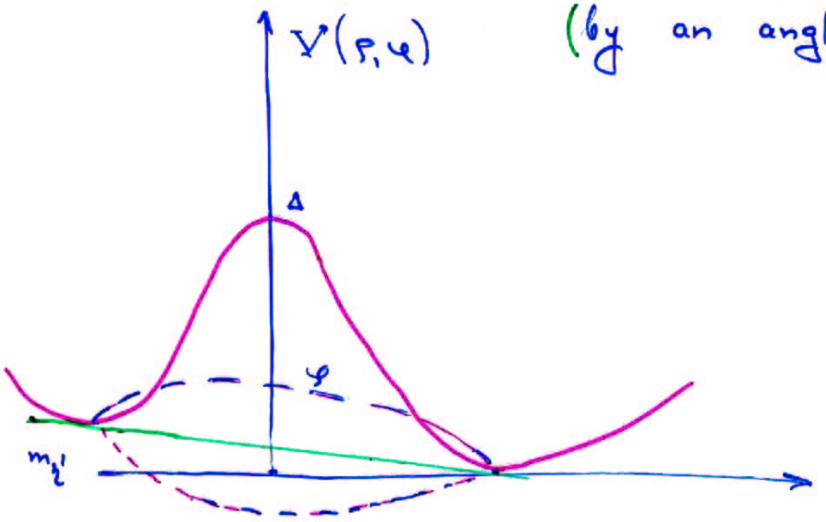


3. If φ is the only degree of freedom, the soliton is also absolutely stable soliton, the Sine-Gordon model.

$L = (\partial_\mu\varphi)^2 + \lambda\cos\varphi$

4. If the heavy degrees of freedom (which have $m \simeq \Delta$) are taken into account, this domain wall becomes metastable.

Tilted Mexican hat
(by an angle ~ "a")



$$\Gamma \sim e^{-\frac{\pi^4}{3}\frac{u^3}{a}\frac{f^4}{\mu^2\Delta^2}\ln^3\frac{1}{\sqrt{a}}} \xrightarrow[(a\to 0)]{} 0$$

(similar picture takes place in large N_c limit in QCD with $\mu = 0$)

Vortices in CFL phase.

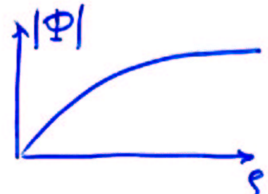
1. $U(1)_B$ is spontaneously broken.
2. $U(1)_A$ is spontaneously and explicitly broken

$\boxed{\pi_1(U(1)) \neq 0}$ ⇒ VORTICES

3. String tension α.

$\alpha \sim 4\pi f^2 u^2 \ln\frac{R}{\Delta}$ R - infrared cutoff

4.



$\Phi \sim |\varphi| e^{i\theta}$

5. Important: symmetry is restored in the center of the core

6. Vortices in crystalline phase ?

V. K^0-strings.

1. CFL + K^0 phase

$$\langle \Sigma \rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\varphi} \\ 0 & -\sin\theta e^{i\varphi} & \cos\theta \end{pmatrix}$$

$$\cos\theta = \frac{m_0^2}{\mu_{eff}^2} < 1, \quad m_0^2 = \frac{3\Delta^2}{\pi^2 f_\pi^2} m_u (m_d + m_s)$$

$$\mu_{eff} = \frac{m_s^2}{2\mu}$$

If $m_0^2/\mu_{eff}^2 < 1$, K_0 is condensed

2. $$L_{eff}(K_0) = |\partial_0 K^0|^2 - u^2 |\partial_i K^0|^2 - \lambda \left[|K_0|^2 - \frac{\eta^2}{2} \right]^2$$

$$\langle K^0 \rangle^2 = \frac{\eta^2}{2}, \quad \lambda \eta^2 = \mu_{eff}^2 (1 - \cos\theta)$$

3. $U(1)$ is spontaneously broken,

→ φ becomes the Goldstone mode

→ there is K^0 string solution. $\pi_1(U(1)) = \mathbb{Z}$

4. Equation

$$u^2 \nabla^2 K_0 = 2\lambda \left(|K_0|^2 - \frac{\eta^2}{2} \right) K_0$$

$$(K_0)_{string} = \frac{\eta}{\sqrt{2}} e^{im\varphi} f(r)$$

5. String tension

$$\alpha \sim 2\pi u^2 f_\pi^2 \int^R \frac{dr}{r} \sim 2\pi u^2 f_\pi^2 \ln R$$

a) $f_\pi \sim \mu$

b) R is an upper cutoff determined by the environment of the string (presence of other strings, K-domain walls, ...)

c) Symmetry is restored in the core

VI. Superconducting K-strings.

1. Q: What happens in the isotopical limit $m_u = m_d$?

2. A: In the $SU(2)_I$ limit $(m_u = m_d)$ K-strings can not exist.
Indeed, $\pi_1(SU(2))$ - is trivial

3. If $m_u \neq m_d$, K^0-strings are stable,
If $m_u = m_d$ - - - - unstable

Q: What happens when $m_d - m_u$ changes?
A: We anticipate that some transition occurs when $m_d - m_u$ changes

4. K-string solution

$$\begin{cases} K^0_{string} = \frac{\xi}{\sqrt{2}} f(r) e^{im\varphi} \\ K^+ = 0 \end{cases} \qquad \langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{\xi}{\sqrt{2}} \end{pmatrix}$$

$$L = |\partial_0 \Phi|^2 - u^2 |\partial_i \Phi|^2 - \lambda \left[|\Phi|^2 - \frac{\xi^2}{2} \right]^2, \quad \Phi = \begin{pmatrix} K_+ \\ K_0 \end{pmatrix}$$

5. Stability analysis. Exact $SU(2)_I$

$$E\left(K^0 = K^0_{string} + \delta K^0, K^+\right) = E_{string} + \delta E$$

a) K^0_{string} - is stable configuration, therefore δK^0 can not destroy stability

b) Keep δK^+ :

$$\delta E = \int d^2 r \left(u^2 |\nabla K^+|^2 + \mu_{eff}^2 (1 - \cos\theta) (f^2 - 1) |K^+|^2 \right)$$

$$K^+ = \frac{\xi}{\sqrt{2}} \sum_m g_m(r) e^{im\varphi}$$

Lowest contribution corresponds to $m = 0$

c) $$\delta E = \frac{\xi^2 u^2}{2} \int d\tilde{r}\, g_0(\tilde{r})\, \hat{O}\, g_0(\tilde{r})$$

$$\hat{O} = -\frac{1}{\tilde{r}} \frac{d}{d\tilde{r}} \left(\tilde{r} \frac{d}{d\tilde{r}} \right) + (1 - \cos\theta) \left(f^2(\tilde{r}) - 1 \right)$$

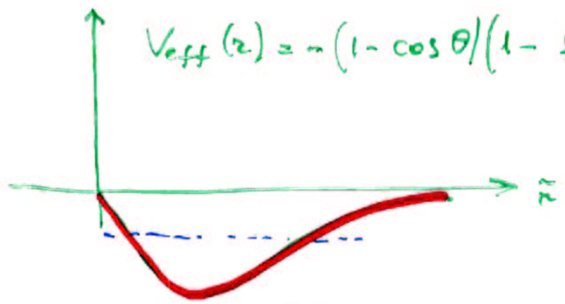
$\tilde{r} = \frac{\mu_{eff}}{u} r$ - dimensionless variable

d) The problem is reduced to the analysis of the 2d Schrödinger equation for a particle in an attractive potential

$$V(\tilde{r}) = -(1-\cos\theta)(1-f^2(\tilde{r})), \quad 0 \le f \le 1$$

where $f(r)$ is the solution of the equation (string profile function)

e) $V_{eff}(r) = -(1-\cos\theta)(1-f^2(r))$



In 2d an arbitrary weak potential has always a negative bound state

f) Operator $\hat{O}$ has always negative mode irrespective of the properties $f(r)$.

g). String is unstable - the result we expected from topology.

6. Stability analysis. Explicitly broken SU(2)

a) $$L_{eff} = |\partial_0\Phi|^2 - u^2|\partial_i\Phi|^2 - \lambda\left[|\Phi|^2 - \frac{\xi^2}{2}\right]^2 - \frac{3\Delta^2}{2\pi^2 f_\pi^2} m_s(m_d - m_u)\,\Phi^+\tau^3\Phi$$

b) $$\delta E = \frac{\xi^2 u^2}{2}\int d^2\tilde{r}\; g_0(\tilde{r})\left[\hat{O} + \varepsilon\right] g_0(\tilde{r})$$

$$\hat{O} = -\frac{1}{\tilde{r}}\frac{d}{d\tilde{r}}\left(\tilde{r}\frac{d}{d\tilde{r}}\right) + (1-\cos\theta)(f^2(r)-1)$$

$$\varepsilon \equiv \frac{3\Delta^2}{2\pi^2 f_\pi^2}\,\frac{m_s(m_d-m_u)}{\mu_{eff}^2} \to 0$$

c) $\hat{O} g_0 = \hat{E} g_0$. For the ground state $\hat{E}$ is always negative. However to insure the instability one should require

$$\boxed{\hat{E} + \varepsilon < 0, \quad \varepsilon = \frac{3\Delta^2}{2\pi^2 f_\pi^2}\,\frac{m_s(m_d-m_u)}{\mu_{eff}^2}}$$

d) Solution $\hat{E} + \varepsilon = 0$ does not exist for arbitrary weak $\sim (1 - \cos\theta)$ potential. However, solution does exist for relatively large θ_{crit}.

e) $E(\theta_{crit}) + \varepsilon = 0$

For realistic parameters $\mu \simeq 500$ MeV, $m_u \simeq 5$ MeV, $m_d = 8$ MeV, $m_s \simeq 150$ MeV, $\Delta = 100$ MeV,

$$\sin\frac{\theta_{crit}}{2} \simeq \text{const}\ \frac{\Delta}{\pi f_\pi}\sqrt{\frac{m_s(m_d - m_u)}{\mu_{eff}^2}}$$

$\theta_{crit} \simeq 53^\circ$, $\theta \simeq 70^\circ$

f). $\theta > \theta_{crit} \Rightarrow K^+$ condensation does occur in the Core of K°-strings

10

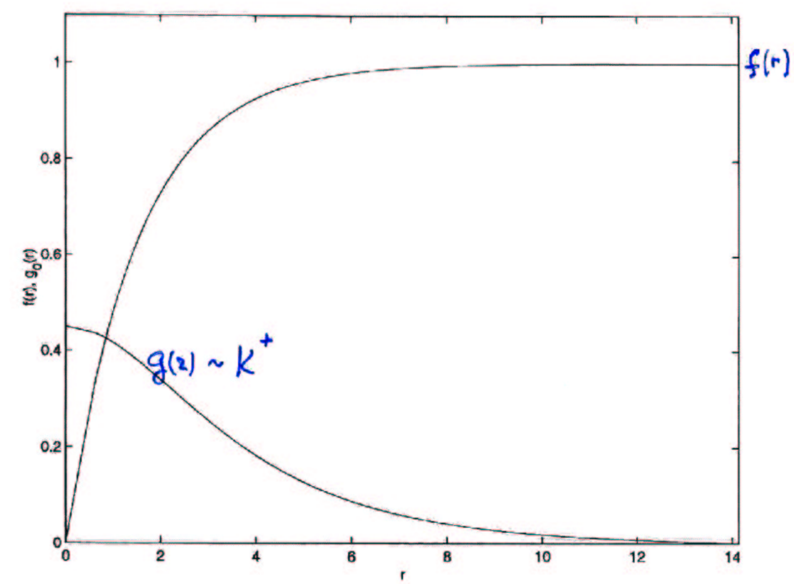
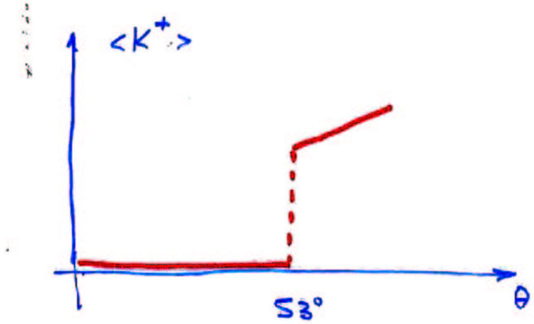


FIG. 2: In this figure we plot the functions $f(r)$ (related to the K^0-string by Eq. (11)) and $g_0(r)$ (related to the K^+-condensate by Eq. (22)) as a function of the dimensionless rescaled coordinate $\tilde{r} \equiv v_\pi r/\mu_{eff}$.



VII Effective Action for Superconducting K-strings.

1. It so happens that K-strings in CFL+K_0 phase is an explicit realization of old idea (Witten, 82) on superconducting cosmic strings.

2. The difference is: a) Our effective lagrangian follows from QCD with specific parameters which can be calculated; b) We do not have to adjust any parameters in the potential for this phenomenon to take place (in contrast with Witten's eff. potential

3. One can follow Witten's paper to analyze such strings.

4. $K^+ = e^{i\theta(z,t)} K^+(x,y)$

$\theta(z,t)$ is an arbitrary slowly varying function

$A_\mu(x,y,z,t) = A_\mu(0,0,z,t)$ whenever $\langle K^+ \rangle \neq 0$

5. $I = \alpha \int dz\, dt\, (\partial_\mu \theta + e A_\mu)^2$

$\alpha \equiv \int dx\, dy\, |K^+(x,y)|^2$ is determined by the condensate in the core

6. In the presence of external z-independent electric field E_z, we have equation of motion

$\partial_\mu(\partial_\mu \theta + e A_\mu) = 0$, $J_\mu = e(\partial_\mu \theta + e A_\mu)$

7. Conserved current J_μ can be represented as

$J_\mu = e\epsilon_{\mu\nu} \partial_\nu \gamma$, $\partial_\nu \gamma = \epsilon_{\nu\lambda}(\partial_\lambda \theta + e A_\mu)$

(property of 2d)

8. Equation of motion becomes

$$\partial^2 \gamma = e\, \epsilon_{\nu\lambda} \partial_\nu A_\lambda = eE$$

$$J_z = e\dot{\gamma} \sim e \frac{d\theta}{dz}$$

$$\ddot{\gamma} = eE$$

$$\frac{dJ_z}{dt} = e^2 E$$

$$N = \frac{1}{2\pi} \oint_{\ell} d\ell \frac{d\theta}{d\ell} \neq 0$$

N is topological invariant

→ persistent current

9. If an electric field E is applied for a time T, a current

$$J_z = e^2 E T$$

builds up. This current remains even if the electric field is turned off after time T (persistent current).

10. If a string moves at velocity v across a magnetic field B_0, then in the rest frame of the string there is an applied electric field $E = \frac{v}{c} B_0$

11. The maximum current the string can carry: $J \simeq \frac{\tilde{e}\Delta}{2\pi}$, $\tilde{e} = \frac{eg}{\sqrt{g^2 + 4h^2 e^2}}$, $h_{CFL} = \frac{1}{\sqrt{3}}$

Conclusion.

Role of these strings and domain walls in physics of neutron stars ???

Analogy with CM physics suggests that the topological defects should play an important role in dynamics.

Quark Dense Matter could be much more common state of matter on the scale of the Universe than the "normal" hadronic phase we know.

Two Seismic Events with the Properties for the Passage of Strange Quark Matter Through the Earth

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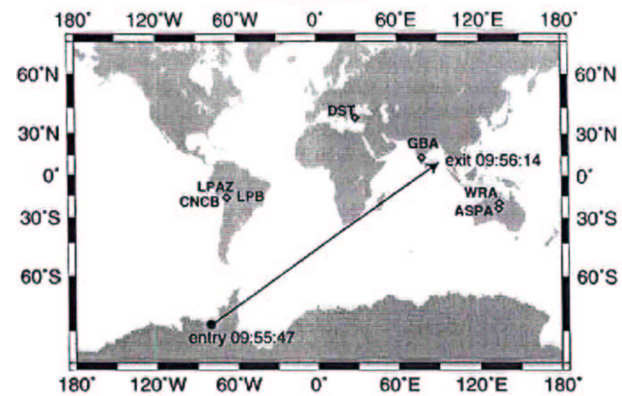

Figure 2. Surface trace for first event.

IV. FIRST EVENT
(10 22 1993)

Two sets of reports were found in the 1993 database that fit all the criteria for a linear source. The model and residuals (observed minus predicted arrival times) for the first event are given in Table 1. The RMS residual is 0.73 sec. This RMS value would indicate a good fit for a well-located earthquake.

Using the earthquake locator program HYPOSAT [8] we tried to fit a point source model to the seven arrival times and two back azimuth values, the azimuths from station to source, from the two Australian arrays (WRA and ASPA). The iterative location algorithm failed to converge after 81 iterations, showing that the data could not fit a point source model.

The linear source model for this event has an entry near Elsworth Land, Antarctica, and an exit south of India (Figure 2). One problem with this event is that only seven stations reported. Times from six stations define a linear source (Entry latitude and longitude, entry time, exit latitude and longitude, and velocity are the defining parameters). In this case, however, the back azimuths determined at the two Australian arrays (ASPA and WRA) point to their respected POCA's. Thus, there are nine observations used to estimate the parameters of the linear source model.

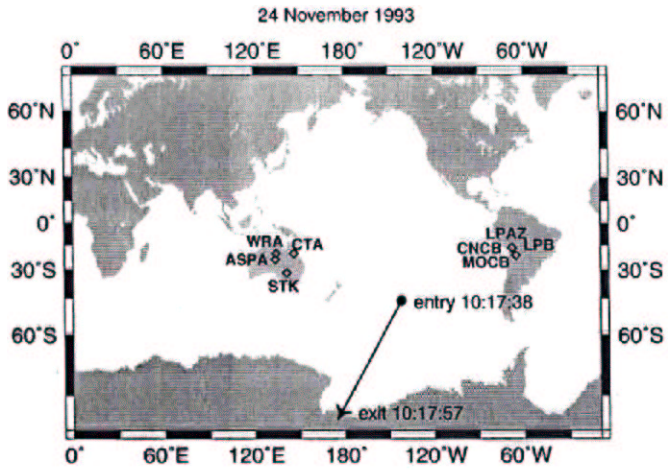

Figure 6. Surface trace for second event.

V. SECOND EVENT
(11 24 1993)

Evidence for a linear source for this event is even more compelling than for the first one. The model and the residuals are given in Table 2. All residuals are less than $\pm$ 0.4 sec., an excellent fit of data to the model. The root-mean-square (RMS) residual is 0.15 sec. This result is better than most obtained for well-located earthquakes.

Using the earthquake location program HYPOSAT [8] we tried to fit the nine arrival times and back azimuth and slowness (the reciprocal of horizontal phase velocity) values from the two Australian arrays (WRA and ASPA) with a point source model. The iterative location process did not converge until depth was fixed at 109 km, a very unlikely depth for an earthquake near the Pacific-Antarctic Ridge. After 158 iterations the program produced a final location with an RMS residual of 2.55 sec. The largest residuals were for STK, WRA and ASPA in Australia where the arrivals were clear and well timed. We consider the point source model to be highly unlikely because its RMS residual was 17 times larger than the RMS value obtained with the linear source model.