Looping through spatially embedded networks

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SCIENTIA · PRO · BONO · HUMANI · GENERIS
1901

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Douglas Daly	(New York Botanical Garden)
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Henrik Ronellenfitsch	(MPI DS)

Outline

Part 1
Characterizing planar degree constrained graphs
(Actual data)

Part 2
Characterizing non-planar degree constrained graphs
(Interesting mathematics)

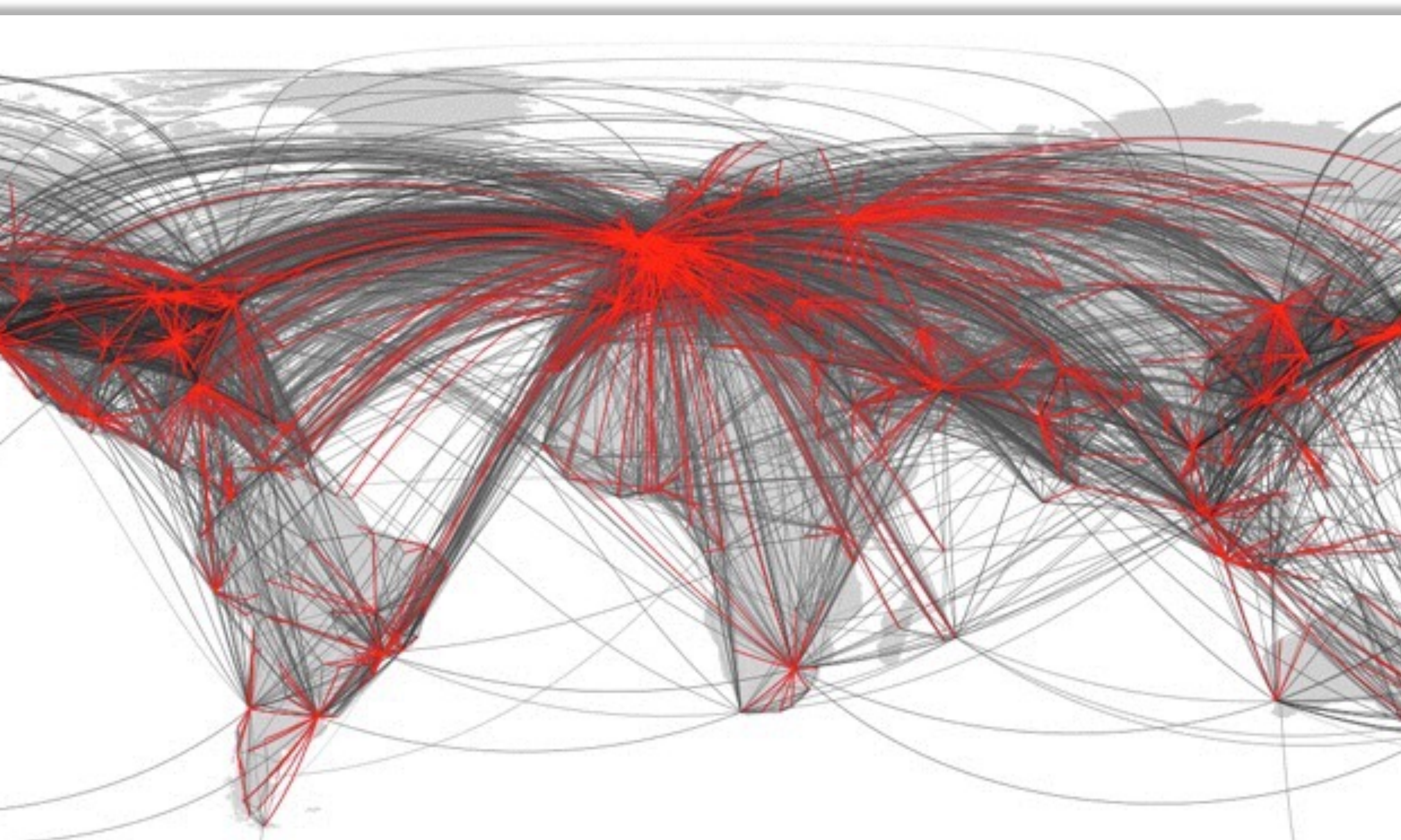
Complex networks

How can one quantify a complex graph?

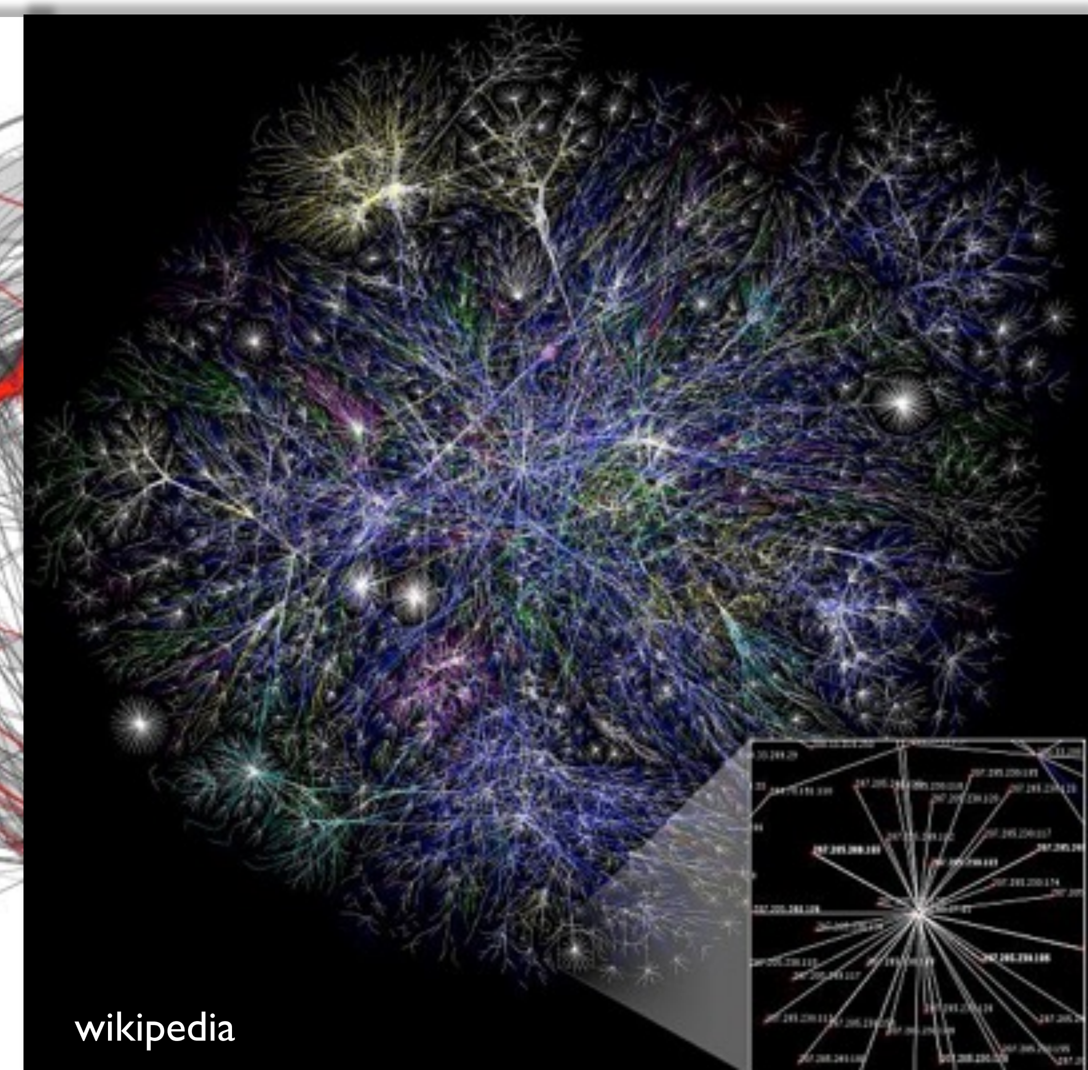
3d Networks with **unrestricted** degree

Internet, airports, neuronal networks in the brain, social networks…

scale free,
clustering coefficients,
degree distributions,
hubs…



http://phys.org/news/2012-06-method-diverse-complex-networks-similar.html



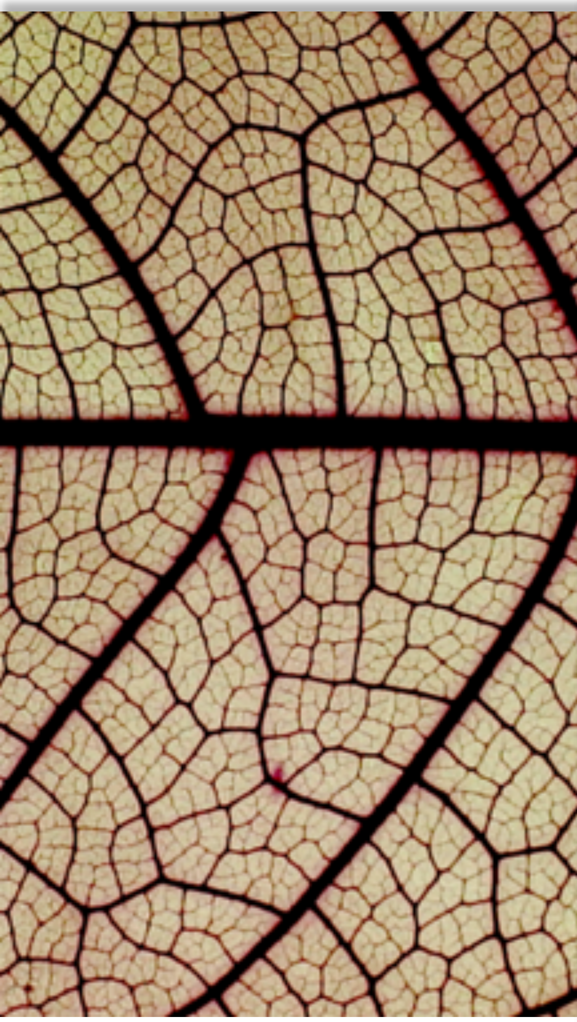
wikipedia

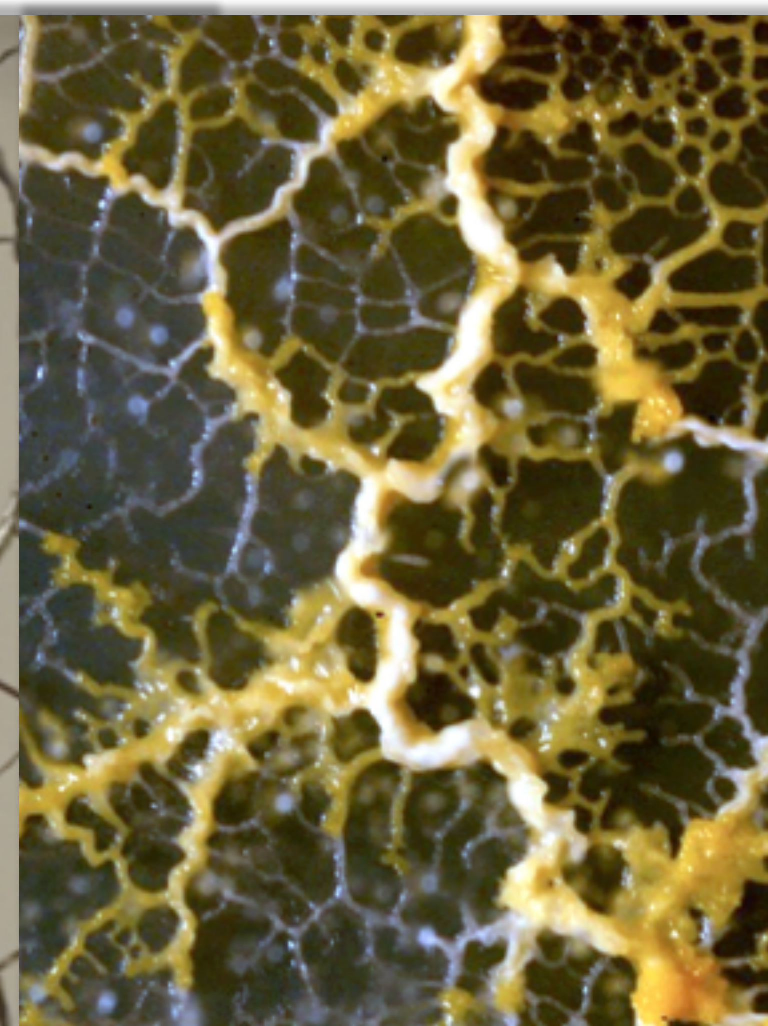
Degree constrained graphs

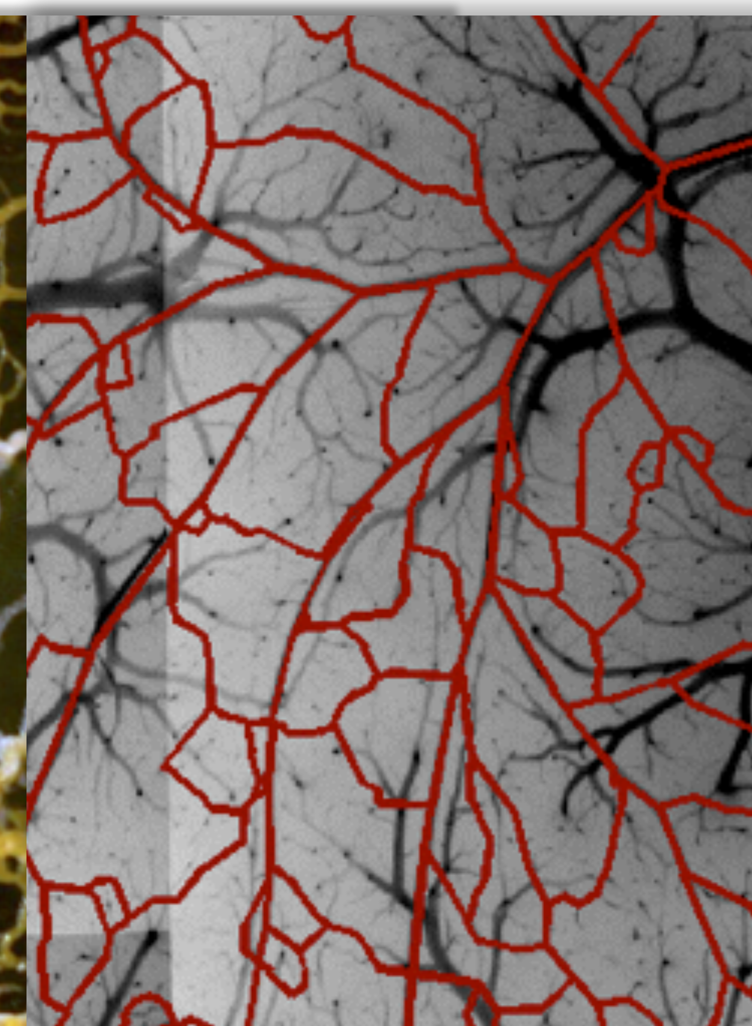
What determines their function?

2d Networks with **restricted** degree

Leaf vascular networks, crack patterns, road networks...



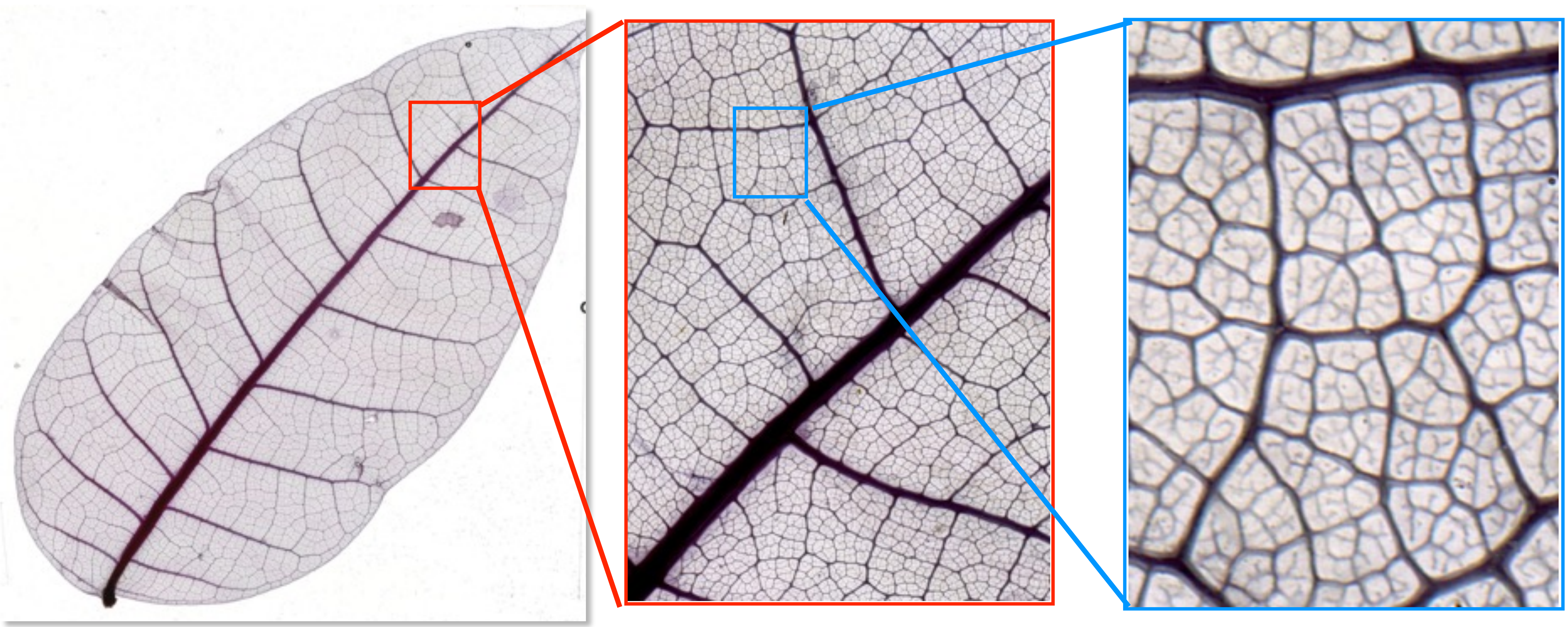




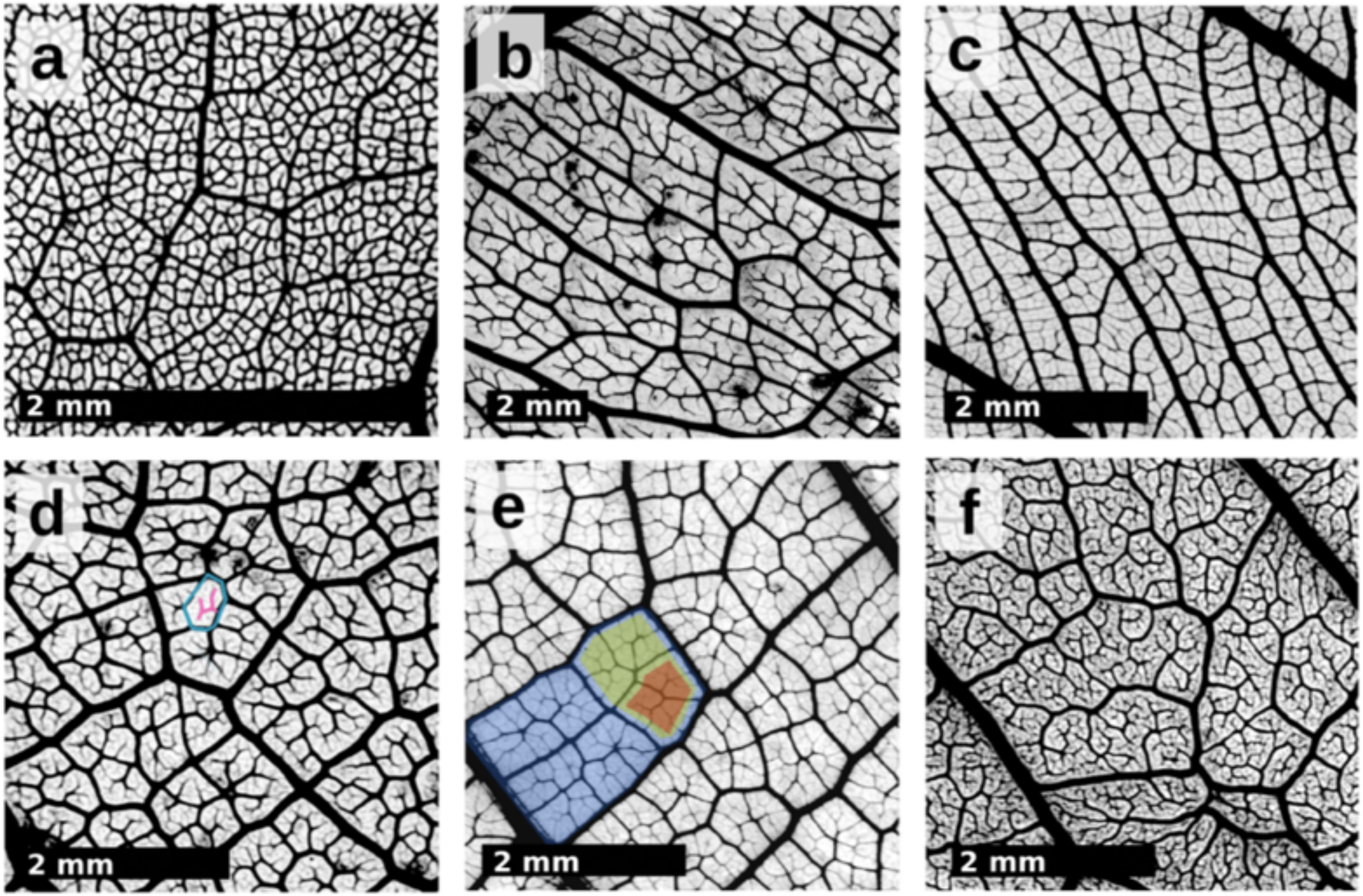
Leaf vascular architecture: Lots of Loops

Leaves are not trees...

Loops within loops within loops!



Evolution of many distinct venation types



a *Protium ovatum*. b *Protium madagascariense*. c *Pouteria filipes*. d *Canarium betamponae*. A single areole is marked in blue, non-anastomosing highest order veins in red. e *Brosimum guianensis*. The hierarchical nesting of loops is highlighted. f *Protium subserratum*.

Ronellenfitsch H. Lasser J., Daly D., and EK, "Topological phenotypes constitute a new dimension in the phenotypic space of leaf venation networks", *in press*

Leaf venation phenotypic traits correlate with climate

nature COMMUNICATIONS OPEN

ARTICLE

Received 23 Mar 2012 | Accepted 10 Apr 2012 | Published 15 May 2012 DOI: 10.1038/ncomms1835

Developmentally based scaling of leaf venation architecture explains global ecological patterns

Lawren Sack[1], Christine Scoffoni[1], Athena D. McKown[1], Kristen Frole[2], Michael Rawls[1], J. Christopher Havran[3], Huy Tran[1] & Thusuong Tran[1]

ECOLOGY LETTERS

Ecology Letters, (2011) **14**: 91–100 doi: 10.1111/j.1461-0248.2010.01554.x

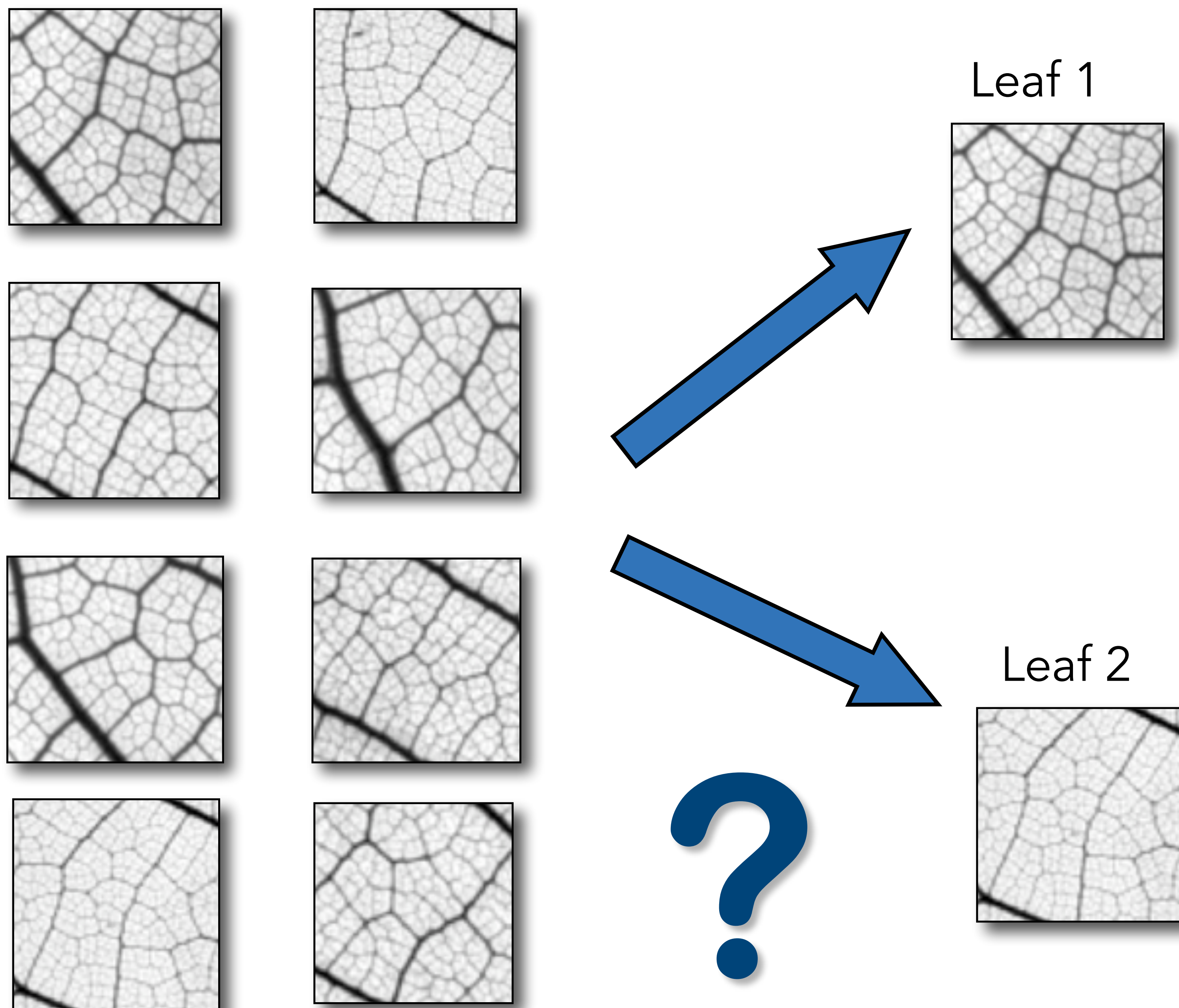
Venation networks and the origin of the leaf economics spectrum

Benjamin Blonder,[1]* Cyrille Violle,[1,2] Lisa Patrick Bentley[1] and Brian J. Enquist[1,3]

Human venation architecture: implications for disease?

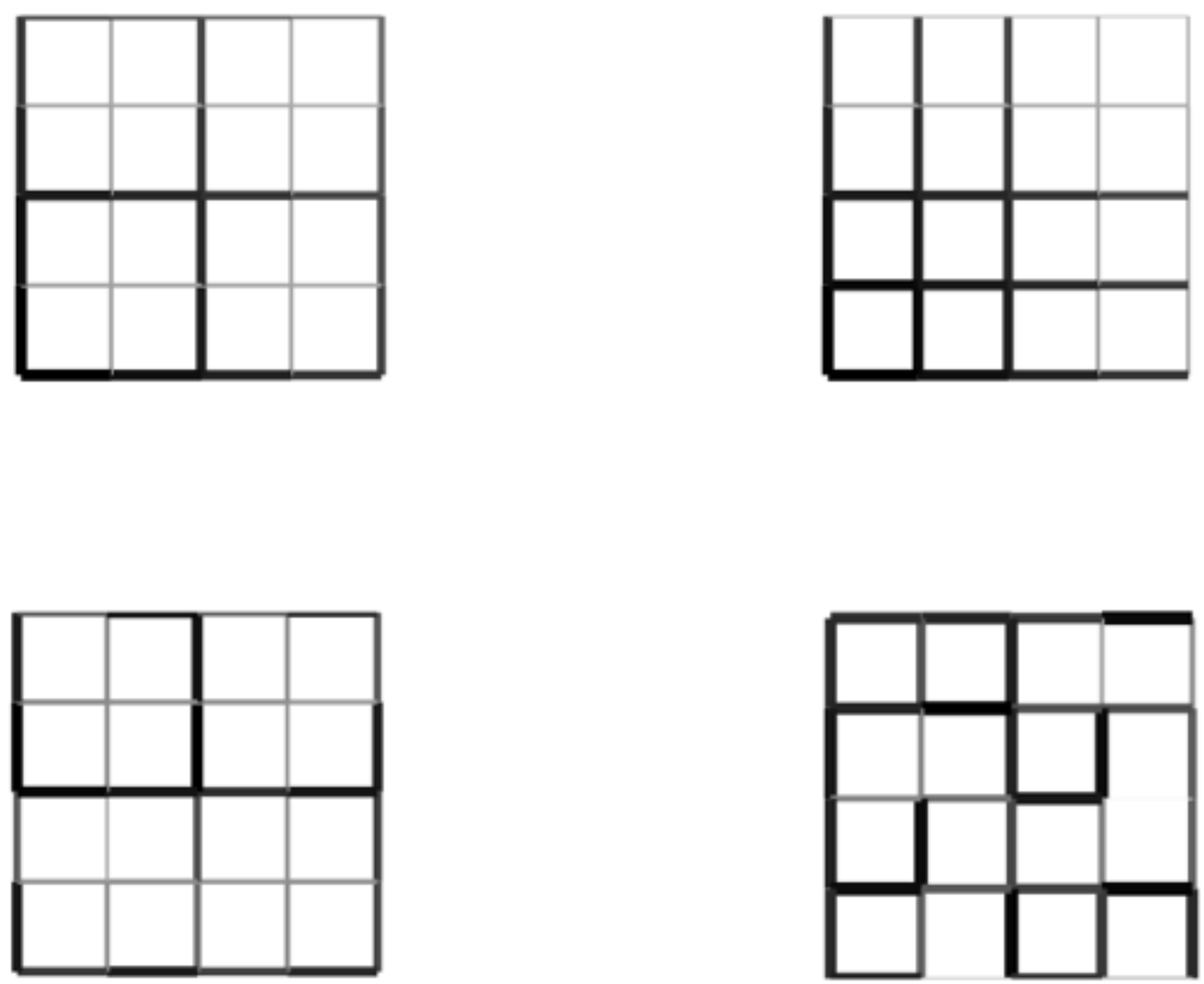
Venation as a quantifiable phenotype

Classify fragments based on feature similarity

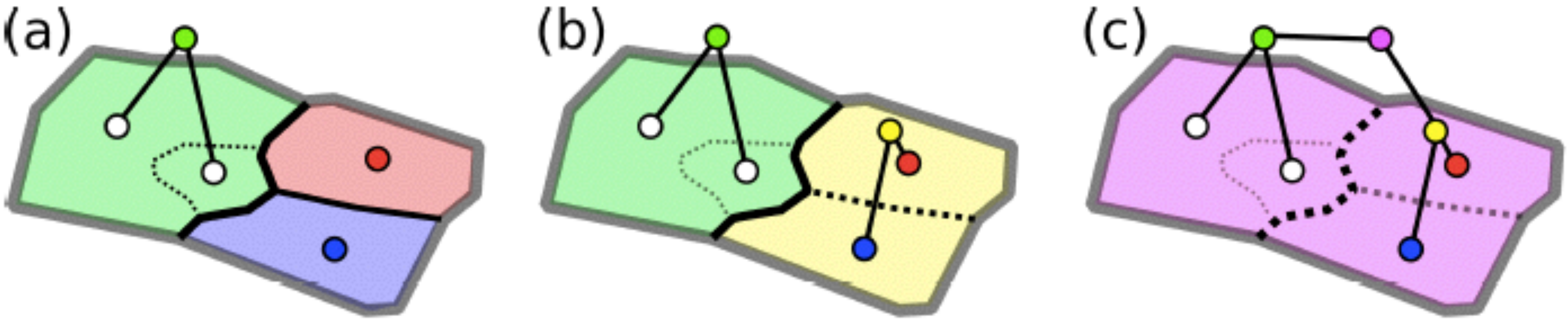


Venation as a quantifiable phenotype

Vein density, vein width distribution, vein density, junction geometry ...
What about connectivity of weighted edges?



Hierarchical decomposition

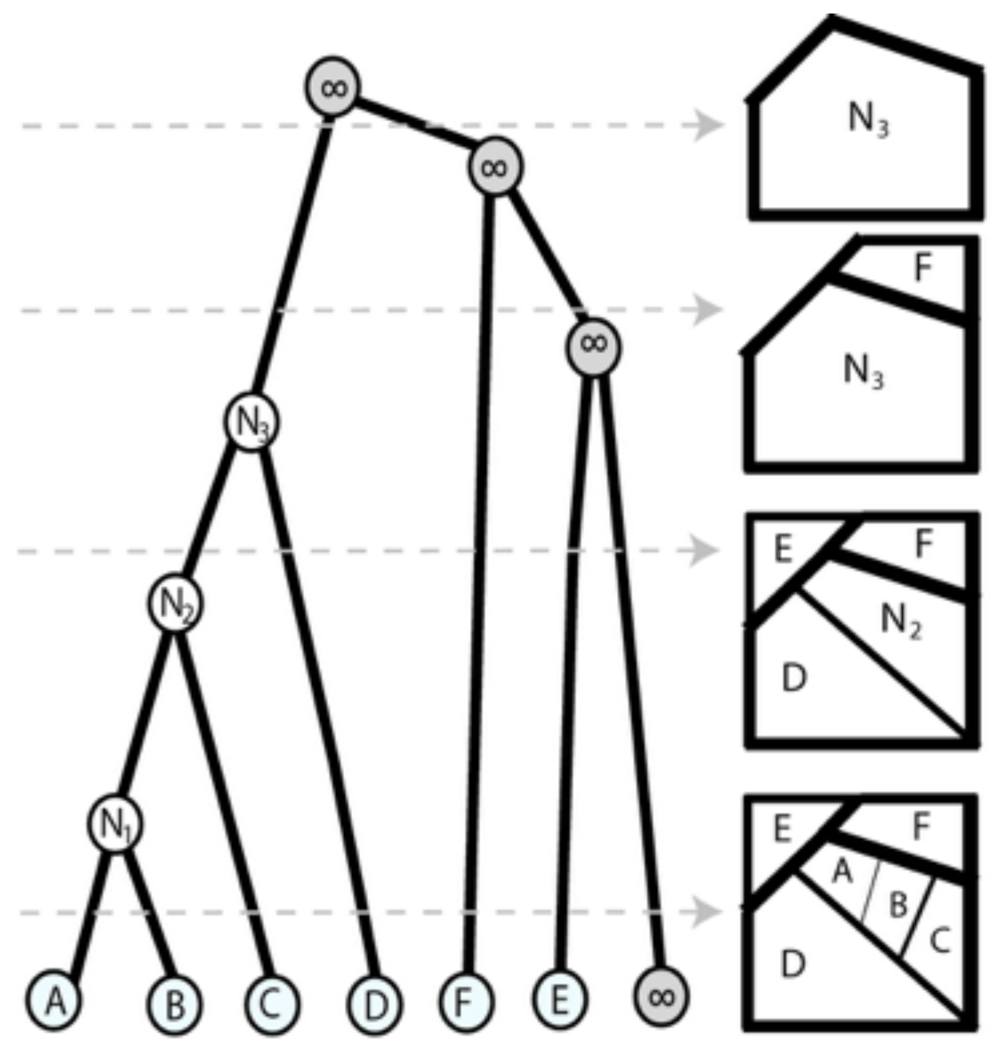


Deciphering the topology

EK. and Magnasco M.O., “Quantifying loopy network architectures”, *PLoS ONE*, 7, e37994 (2012)

Deciphering the topology

Hierarchical loopy network decomposition



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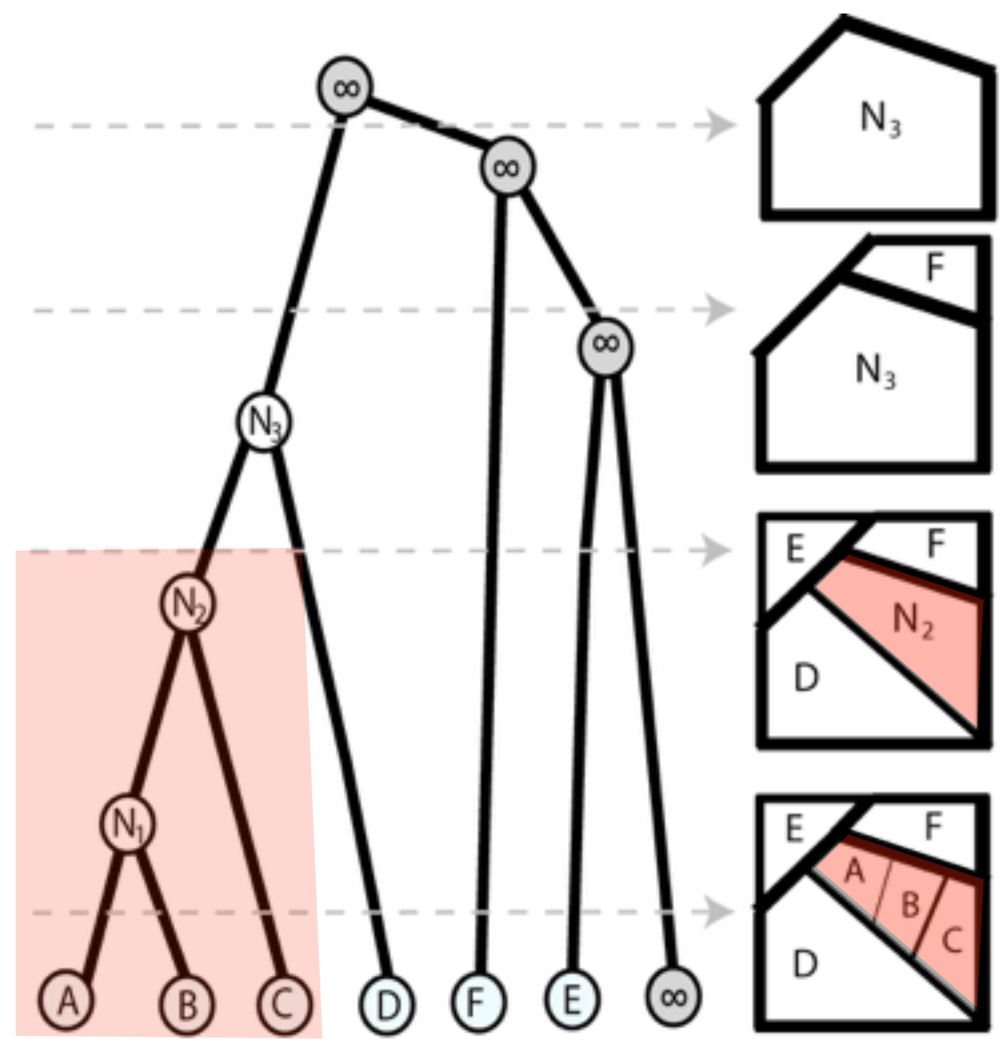

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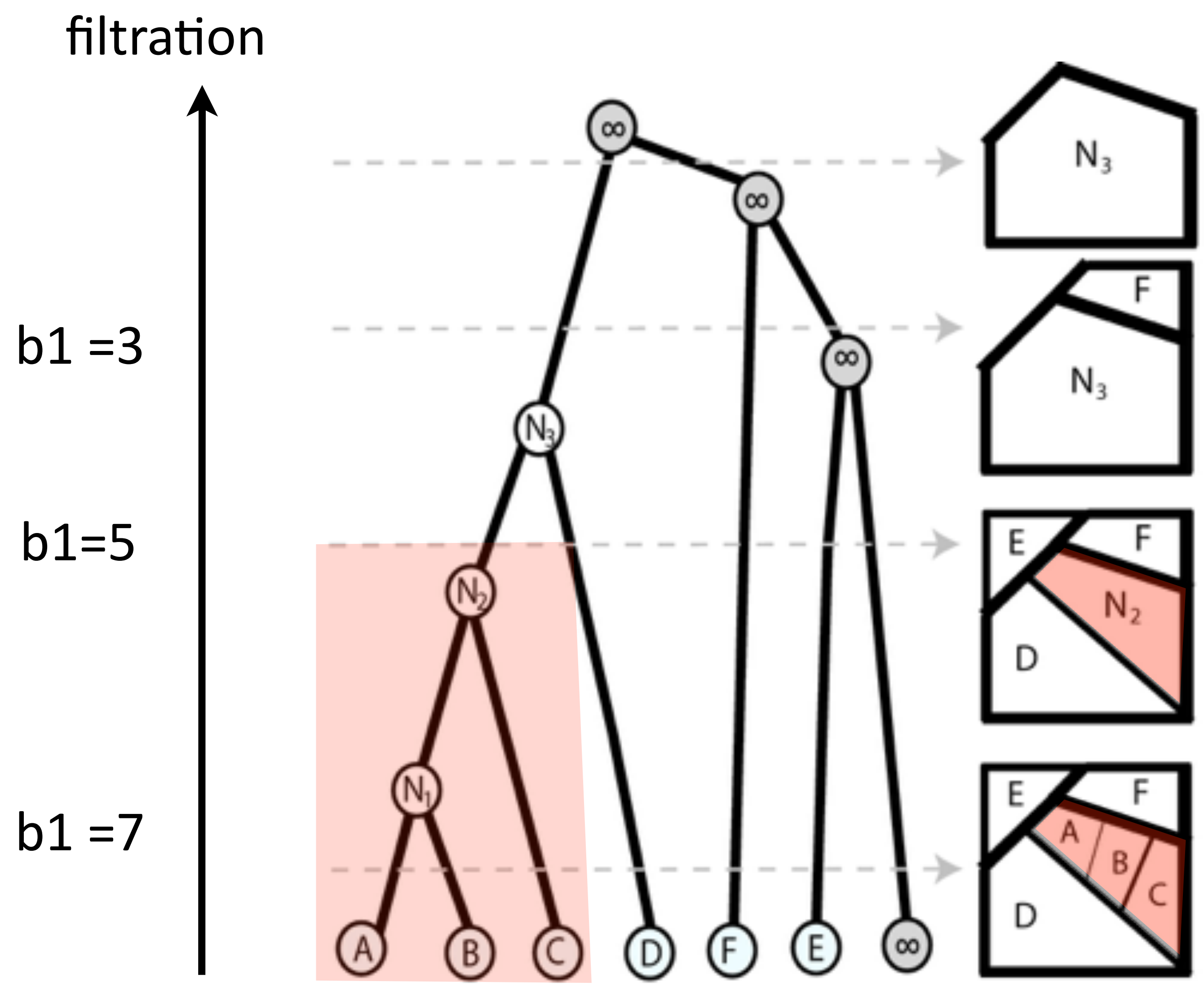
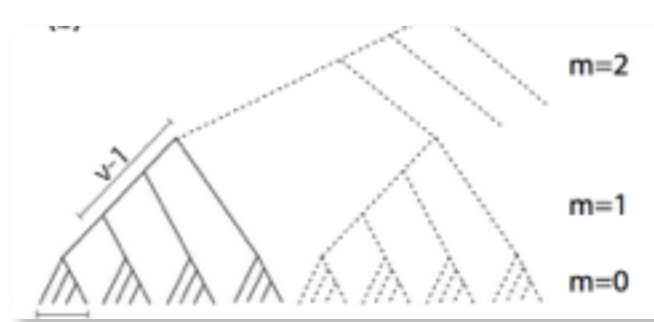
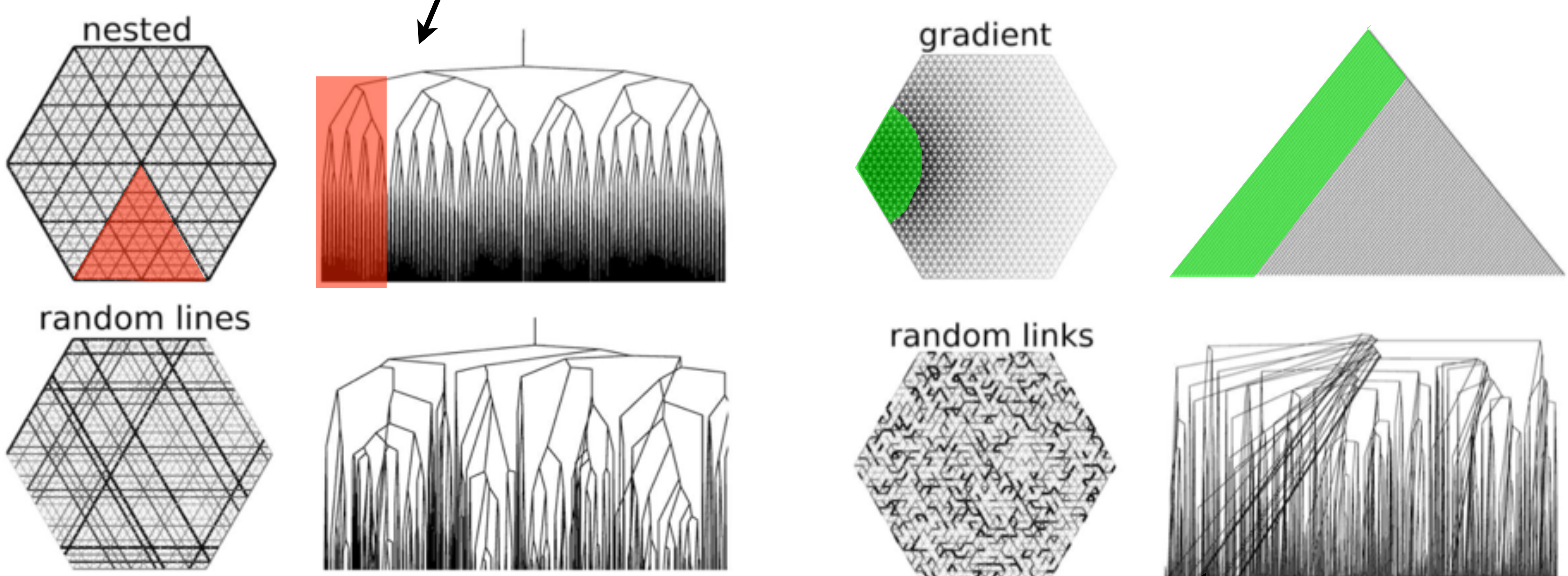


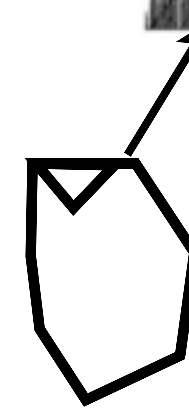
EK. and Magnasco M.O., "Quantifying loopy network architectures", *PLoS ONE*, 7, e37994 (2012)

Deciphering the topology

Assess fractal nature of topological organization







EK. and Magnasco M.O., "Quantifying loopy network architectures", *PLoS ONE*, 7, e37994 (2012)

Deciphering the topology

Asymmetry

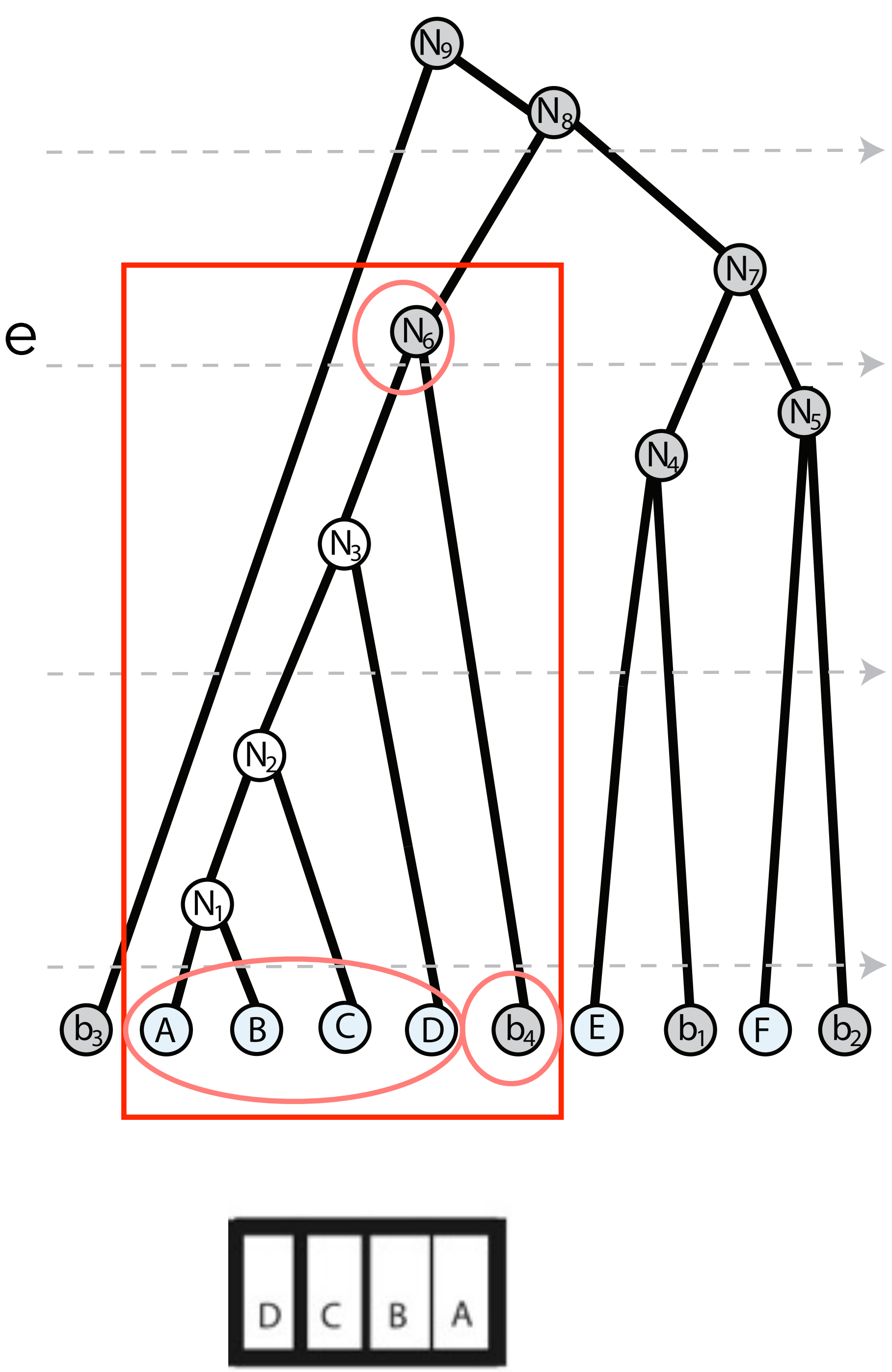
Assigns a number to each node of the tree based on the similarity of the two joining subtrees

Van Pelt at al (1992)

$$q(r_j, s_j) = \frac{s_j - r_j}{s_j}$$

$$Q_T(t_n) = \frac{1}{w(t_n)} \sum_{j=1}^{d(n)-1} w_j q(r_j, s_j)$$

$$w(t_n) = \sum_{j=1}^{d(n)-1} w_j.$$



Deciphering the topology

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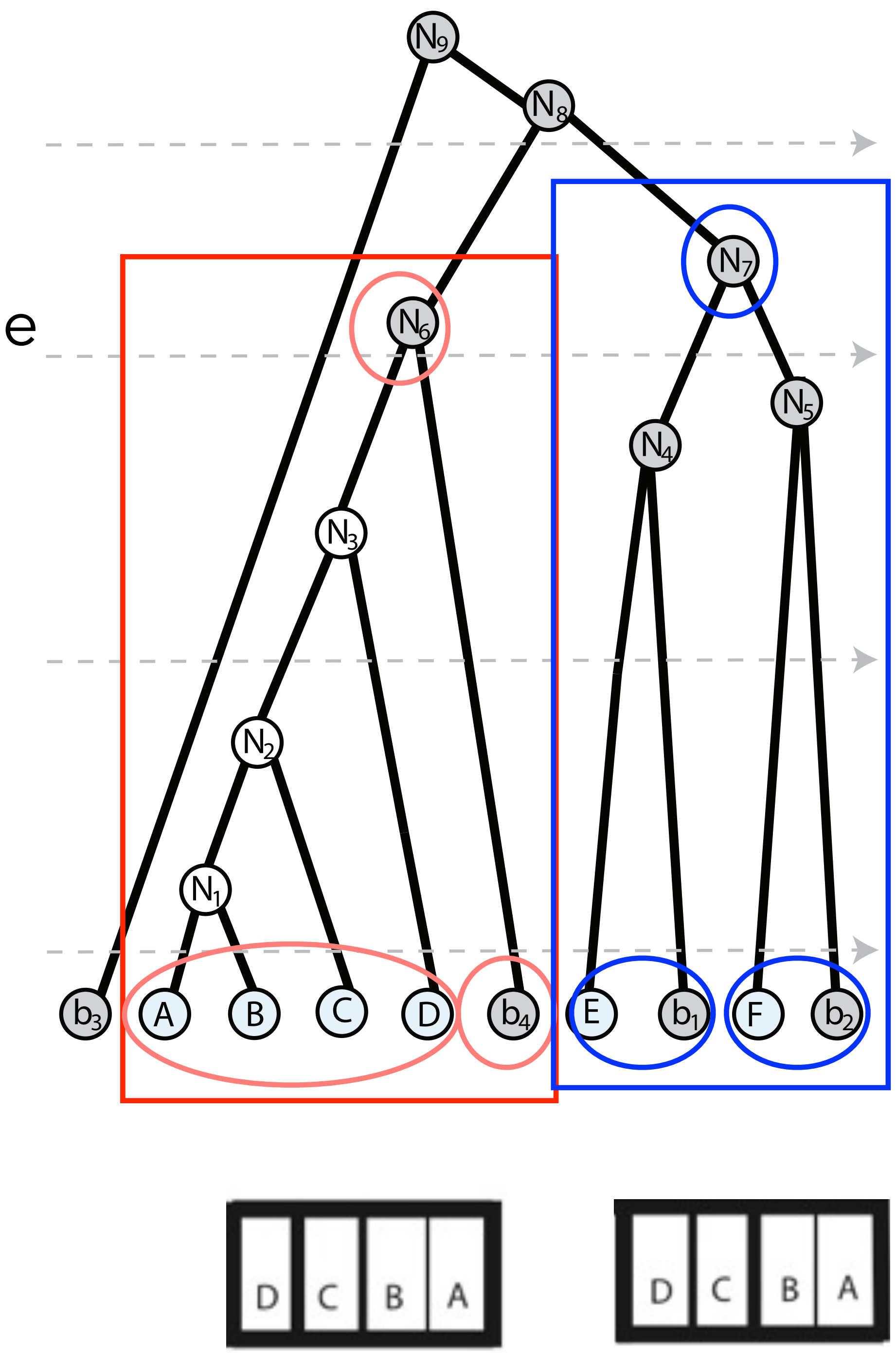
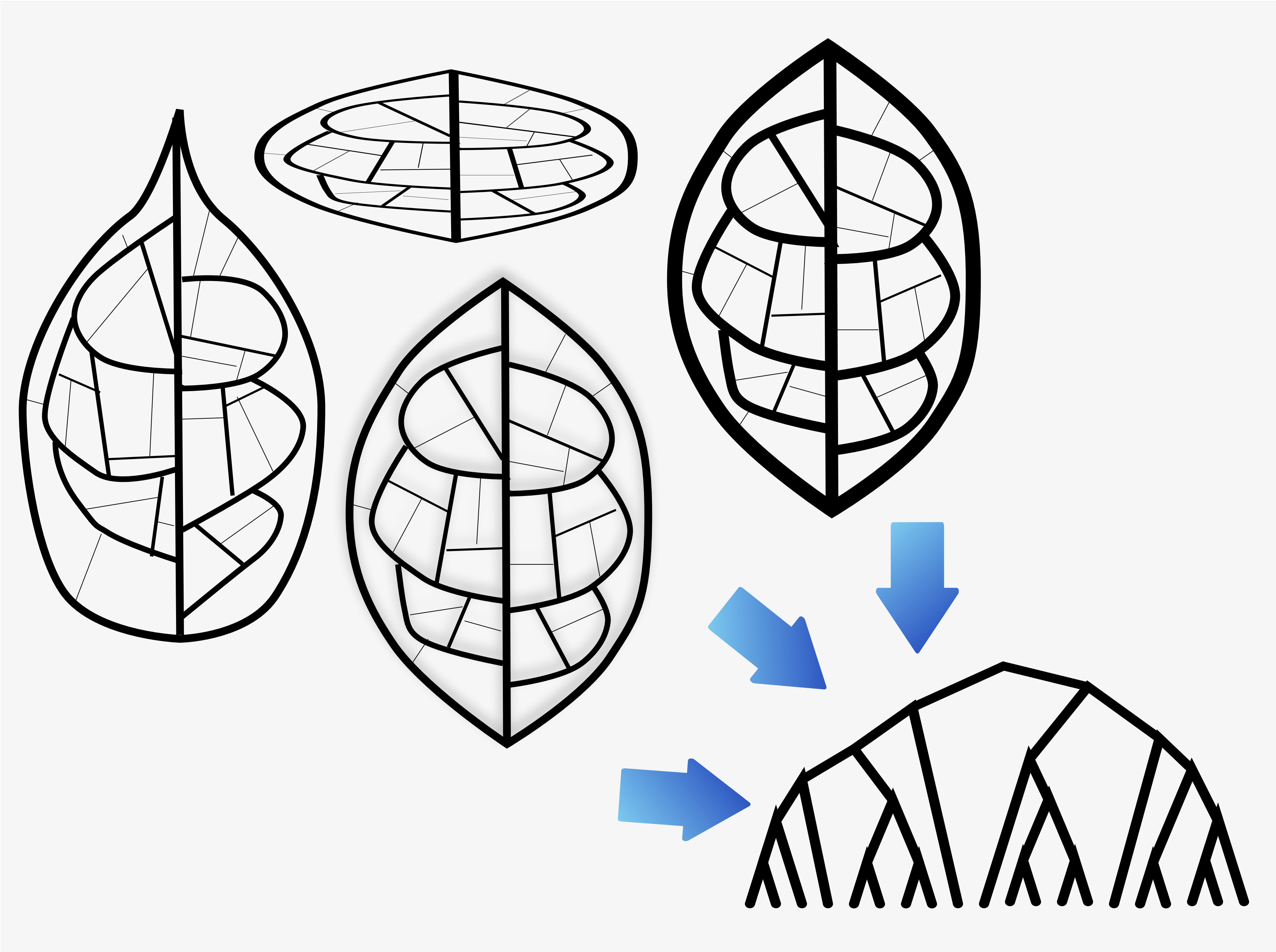
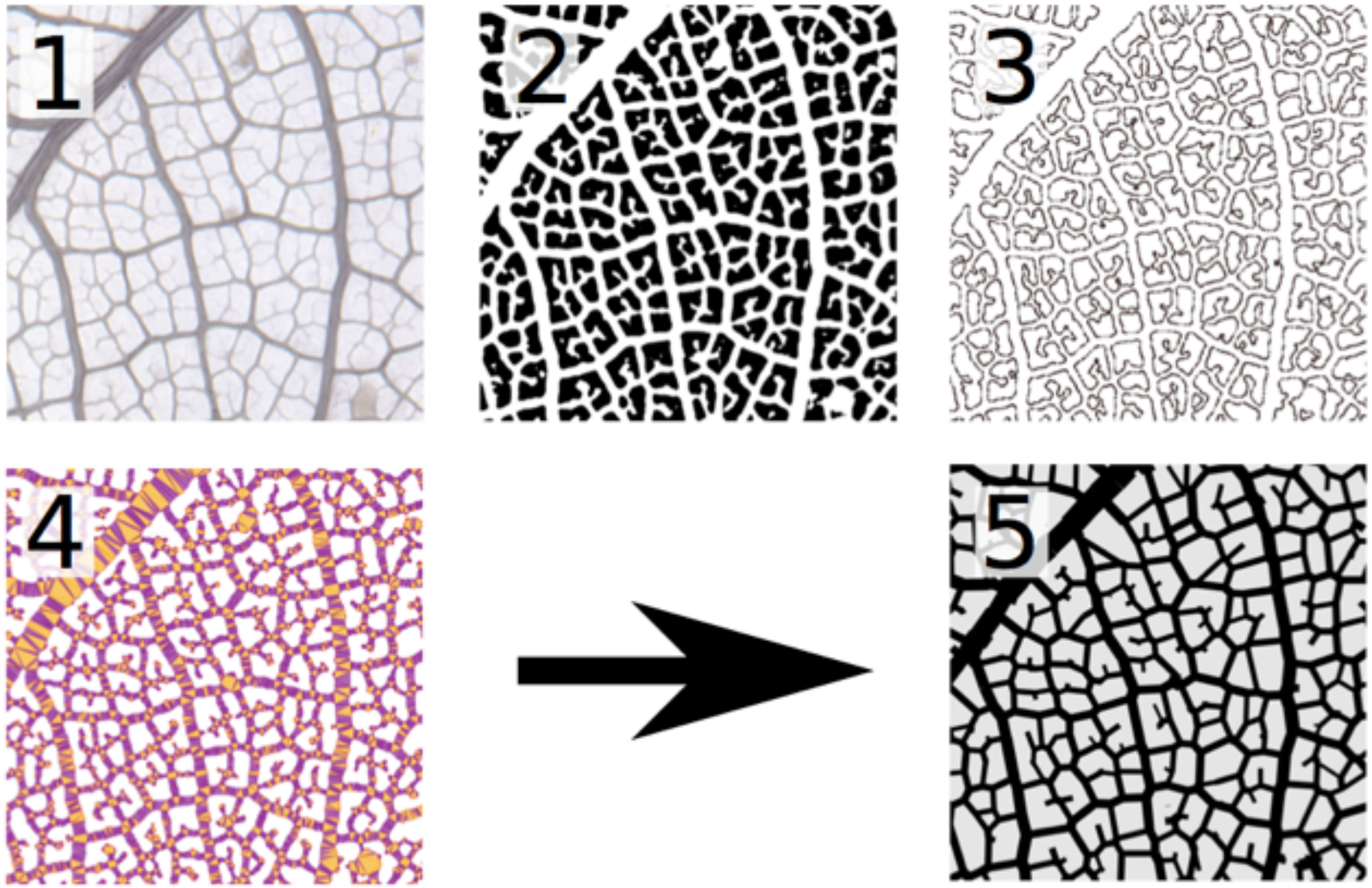


Deciphering the topology

Information about geometry and weight is decoupled
only topology and sort order of edges matters



Leaf fingerprinting

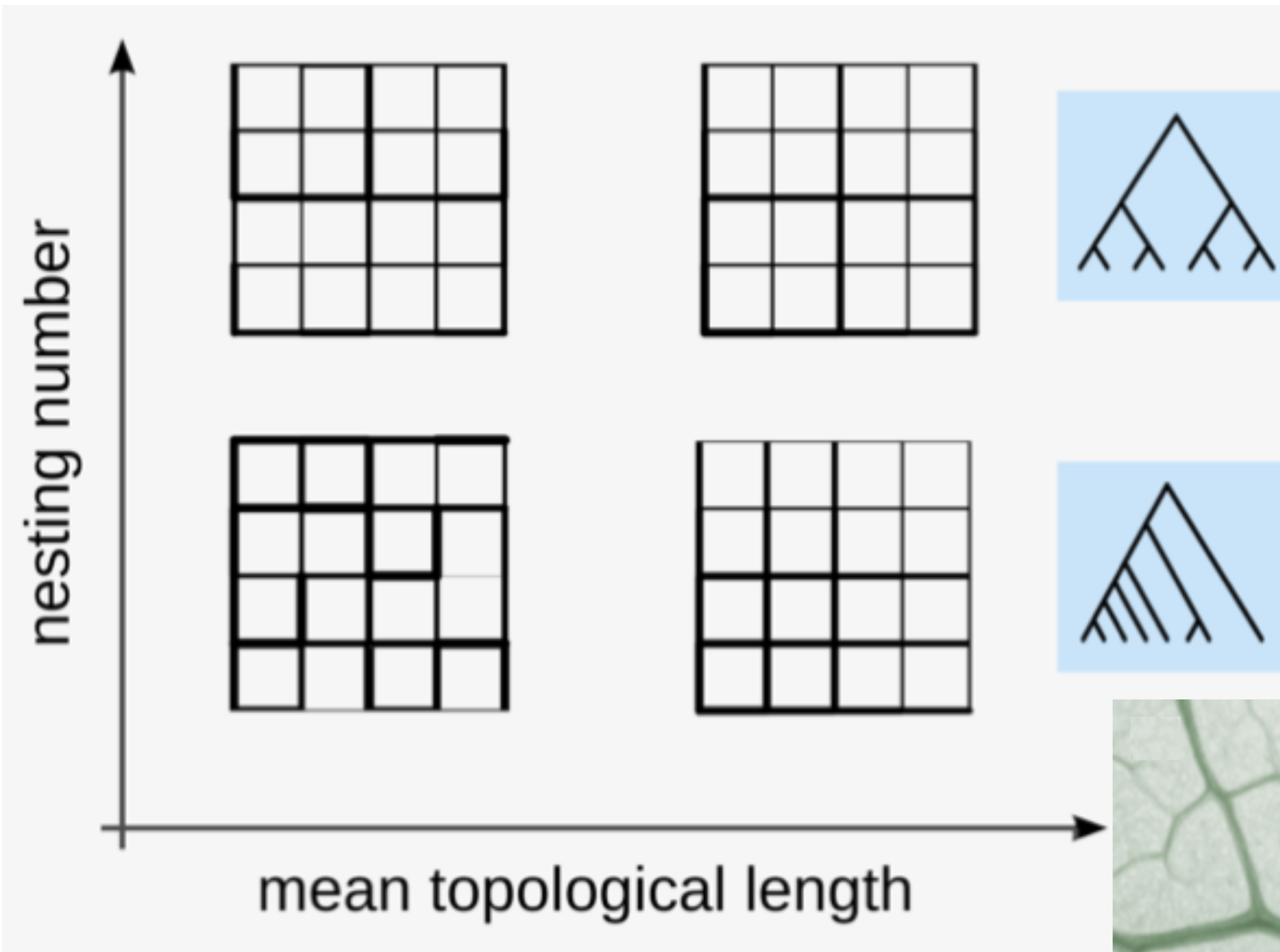


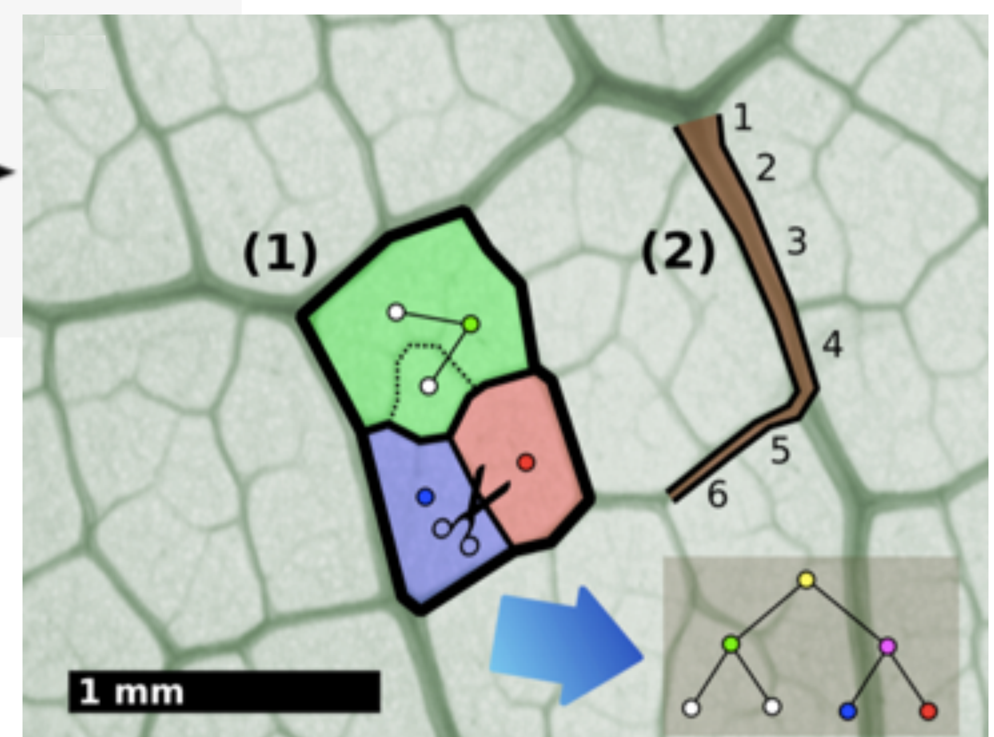
The vectorization process.
(1) start from a high resolution scanned image (6400 dpi) (2) binarize using a combination of blurring, local histogram equalization and finally Otsu thresholding. (3) Teh- Chin dominant point detection to obtain a set of contour points. (4) Constrained Delaunay triangulation of the contour points. (5) The final graph representation of the vascular network.

Lasser J., EK, "NEAT: A new framework for the vectorization and examination of network data", i*n preparation*, *https://github.com/JanaLasser/network_extraction*.

Leaf fingerprinting

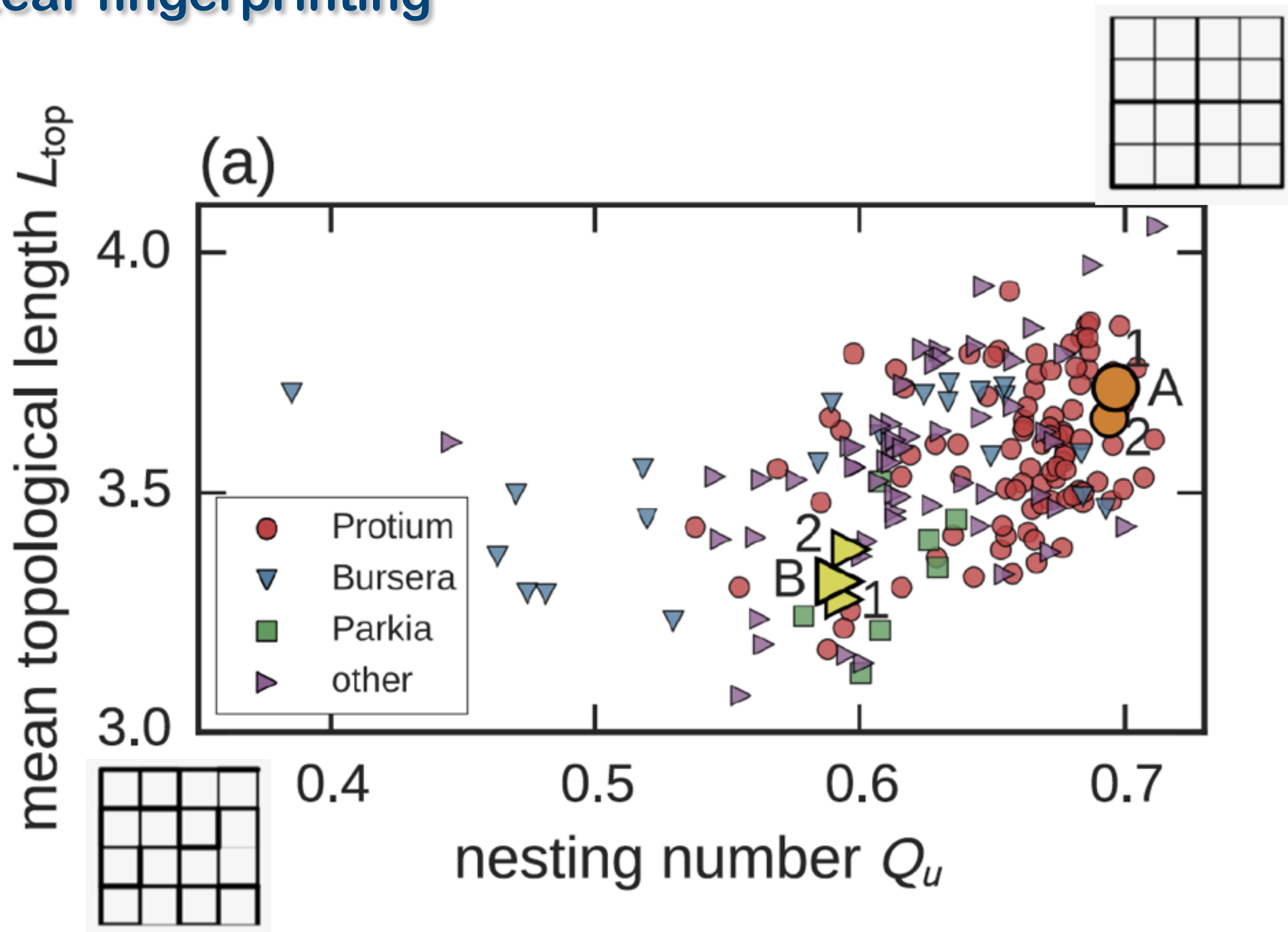
Topological phenotypes





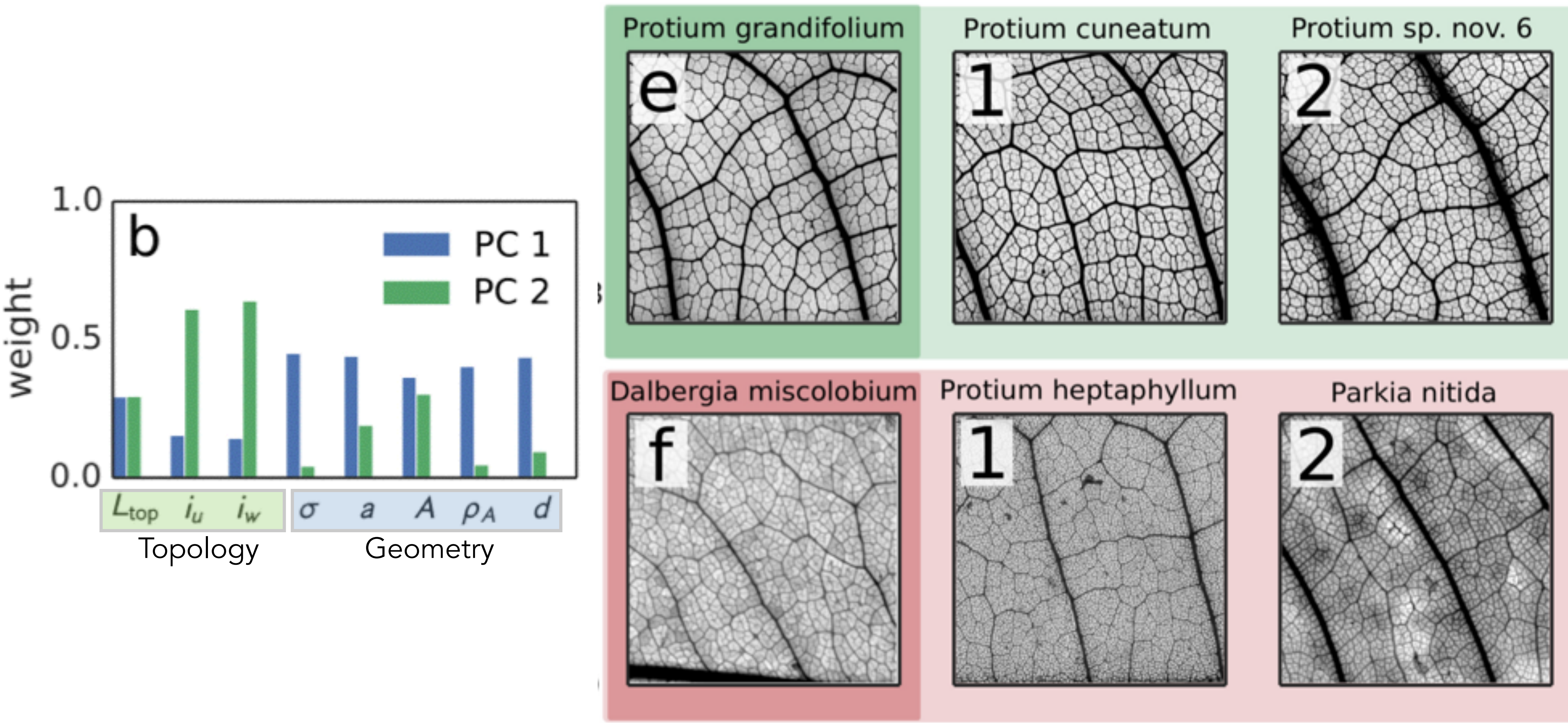
Ronellenfitsch H. Lasser J., Daly D., and EK, "Topological phenotypes constitute a new dimension in the phenotypic space of leaf venation networks", *PloS Comp Biol*

Leaf fingerprinting



Leaf fingerprinting

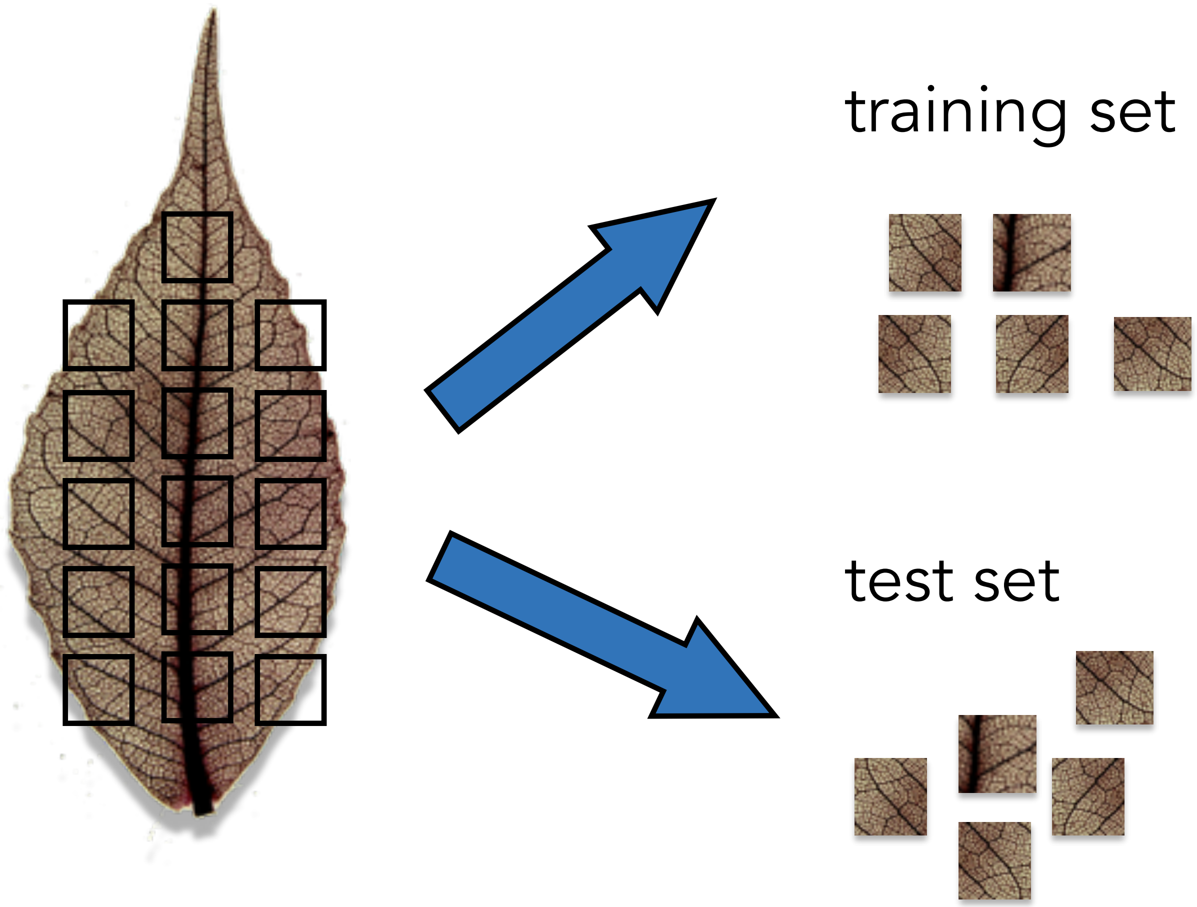
Geometry carries information orthogonal to topology
Geometry and topology are distinct phenotypes



σ vein density
a mean distance between veins
A mean areole area
ρ areole density
d average vein diameter weighted by length of venation between junctions

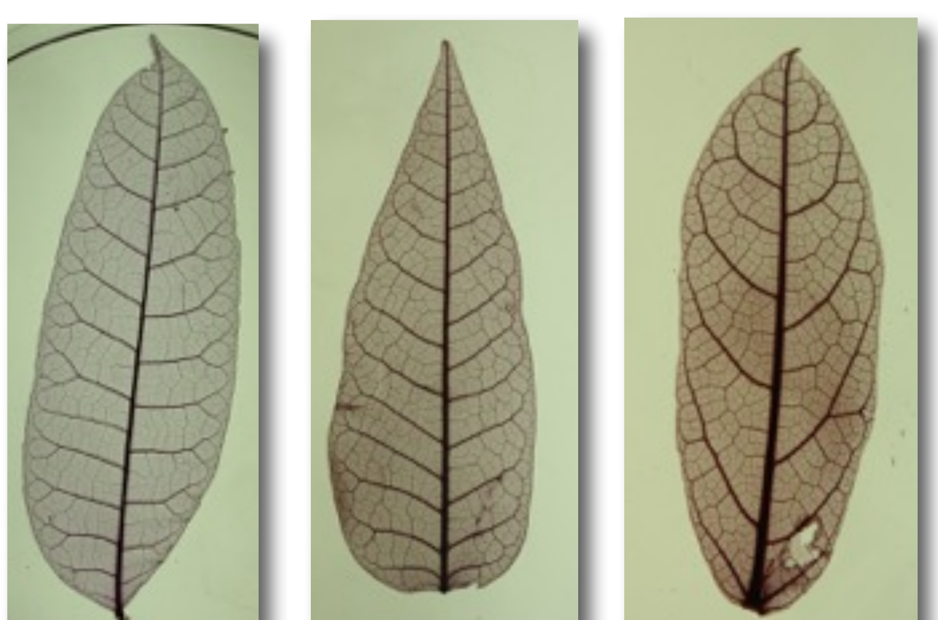
Ronellenfitsch H. Lasser J., Daly D., and EK, “Topological phenotypes constitute a new dimension in the phenotypic space of leaf venation networks”, *in press*

Leaf fingerprinting



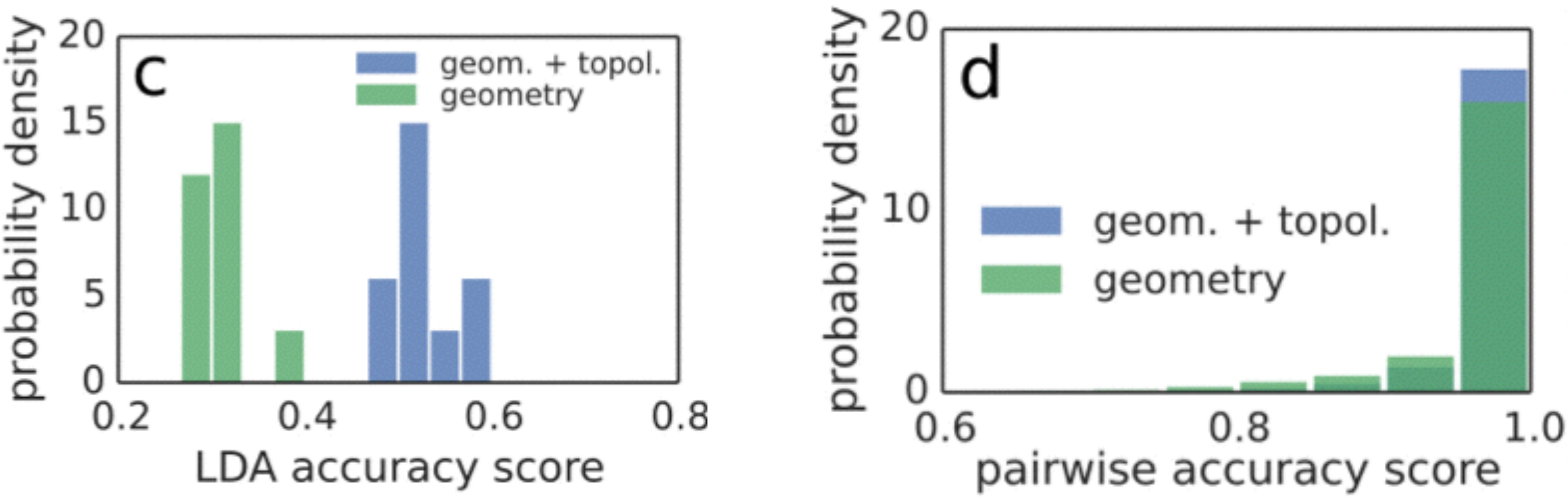
Linear Discriminant Analysis
to identify fragments

supervised learning (attempts to find a set of hyperplanes optimally separating sets of points in a high dimensional space whose class membership is known)



Two tests:
(1) identify fragment based on leaf membership (all 186)
(2) pairwise comparison test

Leaf fingerprinting



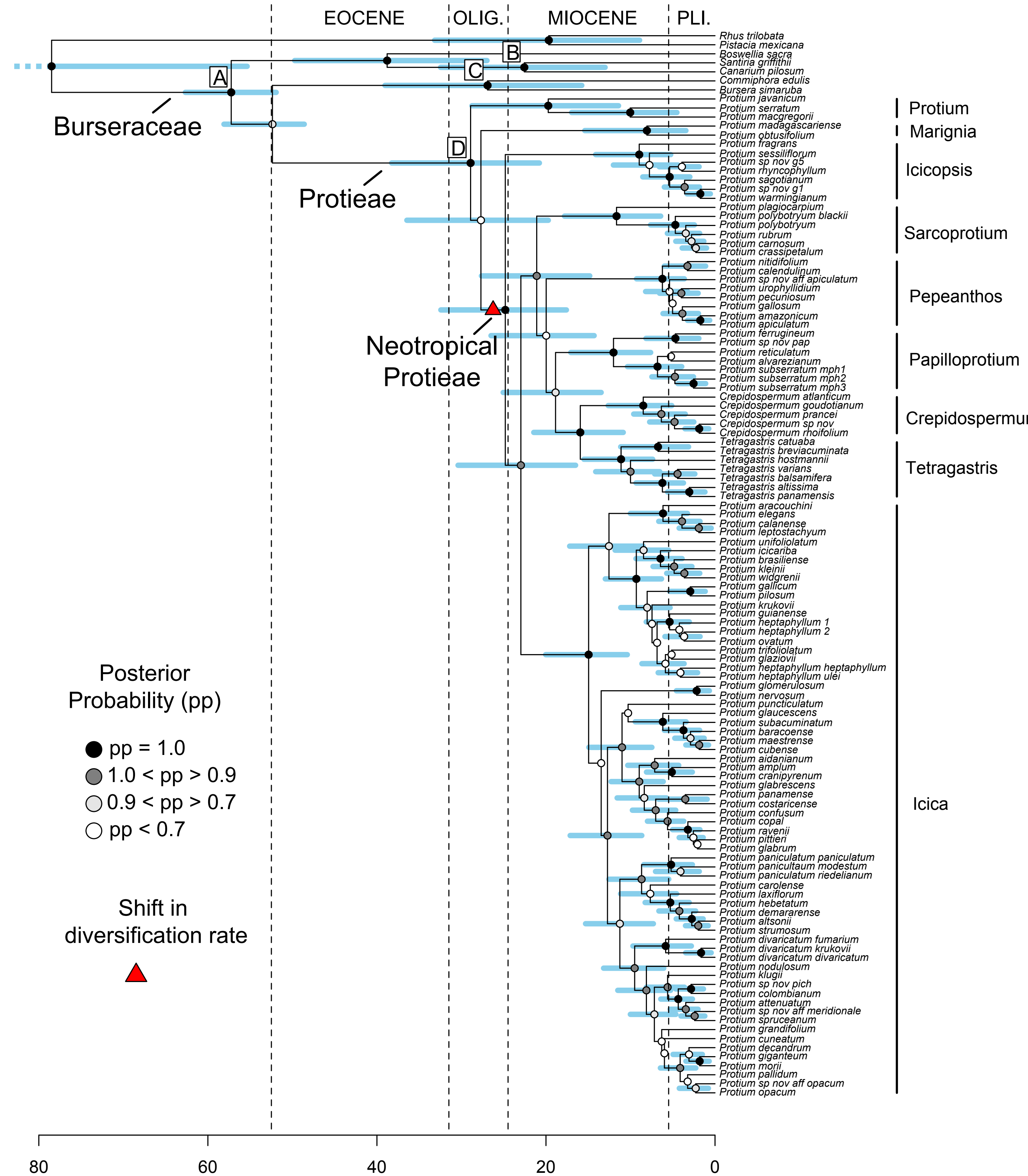
Topological information doubles correct leaf identification probability

Ronellenfitsch H. Lasser J., Daly D., and EK, "Topological phenotypes of leaf vascular networks", *submitted*

Leaf fingerprinting

Comparison to phylogenetic trees?

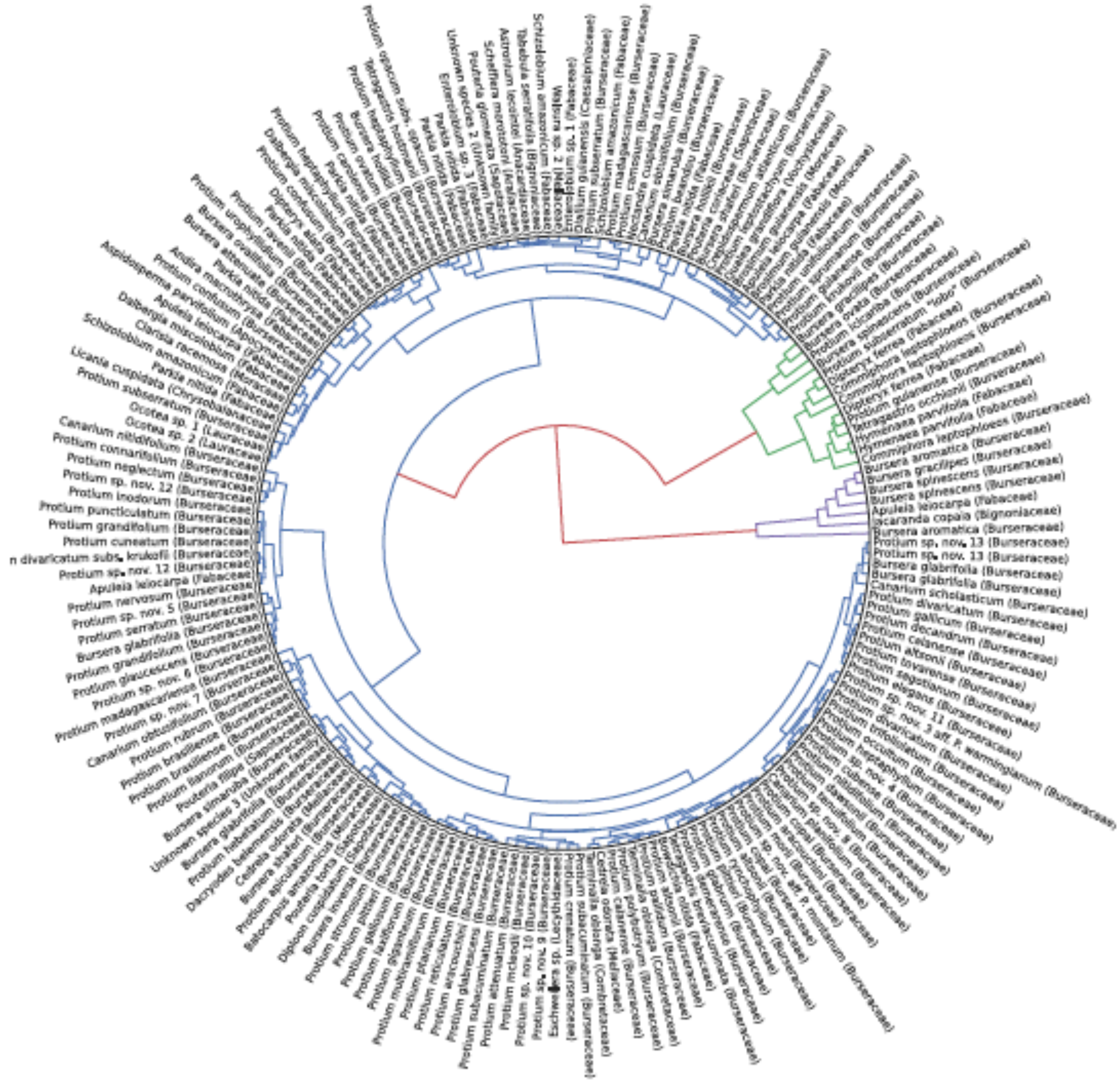
What can we learn about the evolution of land plants?



Leaf fingerprinting

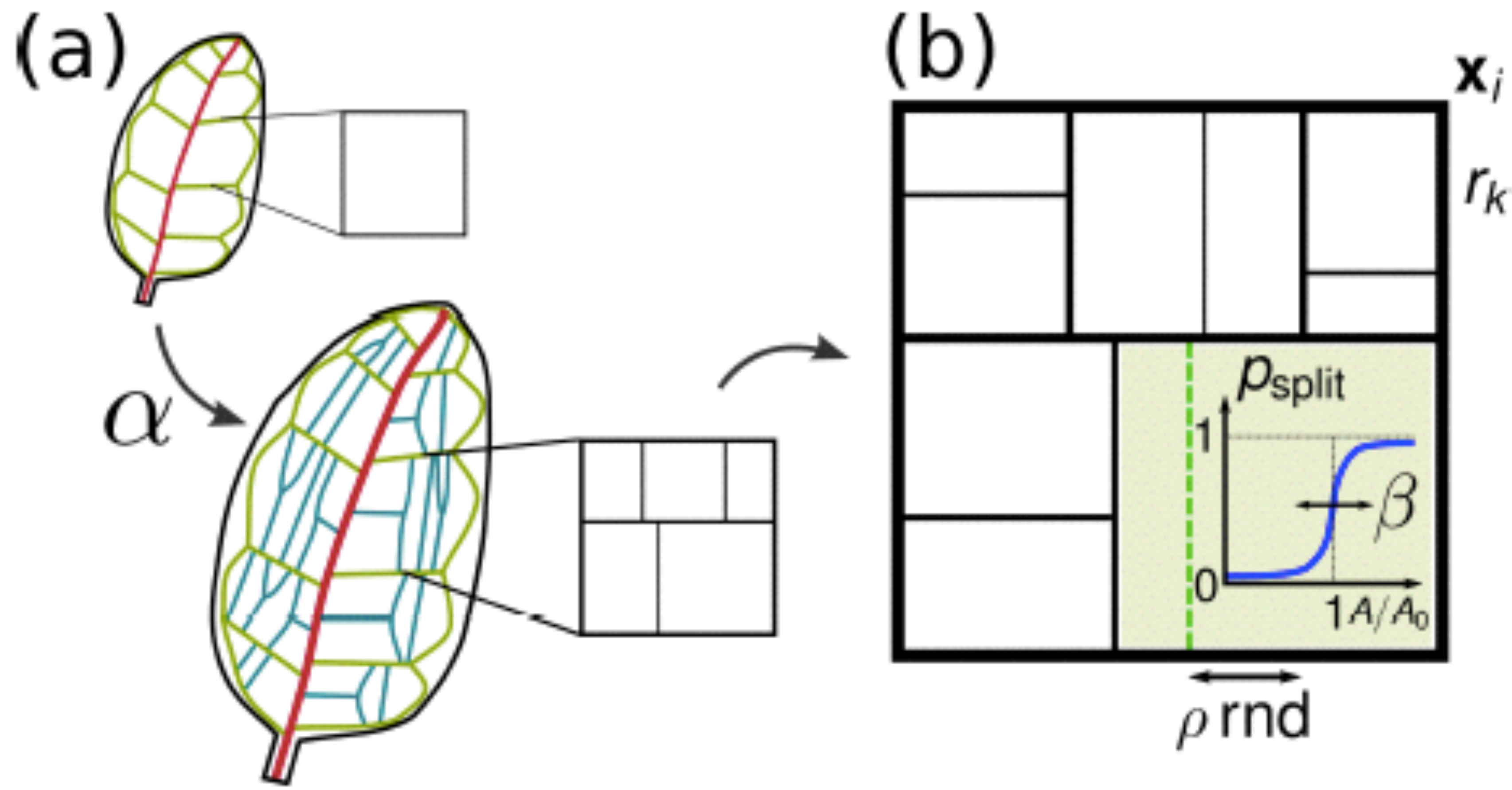
Specimens belonging to the same species tend to be clustered closely together, but no clear correlation with phylogeny

?



The dendrogram was produced using the KS distances between nesting ratio statistics and the complete-linkage (maximum distance) clustering method. The height of the U-shaped links corresponds to the distance between clusters, different colors were used when the distance between clusters was larger than 0.5 times the maximum distance.

Empirical growth model: Noise in development



$$\frac{d\mathbf{x}_i(t)}{dt} = \mathbf{x}_i(t)$$ leaf expansion

$$\frac{dr_k}{dt} = \alpha$$ vein growth + gaussian noise with 0 mean and standard deviation $f_n \mu_r$

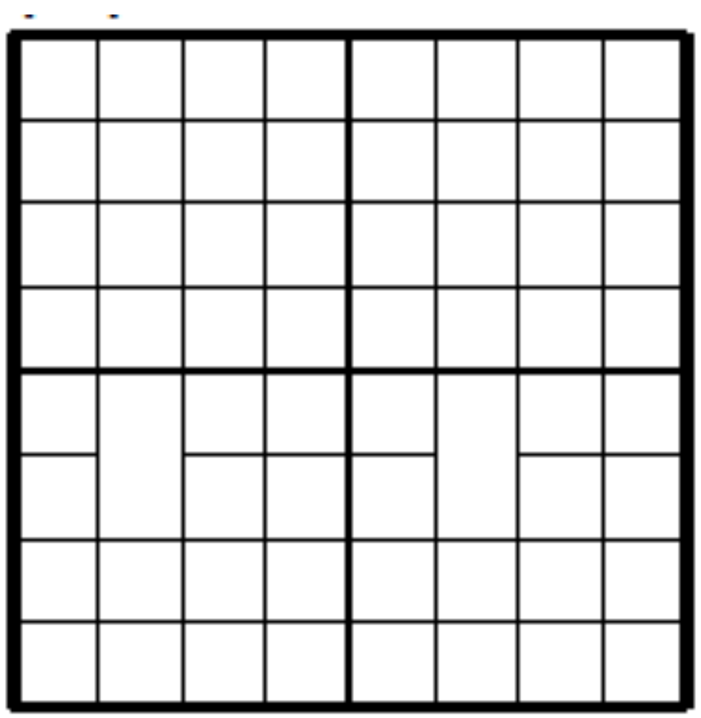
$$p_{\text{split}}(A) = wf\left(\frac{A-1}{\beta}\right)$$ introduction of new veins

constraint: areole size distribution

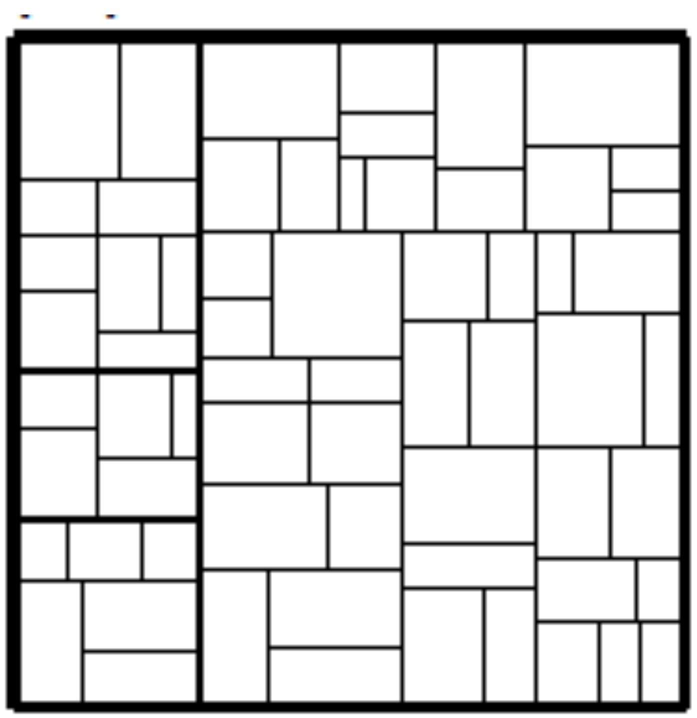
$$x_{\text{rel}} = \frac{1}{2} + \rho \text{ rnd},$$ position of new veins

$dt = 0.01$ and $w = 0.1$

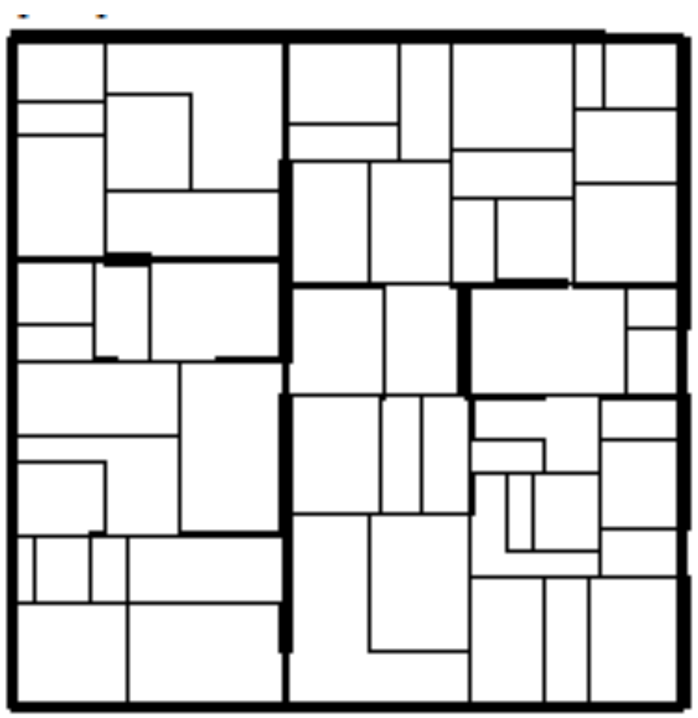
Noise in development



No noise, $\alpha = 0.5, \beta = 0, \rho = 0, f_n = 0.$

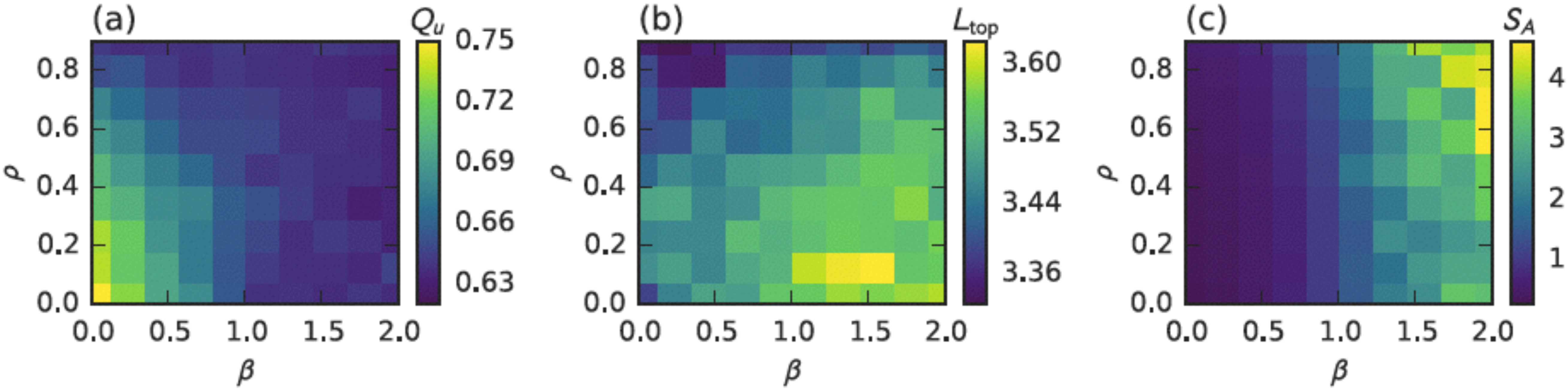


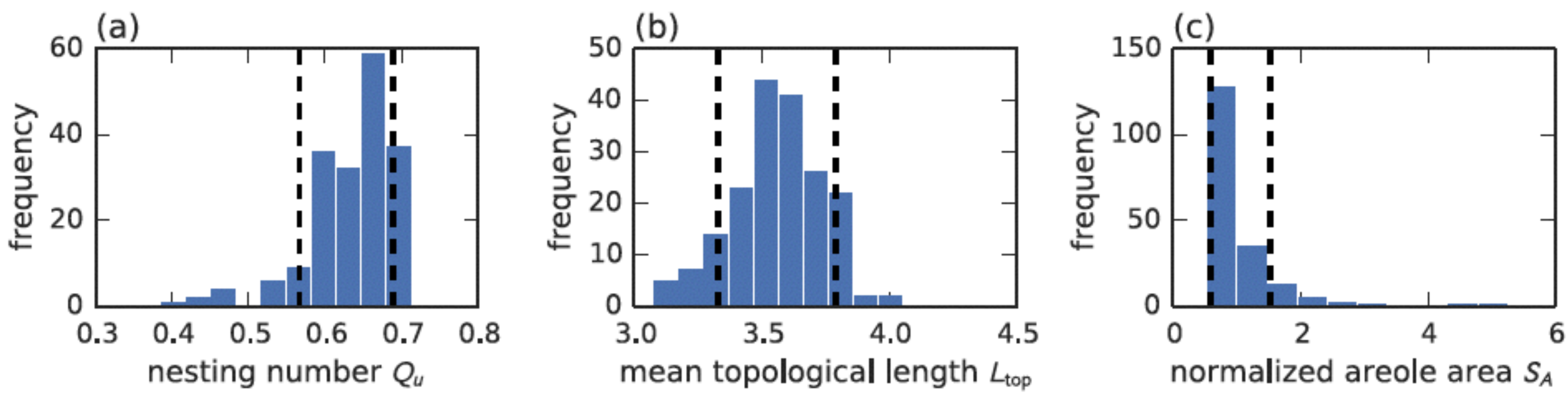
$\alpha = 0.5, \beta = 0.5, \rho = 0.5, f_n = 0$



$\alpha = 0.5, \beta = 0.5, \rho = 0.5, f_n = 0.2.$

Noise in development





Noise in development

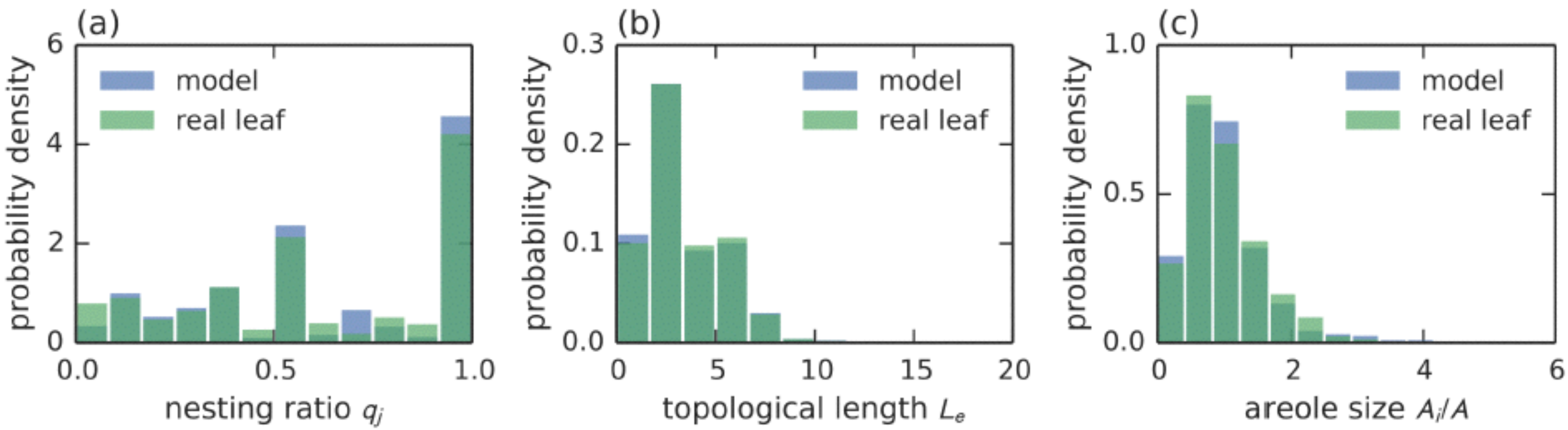


FIGURE 6.7.: Comparison of topological length, nesting ratio, and areole area statistics between model and a real leaf with comparatively low nesting number (*Dalbergia miscolobium*, see Figure 5.4 (b)). The parameters used are $\alpha = 0.2$, $\beta = 0.2$, $\rho = 0.75$, $f_n = 0.35$; the final network had 896 loops. This is a high noise setting. All distributions fit rather well, the very lowest nesting ratios are not reproduced. (a) $D_{KS} = 0.04$, $p = 0.17$. (b) $D_{KS} = 0.02$, $p = 0.31$. (c) $D_{KS} = 0.03$, $p = 0.40$.

Noise in development

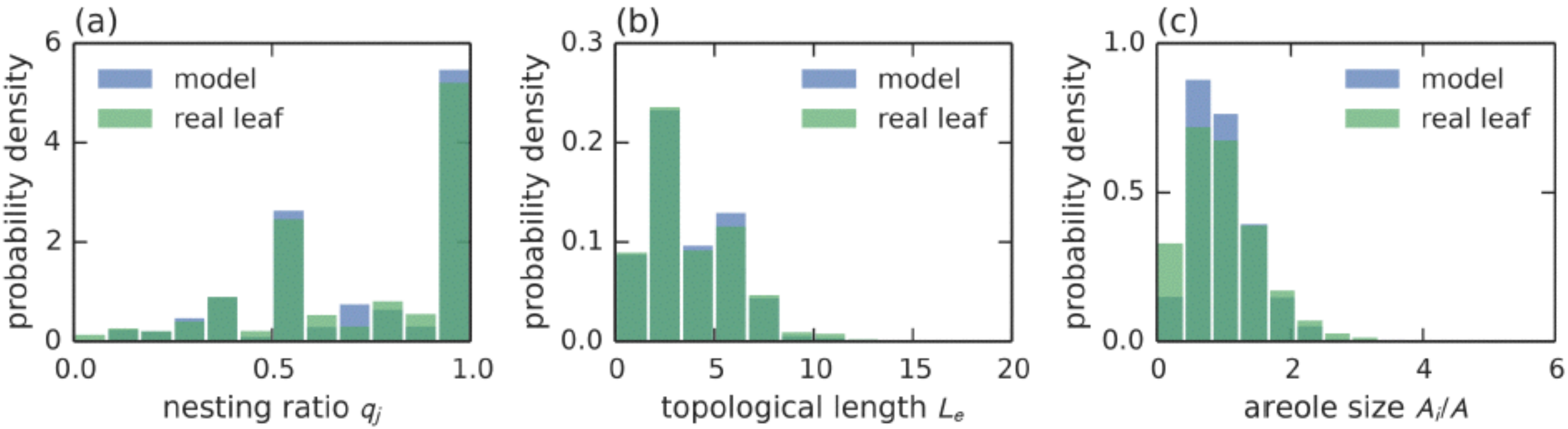


FIGURE 6.6.: Comparison of topological length, nesting ratio, and areole area statistics between model and a real leaf with comparatively high nesting number (*Protium grandifolium*, see Figure 5.4 (a)). The parameters used are $\alpha = 0.2$, $\beta = 0.4$, $\rho = 0.25$, $f_n = 0.1$; the final network had 1038 loops. This is a low noise setting. Except for the areole sizes where the real leaf contains more small areoles, the distributions fit quite well, in particular the nesting ratios. This is quantified by KS tests. (a) $D_{KS} = 0.03$, $p = 0.33$. (b) $D_{KS} = 0.02$, $p = 0.12$. (c) $D_{KS} = 0.08$, $p < 0.001$.

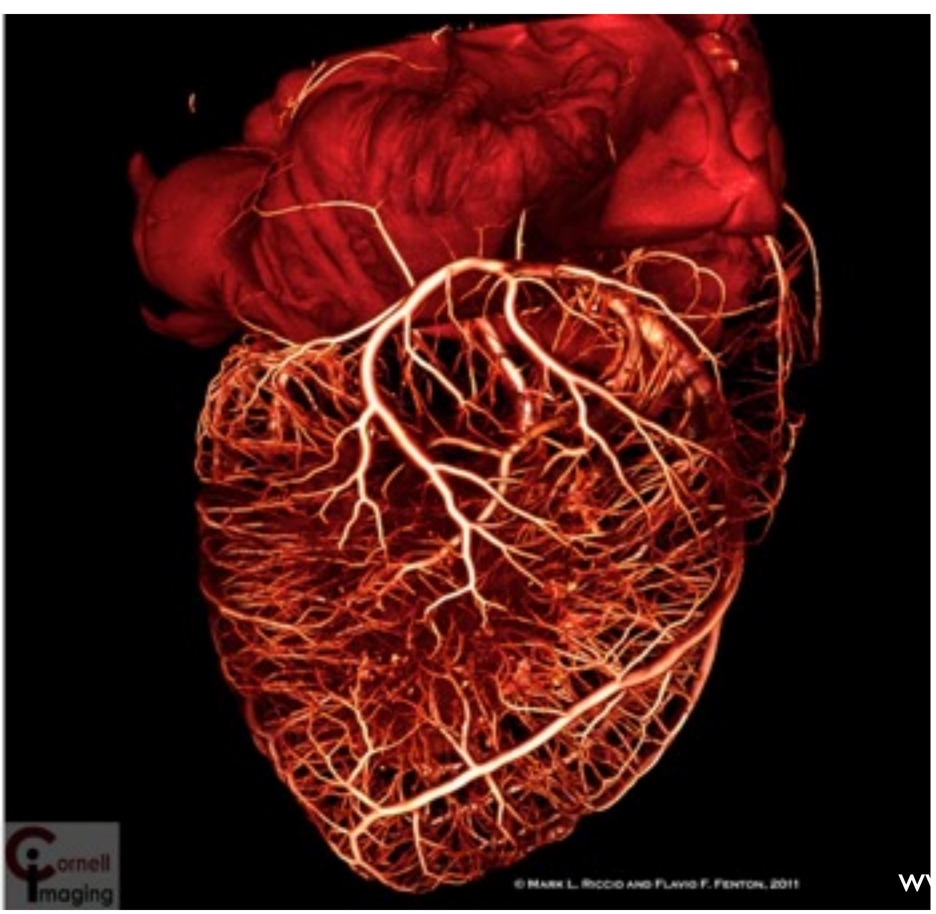
Degree constrained graphs

How can one quantify a complex graph?

3d Networks with **restricted** degree?

Vasculature of animal organs, foams etc

n-regular graphs



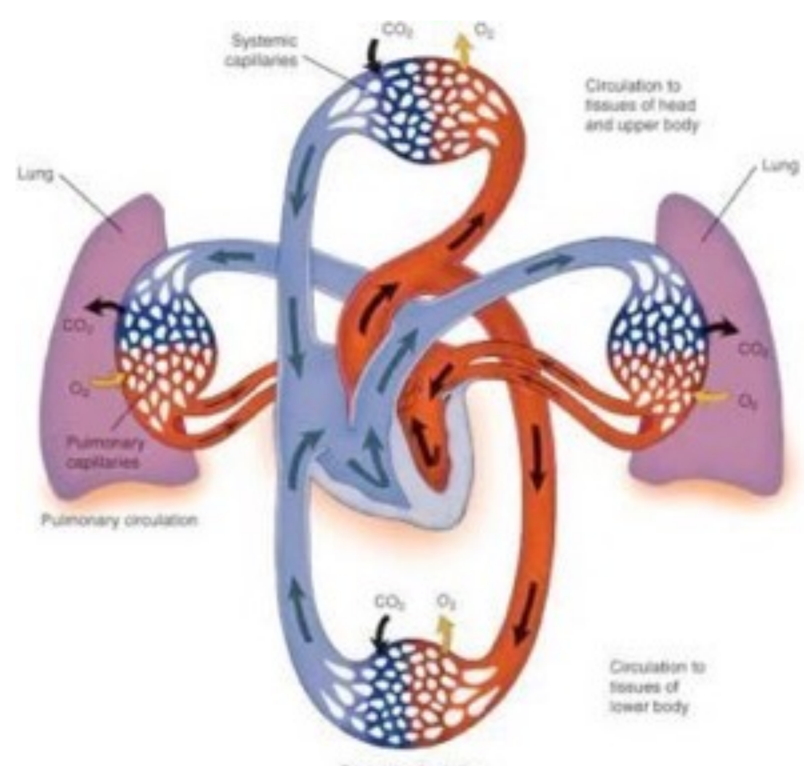
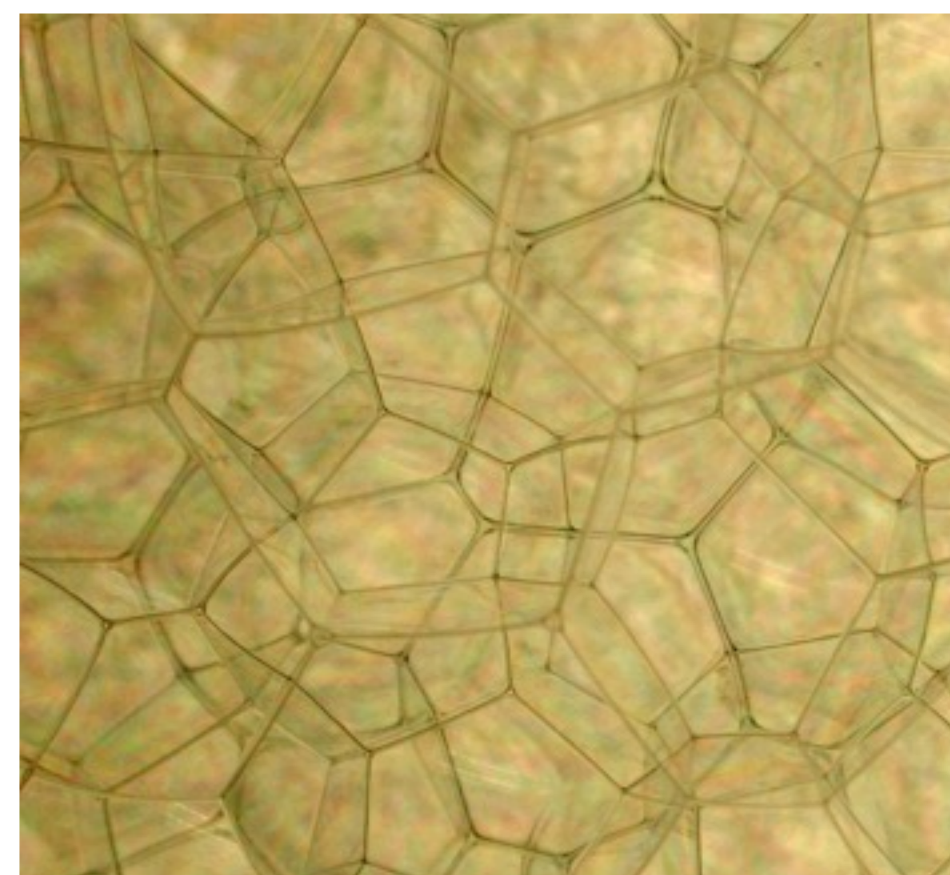
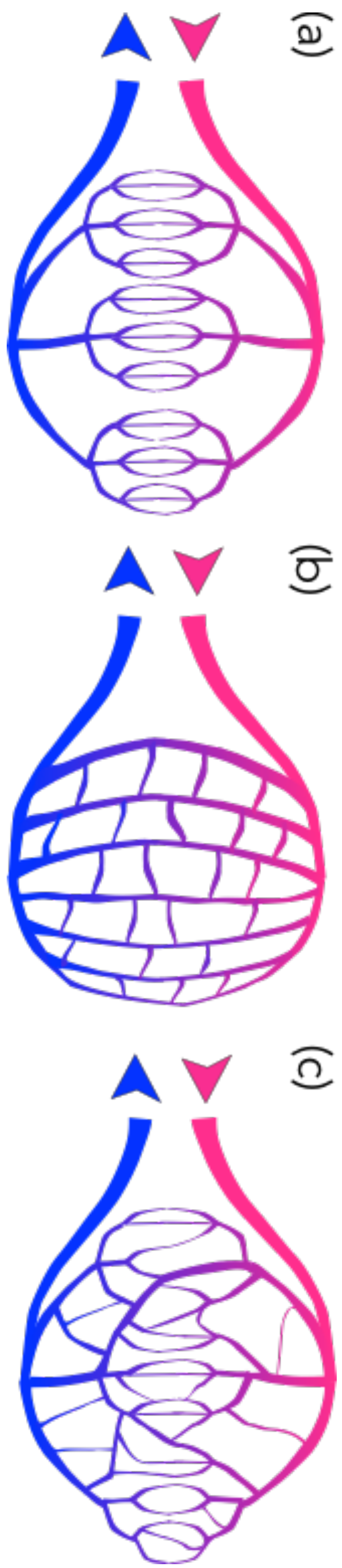


FIGURE 9-6 Generalized circulatory pathways between the heart, lung, and extremities.
Mosby items and derived items © 2009 by Mosby, Inc., an affiliate of Elsevier Inc.

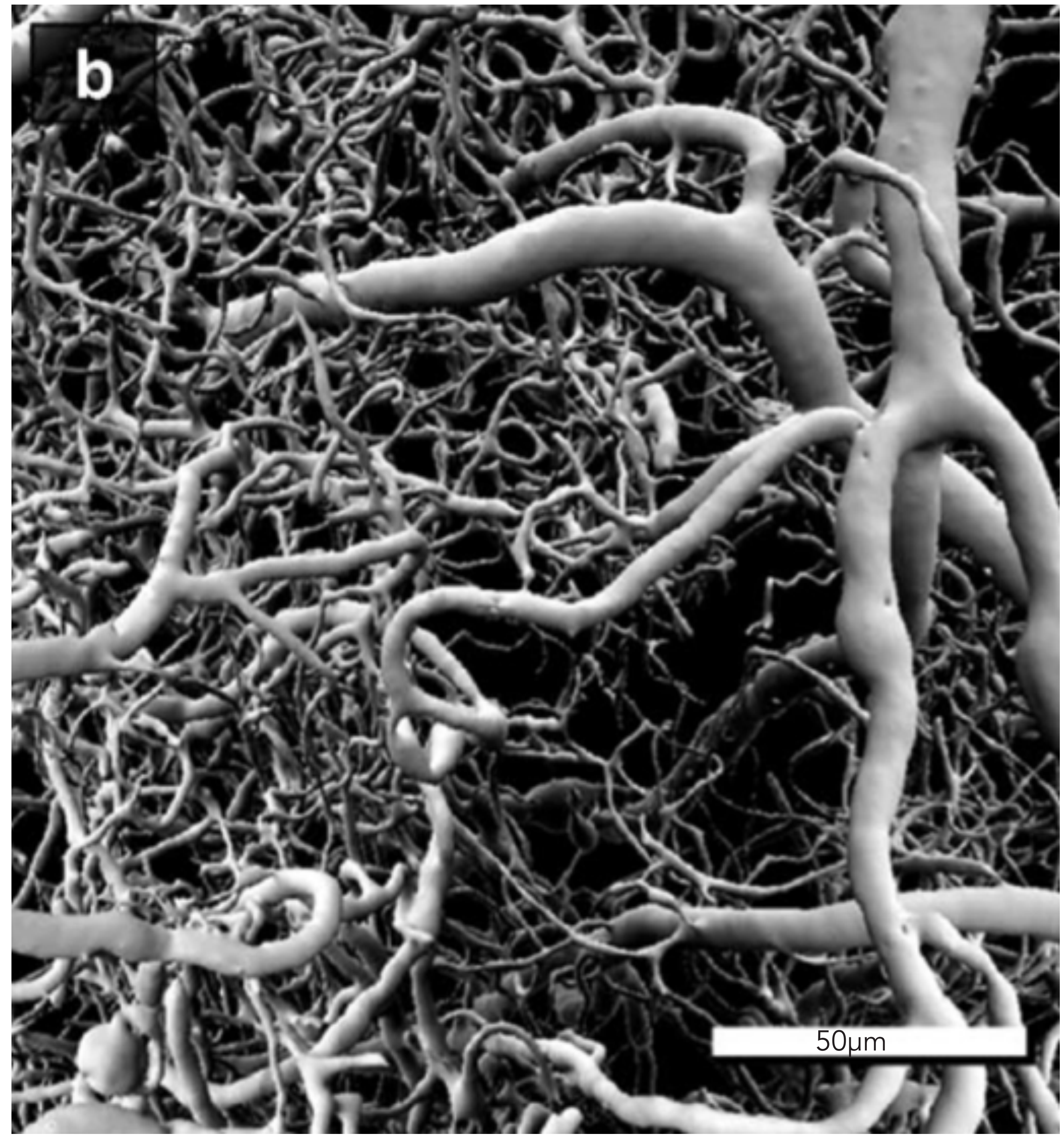


http://www.maths.tcd.ie/~foams/gallery.htm

Applications to vasculature

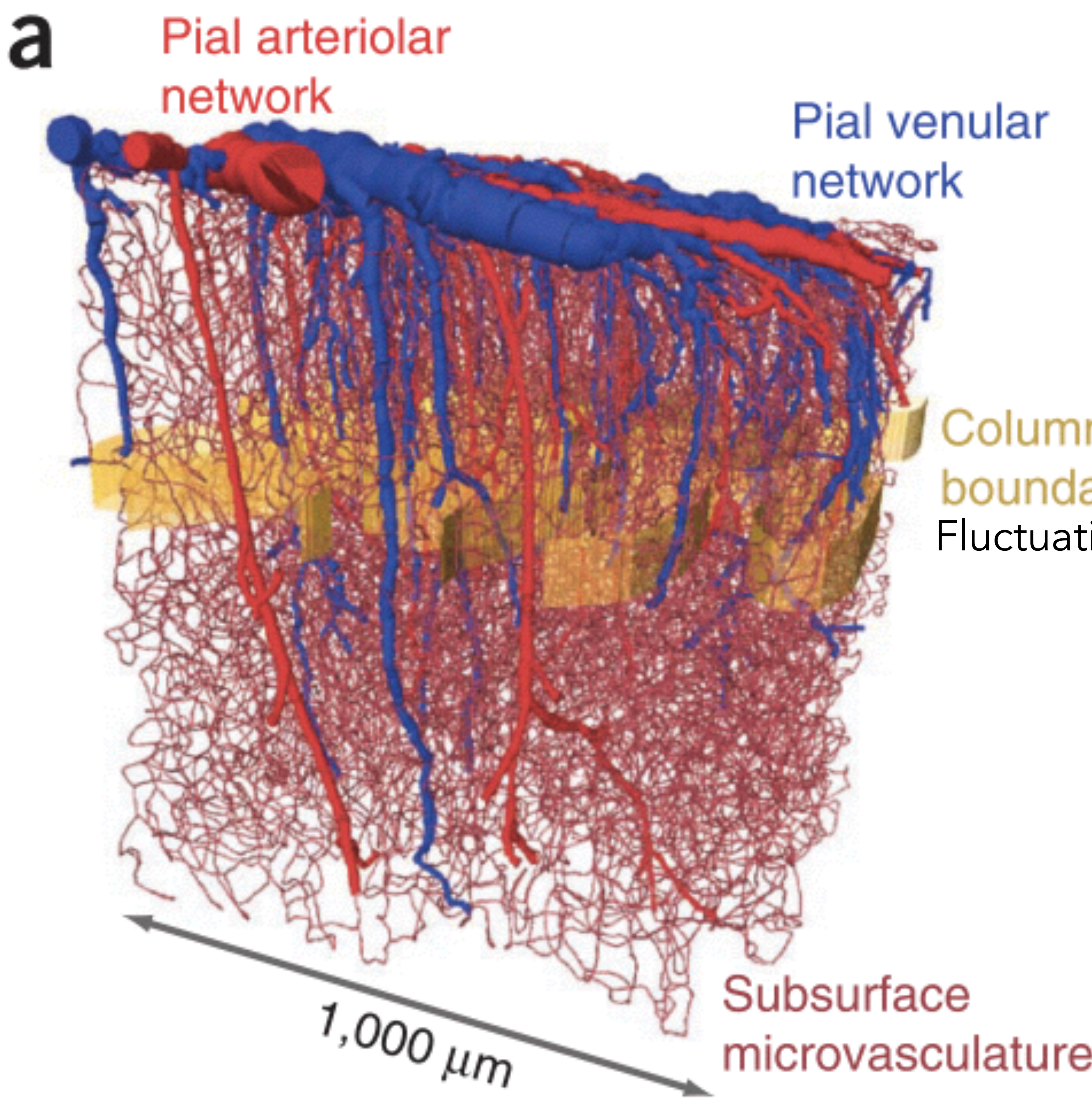


What are the right metrics to identify disease?



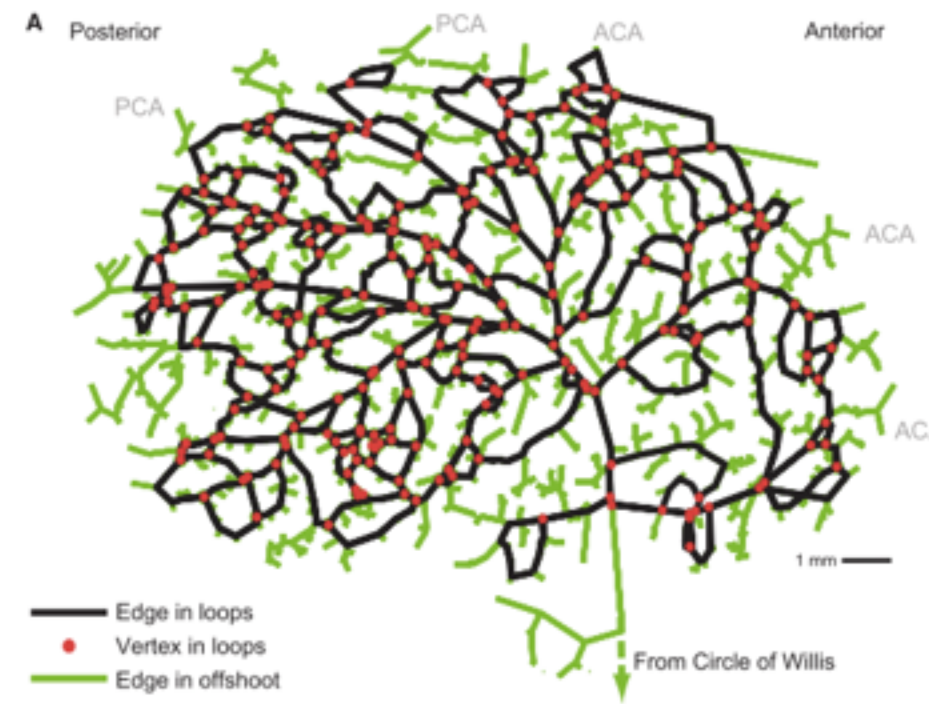
S. Heinzer et al. / NeuroImage 32 (2006) 626–636

The three tiers of the cortex vasculature

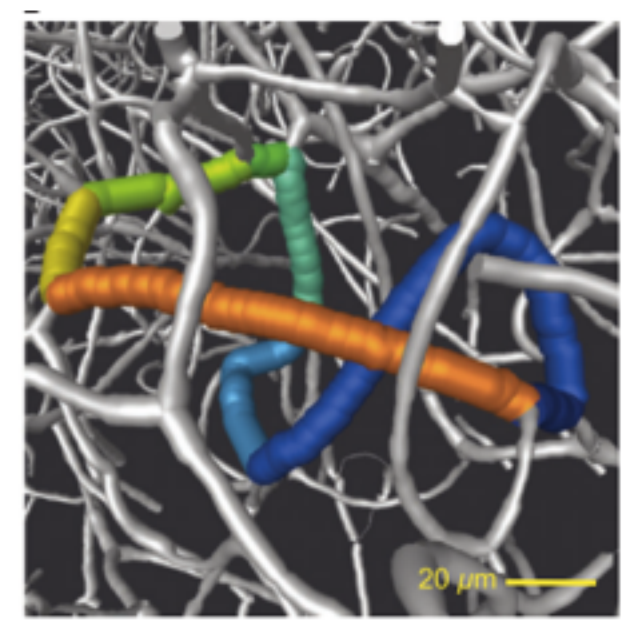


Fluctuations!

Surface arterial network



Subsurface microvasculature



Blinder et al, Nature neuroscience 2013

Shih et al, Microcirculation 2015

Outline

Part 1
Characterizing planar degree constrained graphs
(Actual data)

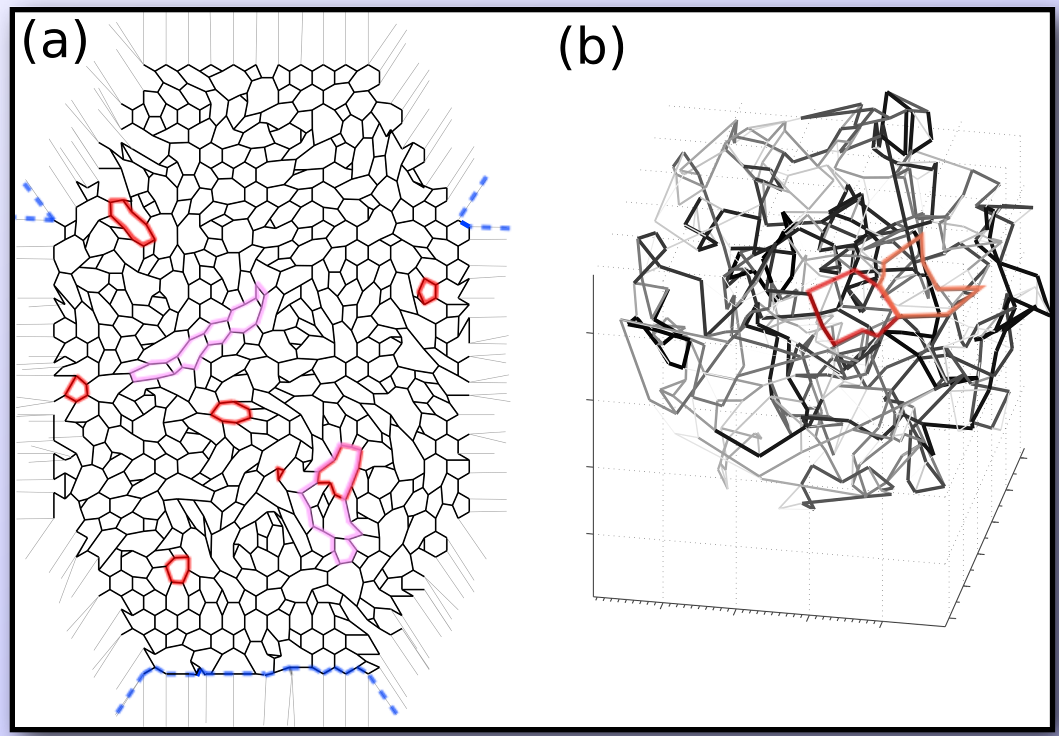
Part 2
Characterizing non-planar degree constrained graphs
(Interesting mathematics)

Intermission

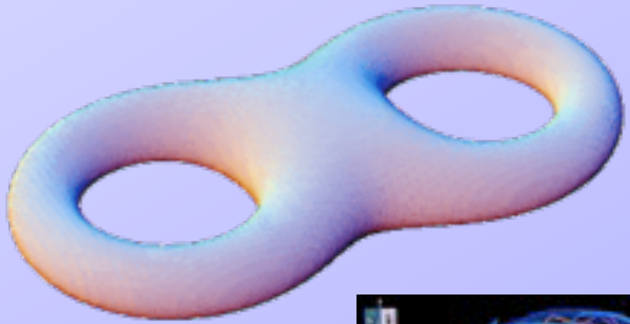
Intermission

Choosing Test Network Topologies and Weights

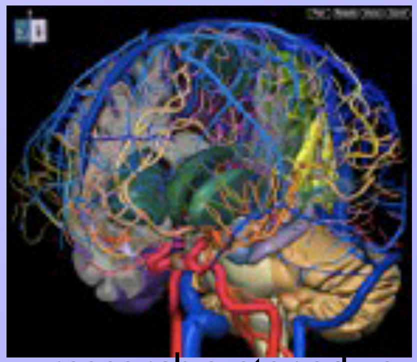
Topology Choices



2-Torus



Random, Spatially Embedded 3-Regular

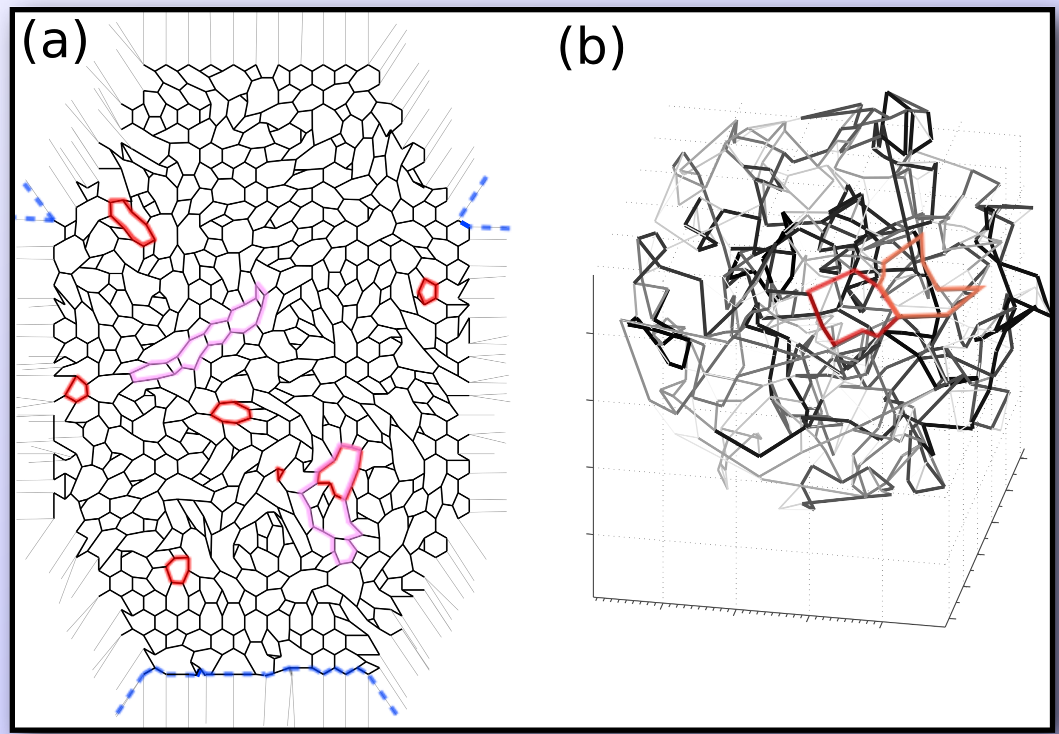


www.research.a-star.edu.sg

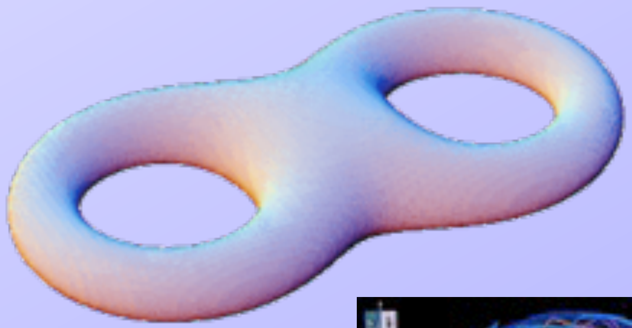
CDM, M.O. Magnasco, and E. Katifori, *arXiv/1410.3951*

Choosing Test Network Topologies and Weights

Topology Choices



2-Torus



Random, Spatially Embedded 3-Regular

www.research.a-star.edu.sg

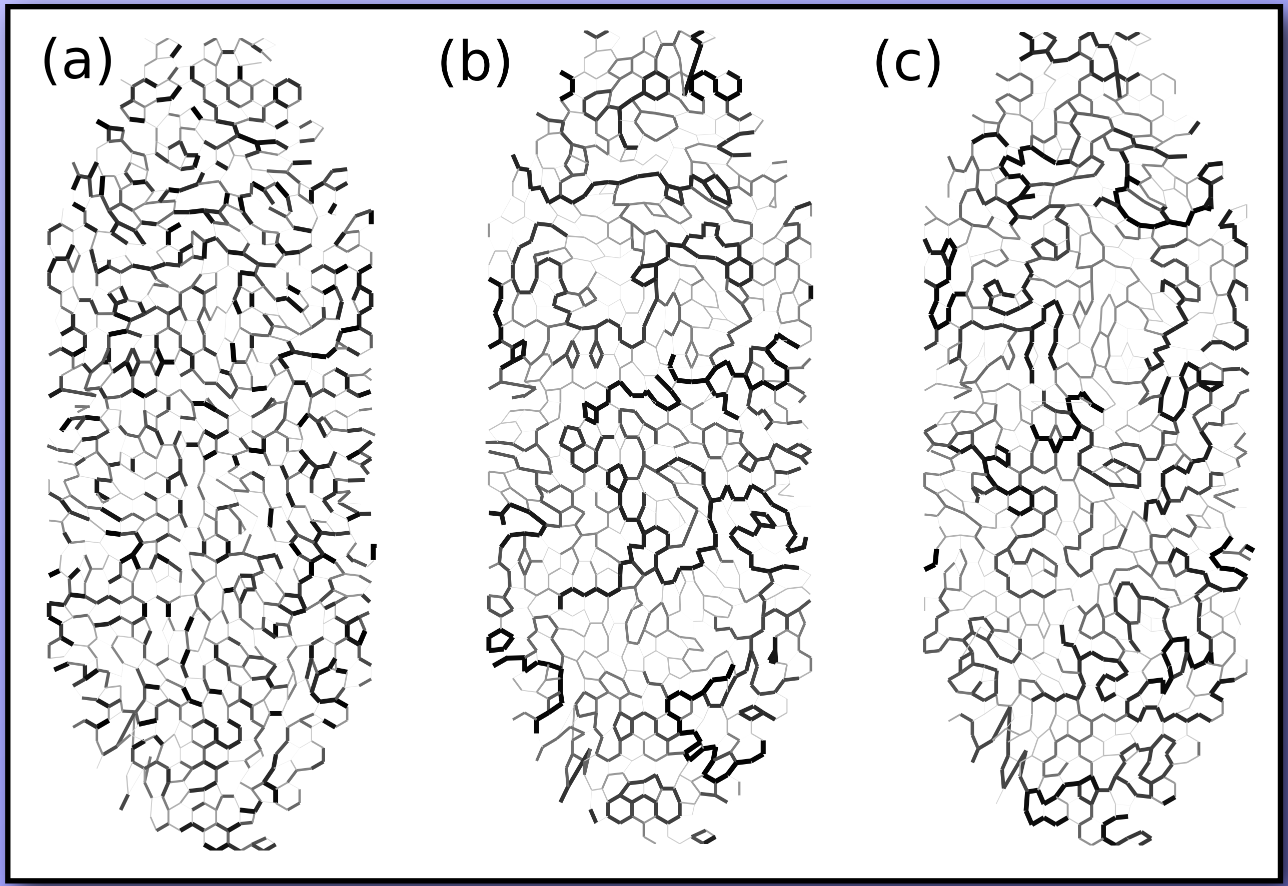
Edge Weight Assignments

Random

Linear

Self-Avoiding Linear

"Evolved"



CDM, M.O. Magnasco, and E. Katifori, *arXiv/1410.3951*

Network Evolution

Local Update Adaptation

$$Q_{ij}^{kl} = \frac{C_{ij}}{l_{ij}} \cdot \left(p_i^{kl} - p_j^{kl}\right)$$

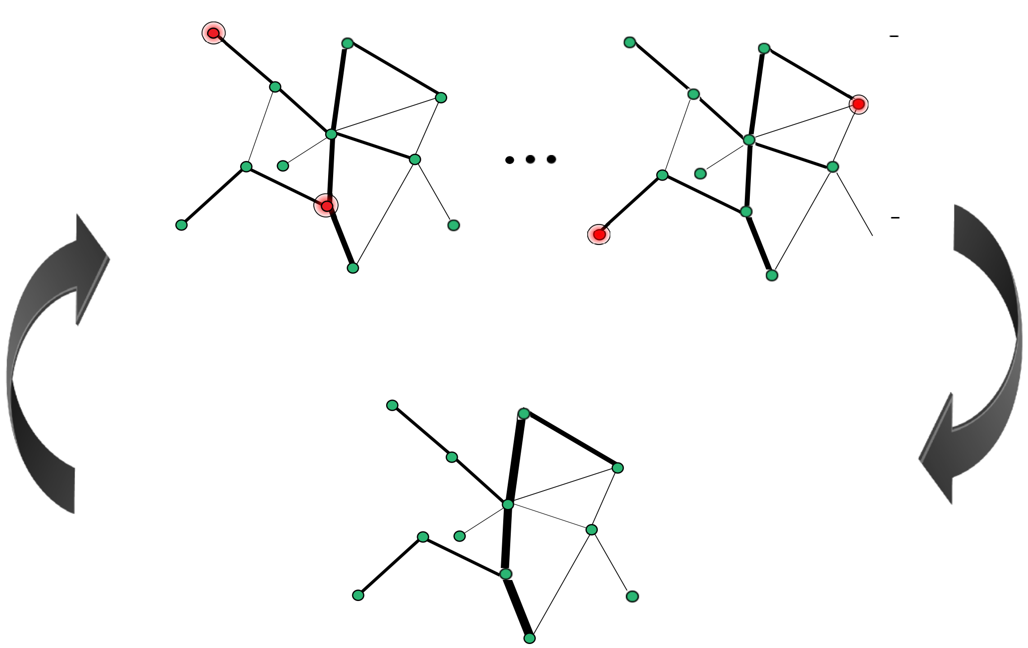
$$\langle |Q_{ij}| \rangle := \frac{1}{\frac{N\cdot(N-1)}{2}} \sum_{(k,l)\in\mathbb{P}} |Q_{ij}^{kl}|$$

decay term

$$\frac{\mathrm{d}C_{ij}(t)}{\mathrm{d}t} = \beta \cdot f\left(\frac{\langle |Q_{ij}(t)| \rangle}{\epsilon}\right) - \alpha \cdot C_{ij}(t)$$

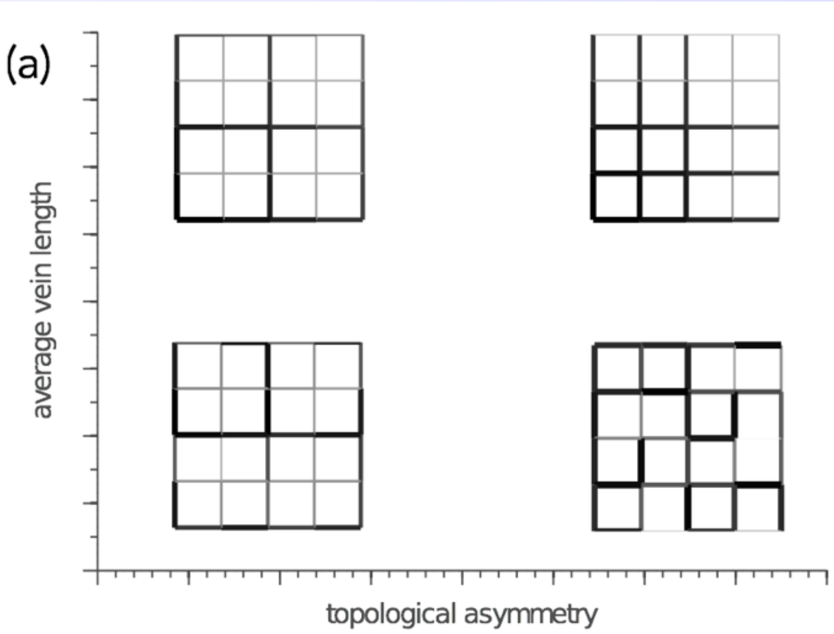
Local positive feedback

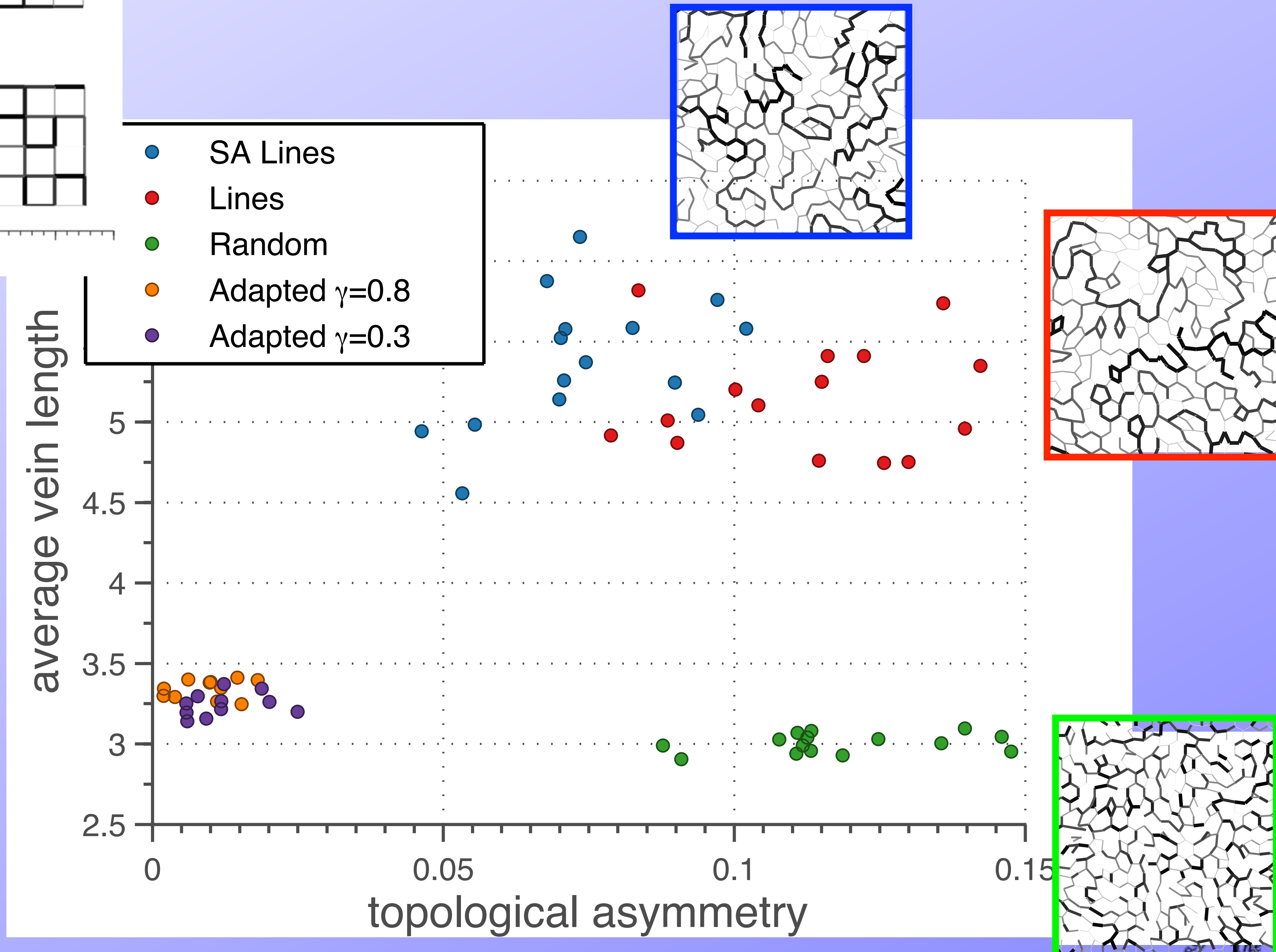
Update model

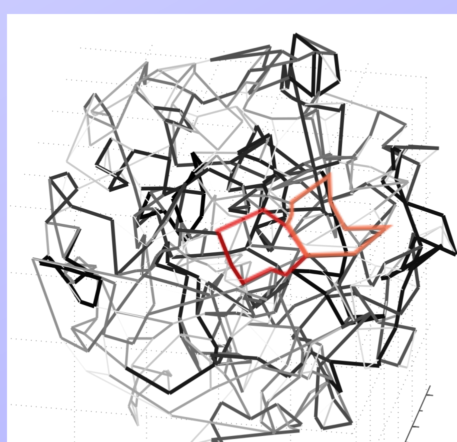


CDM, M.O. Magnasco, and E. Katifori, *arXiv/1410.3951*

Network Identification

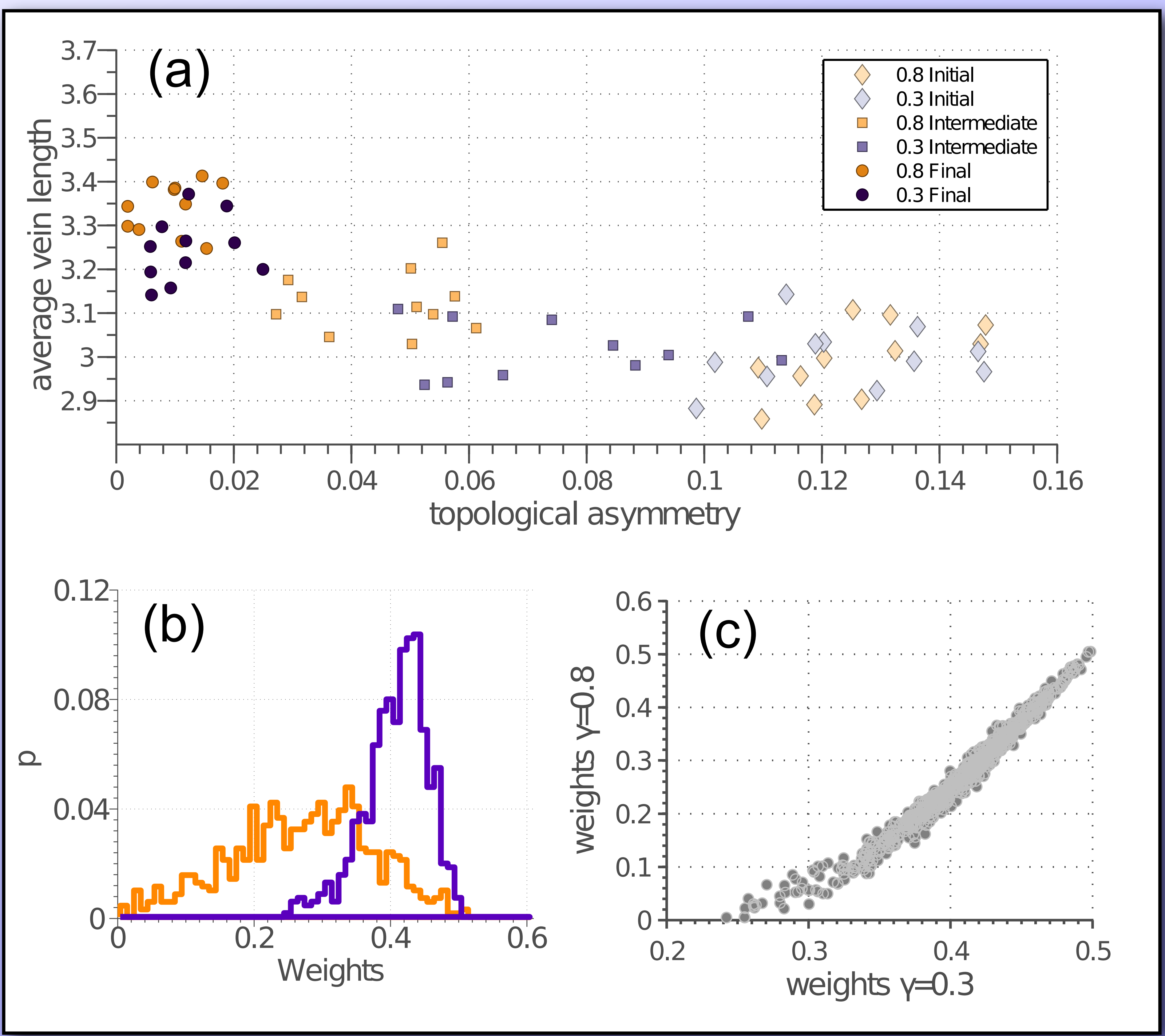






CDM, M.O. Magnasco, and E. Katifori, *arXiv/1410.3951*

Adaptive Networks and Time Sensitivity



Sand Piles, Dunes, & Granular Matter

Sand Piles, Dunes, & Granular Matter

Force Chains in Photoelastic Beads

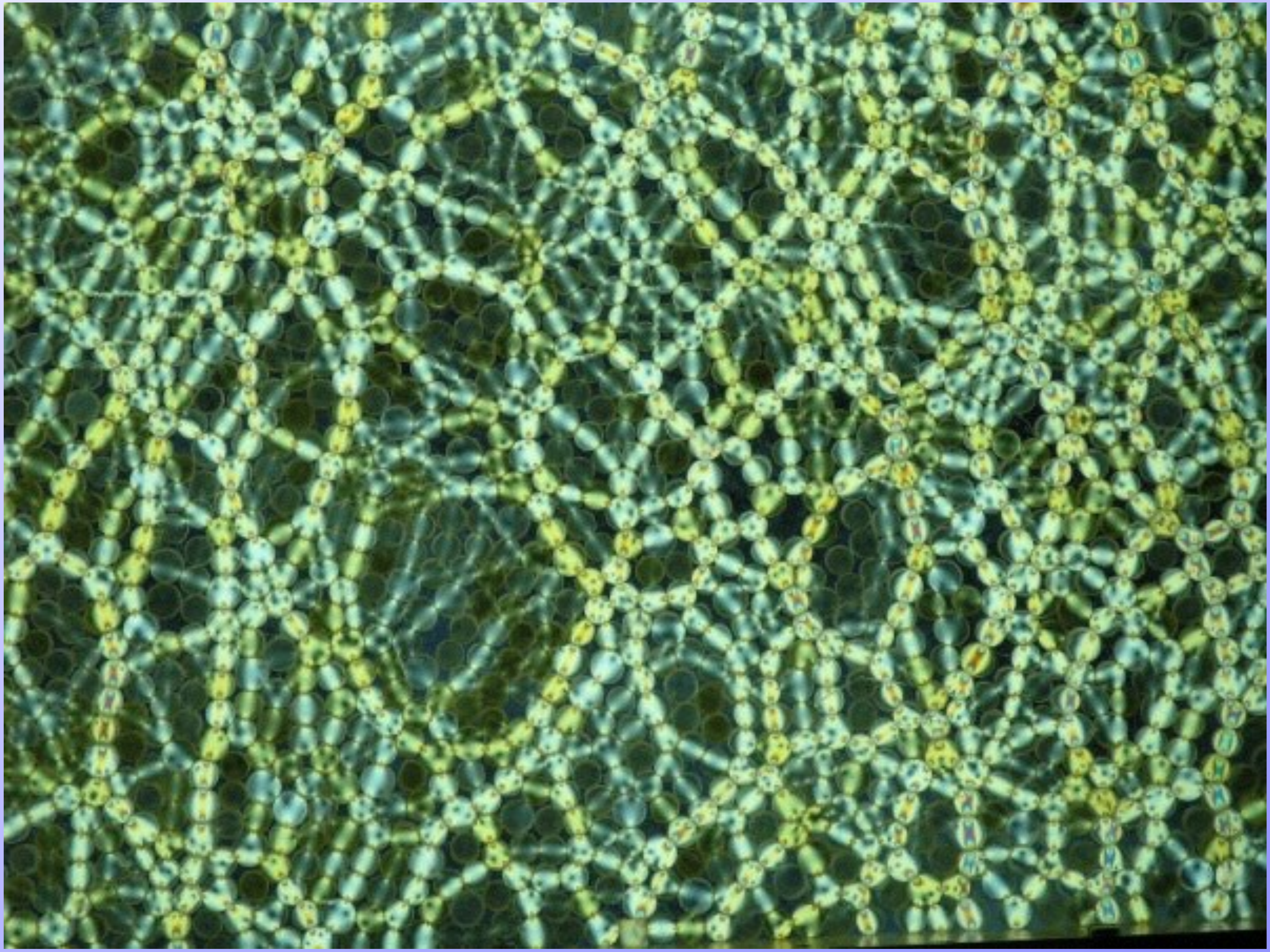
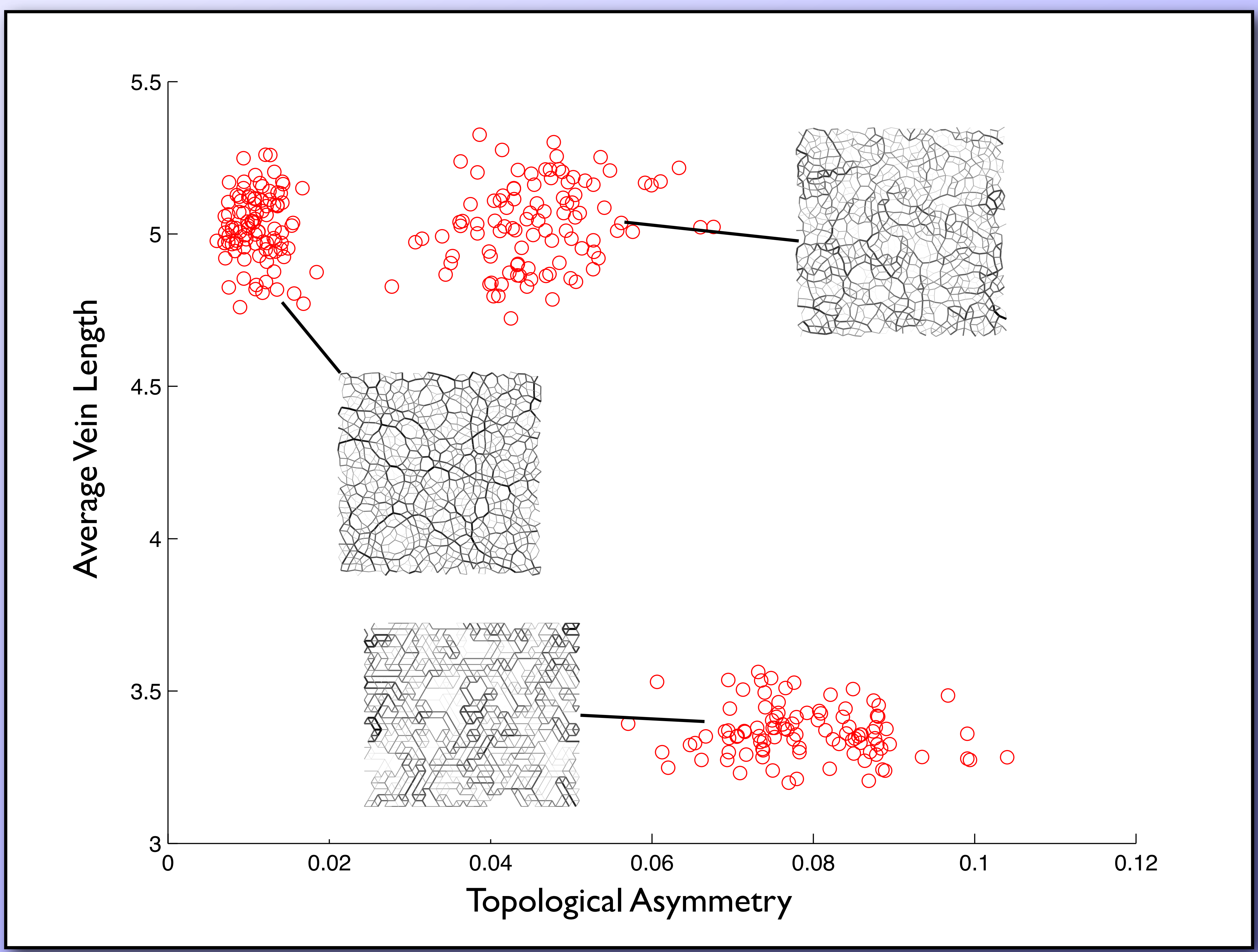


Image from Bob Behringer's home page: phy.duke.edu/~bob

Jammed Granular Matter

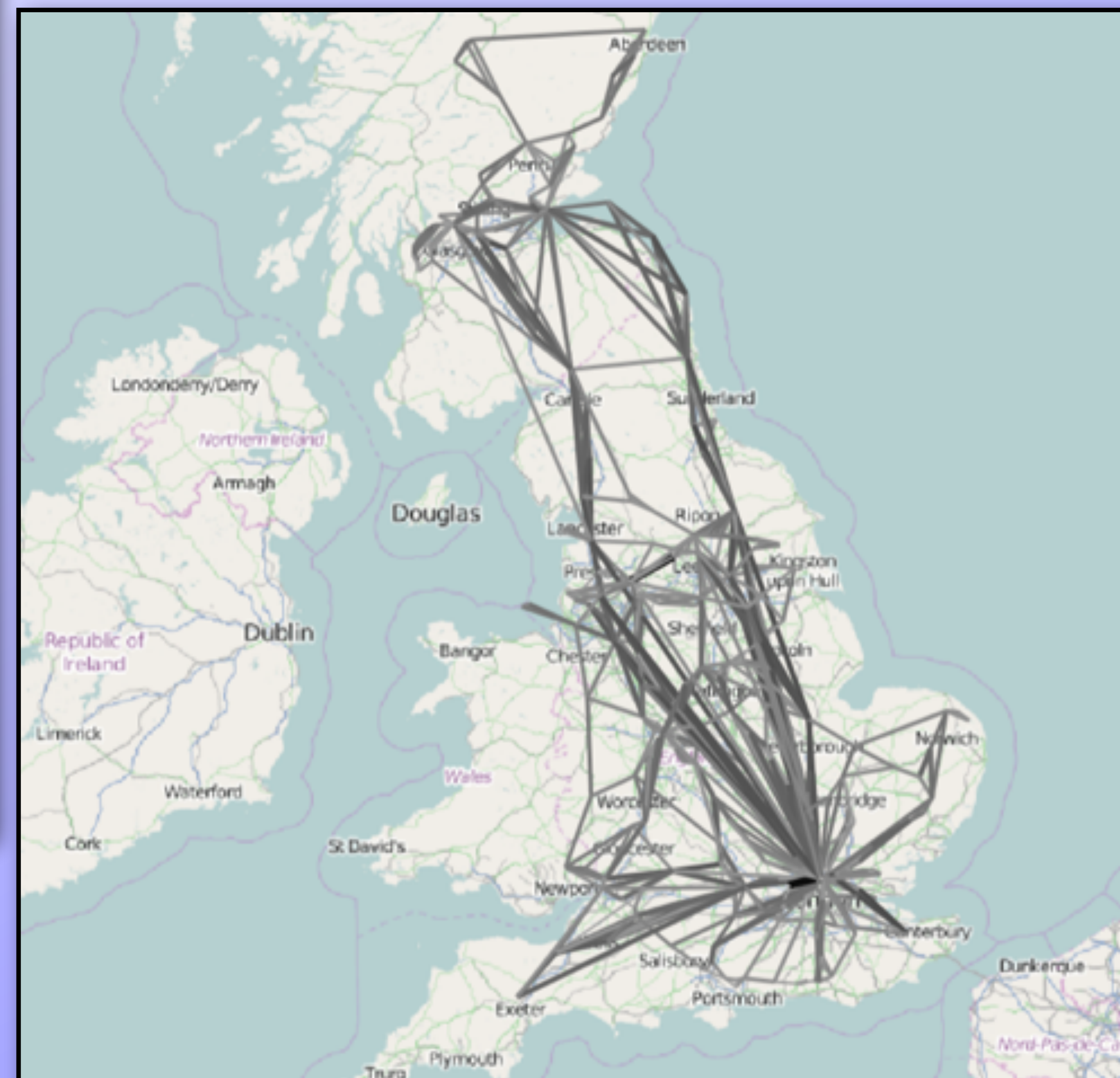


CDM, D. Sussman, and E. Katifori, *in preparation*

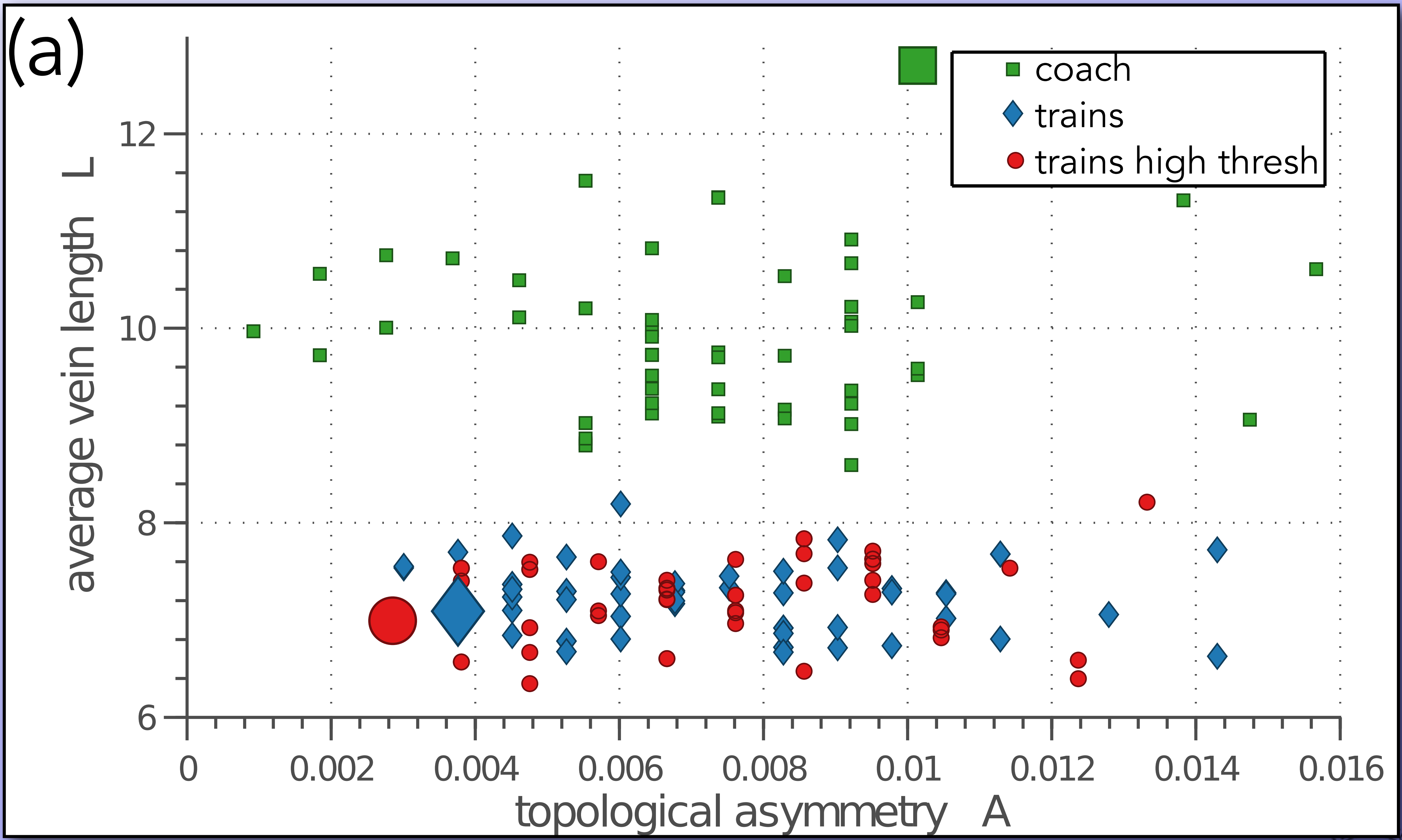
Coach and Rail Networks

Coach Routes by Speed

Train Routes by Speed



Coach and Rail Networks



Bonus material