FROM COOL CGC
TO HOT QGP

Raju Venugopalan
BNL & RBRC.

COLLABORATORS:

* Alex Krasnitz
* Yasushi Nara.
* Dima Kharzeev (on topological charge fluctuations)

Outline Of Talk:

* Introduction.
* Classical Approach To Nuclear Collisions.
* Computing All Possible Tree Level Graphs Numerically.
* The "Late" Stages: Thermalization, Hydrodynamics,...
* The CGC & RHIC: v_2
* Outlook

* The Physics Of Multi-Particle Production Is Small x Physics

$$|A\rangle = c_1\,|qqq\cdots q\rangle + c_2\,|qqq\cdots qg\rangle + \cdots c_n\,|qqq\cdots qg\bar{q}qgg\cdots g\rangle$$

Each Parton Carries Small Fraction x Of Nuclear Momentum

* EFT

Bj Kogut Soper

$\boxed{A^+ = 0}$

$$Z \propto \int [d\rho][dA]\, e^{-S[A,\rho]}$$

$$S[A,\rho] = F_{\mu\nu}^2 + J^+A^- - i\int d^2x_\perp \frac{\mathrm{Tr}(\rho^2)}{\Lambda_s^2}$$

Static Light Cone Sources $J^{+,a} = \rho^a(x_\perp)\,\delta(x^-)$

McLerran + R.V.

Color Charge Squared / Area

* RG a la Wilson JIMWLK

* For $\Lambda_s^2 \gg \Lambda_{QCD}^2$, $\alpha_s(\Lambda_s^2) \ll 1$

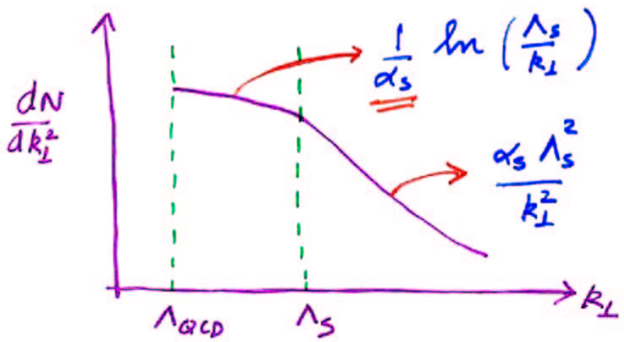
Weak Coupling Methods Applicable!

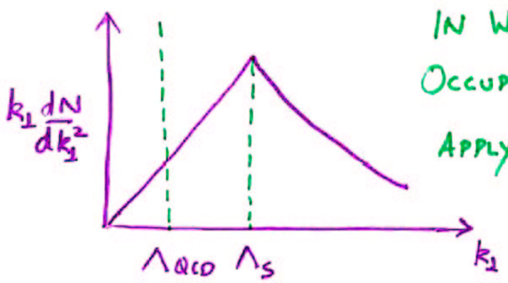
$$\left(\Lambda_s^2 \simeq \frac{A^{1/3}}{x^{\delta}}\, \text{fm}^{-2} \simeq 1\text{–}2\ \text{GeV}^2 \text{ for RHIC};\ \alpha_s \simeq 0.3 \right)$$

$\delta \simeq 0.3$ at HERA

* The Classical Field Of A Nucleus.

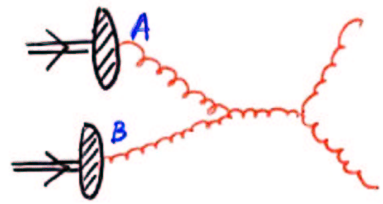
MV
Kovchegov
JKMW





In Weak Coupling,
Occupation # $\propto \frac{1}{\alpha_s} \gg 1$
Apply Classical Methods

* At Small x, Coherence Length $l_c \sim \frac{1}{2 m_N x}$ Is Large (For $x \sim 10^{-2}$, partons from several nucleons interact coherently – No "binary" scaling!)



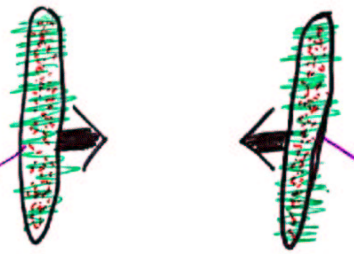
$$\frac{d\sigma}{dp_\perp^2} \neq f_g^A(x_1,\mu^2) \otimes f_g^B(x_2,\mu^2) \otimes \frac{d\hat{\sigma}^{gg\to gg}}{d\hat{t}}$$

* Description At Level Of Amplitudes (Fields) Necessary To Include Effects Of Coherence

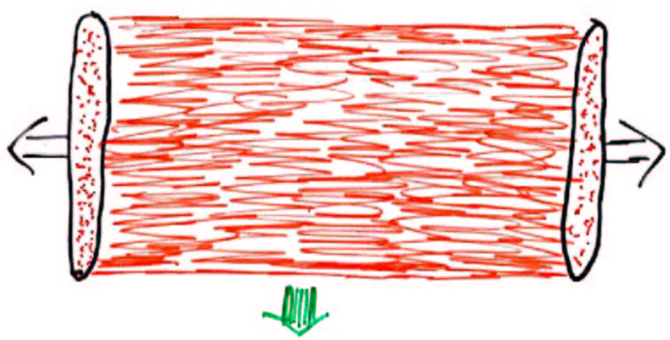
* Large Occupation # Makes Classical Field Approach Plausible

$$\frac{dN}{d^2k_\perp d^2x_\perp} \propto \frac{1}{\alpha_s} > 1$$

CLASSICAL APPROACH TO NUCLEAR COLLISIONS



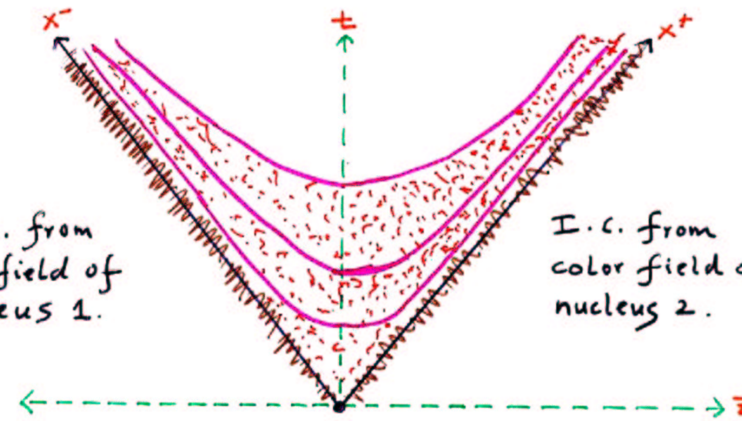
THE SCALE Λ_s CONTROLS THE DISTRIBUTION OF PARTONS IN THE INCOMING NUCLEI



Λ_s ALSO DETERMINES THE INITIAL MULTIPLICITY AND TRANSVERSE ENERGY OF PRODUCED GLUE

$\tau = \sqrt{2x^+x^-}$

$\eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$



Kovner, McLerran, Weigert

- NUCLEAR COLLISIONS – SOLVE YANG-MILLS EQUATIONS FOR TWO SOURCES.

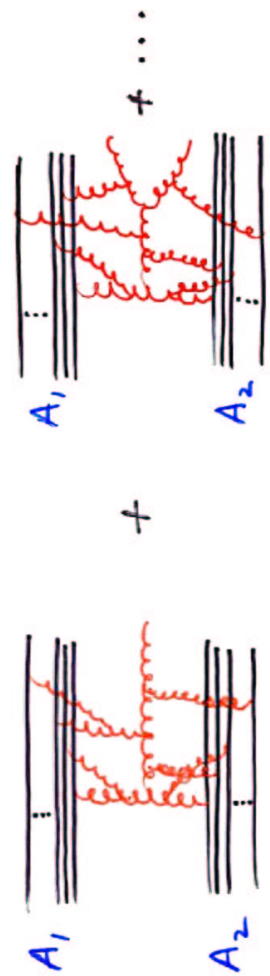
$$D_\mu F^{\mu\nu,a} = \rho_1^a(x_\perp)\,\delta(x^-)\,\delta^{\nu+} + \rho_2^a(x_\perp)\,\delta(x^+)\,\delta^{\nu-}$$

color charge density of nucleus 1

color charge density of nucleus 2.

- GAUSSIAN AVERAGE (with weight Λ_s) OVER COLOR CHARGES OF DIFFERENT CONFIGS.

* COMPUTE ALL TREE LEVEL GRAPHS (NO LOOPS!)



$\boxed{A^\tau = 0}$

* NON-PERTURBATIVE NUMERICAL SOLUTION

Krasnitz + R.V.

ONLY PARAMETERS : Λ_s & R

FOR RHIC : $\Lambda_s \simeq 1\text{-}2$ GeV ; $R \simeq 6.5$ fm $\Rightarrow \Lambda_s R \sim 32.5 - 65$

FOR LHC : $\Lambda_s \simeq 2\text{-}4$ GeV ; $R \simeq 6.5$ fm $\Rightarrow \Lambda_s R \sim 65 - 130$

KRASNITZ + R.V.

* LATTICE FORMULATION

LATTICE HAMILTONIAN IN $A^\tau = 0$ GAUGE.

$$H = \frac{\tau}{2} \int d\eta\, d^2 r_\perp \left[p^\eta p^\eta + \frac{1}{\tau^2} p^r p^r + \frac{1}{\tau^2} F_{\eta r} F_{\eta r} + F_{xy} F_{xy} \right]$$

FOR PERFECT "PANCAKES" ONLY BOOST INVARIANT CONFIGS.

$$A_r(\tau, \eta, r_\perp) = A_r(\tau, r_\perp)\ ;\ A_\eta(\tau, \eta, r_\perp) = \Phi(\tau, r_\perp)$$

(RESEMBLES FINITE-T DIMENSIONAL REDUCTION — AN ADJOINT SCALAR EMERGES)

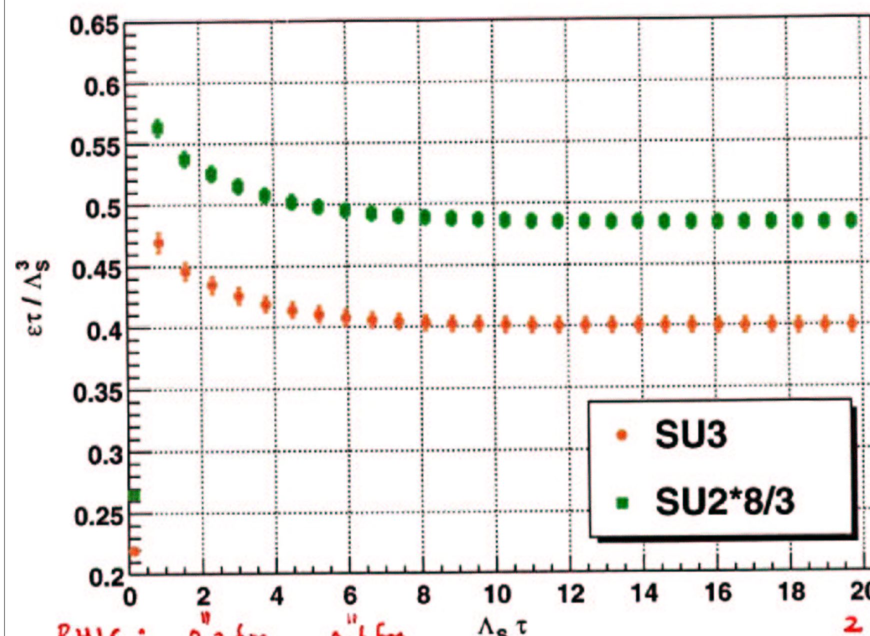
$$\frac{dH}{d\eta} = \frac{\tau}{2} \int d^2 r_\perp \left[p^\eta p^\eta + \frac{1}{\tau^2} E_r E_r + \frac{1}{\tau^2} (D_r \Phi)(D_r \Phi) + F_{xy} F_{xy} \right]$$

DISCRETIZE ON 2-D LATTICE

$$H_L = \frac{1}{2\tau} \sum_{\ell} E_\ell E_\ell + \tau \sum_{\square} \left(1 - \frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U_\square \right) + \frac{\tau}{2} \sum_j p_j p_j + \frac{1}{4\tau} \sum_{j,n} \mathrm{Tr} \left(\Phi_j - U_{j,n} \Phi_{j+n} U_{j,n}^\dagger \right)^2$$

- REAL TIME GLUODYNAMICS OF A NUCLEAR COLLISION:
    - 2+1-D CLASSICAL HAMILTONIAN.
    - SOLVE HAMILTON'S EQNS. AS FUNCTION OF PROPER TIME τ.
    - SPACE-TIME EVOLUTION OF GAUGE FIELDS AT $\eta = 0$

Formation time of gluons



RHIC: 0.2 fm 0.6 fm 2 fm

$\tau_D = 1/(\Lambda_S \gamma)$, $\epsilon\tau/\Lambda_S^3 = \alpha + \beta \exp(-\gamma\tau)$.

Formation time of SU2 and SU3 is the same. $\Rightarrow$ $\tau_D^{RHIC} \simeq 0.3$ fms
$\tau_D^{LHIC} \simeq 0.13$ fms.

* DETERMINING Λ_s:

From HERA:

$$\Lambda_s^2 = A^{1/3}\left(\frac{3*10^{-4}}{x}\right)^{\delta} \text{GeV}^2 \qquad (\Lambda_s \simeq Q_s)$$

Golec-Biernat-Wusthoff parametrization of ALL HERA data for $x \le 10^{-2}$

$\delta \simeq 0.3$

$x \simeq \frac{2\Lambda_s}{\sqrt{s}}$

* Solving self-consistently,

$\Lambda_s \simeq 1.4$ GeV for RHIC

$\quad \simeq 2.2$ GeV for LHC.

* FORMATION TIME:

$$\frac{\varepsilon\tau}{\Lambda_s^3} = \alpha + \beta e^{-\tau/\tau_F}$$

$\tau_F \to \#/\Lambda_s$

$\alpha \to \frac{1}{\pi R^2}\frac{1}{\Lambda_s^3}\frac{dE_T}{d\eta}$

$$\tau_F^{RHIC} = (0.39 \pm 10\%)\ \text{fm}$$

$$\tau_F^{LHC} = (0.25 \pm 10\%)\ \text{fm}$$

* INITIAL TRANSVERSE ENERGY

$$\boxed{\frac{1}{\pi R^2}\frac{dE_T}{d\eta}\bigg|_{\eta=0} = \frac{1}{g^2} f_E \Lambda_s^3}$$

$f_E \equiv f_E(\Lambda_s R) = \{ 0.5 - 0.54$ for RHIC-LHC range.

$$\frac{dE_T}{d\eta}\bigg|_{\eta=0} = \begin{cases} (1140 \pm 10\%)\ \text{GeV for RHIC} & (\Lambda_s \simeq 1.4\ \text{GeV}) \\ (4400 \pm 10\%)\ \text{GeV for LHC} & (\Lambda_s \simeq 2.2\ \text{GeV}) \end{cases}$$

Gluon multiplicity estimates

Employ 2 methods to estimate the gluon number, each extrapolating a definition of the particle number from a free theory:

$$H_f = \frac{1}{2}\sum_k \left(|\pi(k)|^2 + \omega^2(k)\,|\phi(k)|^2\right).$$

1. In a free theory

$$n(k) = \omega(k)\langle|\phi(k)|^2\rangle = \sqrt{\langle|\phi(k)|^2|\pi(k)|^2\rangle}.$$

   Use this expression with fields and momenta determined in the Coulomb gauge. As a by-product, determine also the dispersion relation

$$\omega(k) = \sqrt{\frac{\langle|\pi(k)|^2\rangle}{\langle|\phi(k)|^2\rangle}}.$$

2. If a free field is subject to relaxation (cooling)

$$\partial_t\phi(x) = -\partial H/\partial\phi(x),$$

   then

$$N = \sqrt{\frac{8}{\pi}}\int_0^\infty \frac{\mathrm{d}t}{\sqrt{t}}\,V(t).$$

   where V is the potential part of H. Generalize to full interactive V.

Assuming (as for the energy) the $(N_c^2-1)/N_c$ dependence, we obtain (from the relaxation method).

- $\mathrm{d}N/\mathrm{d}\eta_{\mathrm{RHIC}} \approx 10^3$
- $\mathrm{d}N/\mathrm{d}\eta_{\mathrm{LHC}} \approx 4300$

* INITIAL MULTIPLICITY DISTRIBUTION:

$$\boxed{\left.\frac{dN}{d\eta}\right|_{\eta=0} = \pi R^2 \frac{1}{g^2} f_N \Lambda_s^2}$$

$f_N(\Lambda_s R) = 0.3 \pm 10\%$ for wide range in $\Lambda_s R$.

$$\left.\frac{dN}{d\eta}\right|_{\eta=0} = \begin{cases} 488 \pm 10\% & \text{for RHIC} \\ 1200 \pm 10\% & \text{for LHC} \end{cases}$$

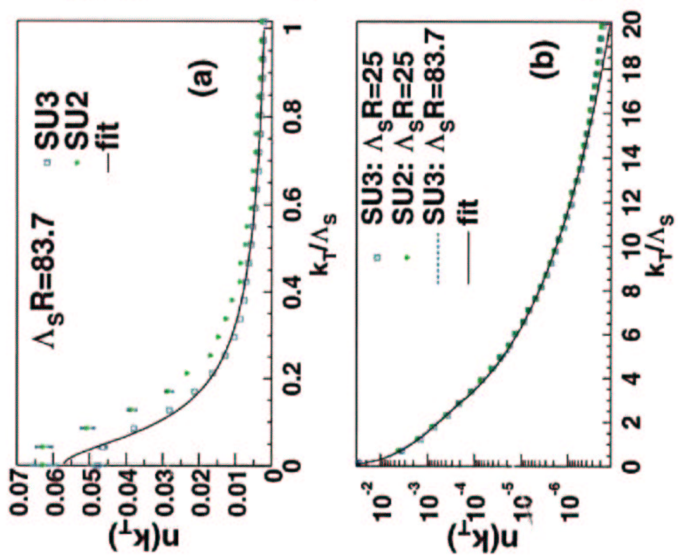
* INITIAL ENERGY DENSITY:

$$\boxed{\varepsilon = \frac{0.17}{g^2}\Lambda_s^4} \quad \text{at } \tau = \tau_F$$

$$\varepsilon^{RHIC} \simeq 20\ \mathrm{GeV/fm^3}$$

$$\varepsilon^{LHC} \simeq 120\ \mathrm{GeV/fm^3}$$

Transverse momentum distribution of gluons



$n(k_T) = \tilde{f}_n/(N_c^2 - 1)$
The SU(3) gluon momentum distribution can be fitted by the following function,

$$\frac{1}{\pi R^2}\frac{dN}{d\eta d^2k_T} = \frac{1}{g^2}\tilde{f}_n(k_T/\Lambda_s)\,, \qquad (1)$$

where $\tilde{f}_n(k_T/\Lambda_s)$ is

$$\tilde{f}_n = \begin{cases} a_1\left[\exp\left(\sqrt{k_T^2+m^2}/T_{\rm eff}\right)-1\right]^{-1} \\ (k_T/\Lambda_s \le 3) \\ \\ a_2\,\Lambda_s^4\log(4\pi k_T/\Lambda_s)k_T^{-4} \\ (k_T/\Lambda_s > 3) \end{cases} \qquad (2)$$

with $a_1 = 0.0295$, $m = 0.067\Lambda_s$, $T_{\rm eff} = 0.93\Lambda_s$, and $a_2 = 0.0343$.

5

* "BOTTOM UP" THERMALIZATION

Baier
Mueller
Schiff
Son.

$Q_s \tau$

HIGHLY VISCOUS – NON-IDEAL – NON-ADIABATIC HYDRODYNAMICS.
$T_i = \alpha_s^{2/5} Q_s \;:\; \frac{n_f}{n_i} = \frac{\#}{\alpha_s^{7/5}}$

$\frac{1}{\alpha_s^{13/5}}$

SOFT GLUONS THERMALIZE. HARD GLUONS ADD ENERGY TO BATH BEFORE THERMALIZING

$\frac{1}{\alpha_s^{5/2}}$

OCCUPATION # $f < 1$ – CLASSICAL APPROACH BREAKS DOWN. SOFT GLUONS SCREEN HARD EMISSION – LPM EFFECT RELEVANT HERE

$\frac{1}{\alpha_s^{3/2}}$

FIELDS LINEARIZE – GLUONS WITH MOMENTUM Q_s – BEGIN TO SCATTER OFF TRANSVERSE PLANE.

1

STRONGLY INTERACTING "NON-LINEAR" CLASSICAL GLUON FIELDS

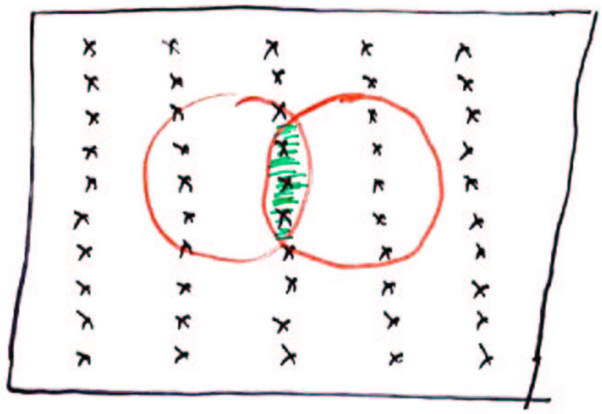
0

NUCLEI COLLIDE

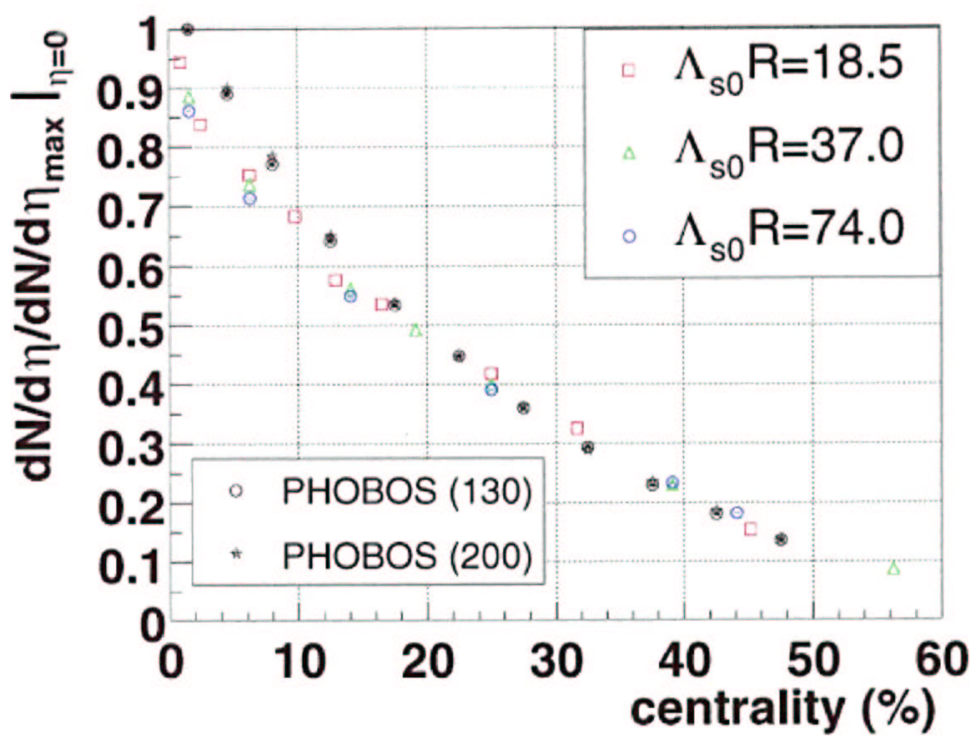
* **Technical Improvements:**

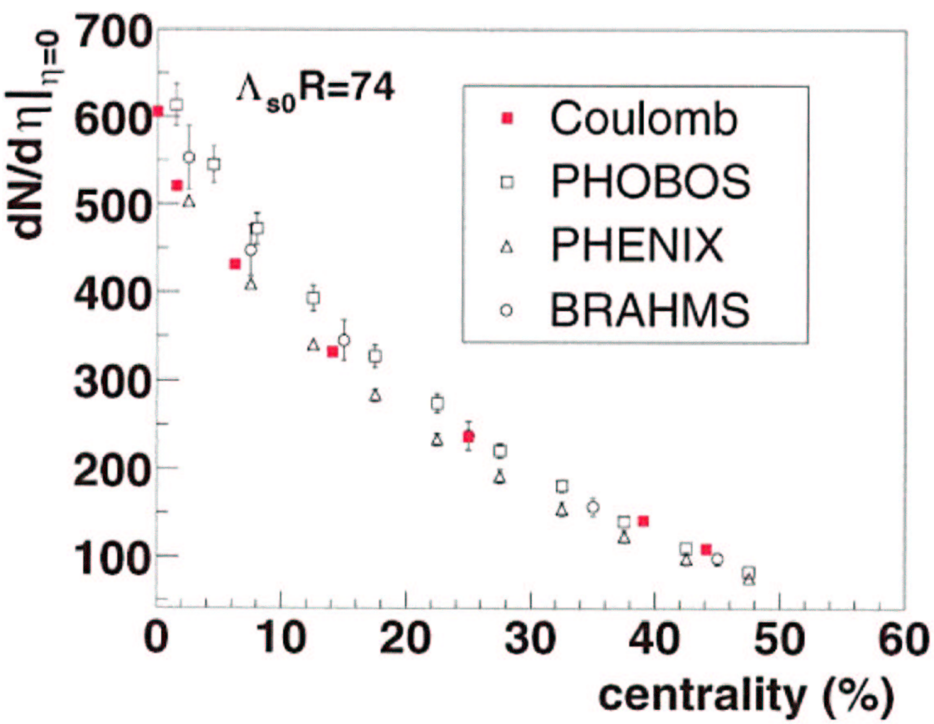
Periodic Boundary Conditions → Open Boundary Conditions

Impose Color Neutrality (Lam & Mahlon)



$$\Lambda_s \equiv \Lambda_s(x_\perp)$$





Elliptic flow in the early stage of the collisions?

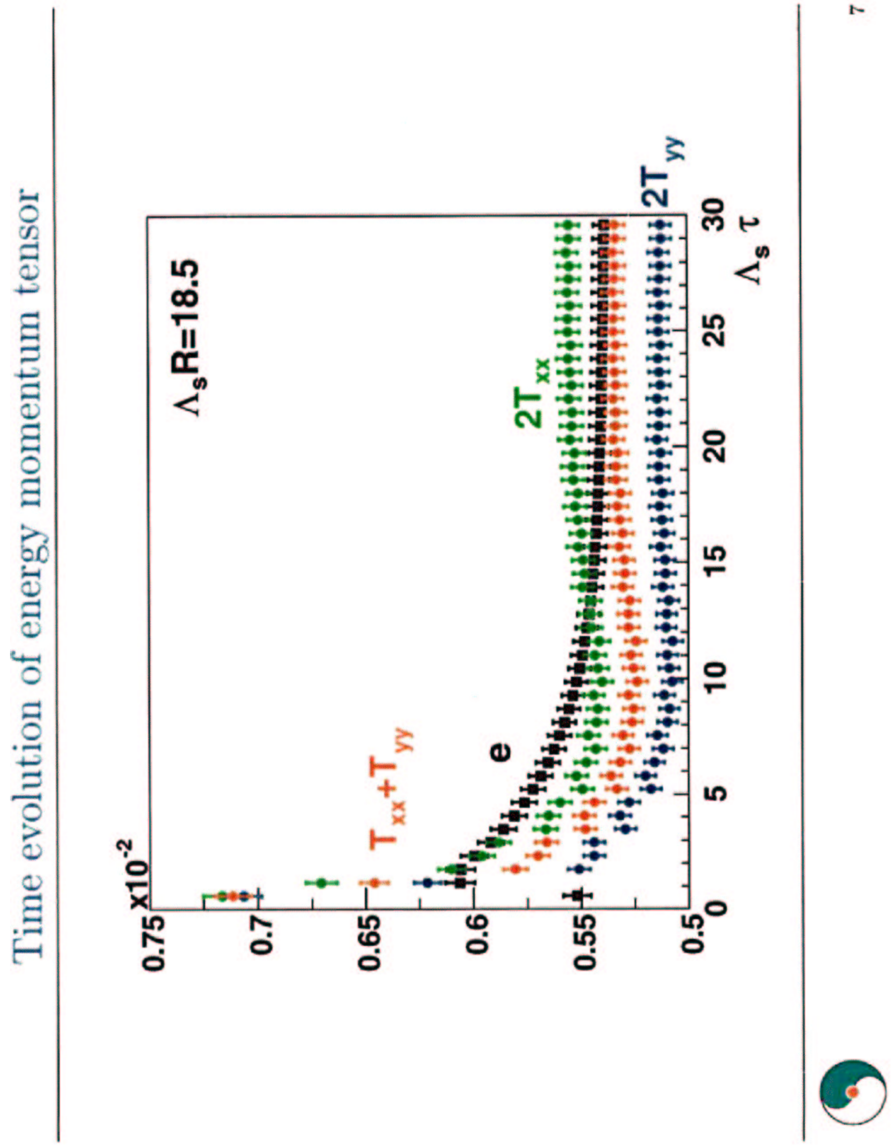
Elliptic flow parameter v_2 is defined by the second Fourier coefficient:

$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \frac{\int_{-\pi}^{\pi} d\phi \cos(2\phi) \int d^2p_T \frac{d^3N}{dy d^2p_T d\phi}}{\int_{-\pi}^{\pi} d\phi \int d^2p_T \frac{d^3N}{dy d^2p_T d\phi}}.$$

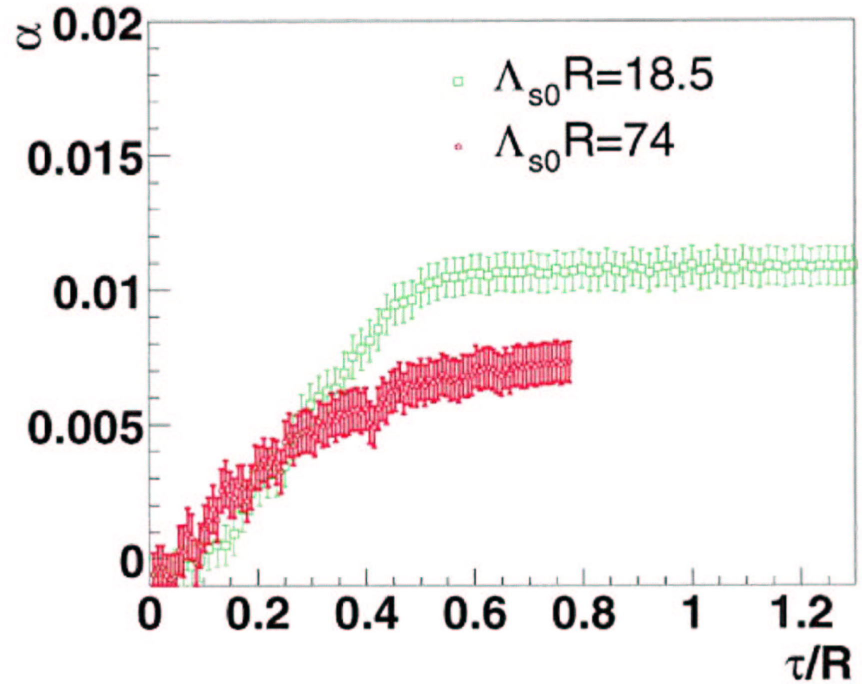
1. Elliptic flow is expected to be generated at early times in heavy ion collisions. (It reflects spatial anisotropy to momentum anisotropy due to interaction)
2. Hydrodynamics works well at RHIC (mid-rapidity and small impact parameters).
3. How much elliptic flow is produced before thermalization?
4. classical Yang-Mills field theory is proposed to describe early stage of nucleus-nucleus collisions.

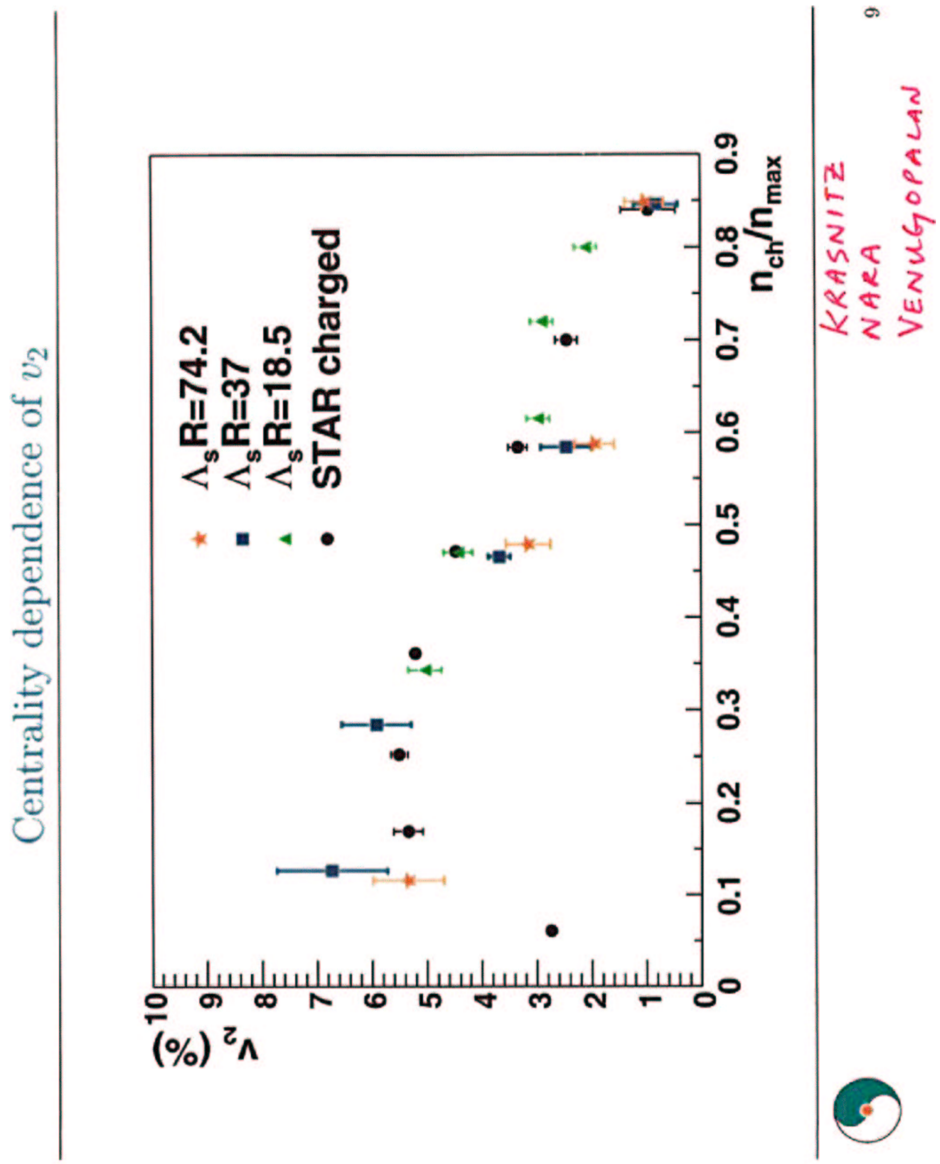
Purpose: compute elliptic flow of gluons from the CGC

17

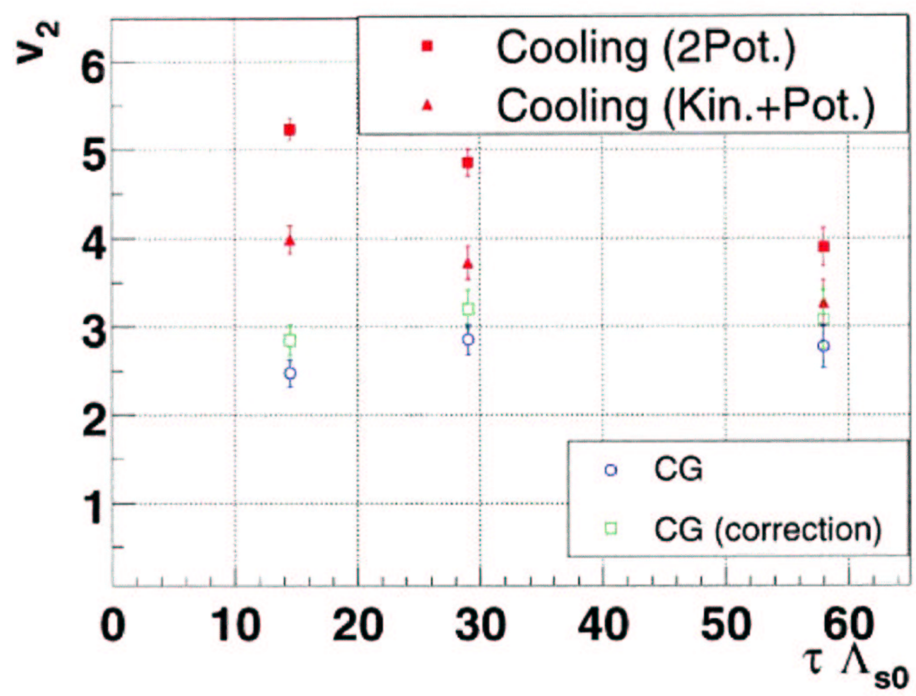


$$\alpha = \frac{T_{xx} - T_{yy}}{T_{xx} + T_{yy}}$$

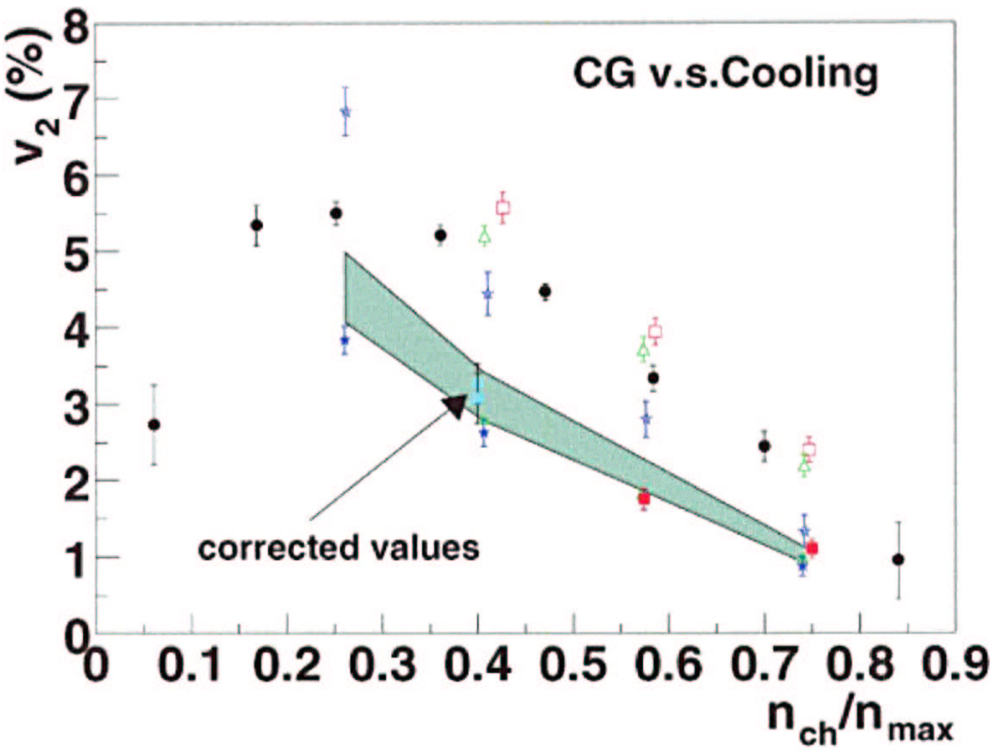


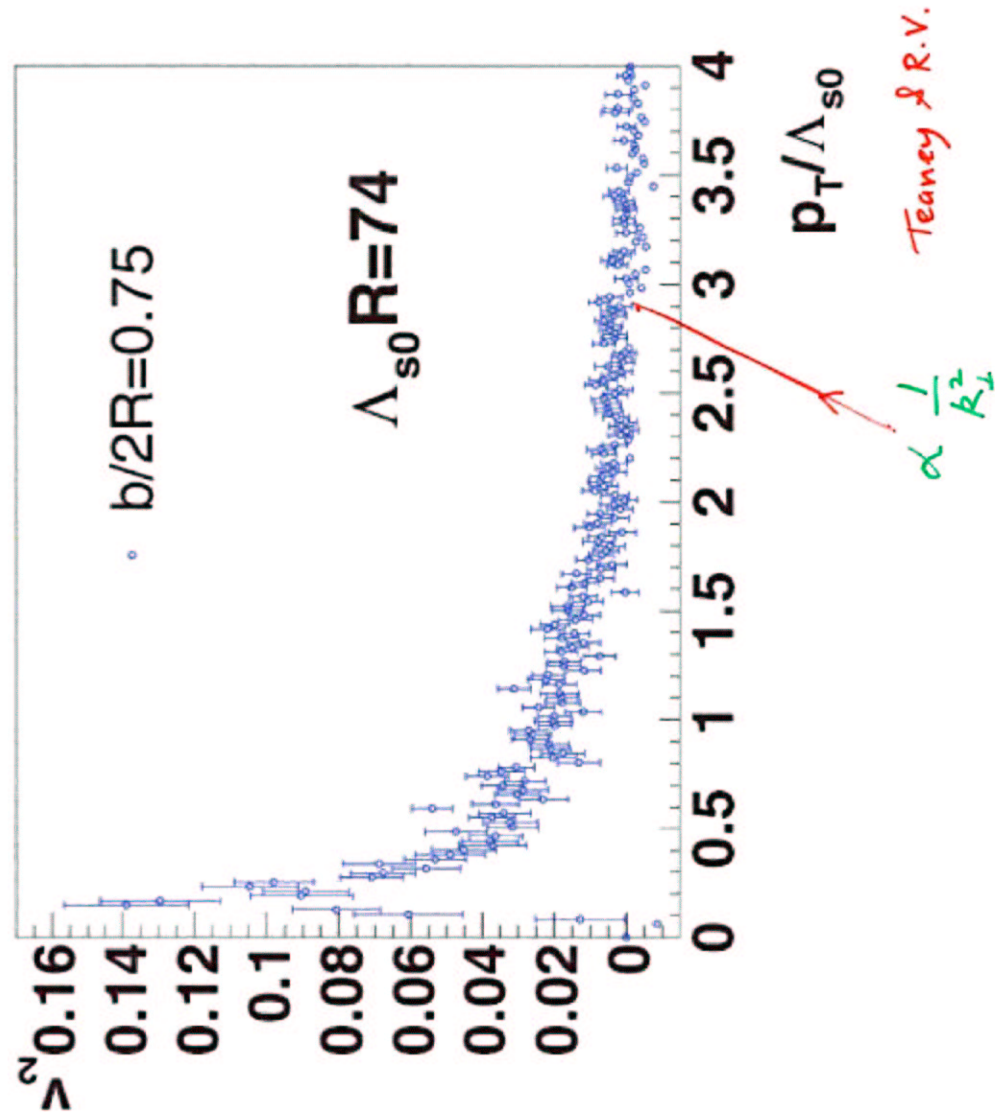


Centrality dependence of v_2
$\Lambda_s R$=74.2
$\Lambda_s R$=37
$\Lambda_s R$=18.5
STAR charged
v_2 (%)
n_{ch}/n_{max}
KRASNITZ
NARA
VENUGOPALAN
9



v_2
Cooling (2Pot.)
Cooling (Kin.+Pot.)
CG
CG (correction)
$\tau \Lambda_{s0}$


CG v.s.Cooling
corrected values
v_2 (%)
n_{ch}/n_{max}


b/2R=0.75
$\Lambda_{s0}R=74$
p_T/Λ_{s0}
v_2
Teaney & R.V.
$\propto \frac{1}{k_\perp^2}$

* CAN ALSO COMPUTE 2-particle cumulant J.-Y. Ollitrault

  $\langle\langle \cos 2(\phi_1-\phi_2)\rangle\rangle \neq v_2^2$

  COMPUTE EVENT-BY-EVENT FLUCTUATIONS IN V_2 ? S. Voloshin.

* INTERESTING RECENT WORK Kovchegov + Tuchin.

  - CLAIMS NON-FLOW CORRELATIONS EXPLAIN HIGH P_t -BEHAVIOR OF V_2.

* FOUR PARTICLE CUMULANTS – POSSIBLE NON-FLOW BIAS. Voloshin.

* ON GOING DISCUSSIONS Filiminov, Rak, Voloshin, Nu Xu

* OUTLOOK

* MANY IMPROVEMENTS FEASIBLE (AND UNDERWAY) IN NUMERICAL CLASSICAL GLUODYNAMICS.

* THERMALIZATION STILL A MYSTERY. MUCH ROOM FOR PROGRESS.