

Numerical simulations of general  
gravitational singularities

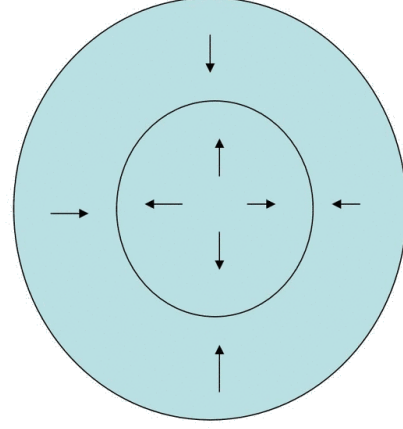
D. Garfinkle, PRL 93, 161101 (2004)

C. Uggla, H. Elst, J. Wainwright, G. Ellis,  
PRD 68, 103502 (2003)

J. Curtis and D. Garfinkle, PRD 72, 064003  
(2005)

- Gravitational collapse
- Singularity theorems
- BKL conjecture
- Gowdy spacetimes
- General spacetimes

Star in equilibrium between gravity and pressure. Gravity can win and the star can collapse



## Singularity theorems

Once a trapped surface forms  
(given energy and causality conditions)  
Some observer or light ray ends in a  
Finite amount of time

Very general circumstances for  
Singularity formation

Very little information about  
The nature of singularities

## Approach to the singularity

As the singularity is approached, some  
terms in the equations are blowing up

Other terms might be negligible in  
comparison

Therefore the approach to the singularity  
might be simple

## BKL Conjecture

As the singularity is approached time derivatives become more important than spatial derivatives. At each spatial point the dynamics approaches that of a homogeneous solution.

Is the BKL conjecture correct? Perform numerical simulations and see

## Gowdy spacetimes

$$ds^2 = e^{(\lambda+t)/2} (-e^{-2t} dt^2 + dx^2) + e^{-t} [e^P (dy+Qdz)^2 + e^{-P} dz^2]$$

P, Q and  $\lambda$  depend only on t and x

The singularity is approached as t goes to infinity

x,y and z are periodic, space is a 3-torus

## Einstein field equations

$$P_{tt} - e^{2P} Q_t^2 - e^{-2t} P_{xx} + e^{2(P-t)} Q_x^2 = 0$$

$$Q_{tt} + 2 P_t Q_t - e^{-2t} (Q_{xx} + 2 P_x Q_x) = 0$$

(subscript means coordinate derivative)

Note that spatial derivatives are multiplied by decaying exponentials

Numerical simulations

Centered differences for spatial derivatives

$$f_x = (f_{i+1} - f_{i-1}) / (2dx)$$

$$f_{xx} = (f_{i+1} + f_{i-1} - 2f_i) / (dx^2)$$

Evolution through ICN

$$d_t S = W$$

is implemented as

$$S^{n+1} = S^n + (dt/2)(W(S^n) + W(S^{n+1}))$$

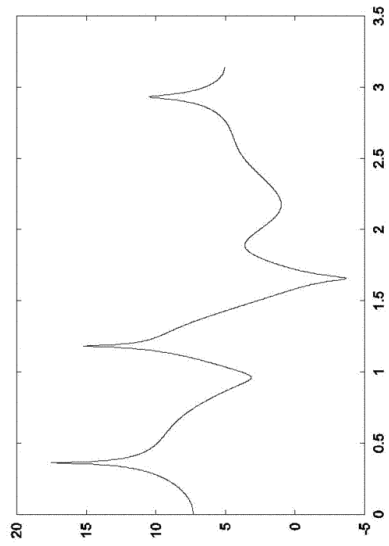
As  $t$  goes to infinity

$$P = P_0(x) + tv_0(x)$$

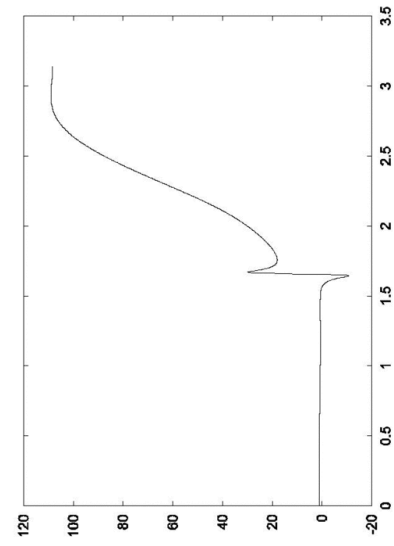
$$Q = Q_0(x)$$

But there are spikes

P



Q



## General case

Variables are scale invariant commutators of tetrad

$$\mathbf{e}_0 = N^{-1} \mathbf{d}_t \quad \mathbf{e}_\alpha = \mathbf{e}_\alpha^i \mathbf{d}_i$$

$$[\mathbf{e}_0, \mathbf{e}_\alpha] = u_\alpha \mathbf{e}_0 - (H\delta_\alpha^\beta + \sigma_\alpha^\beta) \mathbf{e}_\beta$$

$$[\mathbf{e}_\alpha, \mathbf{e}_\beta] = (2 a_{[\alpha} \delta_{\beta]}^\gamma + \varepsilon_{\alpha\beta\delta} n^{\delta\gamma}) \mathbf{e}_\gamma$$

## Scale invariant variables

$$\{\mathbf{d}_t, E_\alpha^i \mathbf{d}_i\} = \{\mathbf{e}_0, \mathbf{e}_\alpha\}/H$$

$$\{\Sigma_{\alpha\beta}, A^\alpha, N_{\alpha\beta}\} = \{\sigma_{\alpha\beta}, \mathbf{a}^\alpha, n_{\alpha\beta}\}/H$$



The vacuum Einstein equations become evolution equations and constraint equations for the scale invariant variables.

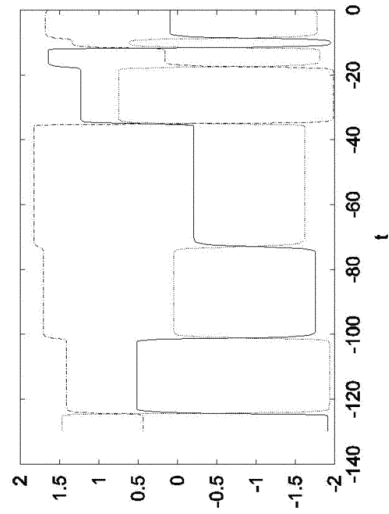
Choose slicing to asymptotically approach the singularity, CMC or inverse mean curvature flow.

## Results of simulations

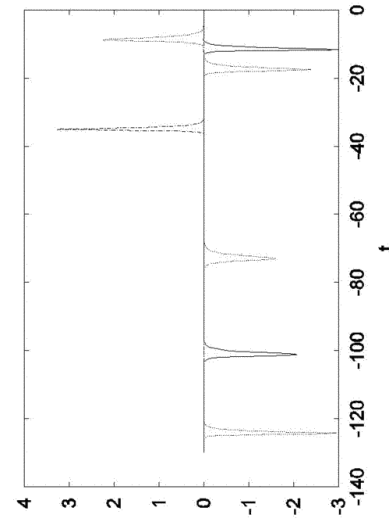
Spatial derivatives become negligible

At each spatial point the dynamics of the scale invariant variables becomes a sequence of “epochs” where the variables are constant, punctuated by short “bounces” where the variables change rapidly

$\Sigma$

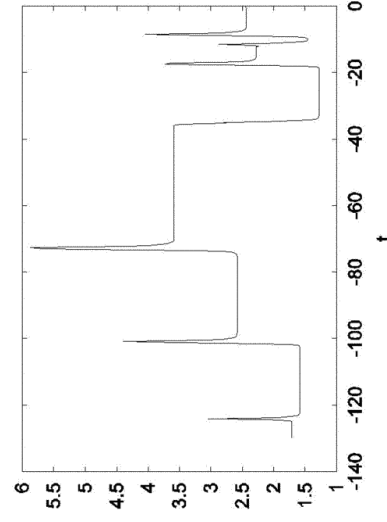


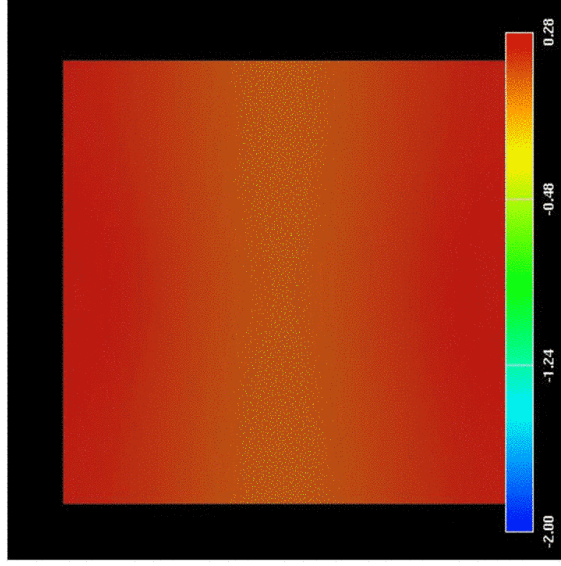
$N$



$\Sigma_{\alpha\beta}$  between bounces is characterized by a single number  $u$ . BKL conjecture that at a bounce  $u$  goes to  $u-1$  if  $u > 2$  and to  $1/(u-1)$  if  $u < 2$ .

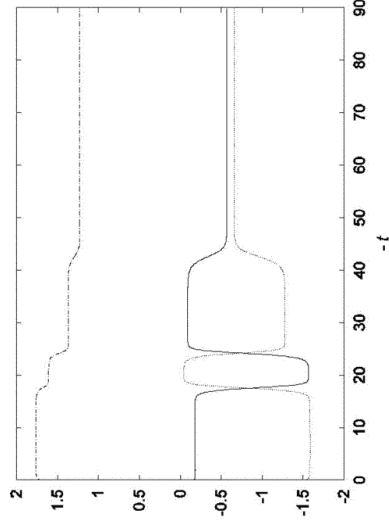
$u$





For stiff fluid the dynamics is similar, but  
there is a last bounce

$\Sigma$  for stiff fluid



## results

BKL conjecture appears to be correct

*What remains to be done*

- (1) Non-stiff Matter
- (2) Small scale structure
- (3) Asymptotically flat case

## Implications for the quantum nature of singularities

- FRW doesn't tell us about gravitational collapse
- Schwarzschild is only a special case
- Don't worry about the complications of Mixmaster
- Concentrate on Kasner

## Question 5

Is there an analog of the BKL ansatz for quantum gravity?

## Conclusions

- BKL ansatz is verified by numerics and shows that classical singularities are tractable
- If something similar holds in quantum theory, then quantum singularities may be tractable too.