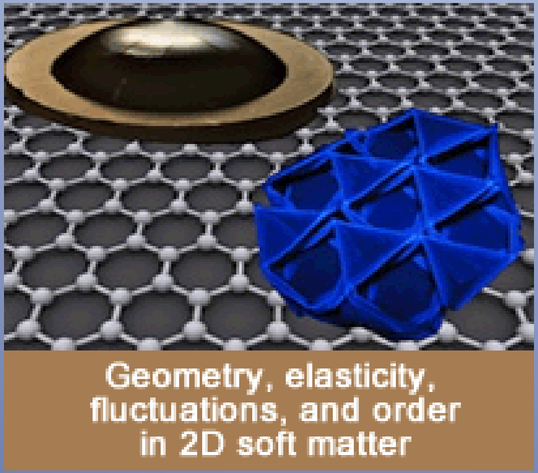
Elastic properties of graphene and other 2D materials

M. I. Katsnelson (Nijmegen)
J. Gonzalez (Madrid)
P. San-Jose (Madrid)
V. Parente (Madrid)
B. Amorim (Madrid)
R. Roldan (Madrid)
P. Le Doussal (Paris)
B. Horowitz (Beersheva)
K. Wiese (Paris)
C. Gomez-Navarro (Madrid)
J. Gomez (Madrid)
G. Lopez-Polin (Madrid)
F. Perez-Murano (Madrid)
A. Morpurgo (Geneva)
N. Couto (Geneva)
C. Stampfer Aachen)
E. Khestanova (Manchester)
I. V. Grigorieva (Manchester)
A. K. Geim (Manchester)


Geometry, elasticity, fluctuations, and order in 2D soft matter

KITP, January 15th 2016

instituto imdea nanociencia

MANCHESTER 1824
The University of Manchester

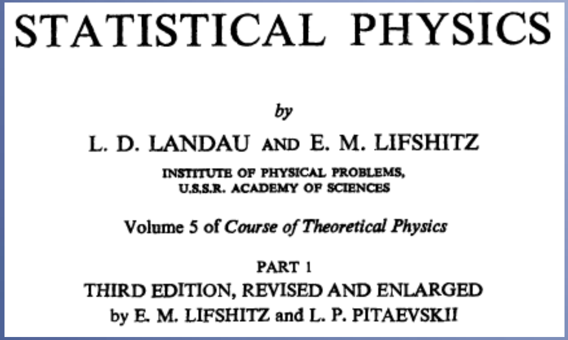
Outline
- Graphene as a membrane
- Defects and elastic constants
- Graphene under pressure
- Strains and transport

Future directions

GRAPHENE’S SUPERLATIVES

- Thinnest imaginable material
- largest surface area (~2,700 m^2 per gram)
- strongest material ‘ever measured’ (theoretical limit)
- stiffest known material (stiffer than diamond)
- most stretchable crystal (up to 20% elastically)
- record thermal conductivity (outperforming diamond)
- highest current density at room T (106 times of copper)
- completely impermeable (even He atoms cannot squeeze through)
- highest intrinsic mobility (100 times more than in Si)
- conducts electricity in the limit of no electrons
- lightest charge carriers (zero rest mass)
- longest mean free path at room T (micron range)

Why are there two dimensional crystals?



STATISTICAL PHYSICS

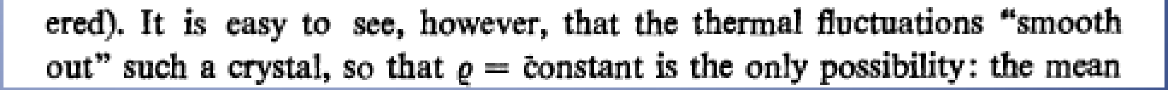
by

L. D. LANDAU AND E. M. LIFSHITZ

INSTITUTE OF PHYSICAL PROBLEMS,
U.S.S.R. ACADEMY OF SCIENCES

Volume 5 of *Course of Theoretical Physics*

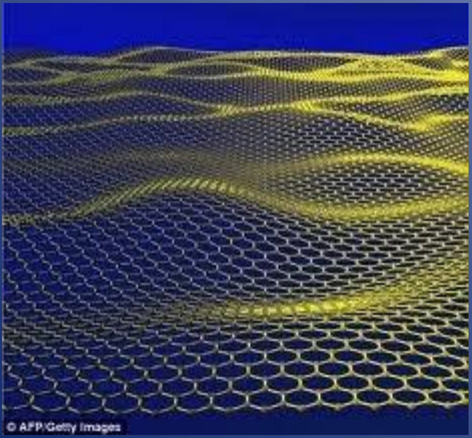
PART 1
THIRD EDITION, REVISED AND ENLARGED
by E. M. LIFSHITZ and L. P. PITAEVSKII



ered). It is easy to see, however, that the thermal fluctuations "smooth out" such a crystal, so that ϱ = constant is the only possibility: the mean

Thermal fluctuations:

$$\langle \vec{u}(L)\vec{u}(0)\rangle \approx \frac{k_B T}{B}\log\left(\frac{L}{d}\right)$$

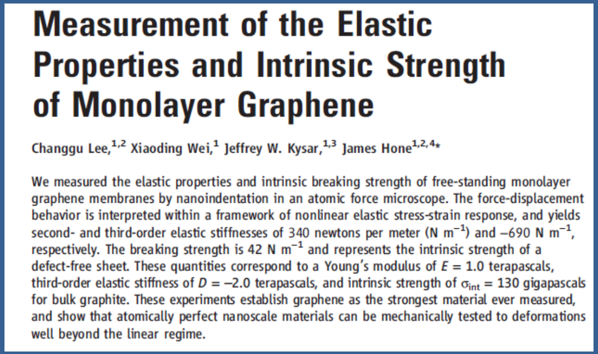


© AFP/Getty Images

$B_{graphene}$ =22 eV Å^{-2} = 352 N/m
$B_{diamond}$ x d=52.4 N/m

T=300K
L=1Km
$\langle \vec{u}(L)\vec{u}(0)\rangle \approx 0.03\text{Å}^2$

Elastic properties of graphene



Measurement of the Elastic Properties and Intrinsic Strength of Monolayer Graphene

Changgu Lee,[1,2] Xiaoding Wei,[1] Jeffrey W. Kysar,[1,3] James Hone[1,2,4]*

We measured the elastic properties and intrinsic breaking strength of free-standing monolayer graphene membranes by nanoindentation in an atomic force microscope. The force-displacement behavior is interpreted within a framework of nonlinear elastic stress-strain response, and yields second- and third-order elastic stiffnesses of 340 newtons per meter ($N\ m^{-1}$) and –690 $N\ m^{-1}$, respectively. The breaking strength is 42 $N\ m^{-1}$ and represents the intrinsic strength of a defect-free sheet. These quantities correspond to a Young's modulus of $E = 1.0$ terapascals, third-order elastic stiffness of $D = -2.0$ terapascals, and intrinsic strength of $\sigma_{int} = 130$ gigapascals for bulk graphite. These experiments establish graphene as the strongest material ever measured, and show that atomically perfect nanoscale materials can be mechanically tested to deformations well beyond the linear regime.

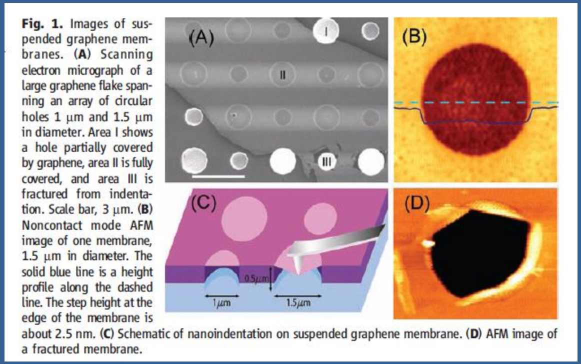


Fig. 1. Images of suspended graphene membranes. (**A**) Scanning electron micrograph of a large graphene flake spanning an array of circular holes 1 μm and 1.5 μm in diameter. Area I shows a hole partially covered by graphene, area II is fully covered, and area III is fractured from indentation. Scale bar, 3 μm. (**B**) Noncontact mode AFM image of one membrane, 1.5 μm in diameter. The solid blue line is a height profile along the dashed line. The step height at the edge of the membrane is about 2.5 nm. (**C**) Schematic of nanoindentation on suspended graphene membrane. (**D**) AFM image of a fractured membrane.

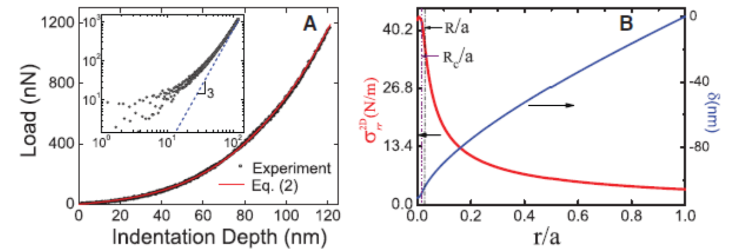
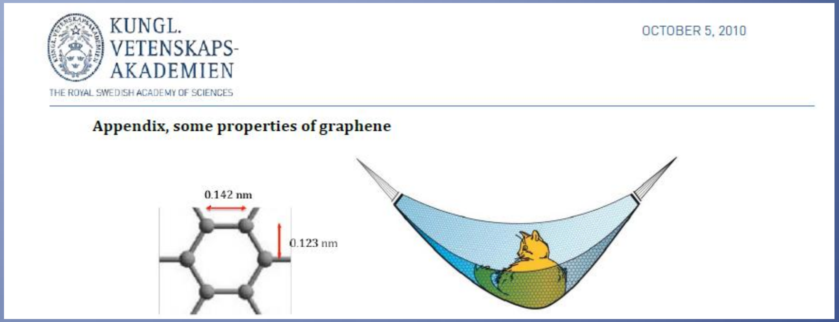


Fig. 2. (**A**) Loading/unloading curve and curve fitting to Eq. 2. The curve approaches cubic behavior at high loads (inset). (**B**) Maximum stress and deflection of graphene membrane versus normalized radial distance at maximum loading (simulation based on nonlinear elastic behavior in Eq. 1). The dashed lines indicate the tip radius R and contact radius R_c.



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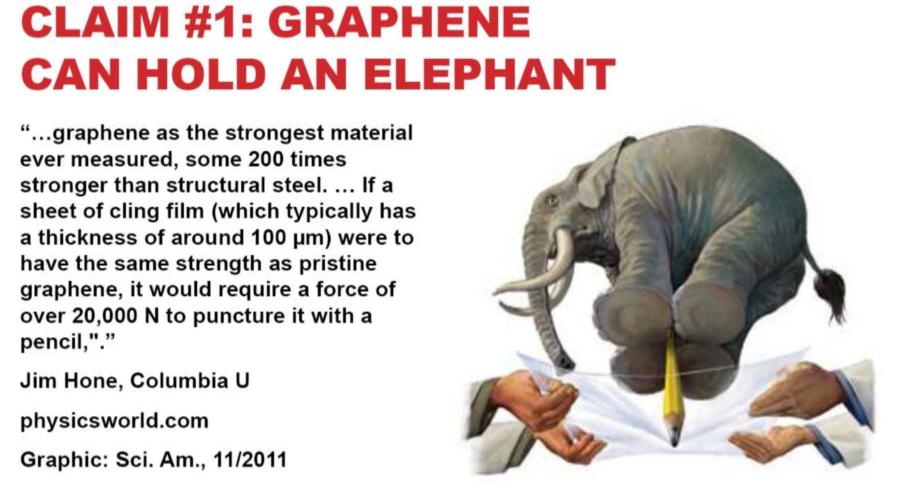
OCTOBER 5, 2010

Appendix, some properties of graphene



CLAIM #1: GRAPHENE CAN HOLD AN ELEPHANT

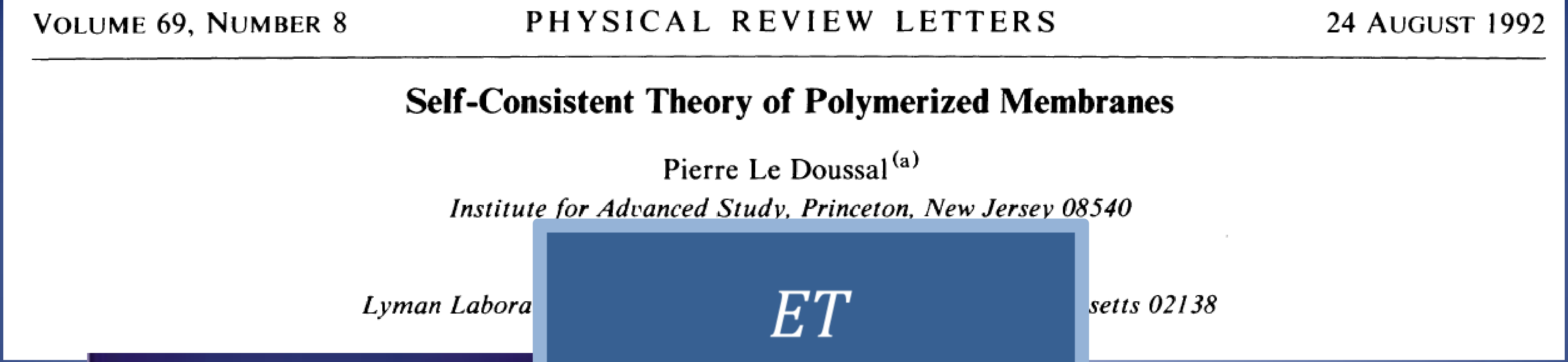
"…graphene as the strongest material ever measured, some 200 times stronger than structural steel. … If a sheet of cling film (which typically has a thickness of around 100 μm) were to have the same strength as pristine graphene, it would require a force of over 20,000 N to puncture it with a pencil,"."

Jim Hone, Columbia U

physicsworld.com

Graphic: Sci. Am., 11/2011

courtesy of M. M. Fogler



VOLUME 69, NUMBER 8 PHYSICAL REVIEW LETTERS 24 AUGUST 1992

Self-Consistent Theory of Polymerized Membranes

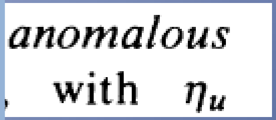
Pierre Le Doussal[a]
Institute for Advanced Study, Princeton, New Jersey 08540

Lyman Labora *setts 02138*

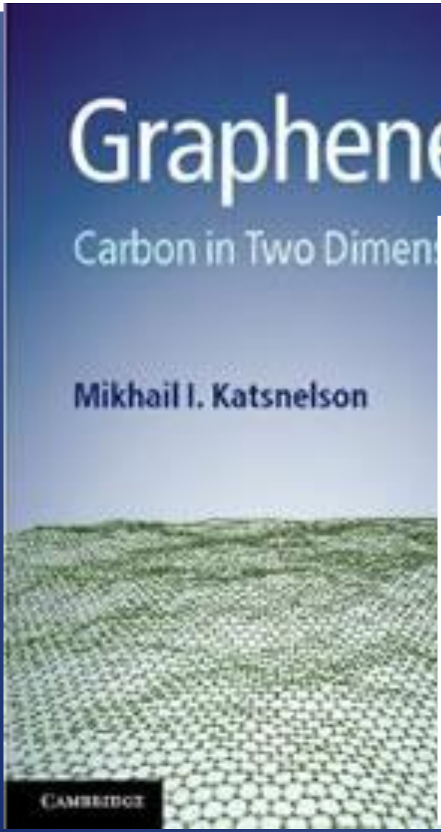
$$\frac{ET}{\kappa^2 q^2}$$

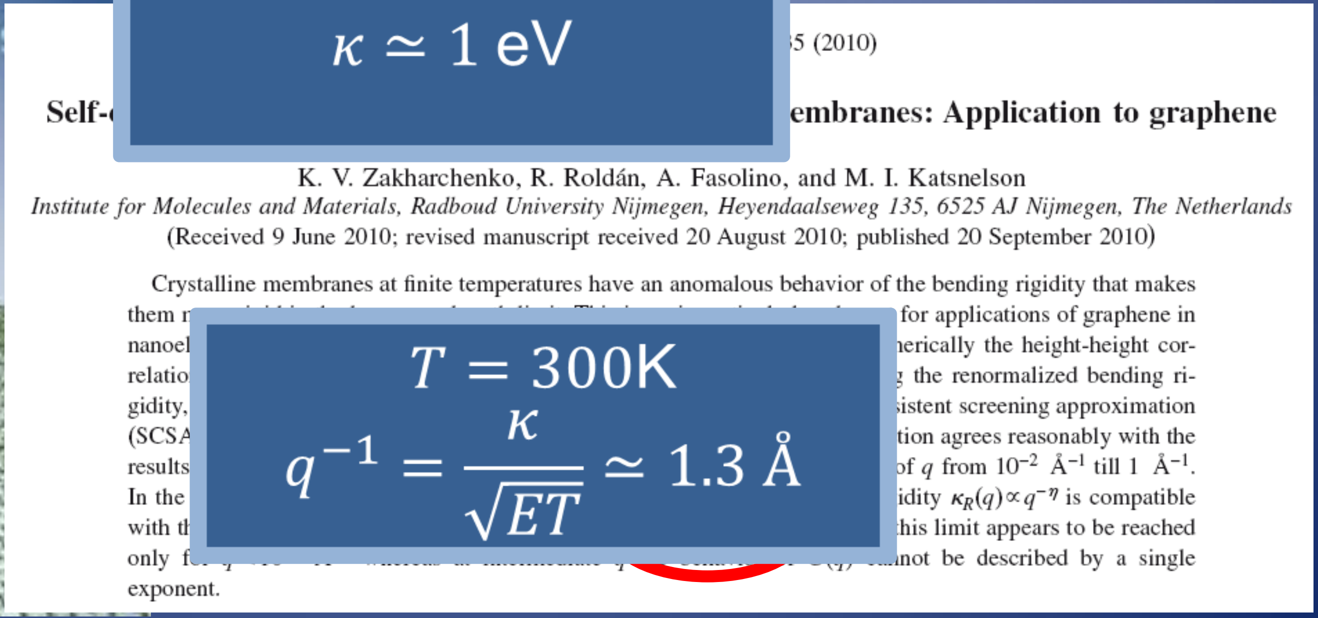
$$E \simeq 22\ \mathrm{eV\AA}^{-2}$$

$$\kappa \simeq 1\ \mathrm{eV}$$



anomalous
, with η_u





5 (2010)

Self- **embranes: Application to graphene**

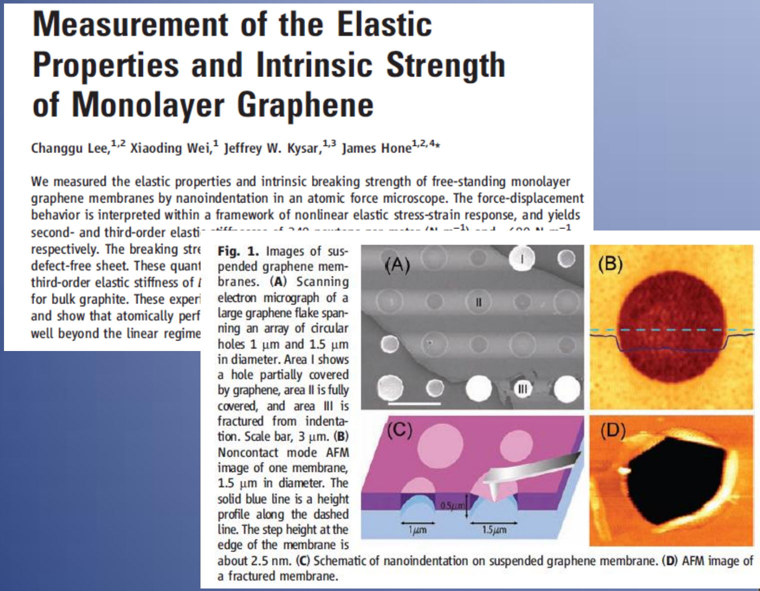
K. V. Zakharchenko, R. Roldán, A. Fasolino, and M. I. Katsnelson
Institute for Molecules and Materials, Radboud University Nijmegen, Heyendaalseweg 135, 6525 AJ Nijmegen, The Netherlands
(Received 9 June 2010; revised manuscript received 20 August 2010; published 20 September 2010)

Crystalline membranes at finite temperatures have an anomalous behavior of the bending rigidity that makes
them r for applications of graphene in
nanoel nerically the height-height cor-
relatio the renormalized bending ri-
gidity, sistent screening approximation
(SCSA tion agrees reasonably with the
results of q from 10^{-2} Å^{-1} till 1 Å^{-1}.
In the idity $\kappa_R(q) \propto q^{-\eta}$ is compatible
with th this limit appears to be reached
only f nnot be described by a single
exponent.

$$T = 300\mathrm{K}$$

$$q^{-1} = \frac{\kappa}{\sqrt{ET}} \simeq 1.3\ \text{Å}$$

Experiments



Measurement of the Elastic Properties and Intrinsic Strength of Monolayer Graphene

Changgu Lee,[1,2] Xiaoding Wei,[1] Jeffrey W. Kysar,[1,3] James Hone[1,2,4]*

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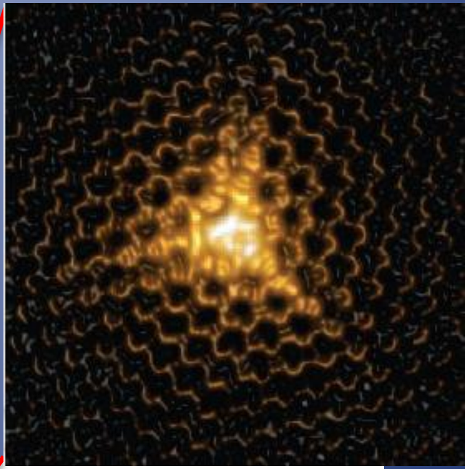
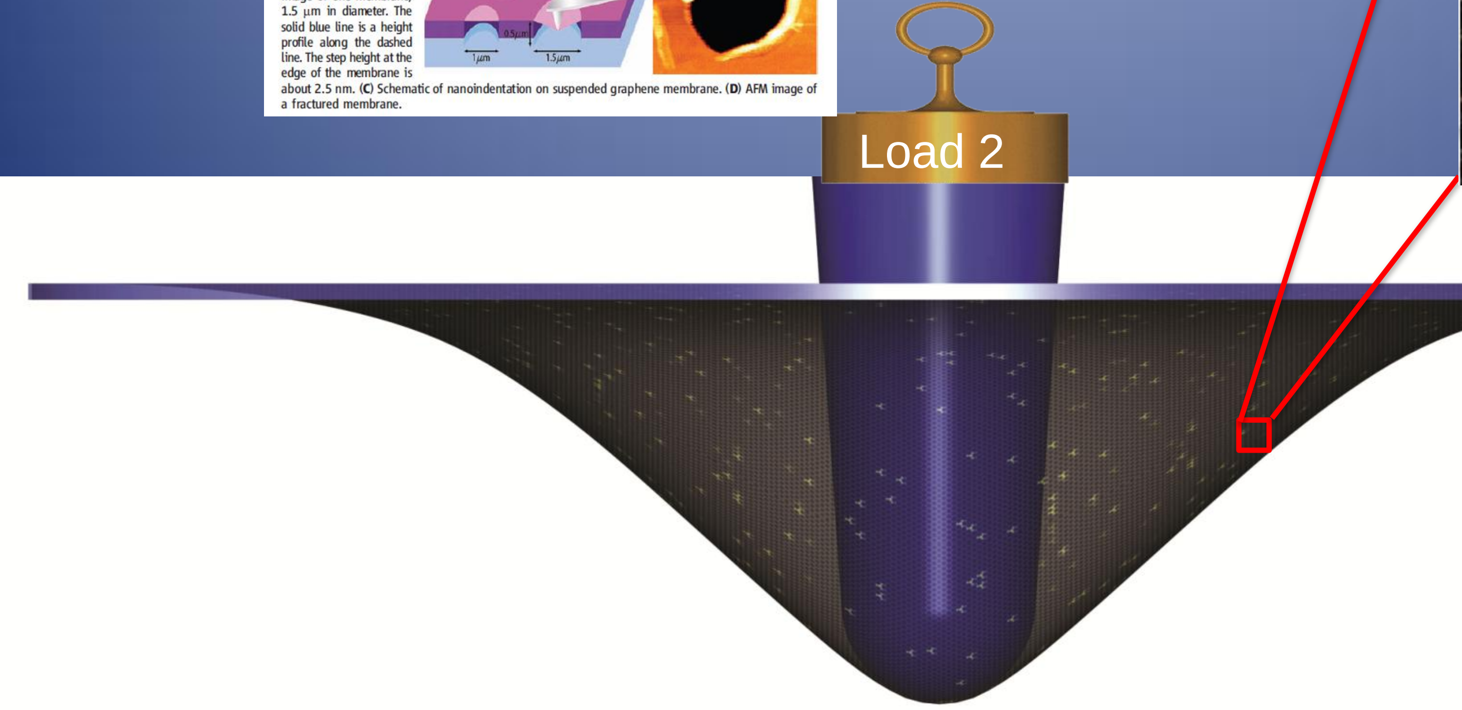
LETTERS
PUBLISHED ONLINE: 15 DECEMBER 2014 | DOI: 10.1038/NPHYS3183
nature physics

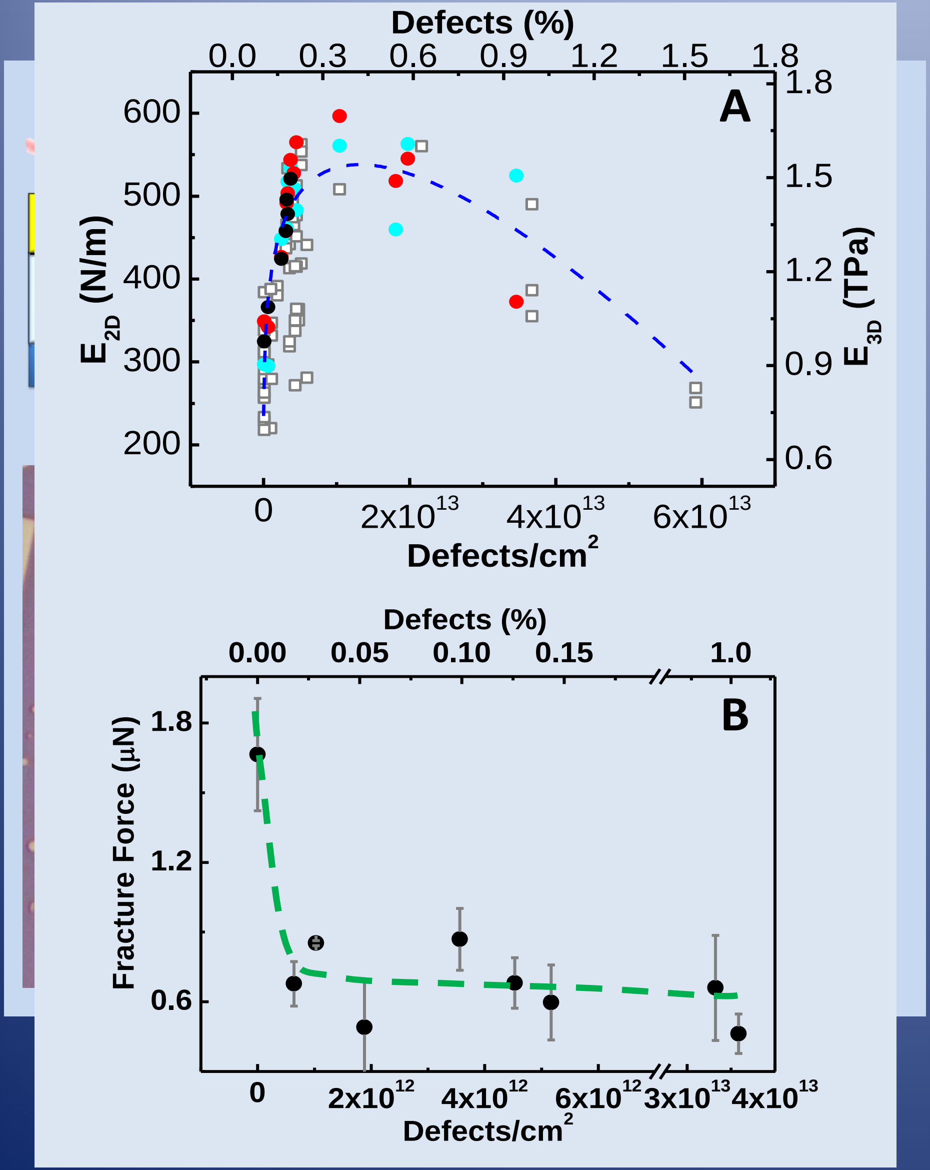
Increasing the elastic modulus of graphene by controlled defect creation

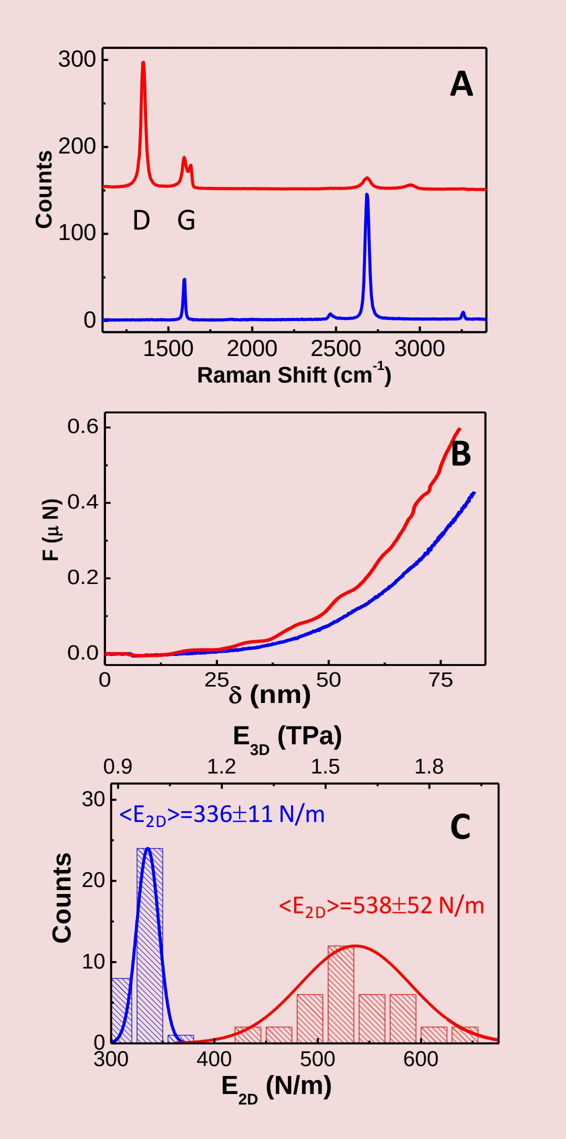
Guillermo López-Polín[1], Cristina Gómez-Navarro[1,2]*, Vincenzo Parente[3], Francisco Guinea[3], Mikhail I. Katsnelson[4], Francesc Pérez-Murano[5] and Julio Gómez-Herrero[1,2]



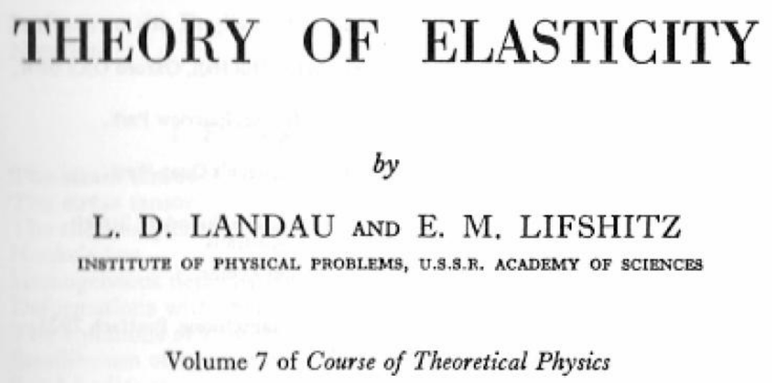


Experiments





Two dimensional membranes



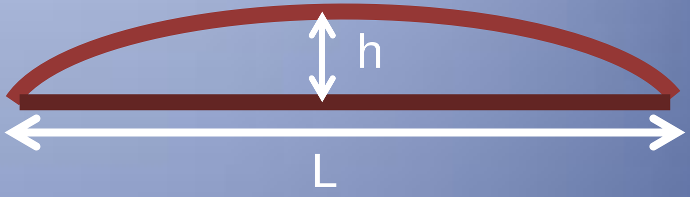
THEORY OF ELASTICITY

by

L. D. LANDAU AND E. M. LIFSHITZ

INSTITUTE OF PHYSICAL PROBLEMS, U.S.S.R. ACADEMY OF SCIENCES

Volume 7 of *Course of Theoretical Physics*



$$\Delta L \approx \frac{h^2}{2L}$$

Out of plane displacements lead to changes in area

Kinetic

Bending

Stretching

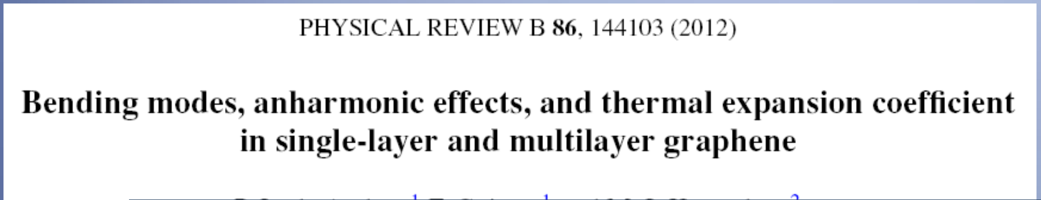
$$H = \frac{\rho}{2}\int d^2\vec{r}\,\frac{\partial^2 h}{\partial t^2} + \frac{\kappa}{2}\int d^2\vec{r}\left(\nabla^2 h\right)^2 + \frac{\lambda}{2}\int d^2\vec{r}\left(\partial_x u_x + \partial_y u_y + \frac{(\partial_x h)^2}{2} + \frac{(\partial_y h)^2}{2}\right)^2 +$$

$$+\mu\int d^2\vec{r}\left[\left(\partial_x u_x + \frac{(\partial_x h)^2}{2}\right)^2 + \left(\partial_y u_y + \frac{(\partial_y h)^2}{2}\right)^2 + \frac{1}{2}\left(\partial_x u_y + \partial_y u_x + \frac{(\partial_x h)(\partial_y h)}{2}\right)^2\right]$$

Shear

Two dimensional crystaline membranes are intrinsically anharmonic

Thermal expansion


PHYSICAL REVIEW B **86**, 144103 (2012)

Bending modes, anharmonic effects, and thermal expansion coefficient in single-layer and multilayer graphene

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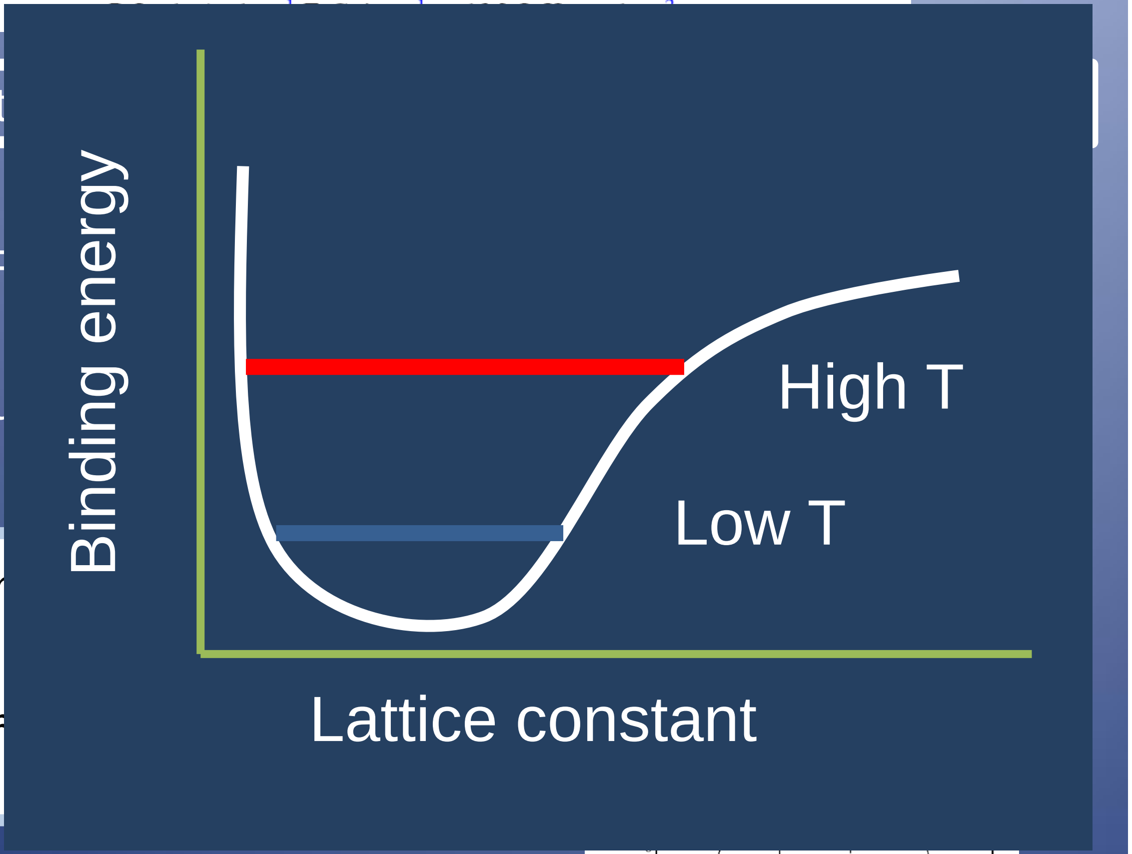
$\alpha \approx$



Negative thermal expansion coefficient

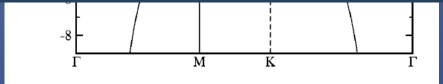
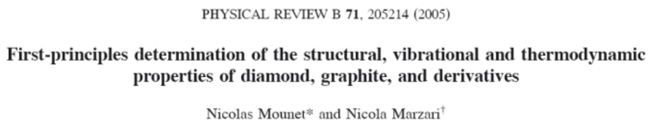
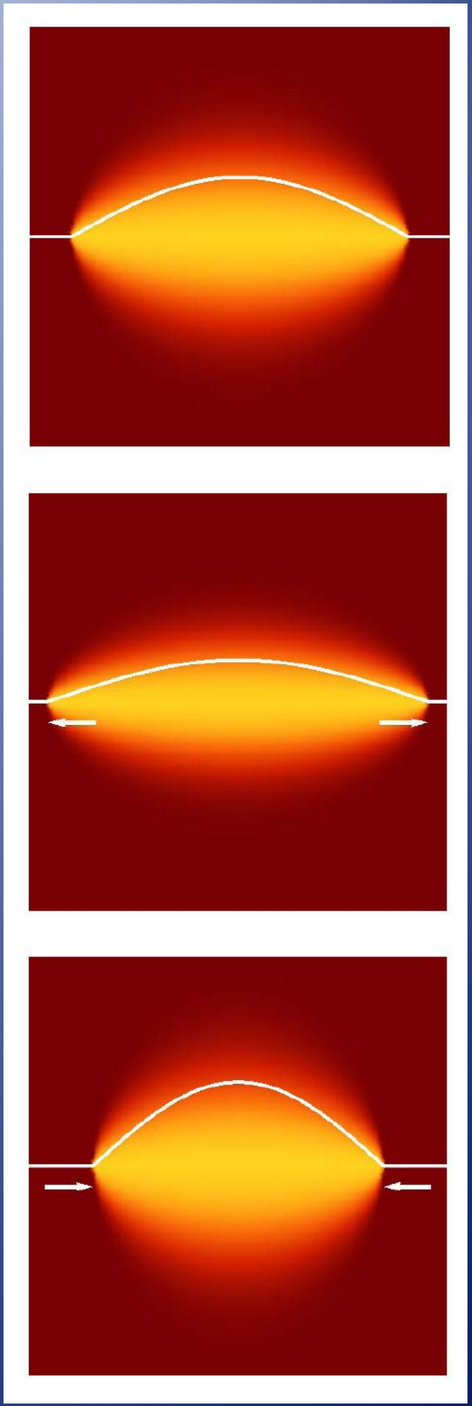


FIG. 17. *Ab initio* mode Grüneisen parameters for graphene.

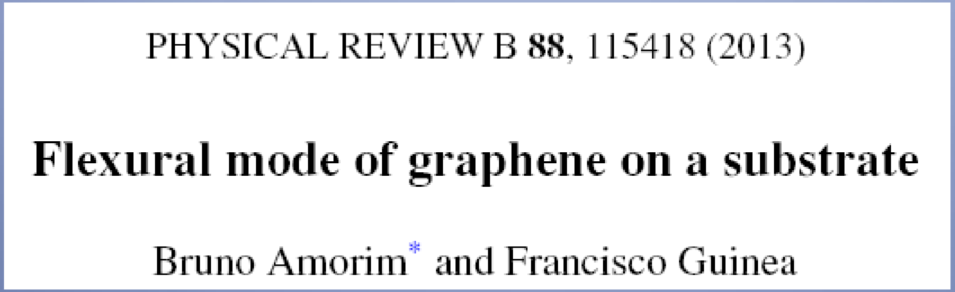

PHYSICAL REVIEW B **71**, 205214 (2005)

First-principles determination of the structural, vibrational and thermodynamic properties of diamond, graphite, and derivatives

Nicolas Mounet* and Nicola Marzari†



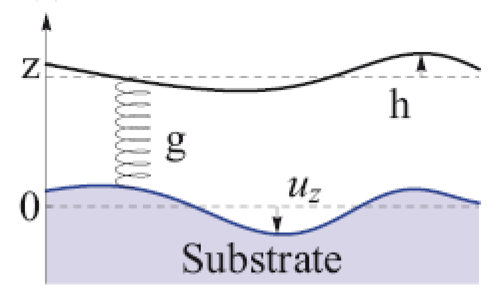
Substrate effects

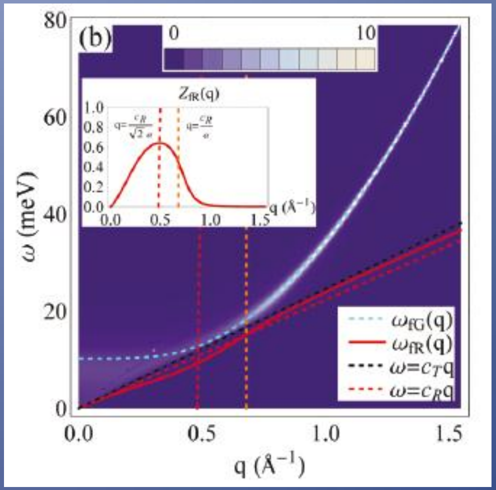


PHYSICAL REVIEW B **88**, 115418 (2013)

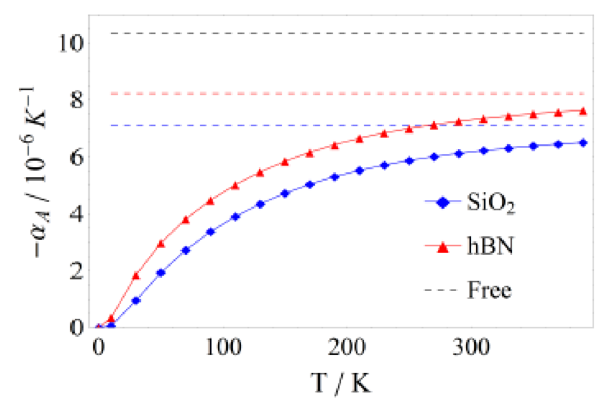
Flexural mode of graphene on a substrate

Bruno Amorim* and Francisco Guinea





Gapped flexural modes



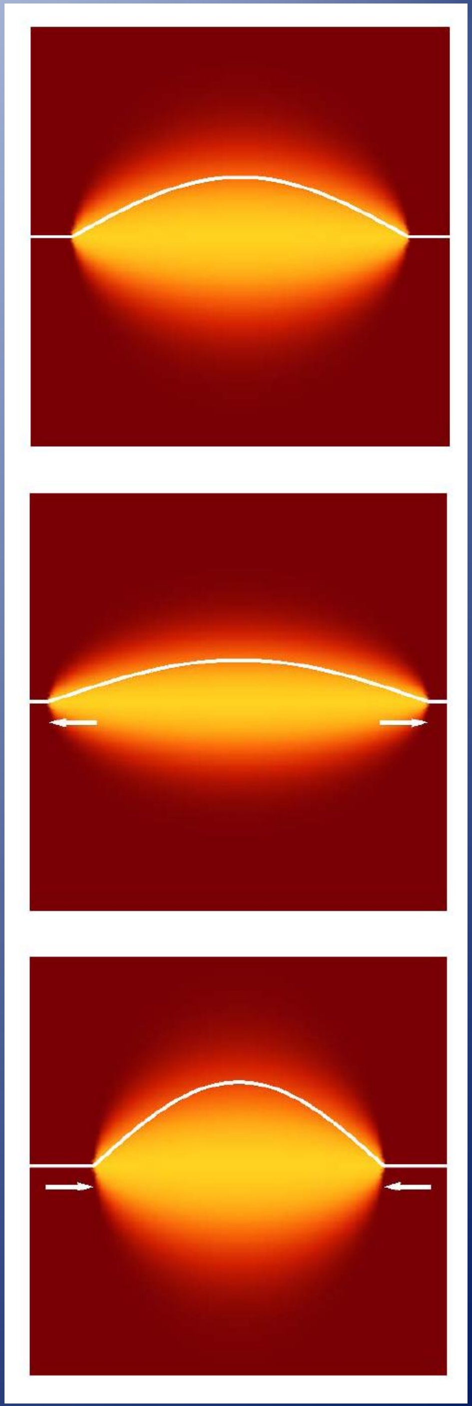
Thermal expansion

Out of plane fluctuations screen the in plane elastic constants

$$E \approx \left(c_1 Y \bar{u} + c_2 \frac{\kappa}{\ell^2} \right) h^2$$

$$F \approx T \log\left(\frac{T}{c_1 Y \bar{u} + c_2 \frac{\kappa}{\ell^2}} \right)$$

$$\delta Y = \frac{1}{\ell^2} \frac{\partial^2 F}{\partial \bar{u}^2} \propto -\frac{Y^2 T \ell^2}{\kappa^2}$$



Numerical results

NANO LETTERS
Letter
pubs.acs.org/NanoLett

Acoustic Phonon Lifetimes and Thermal Transport in Free-Standing and Strained Graphene

Nicola Bonini,*,† Jivtesh Garg,‡ and Nicola Marzari§

PHYSICAL REVIEW B **87**, 214303 (2013)

Anharmonic properties from a generalized third-order *ab initio* approach: Theory and applications to graphite and graphene

Lorenzo Paulatto,* Francesco Mauri, and Michele Lazzeri

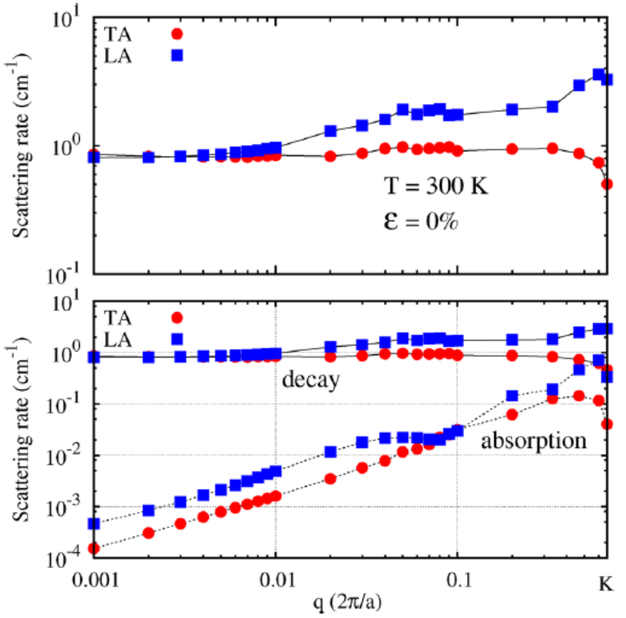
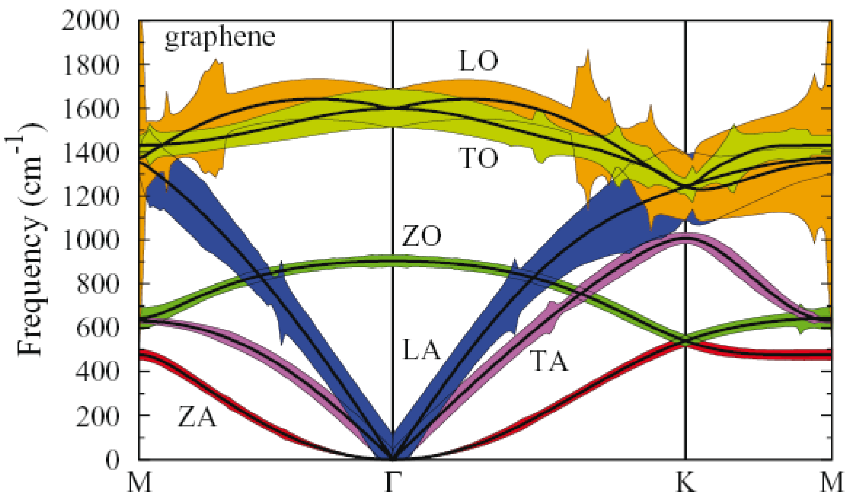


Figure 1. Upper panel: scattering rates for LA and TA modes alon the Γ−*K* direction in unstrained free-standing graphene at 300 K Lower panel: Contributions to the scattering rates due to decay (soli lines) and absorption (dashed line) processes.

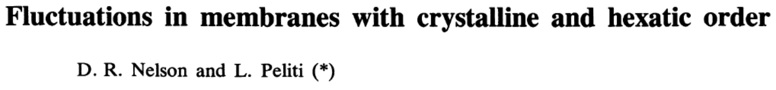


$$\Gamma_L = \frac{(\lambda+\mu)^2 T}{4(\lambda+2\mu)\kappa^{3/2}\rho^{1/2}}$$

$$\Gamma_T = \frac{\mu T}{4\kappa^{3/2}\rho^{1/2}}$$

Theory of elasticity

The self consistent screening approximation

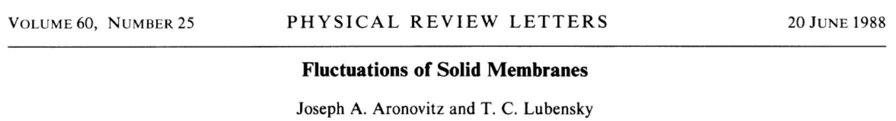


Fluctuations in membranes with crystalline and hexatic order

D. R. Nelson and L. Peliti (*)

J. Physique, **48**, 1085 (1987)

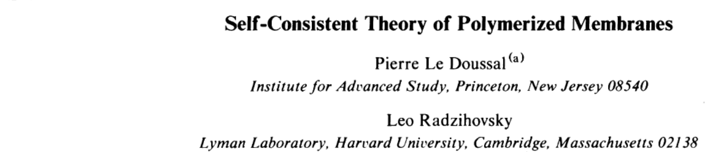
$$\delta\kappa \propto \int d^2\vec{q}\,\frac{TY}{\kappa|\vec{q}|^4}$$



VOLUME 60, NUMBER 25 PHYSICAL REVIEW LETTERS 20 JUNE 1988

Fluctuations of Solid Membranes

Joseph A. Aronovitz and T. C. Lubensky

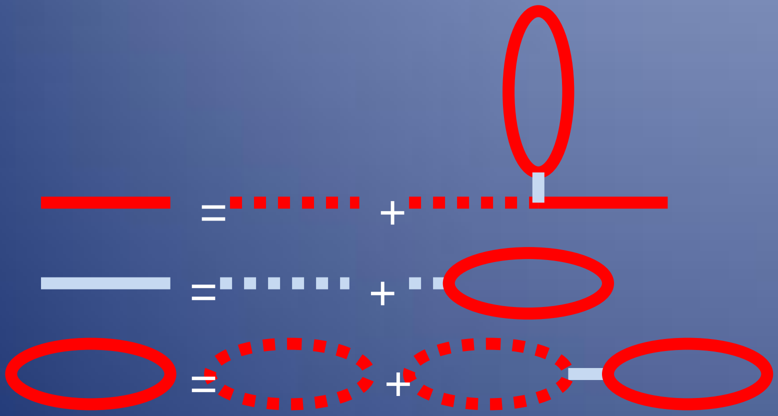


VOLUME 69, NUMBER 8 PHYSICAL REVIEW LETTERS 24 AUGUST 1992

Self-Consistent Theory of Polymerized Membranes

Pierre Le Doussal[(a)]
Institute for Advanced Study, Princeton, New Jersey 08540

Leo Radzihovsky
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138



$$G^{-1}(\vec{q}) = G_0^{-1}(\vec{q}) - \Sigma(\vec{q})$$

$$\Sigma(\vec{q}) = \frac{2}{(2\pi)^2}\int d^2\vec{q}\,b(\vec{q})|\vec{q}P_T(\vec{p})\vec{q}|^2 G(\vec{q}-\vec{p})$$

$$b(\vec{q}) = \frac{b_0}{1+3b_0 I(\vec{q})}$$

$$I(\vec{p}) = \frac{1}{8(2\pi)^2}\int d^2\vec{q}|\vec{q}|^2|\vec{p}-\vec{q}|^2 G(\vec{q})G(\vec{p}-\vec{q})$$

$$\kappa(q) \propto q^{-\eta}$$

$$\lambda(q), \mu(q) \propto q^{\eta_u}$$

$$\eta \approx 0.821$$

$$\eta_u \approx 0.358$$

Power law divergences
Self consistent theory, valid in high dimensions
Agrees well with numerical simulaions

Vacancies and flexural modes

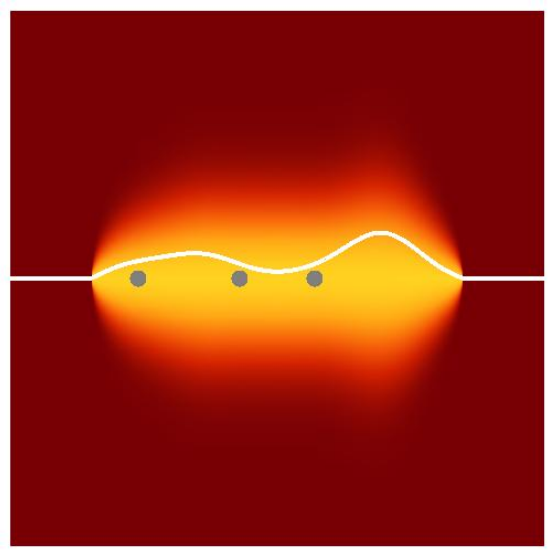
$$G(q,\omega)=\frac{1}{\rho\omega^2-\kappa q^4-\Sigma(q,\omega)}$$

T-matrix approximation

$$\Sigma(\omega)\approx\begin{cases} n_V\sqrt{\kappa\rho\omega^2} & h^2=0 \\ n_V\dfrac{\sqrt{\kappa\rho\omega^2}}{\log\left(\dfrac{\kappa}{a^4\rho\omega^2}\right)} & \left|\nabla h\right|^2=0 \end{cases}$$

infinite mass

vacancies



localization length

$$\frac{\kappa}{\ell^4}\approx\Sigma\left(\sqrt{\frac{\kappa}{\rho\ell^4}}\right)$$

$$\ell\approx n_V^{-1/2}$$

- Vacancies localize flexural modes
- Long wavelength flexural modes do not contribute to the screening of the elastic constants

geometric factor

percolation

$$Y \approx K\left(\frac{1}{R^2}+\frac{1}{\ell_0^2}+n_V\right)^{\frac{\eta_u}{2}}\left[1-c\left(\frac{1}{\ell_0^2}+n_V\right)\right]$$

intrinsic localization length

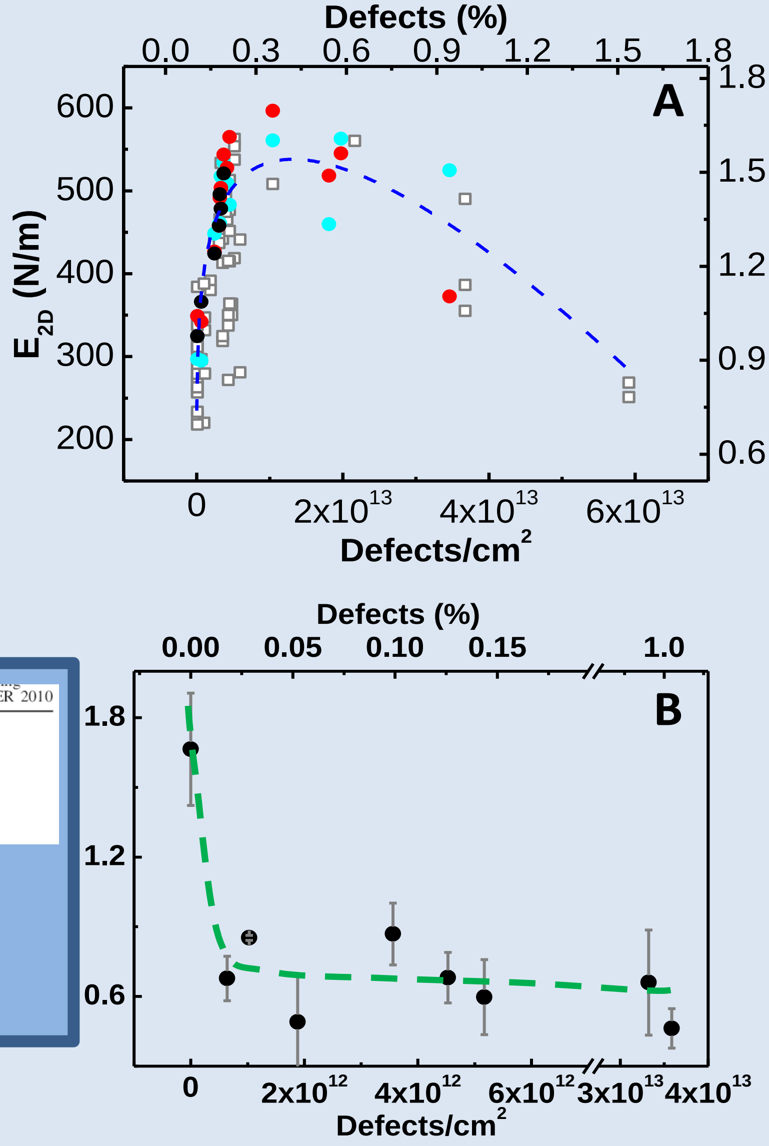
PRL **105,** 266601 (2010) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2010

Limits on Charge Carrier Mobility in Suspended Graphene due to Flexural Phonons

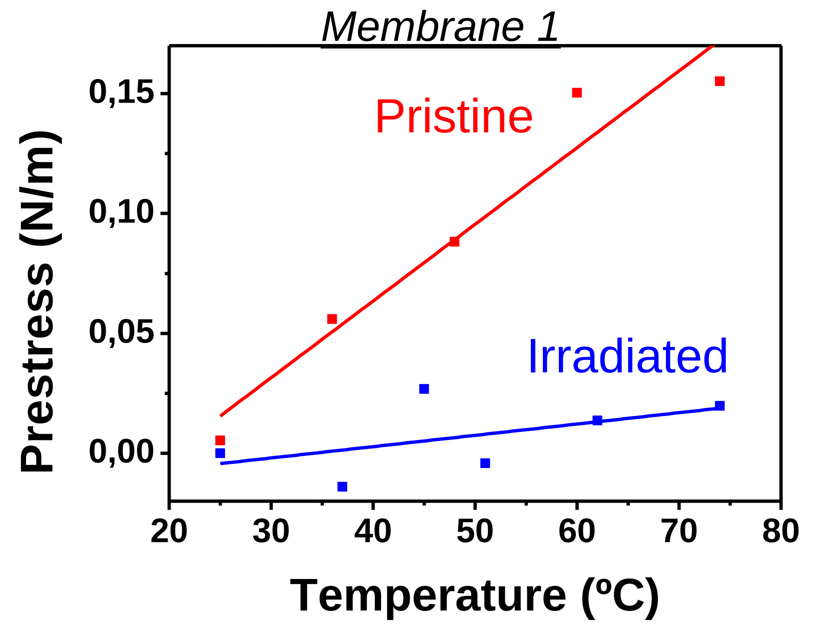
Eduardo V. Castro,[1] H. Ochoa,[1] M. I. Katsnelson,[2] R. V. Gorbachev,[3] D. C. Elias,[3] K. S. Novoselov,[3] A. K. Geim,[3] and F. Guinea[1]

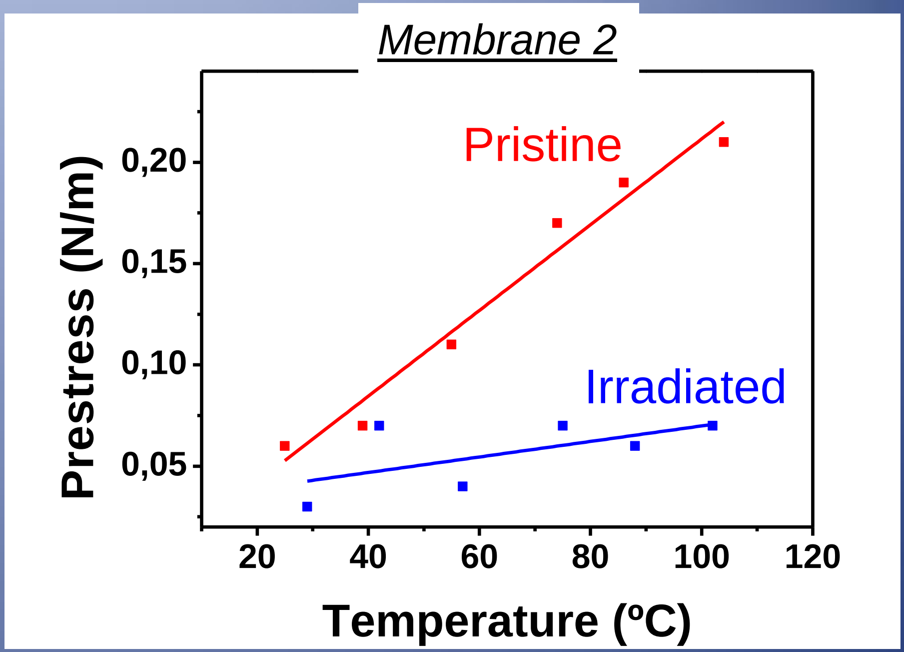
$$\ell_0 \approx 20-100\,\mathrm{nm}$$

$$\ell_0 \geq k_F^{-1}$$



Graphene thermal expansion coefficient





Thermal Expansion Coefficient:

· Pristine: -9.4×10^{-6} K^{-1}

· Irradiated (L_D ~ 5.5 nm): -1×10^{-6} K^{-1}

Thermal Expansion Coefficient:

· Pristine: -6.2×10^{-6} K^{-1}

· Irradiated (L_D ~ 5 nm): -1.1×10^{-6} K^{-1}

L_D : Mean distance between defects as measured by Raman

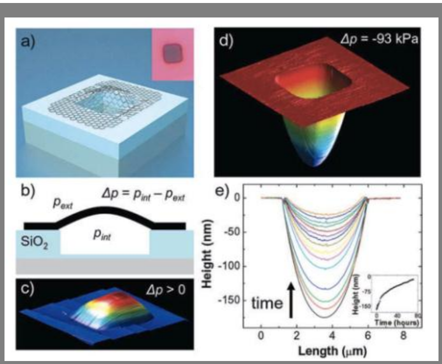
Young modulus and induced strains

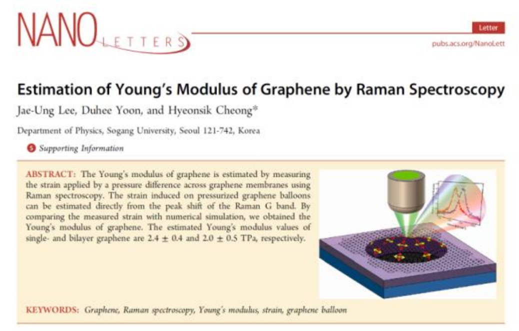
NANO
LETTERS

2008
Vol. 8, No. 8
2458-2462

Impermeable Atomic Membranes from Graphene Sheets

J. Scott Bunch, Scott S. Verbridge, Jonathan S. Alden, Arend M. van der Zande, Jeevak M. Parpia, Harold G. Craighead, and Paul L. McEuen*





NANO LETTERS
Letter
pubs.acs.org/NanoLett

Estimation of Young's Modulus of Graphene by Raman Spectroscopy

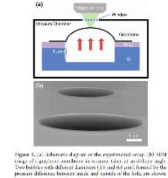
Jae-Ung Lee, Duhee Yoon, and Hyeonsik Cheong*

Department of Physics, Sogang University, Seoul 121-742, Korea

Ⓢ *Supporting Information*

ABSTRACT: The Young's modulus of graphene is estimated by measuring the strain applied by a pressure difference across graphene membranes using Raman spectroscopy. The strain induced on pressurized graphene balloons can be estimated directly from the peak shift of the Raman G band. By comparing the measured strain with numerical simulation, we obtained the Young's modulus of graphene. The estimated Young's modulus values of single- and bilayer graphene are 2.4 ± 0.4 and 2.0 ± 0.5 TPa, respectively.

KEYWORDS: *Graphene, Raman spectroscopy, Young's modulus, strain, graphene balloon*



Young modulus measured by Raman is two times larger than the one measured by indentation

Strain dependent elastic modulus of graphene

G. López-Polín,[1,*] M. Jaafar,[1,2,*] F. Guinea,[3,4] R. Roldán,[2,4] C. Gómez-Navarro,[1,5,†] and J. Gómez-Herrero[1,5]

arXiv:1504.05521

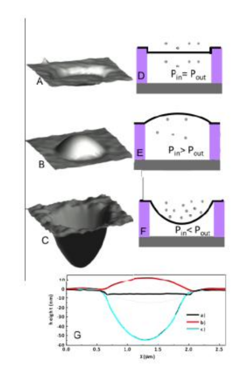
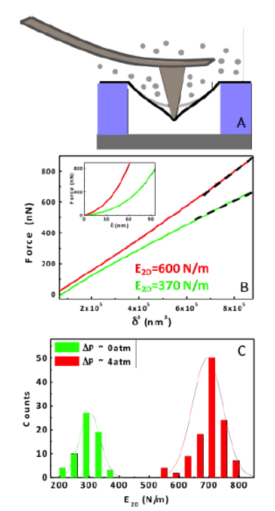
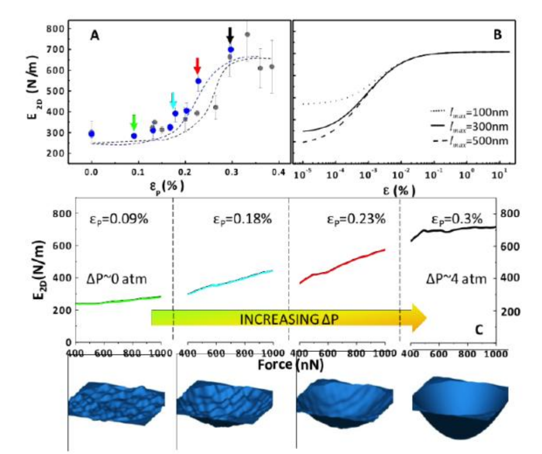
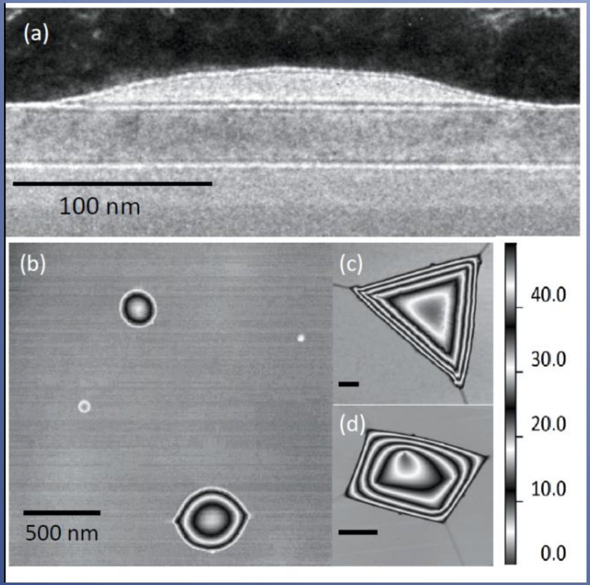


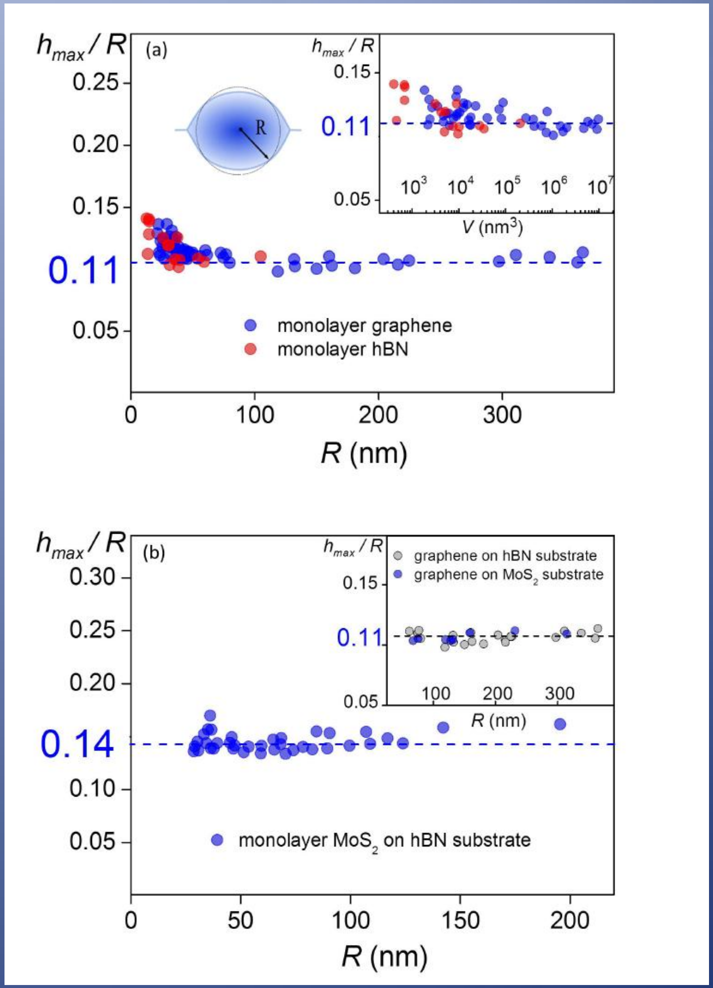
FIG. 1: Panels (A) to (C) display AFM images of a graphene drumhead of 1.5 μm diameter subjected to different ΔP. (A) corresponds to $P_{in} = P_{out} \sim 1$ atm, (B) to $P_{out} \sim 0$ atm, $P_{in} \sim 1$ atm, (C) to $P_{in} \sim 0$ atm, $P_{out} \sim 3$ atm. . Panels (D) to (F) are schematic profiles of the pressurized membrane. Panel (G) shows AFM profiles taken along a great arc in the AFM images shown in panel (A) to (C)





Graphene bubbles





Bubbles: scaling properties

$$\tilde{r} = \frac{r}{R}$$

$$h(r) = h_{max}\tilde{h}(\tilde{r})$$

$$u_r(r) = \frac{h_{max}^2}{R}\tilde{u}_r(\tilde{r})$$

$$\tilde{h}(0) = 1$$

$$\tilde{h}(1) = 0$$

$$E_{tot} = E_{el} + E_{vW} + E_V = c_1[\tilde{h}(\tilde{r})]Y\frac{h_{max}^4}{R^2} + \pi R^2\gamma + E(V)$$

$$V = c_V[\tilde{h}(\tilde{r})]h_{max}R^2$$

$$\frac{\partial E_{tot}}{\partial h_{max}} = c_1[\tilde{h}(\tilde{r})]Y\frac{4h_{max}^3}{R^2} + c_V[\tilde{h}(\tilde{r})]R^2P = 0$$

$$\frac{\partial E_{tot}}{\partial R} = -c_1[\tilde{h}(\tilde{r})]Y\frac{2h_{max}^4}{R^3} + 2\pi R\gamma + 2c_V[\tilde{h}(\tilde{r})]h_{max}RP = 0$$

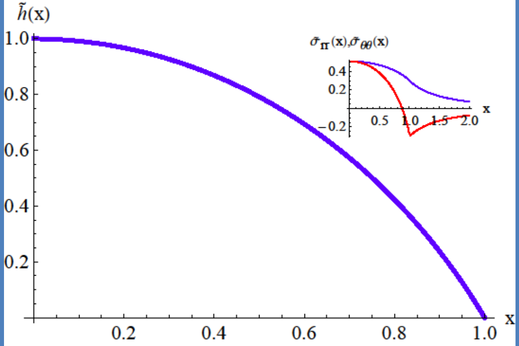
$$P = \frac{\partial E_V}{\partial V}$$

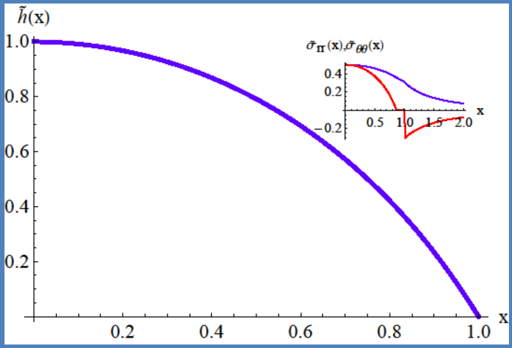
$$\frac{h_{max}}{R} = \left(\frac{\pi\gamma}{5c_1[\tilde{h}(\tilde{r})]Y}\right)^{\frac{1}{4}}$$

$$c_1[\tilde{h}(\tilde{r})] \approx 0.7$$

$$c_V[\tilde{h}(\tilde{r})] \approx 1.7$$

Bubbles: scaling properties

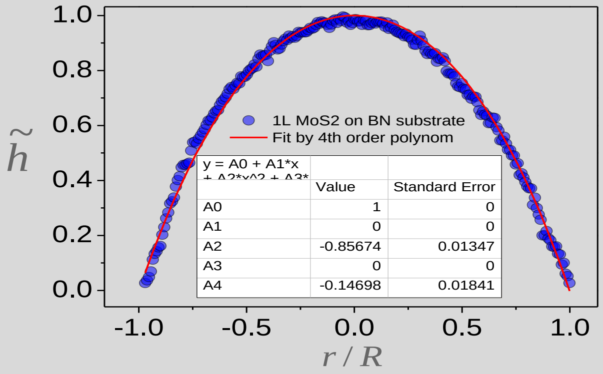




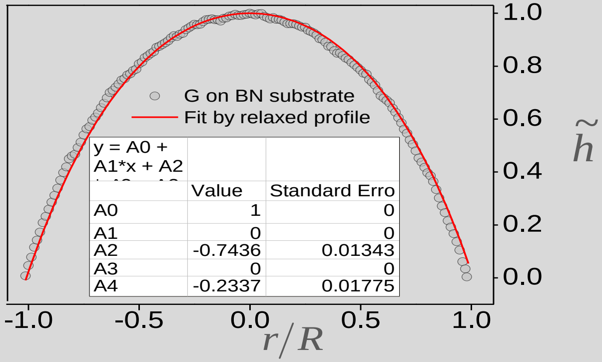
Prototypical model for tensional wrinkling in thin sheets

Benny Davidovitch[a,1], Robert D. Schroll[a], Dominic Vella[b,c], Mokhtar Adda-Bedia[b], and Enrique A. Cerda[d]

PNAS | **November 8, 2011** | vol. 108 | no. 45 | **18227–18232**

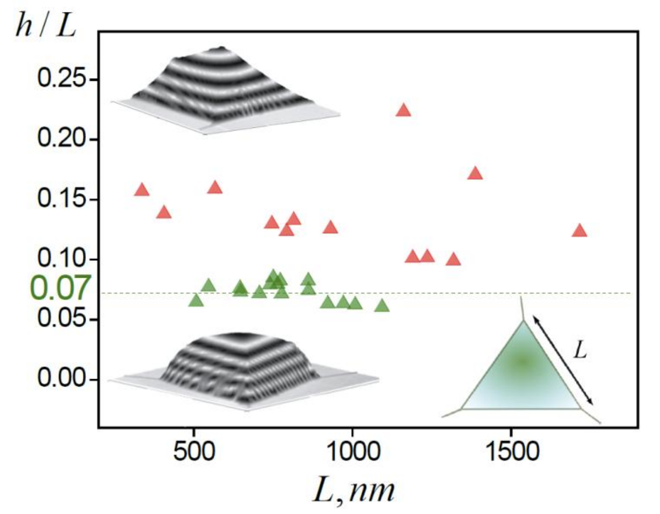


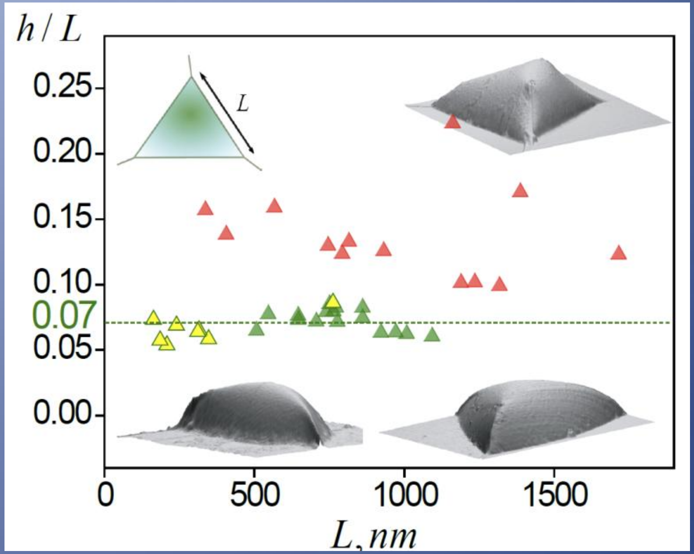
y = A0 + A1*x + A2*x^2 + A3*		
	Value	Standard Error
A0	1	0
A1	0	0
A2	-0.85674	0.01347
A3	0	0
A4	-0.14698	0.01841

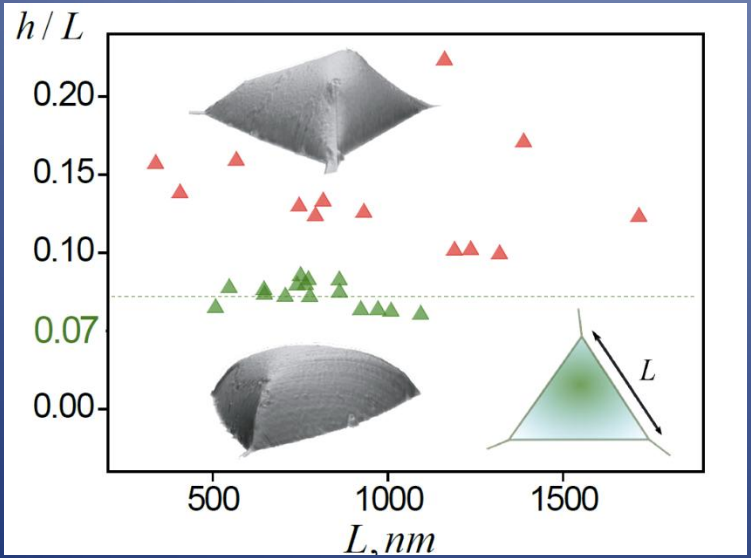


y = A0 + A1*x + A2		
	Value	Standard Erro
A0	1	0
A1	0	0
A2	-0.7436	0.01343
A3	0	0
A4	-0.2337	0.01775

Other bubbles



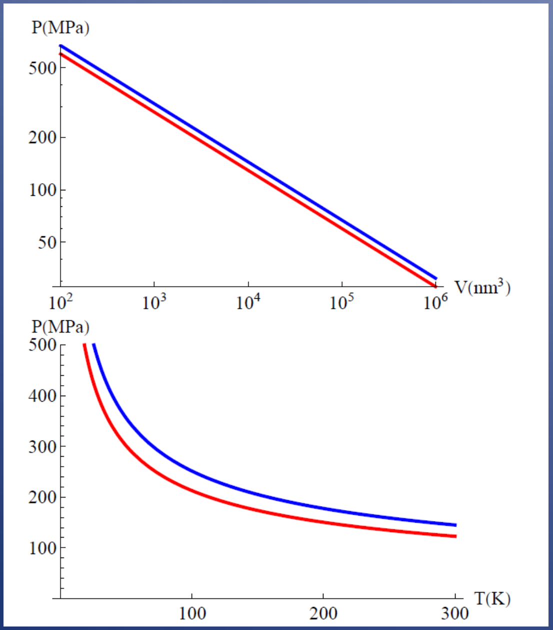




Pressure within bubbles

$$P = \frac{4Yc_1}{c_V h_{max}}\left(\frac{h_{max}}{R}\right)^4 = \frac{4\pi\gamma}{5h_{max}} = \frac{4\pi\gamma}{5}\left(\frac{5c_1Y}{\pi\gamma}\right)^{\frac{1}{6}}\left(\frac{c_V}{V}\right)^{\frac{1}{3}}$$

The pressure is independent of the properties of the fluid inside the bubble



Volume dependence

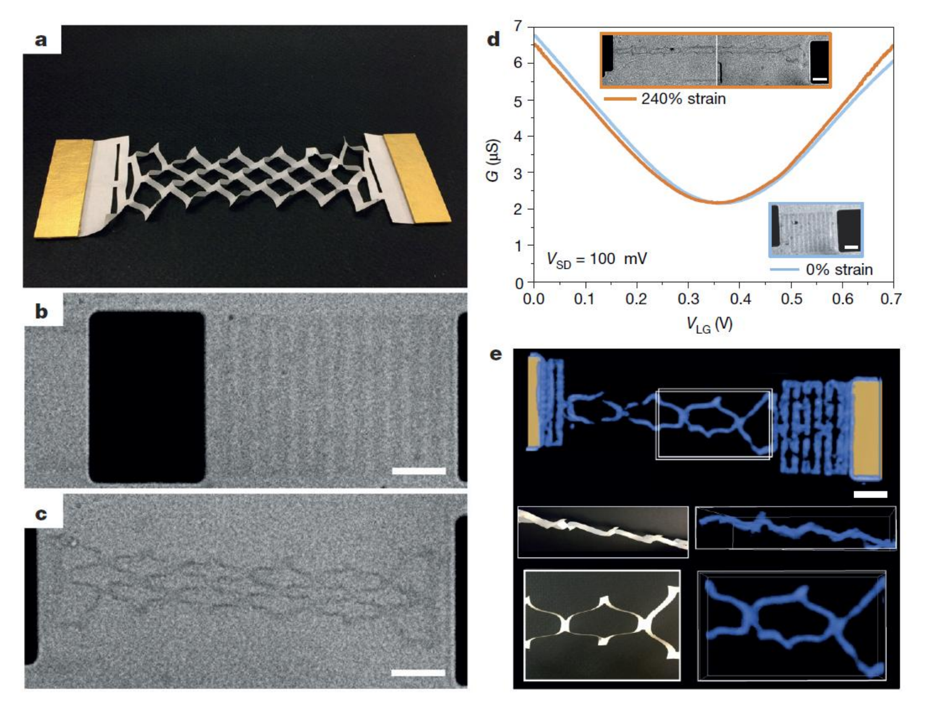
Perfect gas: temperature dependence

LETTER

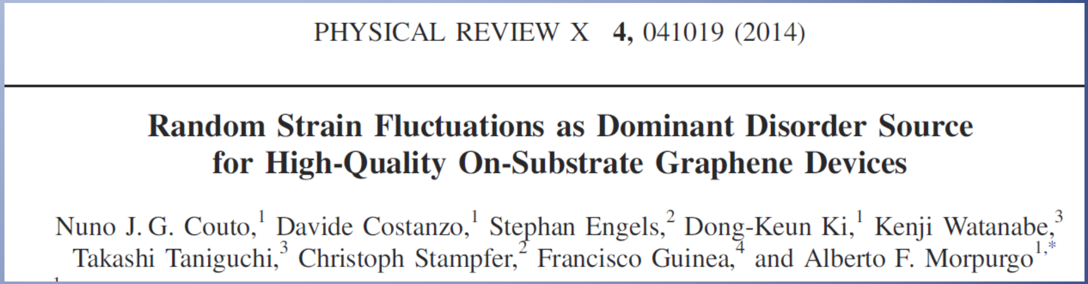
doi:10.1038/nature14588

Graphene kirigami

Melina K. Blees[1], Arthur W. Barnard[2], Peter A. Rose[1], Samantha P. Roberts[1], Kathryn L. McGill[1], Pinshane Y. Huang[2], Alexander R. Ruyack[3], Joshua W. Kevek[1], Bryce Kobrin[1], David A. Muller[2,4] & Paul L. McEuen[1,4]



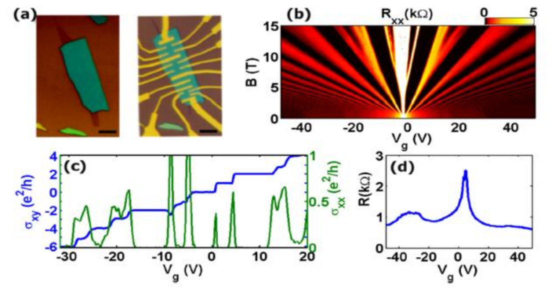
Strains and conductivity in graphene



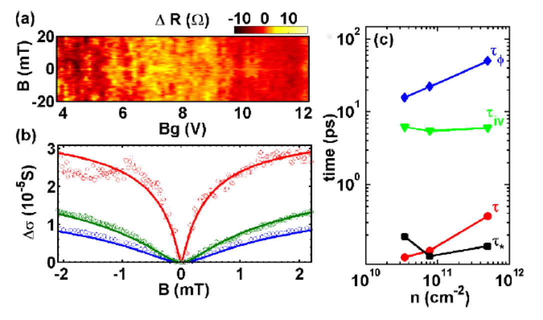
PHYSICAL REVIEW X **4,** 041019 (2014)

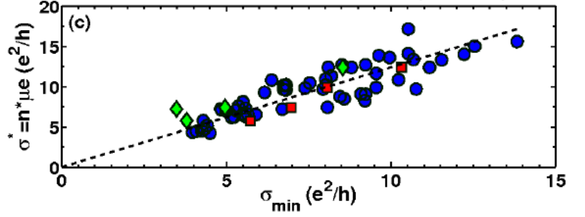
Random Strain Fluctuations as Dominant Disorder Source for High-Quality On-Substrate Graphene Devices

Nuno J. G. Couto,[1] Davide Costanzo,[1] Stephan Engels,[2] Dong-Keun Ki,[1] Kenji Watanabe,[3] Takashi Taniguchi,[3] Christoph Stampfer,[2] Francisco Guinea,[4] and Alberto F. Morpurgo[1,*]



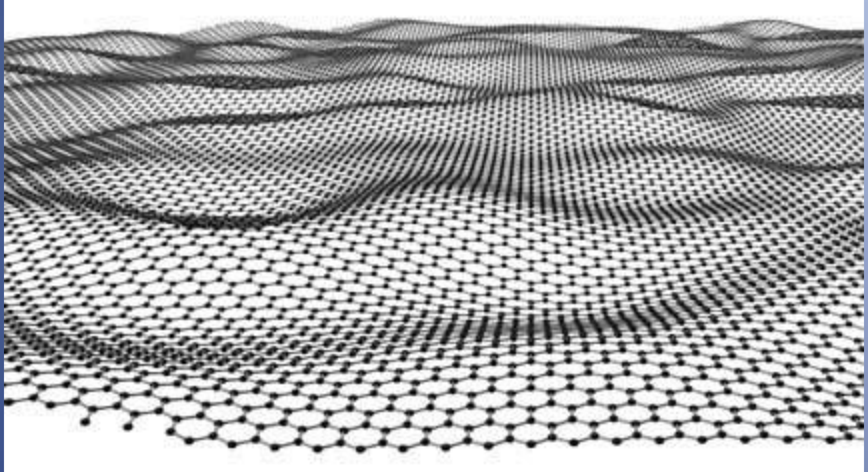
- Study of dc electronic transport in high quality samples
- Weak localization measurements
- Correlation between results at the neutrality point and at high carrier concentrations

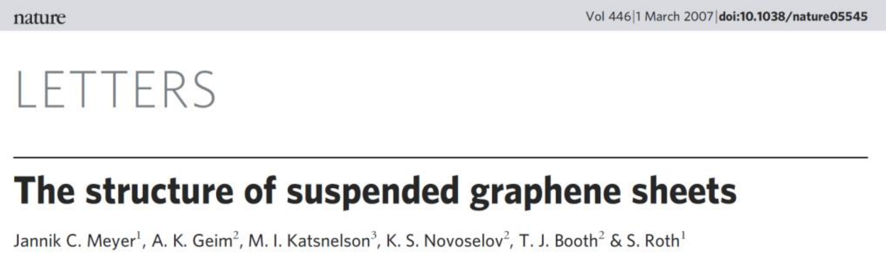




- Scattering is due to intravalley processes
- Interference processes (weak localization) are suppressed
- Puddles and transport are correlated
- Strains are the likely origin of puddles and scattering

Ripples in graphene





nature Vol 446|1 March 2007|**doi:10.1038/nature05545**

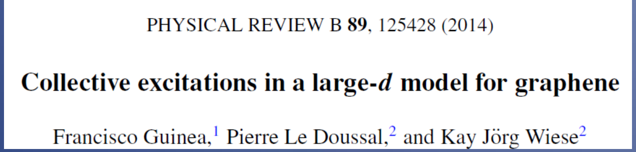
LETTERS

The structure of suspended graphene sheets

Jannik C. Meyer[1], A. K. Geim[2], M. I. Katsnelson[3], K. S. Novoselov[2], T. J. Booth[2] & S. Roth[1]

- Quenched (non thermal) ripples in suspended samples
- Lateral scale $\sim 10^2 - 10^3$ Å
- Vertical scale ~ 10Å

Instability due to the coupling to low energy electron-hole pairs?



PHYSICAL REVIEW B **89**, 125428 (2014)

Collective excitations in a large-*d* model for graphene

Francisco Guinea,[1] Pierre Le Doussal,[2] and Kay Jörg Wiese[2]

PRL **106,** 045502 (2011) PHYSICAL REVIEW LETTERS week ending 28 JANUARY 2011

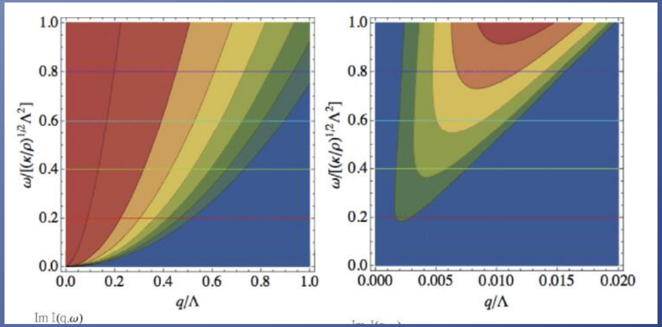
Electron-Induced Rippling in Graphene

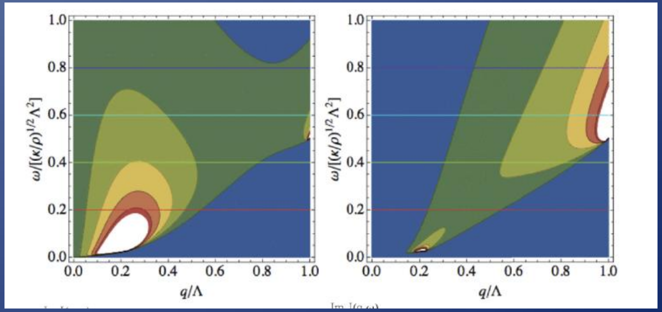
P. San-Jose,[1] J. González,[1] and F. Guinea[2]

PHYSICAL REVIEW B **80**, 161406(R) (2009)

Correlation between charge inhomogeneities and structure in graphene and other electronic crystalline membranes

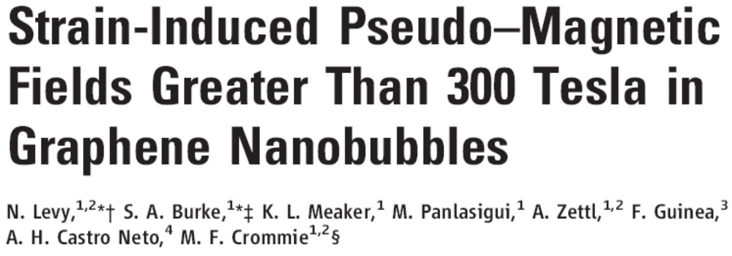
Doron Gazit*





Also: wrinkles induced by absorbates

Strain engineering in graphene

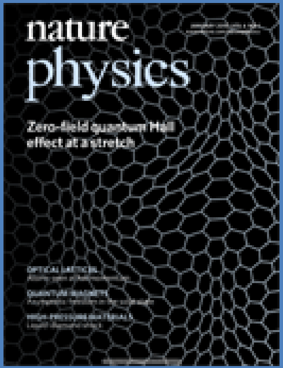


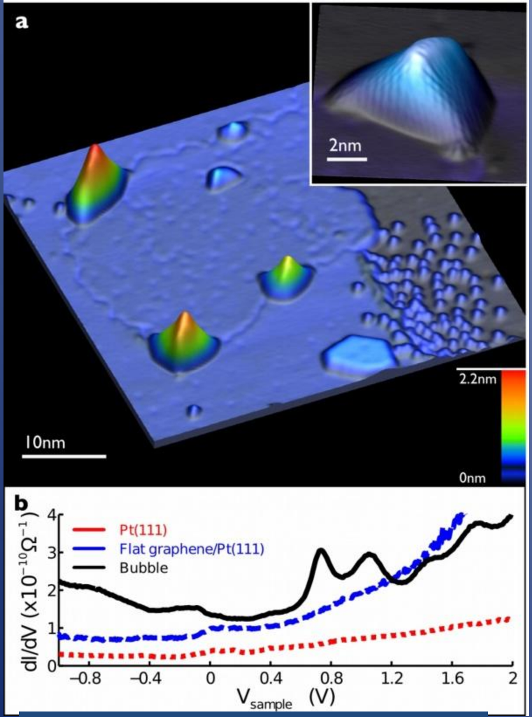
Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§

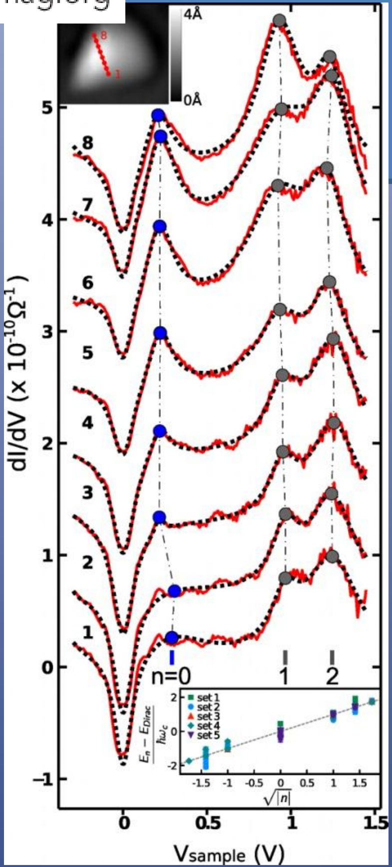
30 JULY 2010 VOL 329 **SCIENCE** www.sciencemag.org

F. G., M. I. Katsnelson, A. K. Geim, Nature Phys. **6**, 30 (2010)

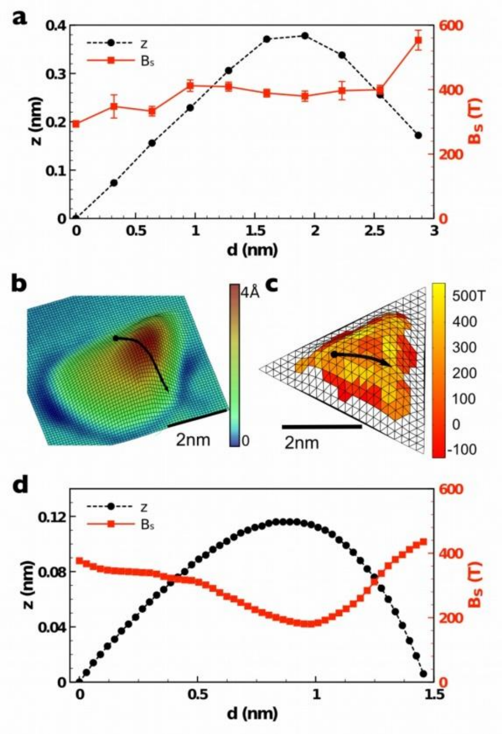




Topography and spectroscopy of bubbles in graphene on Pt



Scaling of resonances observed with STM

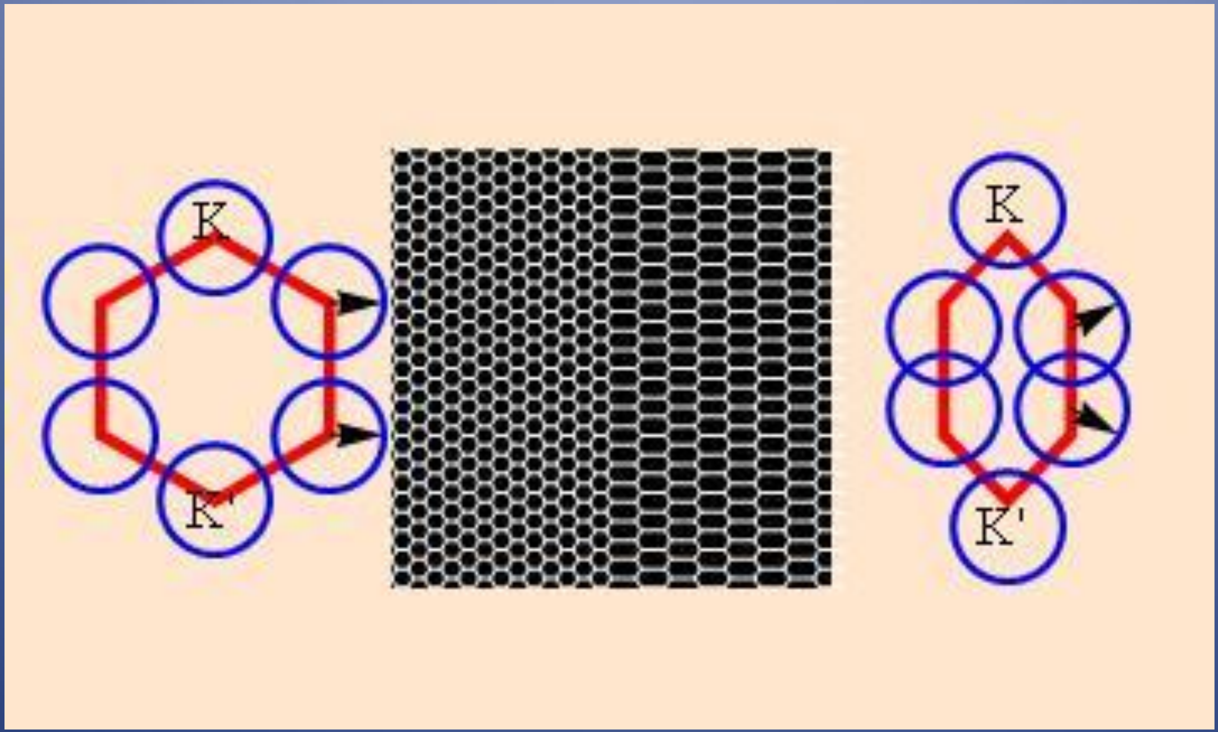


Comparison of theory and experiment

Effective gauge fields

M. A. H. Vozmediano, M. I. Katsnelson, F. G (2010), Physics Reports **496**, 109 (2010)

$$H \equiv \begin{pmatrix} 0 & t_1 e^{i\vec{k}_1\vec{a}_1} + t_2 e^{i\vec{k}_2\vec{a}_2} + t_3 e^{i\vec{k}_3\vec{a}_3} \\ t_1 e^{-i\vec{k}_1\vec{a}_1} + t_2 e^{-i\vec{k}_2\vec{a}_2} + t_3 e^{-i\vec{k}_3\vec{a}_3} & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{3\bar{t}a}{2}\left(k_x + ik_y\right) + \Delta t \\ \frac{3\bar{t}a}{2}\left(k_x + ik_y\right) + \Delta t & 0 \end{pmatrix}$$

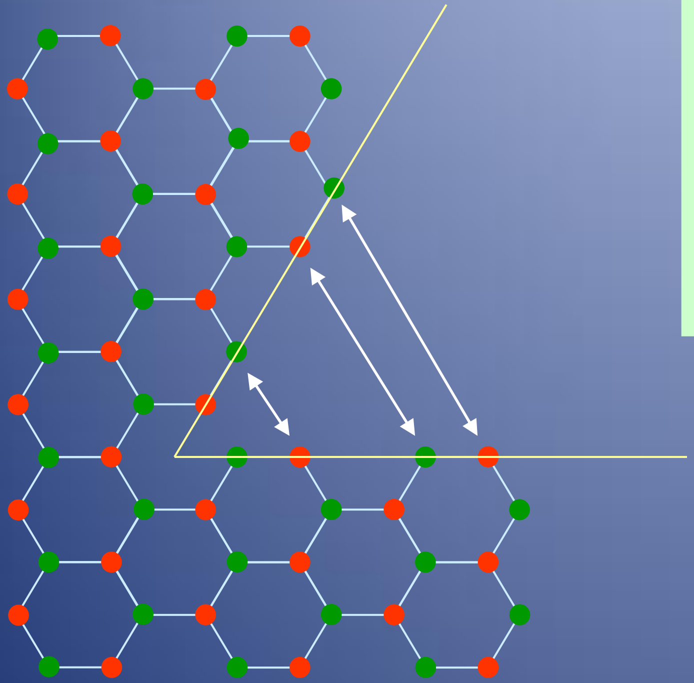


A modulation of the hoppings leads to a term which modifies the momentum: an effective gauge field.

The induced “magnetic” fields have opposite sign at the two corners of the Brillouin Zone.

Lattice frustration as a gauge potential.

J. González, F. G. and M. A. H. Vozmediano, Phys. Rev. Lett. **69**, 172 (1992)



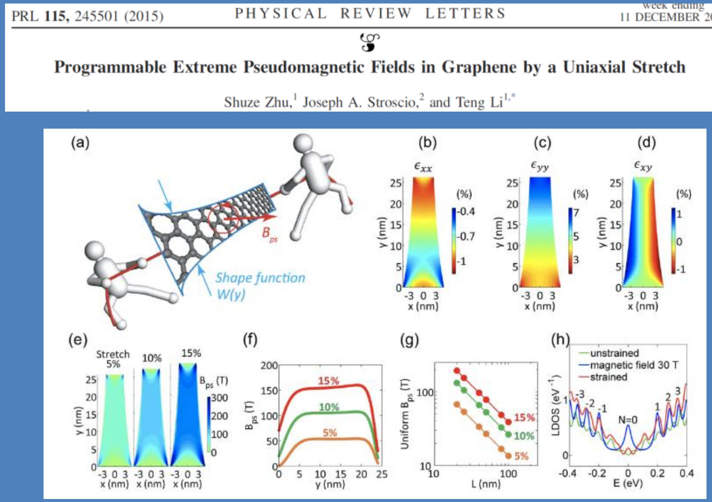
- A fivefold ring defines a disclination.
- The sublattices are interchanged.
- The Fermi points are also interchanged.
- These transformations can be achieved by means of a gauge potential.

$$i\vec{\nabla} \rightarrow i\vec{\nabla} - \vec{A}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Phi = \int \vec{A}\, d\vec{l}$$

The flux Φ is determined by the total rotation induced by the defect.

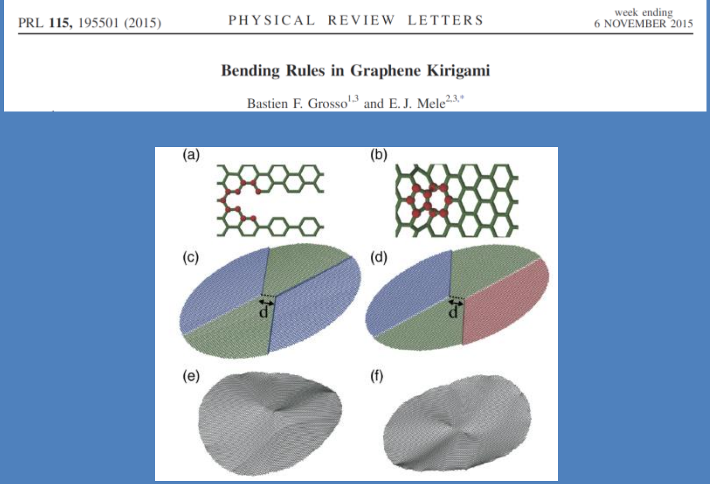
Strain engineering, recent developments



PRL **115,** 245501 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

Programmable Extreme Pseudomagnetic Fields in Graphene by a Uniaxial Stretch

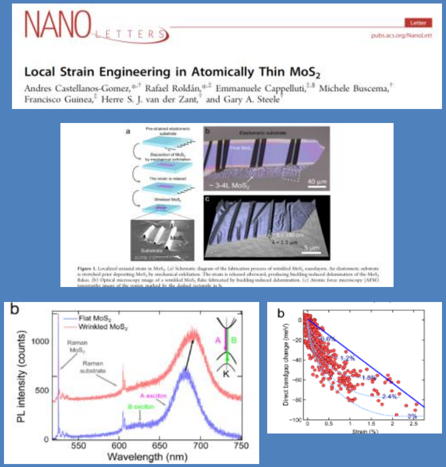
Shuze Zhu,[1] Joseph A. Stroscio,[2] and Teng Li[1,*]



PRL **115,** 195501 (2015) PHYSICAL REVIEW LETTERS week ending 6 NOVEMBER 2015

Bending Rules in Graphene Kirigami

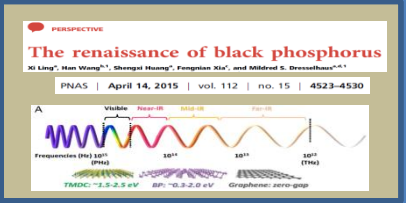
Bastien F. Grosso[1,3] and E. J. Mele[2,3,*]



NANO LETTERS Letter pubs.acs.org/NanoLett

Local Strain Engineering in Atomically Thin MoS_2

Andres Castellanos-Gomez,[*,†] Rafael Roldán,[*,‡] Emmanuele Cappelluti,[‡,§] Michele Buscema,[†] Francisco Guinea,[‡] Herre S. J. van der Zant,[†] and Gary A. Steele[†]



PERSPECTIVE

The renaissance of black phosphorus

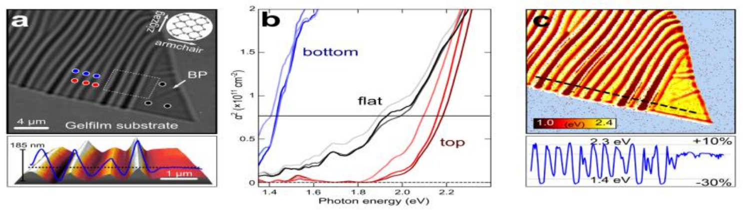
Xi Ling[a], Han Wang[b,1], Shengxi Huang[a], Fengnian Xia[c], and Mildred S. Dresselhaus[a,d,1]

PNAS | April 14, 2015 | vol. 112 | no. 15 | 4523–4530

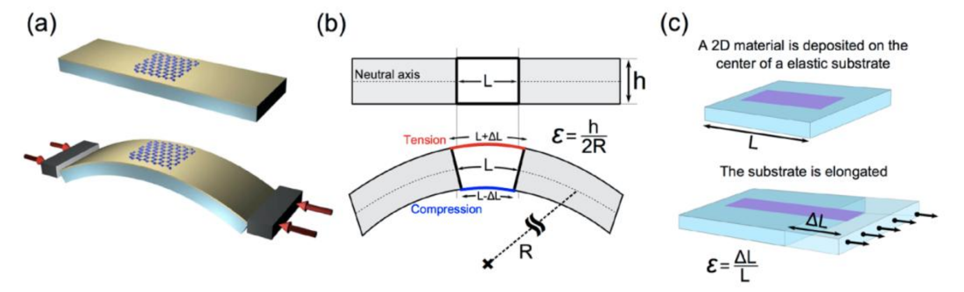
Black phosphorus quantum wire array by periodic strain engineering

Jorge Quereda[1], Vincenzo Parente[2], Pablo San-José[3], Nicolás Agraït[1,2,4], Gabino Rubio-Bollinger[1,4], Francisco Guinea[2], Rafael Roldán[3], Andres Castellanos-Gomez[2*]*

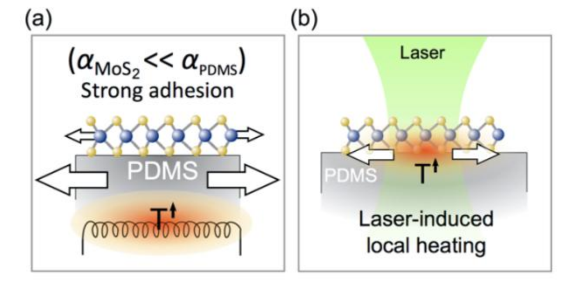
arXiv:1509.01182,
Nano Lett., in press



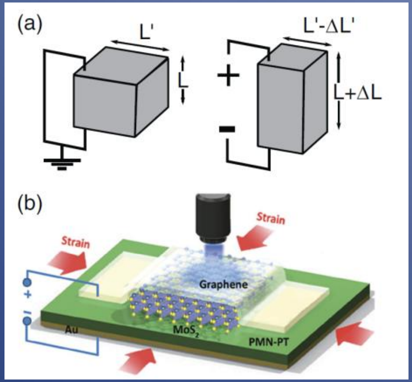
Inducing strain: experimental methods



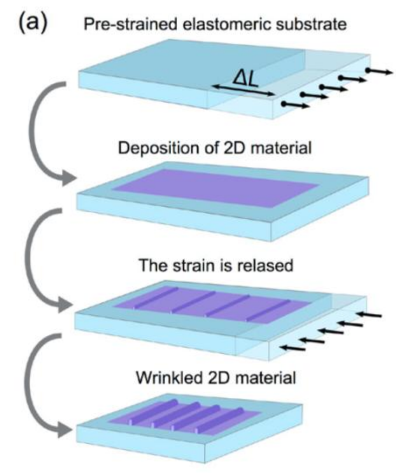
Uniaxial strain by bending



Biaxial strain by heating



Biaxial strain with a piezoelectric substrate



Uniaxial strain by inducing wrinkles

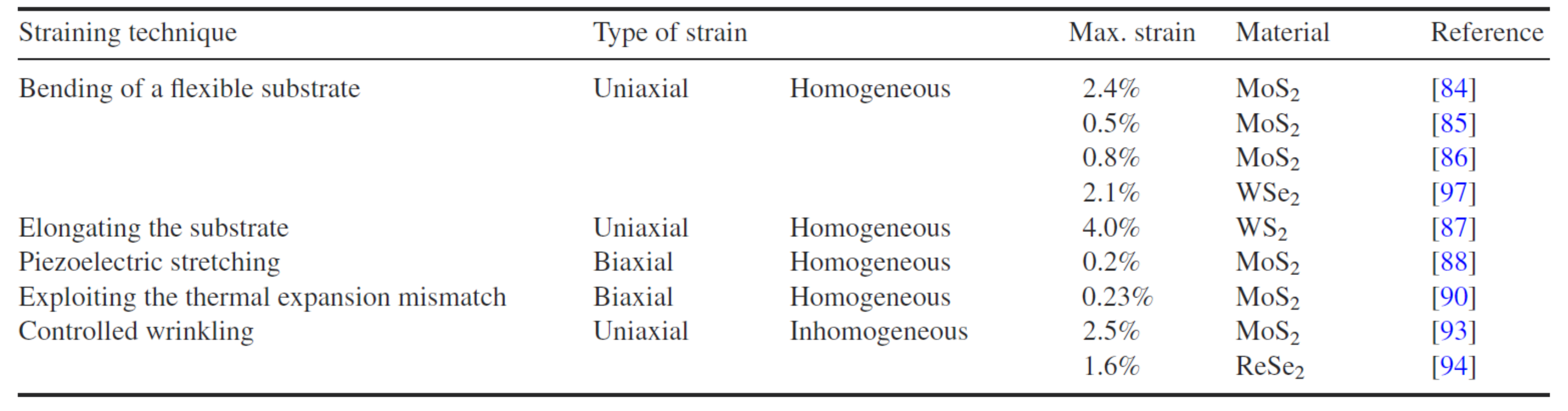
Inducing strain: experimental methods

IOP Publishing
Journal of Physics: Condensed Matter
J. Phys.: Condens. Matter **27** (2015) 313201 (18pp)
doi:10.1088/0953-8984/27/31/313201

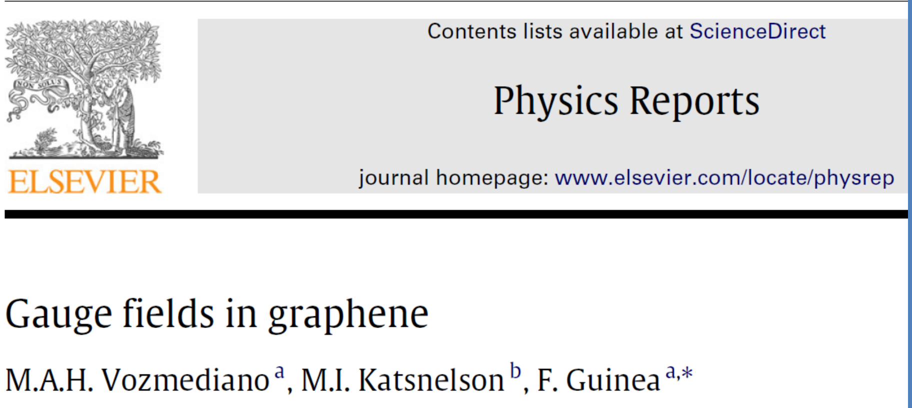
Topical Review

Strain engineering in semiconducting two-dimensional crystals

Rafael Roldán[1,2], Andrés Castellanos-Gomez[2], Emmanuele Cappelluti[3] and Francisco Guinea[2,4]



Straining technique	Type of strain		Max. strain	Material	Reference
Bending of a flexible substrate	Uniaxial	Homogeneous	2.4%	MoS_2	[84]
			0.5%	MoS_2	[85]
			0.8%	MoS_2	[86]
			2.1%	WSe_2	[97]
Elongating the substrate	Uniaxial	Homogeneous	4.0%	WS_2	[87]
Piezoelectric stretching	Biaxial	Homogeneous	0.2%	MoS_2	[88]
Exploiting the thermal expansion mismatch	Biaxial	Homogeneous	0.23%	MoS_2	[90]
Controlled wrinkling	Uniaxial	Inhomogeneous	2.5%	MoS_2	[93]
			1.6%	$ReSe_2$	[94]



Contents lists available at ScienceDirect

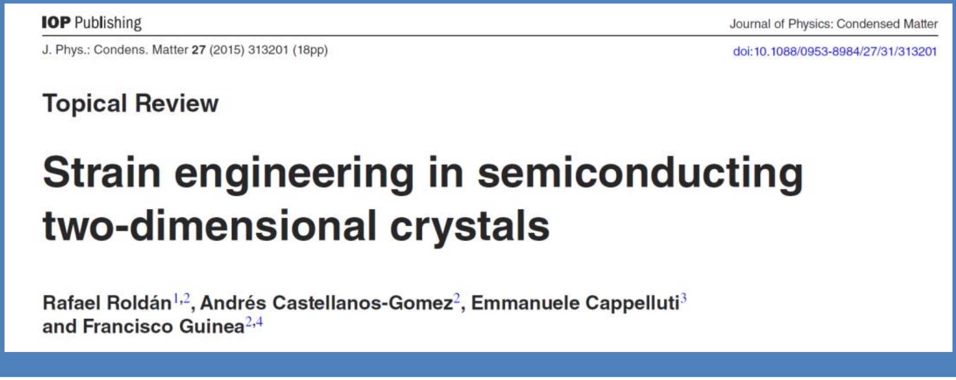
Physics Reports

journal homepage: www.elsevier.com/locate/physrep

ELSEVIER

Gauge fields in graphene

M.A.H. Vozmediano[a], M.I. Katsnelson[b], F. Guinea[a,*]



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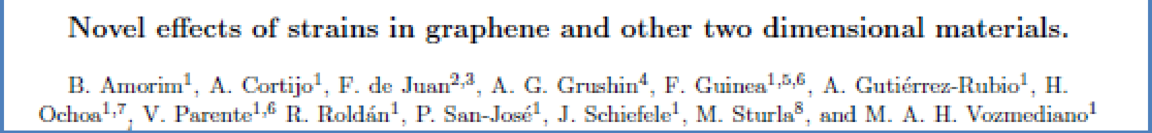


Novel effects of strains in graphene and other two dimensional materials.

B. Amorim[1], A. Cortijo[1], F. de Juan[2,3], A. G. Grushin[4], F. Guinea[1,5,6], A. Gutiérrez-Rubio[1], H. Ochoa[1,7], V. Parente[1,6] R. Roldán[1], P. San-José[1], J. Schiefele[1], M. Sturla[8], and M. A. H. Vozmediano[1]

arXiv:1503:00747, Phys. Rep., in press

Graphene on hBN: a 2D Frenkel-Kontorova model

LETTERS

PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NMAT2968

nature materials

Scanning tunnelling microscopy and spectroscopy of ultra-flat graphene on hexagonal boron nitride

Jiamin Xue[1], Javier Sanchez-Yamagishi[2], Danny Bulmash[2], Philippe Jacquod[1,3], Aparna Deshpande[1†], K. Watanabe[4], T. Taniguchi[4], Pablo Jarillo-Herrero[2] and Brian J. LeRoy[1]*

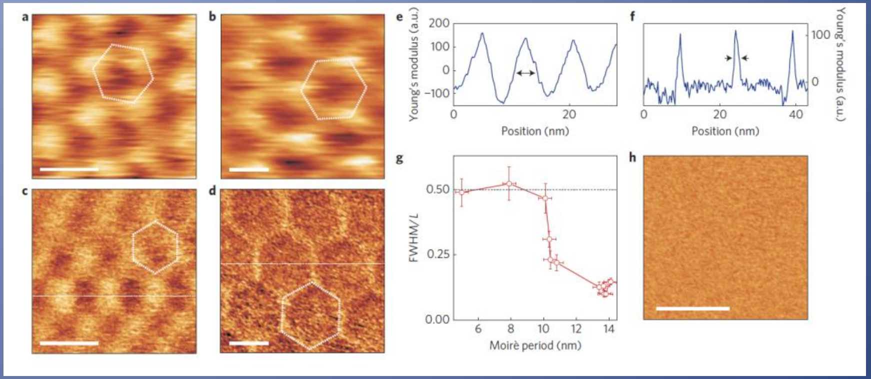
nature physics

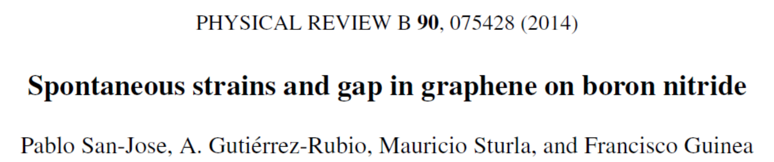
ARTICLES

PUBLISHED ONLINE: 28 APRIL 2014 | DOI: 10.1038/NPHYS2954

Commensurate–incommensurate transition in graphene on hexagonal boron nitride

C. R. Woods[1], L. Britnell[1], A. Eckmann[2], R. S. Ma[3], J. C. Lu[3], H. M. Guo[3], X. Lin[3], G. L. Yu[1], Y. Cao[4], R. V. Gorbachev[4], A. V. Kretinin[1], J. Park[1,5], L. A. Ponomarenko[1], M. I. Katsnelson[6], Yu. N. Gornostyrev[7], K. Watanabe[8], T. Taniguchi[8], C. Casiraghi[2], H-J. Gao[3], A. K. Geim[4] and K. S. Novoselov[1]*





PHYSICAL REVIEW B **90**, 075428 (2014)

Spontaneous strains and gap in graphene on boron nitride

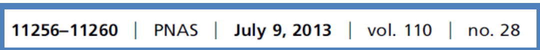
Pablo San-Jose, A. Gutiérrez-Rubio, Mauricio Sturla, and Francisco Guinea

Other topics

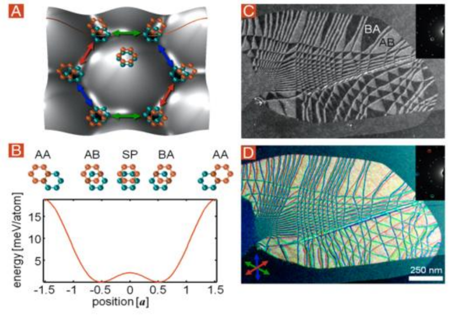
Domain boundaries in bilayer graphene

Strain solitons and topological defects in bilayer graphene

Jonathan S. Alden[a], Adam W. Tsen[a], Pinshane Y. Huang[a], Robert Hovden[a], Lola Brown[b], Jiwoong Park[b,c], David A. Muller[a,c], and Paul L. McEuen[c,d,1]



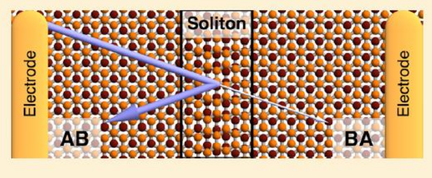
11256–11260 | PNAS | **July 9, 2013** | vol. 110 | no. 28

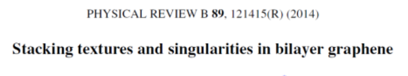


NANO LETTERS

Stacking Boundaries and Transport in Bilayer Graphene

P. San-Jose,*,† R. V. Gorbachev,§ A. K. Geim,§ K. S. Novoselov,‖ and F. Guinea†,‡

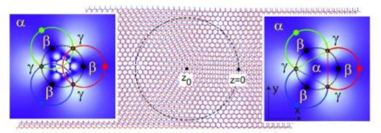




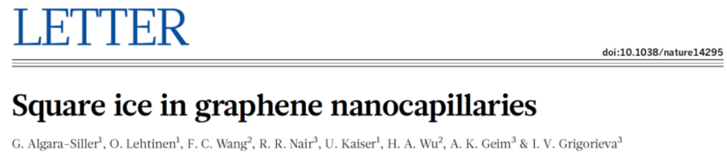
PHYSICAL REVIEW B **89**, 121415(R) (2014)

Stacking textures and singularities in bilayer graphene

Xingting Gong and E. J. Mele*



Square ice
Filtration by monoatomic membranes

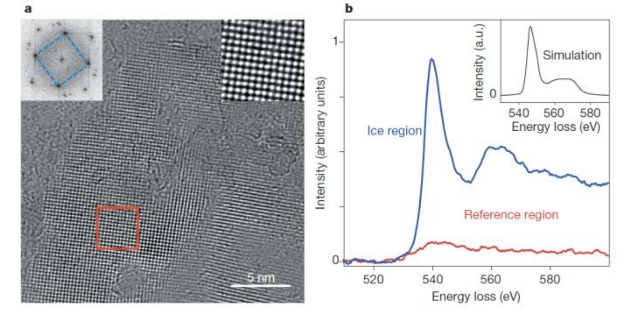


LETTER

doi:10.1038/nature14295

Square ice in graphene nanocapillaries

G. Algara-Siller[1], O. Lehtinen[1], F. C. Wang[2], R. R. Nair[3], U. Kaiser[1], H. A. Wu[2], A. K. Geim[3] & I. V. Grigorieva[3]



MEMBRANES

Sieving hydrogen isotopes through two-dimensional crystals

M. Lozada-Hidalgo,[1]*† S. Hu,[1]† O. Marshall,[1] A. Mishchenko,[1] A. N. Grigorenko,[1] R. A. W. Dryfe,[2] B. Radha,[1] I. V. Grigorieva,[1] A. K. Geim[1]*

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Scrolls

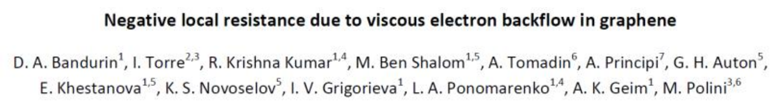
PRL **108,** 166602 (2012) PHYSICAL REVIEW LETTERS week ending 20 APRIL 2012

Quenching of the Quantum Hall Effect in Graphene with Scrolled Edges

Alessandro Cresti,[1] Michael M. Fogler,[2] Francisco Guinea,[3] A. H. Castro Neto,[4] and Stephan Roche[5,6]

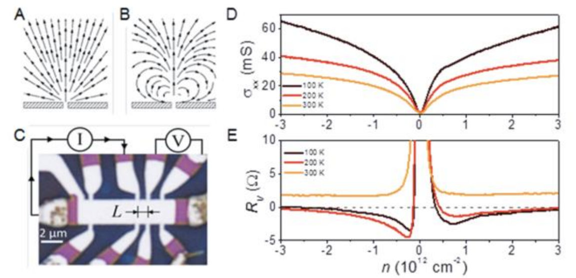
Other topics

Electronic 2D hydrodynamics



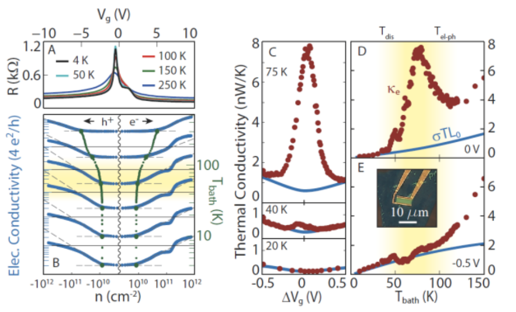
Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



Observation of the Dirac fluid
and the breakdown of the Wiedemann-Franz law in graphene

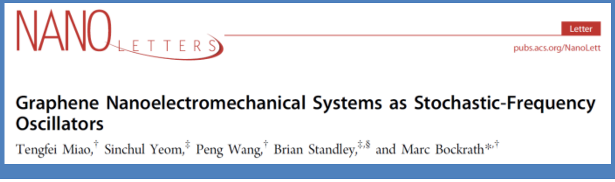
Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]



http://www.condmatjournalclub.org/?p=2687
F. G., October 2015

Fracture strength:
Graphene can withstand stresses of up to 15%

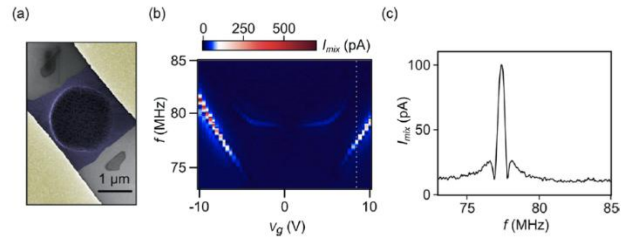
Graphene NEMs

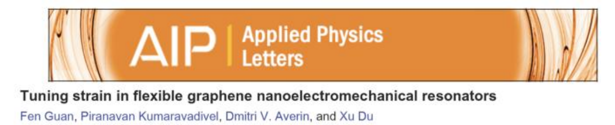


NANO LETTERS
Letter
pubs.acs.org/NanoLett

Graphene Nanoelectromechanical Systems as Stochastic-Frequency Oscillators

Tengfei Miao,[†] Sinchul Yeom,[‡] Peng Wang,[†] Brian Standley,[‡,§] and Marc Bockrath[*,†]

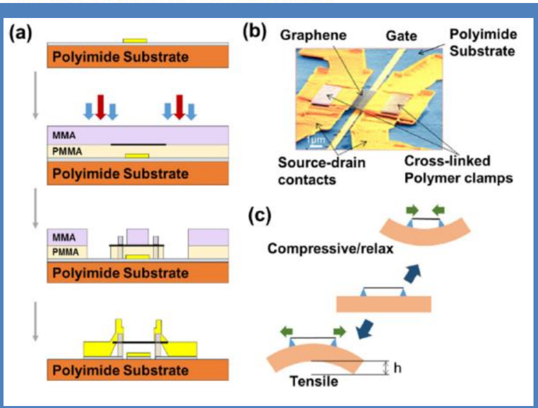




AIP Applied Physics Letters

Tuning strain in flexible graphene nanoelectromechanical resonators

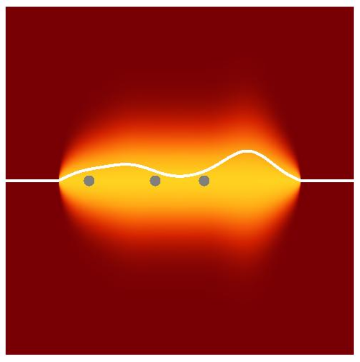
Fen Guan, Piranavan Kumaravadivel, Dmitri V. Averin, and Xu Du



Also: graphene NEMs in the quantum limit, A. Bachold, private communication

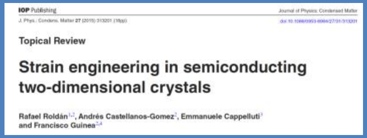
Anharmonic properties of graphene

- Anharmonic effects in membranes
- Negative thermal expansion coefficient
- Screening of the in plane stiffness
- The elastic response of graphene depends on the experimental setup (size, temperature, defects, pre existing strain, …)



Other topics

- Quenched ripples
- Structure and electronic properties: strain engineering
- Random strains and conductivity
- Other 2D materials: dichalcogenides, black phosphorus, …
- Moiré structures: 2D Frenkel-Kontorova model
- Domain walls in bilayer and multilayered graphene
- 2D electron hydrodynamics, NEMs, strains and spins (in dichalcogenides), …



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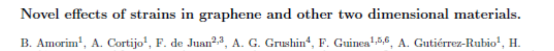

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arXiv:1503:00747, Phys. Rep., in press