Spacetime CFTs from the Riemann sphere

Ricardo Monteiro

Queen Mary University of London

"Scattering Amplitudes: from Gauge Theory to Gravity"

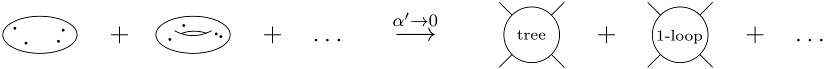
KITP Santa Barbara, 18 April 2017

Based on arXiv:1703.04589,

with Tim Adamo (Imperial) and Miguel Paulos (CERN)

Spacetime physics from 2d physics

String theory: field theory is $\alpha' \to 0$



Target space is spacetime.

Spacetime D fixed by 2d physics.

Spacetime physics from 2d physics

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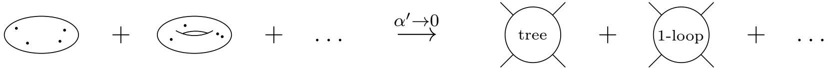

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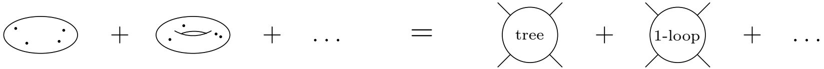
Worldsheet models of (perturbative, massless) QFTs: no α'



Target space is (complexified) space of null rays.

Modular integrals localised by scattering equations.

Worldsheet models $\longleftrightarrow$ Scattering equations

Old story

Twistor string theory [Witten 03] $\longrightarrow$ RSV formula [Roiban, Spradlin, Volovich 04]

$D = 4$. SYM, SUGRA [Hodges, Cachazo, Geyer, Skinner, Mason 12].

Only tree level. [Berkovitz, Witten 04]

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New story

Ambitwistor string theory [Mason, Skinner 13] $\longleftarrow$ CHY formulas [Cachazo, He, Yuan 13-14]

Any d. Many theories of massless particles.

Loop-level progress!

[Adamo, Casali, Skinner, Tourkine, Geyer, Mason, RM, He, Yuan, Cachazo, Feng, Cardona, Gomez, ... 13-16]

$\Longrightarrow$ Riemann sphere is enough (hopefully) see talk by Yvonne Geyer

More applications?

Quantities beyond scattering amplitudes?

Recent work on form factors. [He, Zhang, Liu, Brandhuber, Hughes, Panerai, Spence, Travaglini 16]

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Idea for this talk

Want to study CFT correlation functions perturbatively.

Put together

- use of worldsheet models,
- formalism of the projective null cone to study CFTs.

$$\text{Conformal transf. in } \mathbb{R}^d \quad \longleftrightarrow \quad \text{Lorentz transf. in } \mathbb{R}^{D-1,1}$$
$$SO(d+1,1) \qquad d+2=D \qquad SO(D-1,1)$$

Need worldsheet model with target space based on projective null cone.

Outline

- Review of projective null cone
- Review of ambitwistor strings
- New worldsheet models
- Quantisation of models
- 3-pt functions

Review of projective null cone

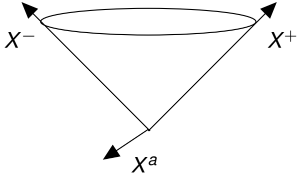
Projective null cone

[Dirac; Mack, Salam; Ferrara, Grillo, Gatto; **Bars**; Weinberg; Cornalba, Costa, Penedones, Schiappa; ...]

"Embedding space" $\mathbb{R}^{D-1,1}$, $ds^2 = dX^\mu dX_\mu = -dX^+ dX^- + dX^a dX_a$

Projective null cone $= \left\{ X \in \mathbb{R}^{D-1,1} \mid X^2 = 0 \right\} / X \cdot \partial_X$ $\qquad (X^\mu \sim \lambda X^\mu)$

Dimension: $d = D - 2$



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Map: $X^\mu = X^+(1, \mathrm{x}^2, \mathrm{x}^a)\,, \quad \mathrm{x}^a \in \mathbb{R}^d$

$SO(D-1,1)$ on $\mathbb{R}^{D-1,1}$: $X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu$ (Lorentz)

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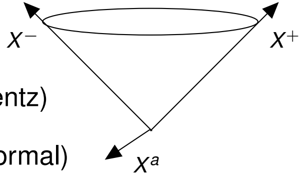

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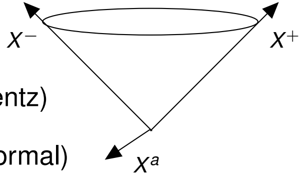

Conformal fields (eg. scalar)

Since $X^\mu \sim \lambda X^\mu$, require $\Phi(\lambda X) = \lambda^{-\Delta} \Phi(X)$

d-dim scalar: $\boxed{\phi(\mathrm{x}) = (X^+)^\Delta \, \Phi(X) \big|_{X^2=0}}$

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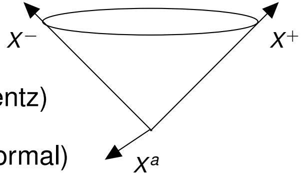

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Eg: $\langle \Phi(X) \Phi(Y) \rangle \sim (X \cdot Y)^{-\Delta} = (-\tfrac{1}{2} X^+ Y^+)^{-\Delta} \, |x - y|^{-2\Delta} \quad \to \langle \phi(\mathrm{x}) \phi(\mathrm{y}) \rangle$

Phase space for massless particles

Seen from $\mathbb{R}^d$,

phase space $= \{ (\mathrm{x}^a, \mathrm{p}_b) \mid \mathrm{p}^2 = 0 \}$. Dimension $= 2d - 1 = 2D - 5$.

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- $X \cdot P = P^2 = 0$ ensure motion is on and tangent to null cone,
- quotients ensure space is projective.

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Goal

Vertex ops in ambitwistor strings: on-shell external states $\sim e^{ik\cdot \mathrm{x}}$.

Vertex ops in our models: CFT single field insertions $\sim \mathcal{O}(X)$.

Review of ambitwistor strings

Geometry of scattering equations

Scattering of n massless particles: $k_i^2 = 0\,,\ \sum_{i=1}^{n} k_{i\,a} = 0$.

Consider 1-form on $\mathbb{CP}^1$, $\quad \mathrm{p}_a(z) = dz \sum_{i=1}^{n} \frac{k_{i\,a}}{z - z_i} \quad SL(2,\mathbb{C})$ invariant.

Scattering equations [Cachazo, He, Yuan 13] :

$$\mathrm{p}(z)^2 = 0 \quad \Leftrightarrow \quad \mathrm{Res}_{z_i}\,\mathrm{p}^2 \;=\; 2\,k_i \cdot \mathrm{p}(z_i) \;=\; 2\,dz \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j} \;=\; 0$$

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Worldsheet model: **ambitwistor strings** [Mason, Skinner 13] .

- $\{\,(\mathrm{x}^a, \mathrm{p}_b) \,|\, \mathrm{p}^2 = 0\,\}\,/\,\mathrm{p}\cdot\partial_{\mathrm{x}} \quad \rightarrow$ space of null rays.
- quotient by $\mathrm{p}\cdot\partial_{\mathrm{x}}$ means $(\mathrm{x}^a, \mathrm{p}_b) \sim (\mathrm{x}^a + \alpha\,\mathrm{p}^a,\, \mathrm{p}_b)$.
- complexification $\rightarrow$ ambitwistor space $\mathbb{A}$.

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- complexification $\quad\rightarrow\quad$ ambitwistor space $\mathbb{A}$.
- if $\mathrm{p}_a = (\mathrm{p}_a)_z\, dz$ is 1-form, $(\mathrm{p}_a)_z$ is defined up to scale.

  $\rightarrow$ *projective* ambitwistor space $\mathbb{PA}$.

Strings in ambitwistor space

[Mason, Skinner 13]

Chiral complexification of worldline action for massless particle:

$$S_B = \frac{1}{2\pi}\int_\Sigma \mathrm{p}_a\,\bar\partial\,\mathrm{x}^a - \frac{1}{2}\,e\,\mathrm{p}^2$$

$$\mathrm{p}_a = (\mathrm{p}_a)_z\,dz, \quad \bar\partial = d\bar z\,\partial_{\bar z}$$

- e enforces $\mathrm{p}^2 = 0$. target space is $\mathbb{PA}$ ✓
- gauge freedom: $\delta\,\mathrm{x}^a = \alpha\,\mathrm{p}^a,\ \delta\,\mathrm{p}_b = 0,\ \delta e = \bar\partial\alpha$.

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Quantisation: $\mathcal{A} = \left\langle \prod_{i=1}^{n} V_i \right\rangle$ Fix $e = 0$, $V_i = \int_\Sigma \bar\delta(k \cdot \mathrm{p}) \, e^{ik \cdot \mathrm{x}} \ldots$

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x^a integration is exact: $\bar\partial\,\mathrm{p}_a = 2\pi i\,dz\wedge d\bar z\,\sum_i k_{i\,a}\,\delta^2(z-z_i)$

$$\Rightarrow \quad \text{on } \mathbb{CP}^1, \quad \mathrm{p}_a = dz\sum_i \frac{k_{i\,a}}{z-z_i} \quad \Rightarrow \quad \text{scattering equations}$$

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No α'. Weight$(e^{ik\cdot \mathrm{x}}) = 0$ $\rightarrow$ **massless spectrum**

Worldsheet matter

[Mason, Skinner 13]

Combine with other chiral CFTs to reproduce CHY formulas:

$$S = S_B + S^\ell + S^r \quad \rightarrow \quad \mathcal{A}_{\text{CHY}} = \int d\mu \;\, \mathcal{I}^\ell\, \mathcal{I}^r$$

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Ambitwistor strings

Bosonic: $S = S_B + S_{\mathfrak{g}} + S_{\tilde{\mathfrak{g}}}$

- two current algebras, $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$ (colours).
- CHY formulas for bi-adjoint scalar $\phi^{a\tilde{a}}$ with cubic vertex $\sim f^{abc} f^{\tilde{a}\tilde{b}\tilde{c}}$

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Heterotic: $S = S_B + S_{\mathfrak{g}} + S_\psi$

- current alg. $\mathfrak{g}$ plus system of d fermions, $S_\psi = \frac{1}{2\pi} \int \psi \cdot \bar{\partial}\psi + \chi \, \mathrm{p} \cdot \psi$.
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Type II: $S = S_B + S_{\psi} + S_{\tilde{\psi}}$

- two decoupled systems of d fermions, S_ψ and $S_{\tilde{\psi}}$.
- CHY formulas for gravity amplitudes.

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Various others [Ohmori 15; Casali, Geyer, Mason, RM, Roehrig 15].

Models on the projective null cone

Bosonic model

Action

$$S = \frac{1}{2\pi}\int_\Sigma \left(P_\mu\, \bar\partial X^\mu - \frac{e^{(1)}}{2}\, P^2 - \frac{e^{(2)}}{2}\, X^2 - e^{(3)}\, P\cdot X \right) + S_{\mathfrak{g}} + S_{\tilde{\mathfrak{g}}}$$

- $e^{(r)}$ enforce null cone phase space. (recall P is a 1-form)
- coupling to worldsheet metric: $\bar\partial \to \bar\partial + e^{(0)}\,\partial$.

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Symmetries

- D-dim Lorentz. 2d diffeos, Weyl. $\rightarrow$ possible conformal anomaly
- $SL(2,\mathbb{C})$ symmetry $\{X, P\}$. $\rightarrow$ possible $SL(2,\mathbb{C})$ anomaly

$$\delta X^\mu = \alpha^{(1)}\,P^\mu + \alpha^{(3)}\,X^\mu \qquad \delta P_\mu = -\alpha^{(2)}\,X_\mu - \alpha^{(3)}\,P_\mu$$

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$$\Rightarrow \quad A_{\alpha\beta} = \begin{pmatrix} e^{(2)} & e^{(3)} \\ e^{(3)} & e^{(1)} \end{pmatrix}$$

is non-dynam. $SL(2,\mathbb{C})$ gauge field, mixed conformal weights.

Heterotic and type II models

Consider SUSY extensions of null cone.

Heterotic

$$S = \frac{1}{2\pi}\int_\Sigma \left(P_\mu\,\bar\partial X^\mu - \frac{e^{(1)}}{2}\,P^2 - \frac{e^{(2)}}{2}\,X^2 - e^{(3)}\,P\cdot X\right) + S_{\mathfrak{g}}$$
$$+\,\frac{1}{2\pi}\int_\Sigma \left(\frac{1}{2}\psi_\mu\,\bar\partial\psi^\mu + \chi^{(1)}\,\psi\cdot P + \chi^{(2)}\,\psi\cdot X\right)$$

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$$S = \frac{1}{2\pi}\int_\Sigma \left(P_\mu\,\bar\partial X^\mu - \frac{e^{(1)}}{2}\,P^2 - \frac{e^{(2)}}{2}\,X^2 - e^{(3)}\,P\cdot X + \frac{1}{2}\psi_\mu\,\bar\partial\psi^\mu + \frac{1}{2}\tilde\psi_\mu\,\bar\partial\tilde\psi^\mu\right.$$
$$\left. +\,\chi^{(1)}\,\psi\cdot P + \chi^{(2)}\,\psi\cdot X + \tilde\chi^{(1)}\,\tilde\psi\cdot P + \tilde\chi^{(2)}\,\tilde\psi\cdot X + \xi\,\psi\cdot\tilde\psi\right)$$

Symmetries include SUSY versions of $SL(2,\mathbb{C})$.

Quantisation of models

Anomalies: bosonic

Gauge-fixed action

$$S = \frac{1}{2\pi}\int_\Sigma \left(P_\mu\, \bar\partial X^\mu + \sum_{a=0}^{3} b^{(a)}\, \bar\partial c^{(a)} \right) + S_{\mathfrak{g}} + S_{\tilde{\mathfrak{g}}}$$

- $b^{(0)}, c^{(0)}$ related to worldsheet gravity.
- $b^{(r)}, c^{(r)},\ r = 1, 2, 3$ related to $SL(2, \mathbb{C})$ symmetry.

Free OPEs: $P_\mu(z)\, X^\nu(0) \sim \frac{\delta^\nu_\mu}{z}\,,\ \ b^{(a)}(z)\, c^{(b)}(0) \sim \frac{\delta^{ab}}{z}\,.$

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BRST charge: $Q = \oint j$.

- $SL(2,\mathbb{C})$ anomaly: $Q^2 \supset \dfrac{D-8}{2} \oint \left(\partial c^{(1)} c^{(2)} - c^{(1)} \partial c^{(2)} + 2c^{(3)} \partial c^{(3)}\right)$.

  $\Rightarrow$ $\boxed{D = 8}$ for quantum consistency.

- conformal anomaly: $Q^2 = \dfrac{\mathfrak{c}_{\mathfrak{g}} + \mathfrak{c}_{\tilde{\mathfrak{g}}} - 40}{12} \oint c^{(0)}\, \partial^3 c^{(0)}$.

  Can eliminate by choice of current algebras (central charges $\mathfrak{c}_{\mathfrak{g}}, \mathfrak{c}_{\tilde{\mathfrak{g}}}$).

Anomalies: heterotic and type II

Heterotic (fermions ψ^μ, current algebra $\mathfrak{g}$)

- also ghosts for fermionic symmetries.
- absence of $SL(2,\mathbb{C})$ anomaly requires $\boxed{D=6}$.
- absence of conformal anomaly fixes central charge $\mathfrak{c}_\mathfrak{g}$.

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Summary

Model	Critical dimension	(Classical) CFT
bosonic	$d=6$	bi-adjoint ϕ^3 ?
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Vertex operators: bosonic

Candidate: $\boxed{V = c^{(0)}\, c^{(1)}\, j^{\mathsf{a}}\, \tilde{j}^{\tilde{\mathsf{a}}}\, f(X)}$

where $j^{\mathsf{a}}, \tilde{j}^{\tilde{\mathsf{a}}}$ are the currents of $S_{\mathfrak{g}}, S_{\tilde{\mathfrak{g}}}$, $\quad j^{\mathsf{a}}(z)\, j^{\mathsf{b}}(0) \sim \frac{k\, \delta^{\mathsf{ab}}}{z^2} + i\, \frac{f^{\mathsf{abc}}\, j^{\mathsf{c}}(0)}{z}$.

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Maybe: loop expansion here (inc. ambitwistor) is nodal expansion on sphere.

[Geyer, Mason, RM, Tourkine 15-16]

see talk by Yvonne Geyer

Gauge-fixing and ambitwistor string

Recall basic action:

$$S_B = \frac{1}{2\pi} \int_\Sigma \left(P_\mu \, \bar{\partial} X^\mu - \frac{e^{(1)}}{2} P^2 - \frac{e^{(2)}}{2} X^2 - e^{(3)} \, P \cdot X \right)$$

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Same theories in special d, but null cone adapted to CFT field insertions.

Prescription for 3-pt functions

Prescription (bosonic)

Vertex operators consistent with CFT interpretation.

Sphere correlation function $\langle V_1 \, V_2 \, V_3 \cdots \rangle$?

Hope: tree-level contribution to CFT correlators.

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$$\left\langle \prod_{i=1}^{3} V^{[0]}(z_i) \right\rangle := \int [\mathrm{D}\mathcal{F}]\, c^{(3)}(z_0)\ \left(b^{(2)} | \omega_0 \right)\ \bar{\delta}(X^2(z_0)) \prod_{j=1}^{3} \bar{\delta}\left(\log \mathrm{Hol}_{z_j} X^2 \right)$$

$$\times \prod_{k=1}^{2} \bar{\delta}\left(\mathrm{Res}_{z=z_k} X \cdot P\right) \prod_{i=1}^{3} V^{[0]}(z_i)\ e^{-S}$$

Results

Bosonic Consider $f^{[0]}(X) = \dfrac{1}{(Y \cdot X)^2}$, $Y^\mu = Y^+(1, \mathrm{y}^2, \mathrm{y}^a)$, insertion at y^a.

$$\left\langle \prod_{i=1}^{3} V^{[0]}(z_i) \right\rangle = \frac{1}{(4\pi)^4} \, \frac{f^{\mathrm{a}_1\mathrm{a}_2\mathrm{a}_3} \, f^{\tilde{\mathrm{a}}_1\tilde{\mathrm{a}}_2\tilde{\mathrm{a}}_3}}{(Y_1 \cdot Y_2)(Y_2 \cdot Y_3)(Y_3 \cdot Y_1)}$$

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Heterotic Also as expected.

Type II Zero mode issue for *c*-ghost related to constraint $\psi \cdot \tilde{\psi}$.

Face value: correlator **vanishes**.

Interpretation? $d = 2$ gravity is topological.

Much more to understand!

Conclusion

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- Vertex operators and 3-pt functions support proposal.
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Some open questions

- General (tree-level) correlation functions? Expect analogue of CHY formulas for amplitudes.
- Understand space of 2-dim vertex ops vs. d-dim CFT ops.
- Twistorial $D = 6$ model for $d = 4$ maximal SYM? [Geyer, Lipstein Mason, 14]
- Quantum corrections?