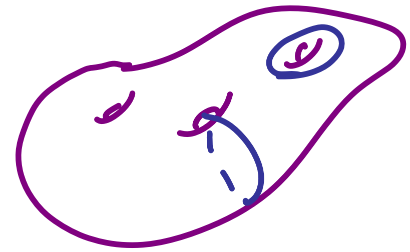
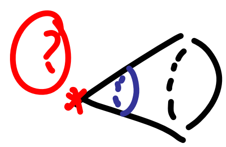
Winding Strings and Spacelike Singularities

String theory contains new degrees of freedom beyond GR. e.g. given nontrivial π_1 in space*; new winding sectors appear

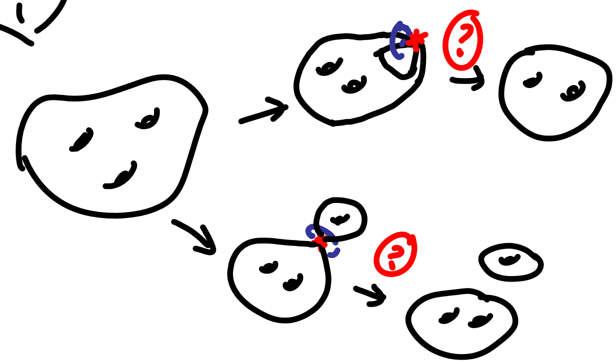


These become important near various basic types of singularities:

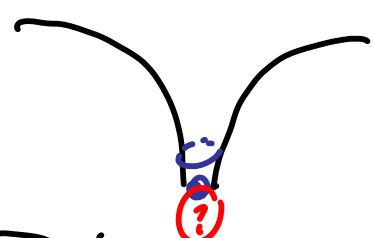
- orbifolds



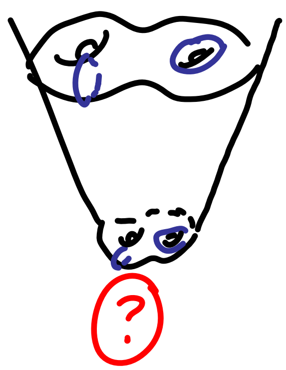
- topology change



- Black holes/branes



- cosmological



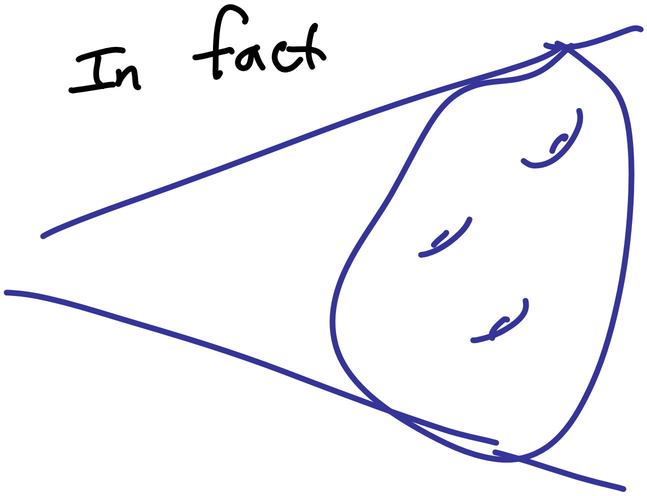
<u>Plan</u>: 2 Cases

I. Winding <u>tachyon</u> <u>condensate</u> dominates, evading GR singularity

II. Winding mode spectrum builds up new effective dimensions

* This specification of nontrivial π_1 is not a strong assumption in the case of cosmological solutions (or compactification geometries).

In fact

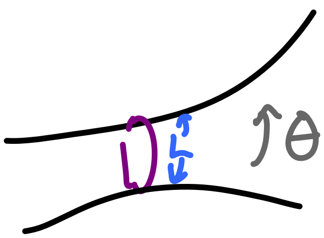


most 3-manifolds have a fundamental group of exponential growth (more later...)

For black holes, it is a strong (simplifying) assumption: Schwarzchild proper not controlled by these methods (at least not yet), though similar effects play a role there.

I. Winding tachyons & Singularities

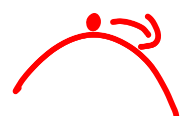
For a string winding around a circle with antiperiodic Fermion boundary conditions, the



string mass^2 includes negative Casimir energy (worldsheet 0-point energy)

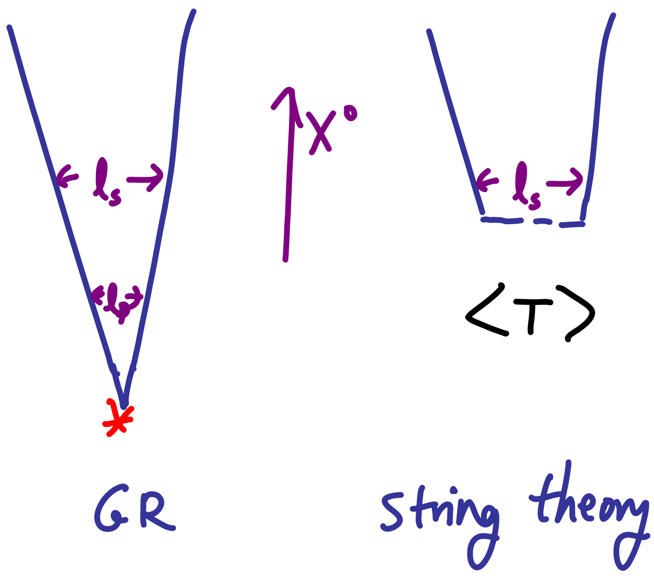
$\rightarrow$ $\text{mass}^2 = \underbrace{-M_s^2}_{\text{Casimir}} + \underbrace{L^2 M_s^4}_{\text{classical energy of stretched string}} + \text{excited modes}$

$L < \frac{1}{M_s} \Rightarrow \text{mass}^2$ negative at short distances $\rightarrow$ "Tachyon" instability



As the circle shrinks below the string scale, the winding string becomes light and then tachyonic $\rightarrow$ condenses

Semiclassically $T \propto e^{\kappa X^0} e^{i(\theta_L - \theta_R) w}$



For control:

- $g_s \ll 1$
- slowly shrinking circle

→ What is the effect of $\langle T \rangle$?

In the worldsheet path integral $\int DX^0 D\tilde{X}\, e^{iS} \prod V$, the integrand has semiclassical action

de Alwis et al '89

$$S \to S_0 + \underbrace{\int d^2\sigma\, \mu^2 e^{-2kX^0}}_{\text{cf mass}^2 \text{ of relativistic particle}} \hat{T}(\tilde{X})$$

$\hat{T}(\tilde{X})$: relevant in worldsheet matter sector

cf mass^2 of relativistic particle Strominger '02

in analogue QFT

suggest degrees of freedom becoming heavy in $\langle T \rangle$ phase.

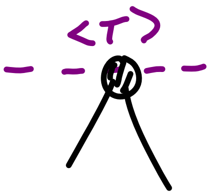
$\langle T \rangle$ acts as potential barrier

This expectation is borne out by explicit calculation:

$\langle T \rangle = \hat{T}\mu e^{\kappa X^0} \Rightarrow$ time-dependent background,

so no a priori preferred vacuum state.

Simplest choice: Out vacuum, related to spatial Liouville theory by Wick rotation. Strominger/Takayanagi '03
McGreevy/ES '05
cf Nakayama
Rey
Sugawara



- Calculate occupation #s of particles in bulk → $N_\omega = \dfrac{1}{e^{\frac{2\pi\omega}{\kappa}} \pm 1}$

- Calculate partition function (quantum correction to stress-energy) → $\mathrm{Re}(Z) = -\dfrac{\ln\mu}{\kappa}\,\hat{Z}_{free}$

not $\delta(0) = \mathrm{Vol}(X^0)$

These results are the same as the corresponding results in a time-dependent field theory where the particles have exponentially growing mass $\propto e^{\kappa X^0}$

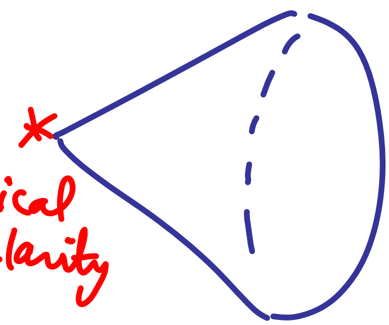
cf higher-pt amplitudes Schomerus, ...

Remarks

- This appears to provide a perturbative string realization of the quantum-cosmo. idea of starting or ending with nothing.
- We are not claiming $|out\rangle$ as the only allowed state: so far we use it to test the hypothesis that $\langle T \rangle$ effectively induces an exponentially growing mass for all modes.
  - Note that $\delta g_{\mu\nu}$, δg_s get heavy, limiting back reaction and interactions
- In specific examples, other tools (D-brane probes, AdS/CFT, ...) apply and corroborate this interpretation.

The tachyonic boundary conditions for Fermions arises naturally in several types of singularities :

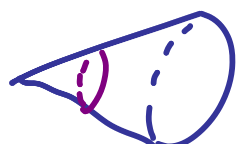
- orbifolds $\mathbb{C}^q/\Gamma$



S^{2q-1}/Γ

Adams Polchinski ES '01

e.g. $\mathbb{C}/\mathbb{Z}_N$



$$\begin{pmatrix} Z \\ S^\alpha \end{pmatrix} \cong \begin{pmatrix} e^{2\pi i\left(\frac{1}{N}+1\right)J} & Z \\ e^{\pm\pi i\left(1+\frac{1}{N}\right)} & S^\alpha \end{pmatrix}$$

N odd

$$g^N = e^{2\pi i(N+1)} = 1$$ also on spinors

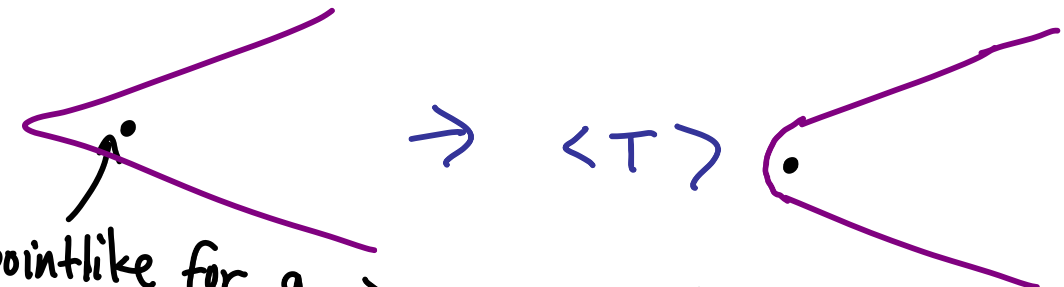
$N \to \infty$

antiperiodic

$$\frac{\alpha'}{4} m^2_{\substack{k\text{th} \\ \text{winding} \\ \text{sector}}} = \begin{cases} -\frac{k}{2N} & k \text{ even} \\ \frac{k-N}{2N} & k \text{ odd} \end{cases} \Rightarrow \text{always get Tachyon}$$

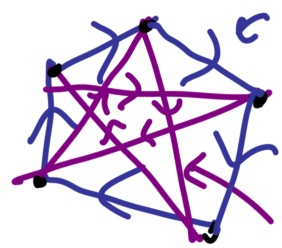
In this case, the initial evolution as the tachyon starts condensing can be understood using

<u>D-brane probes</u>:



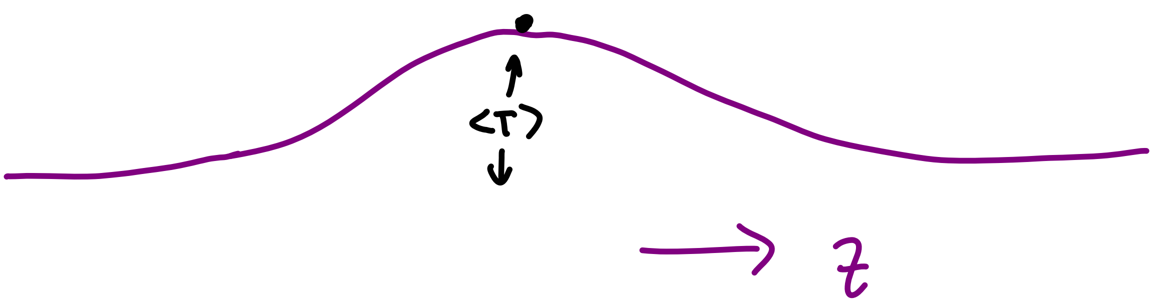
pointlike for $g_s \to 0$: worldvolume gauge theory for $\mathbb{Z}_N$ orbifold is a $U(1)^N$ with bifundamental matter e.g. $N=5$

$$\mathcal{L} = \cdots - \sum_j \left(|Z_{j,j+1}|^2 - |Z_{j-1,j}|^2 - T_j \right)^2$$



As $\langle T \rangle$ turns on, this low energy theory becomes $U(1)$ $\mathcal{N}=4$ SYM, the theory on a D-brane in flat space : the conical singularity is smoothed out.

In particular, a D-brane sitting at the tip of the cone $|z_{i,i+1}|=0$ gets lifted by a worldvolume potential



APS

similarly to the closed strings.

cf Martinec, Moore, Parnachev '02-'05
Minwalla, Takayanagi '03
Karczmarek/Strominger '04
Melnikov, Plessen '05
D. Green '06

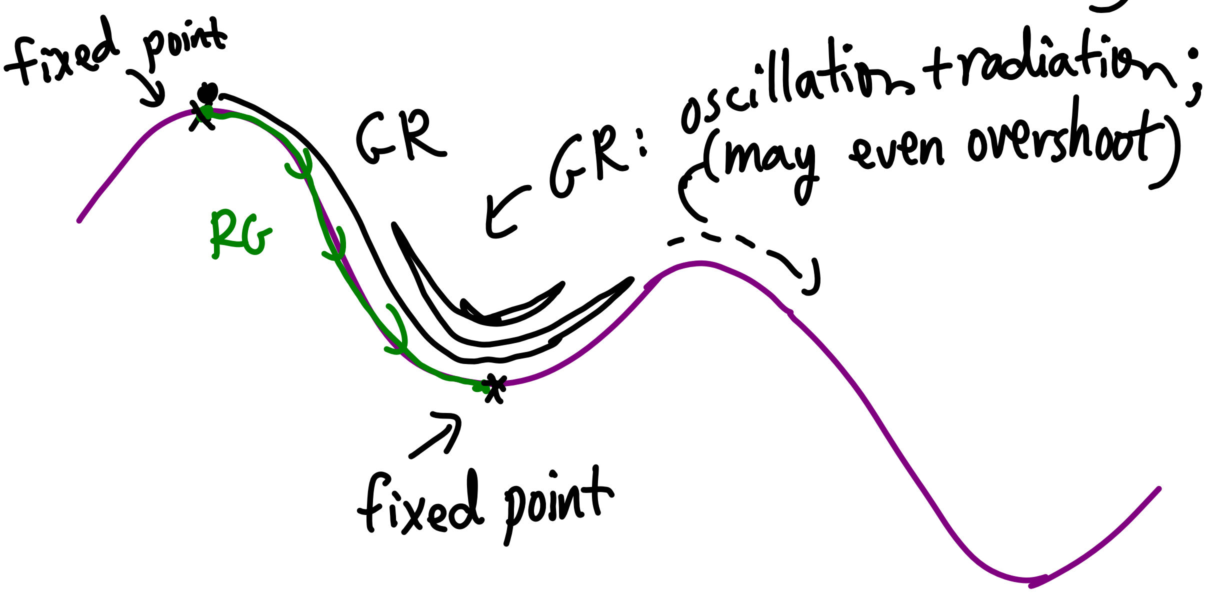
Time evolution vs worldsheet RG flow:

In the worldsheet theory, $\langle T \rangle$ deforms the action by a <u>marginal operator</u> of the form $e^{KX^0}\underbrace{\hat{T}(\vec{X})}_{\text{relevant operator}}$

→ can we conclude spacetime is lifted due to mass gap in $\vec{X}$?

In general, time evolution does not follow RG flow ("GR $\neq$ RG")

e.g.



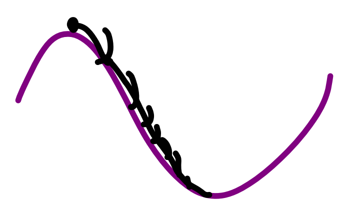
The RG evolution applies in two situations:

① In a background with large friction (e.g. timelike linear dilaton in large-D supercritical regimes of string theory cf Polchinski, Cooper/Susskind, Schmidhuber/Tseytlin, ... Freedman et al, Hellerman/Liu/Swanson, Aharony/ES ...)

the time-dependent evolution approaches

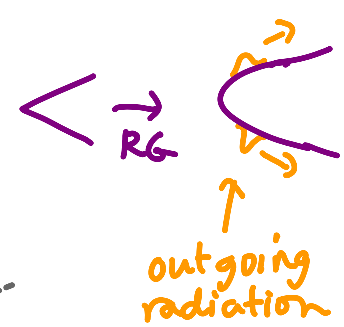
RG: $\ddot{\eta} + Q\dot{\eta} = \beta_\eta$

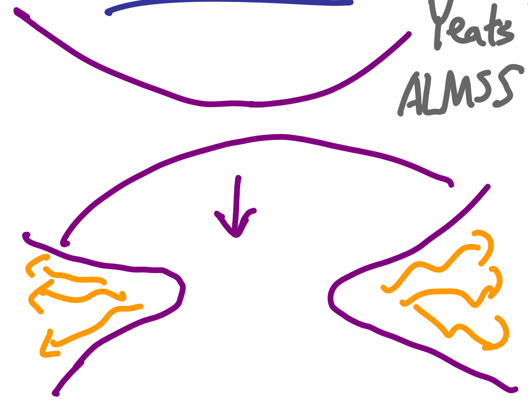
$\uparrow$ friction



② For <u>localized</u> $\langle T \rangle$, the energy escapes in outgoing radiation, so it is plausible that the <u>endpoints</u> coincide.

APS
Vafa
Harvey et al
David et al
Plesser Mumsen
Narayan ...



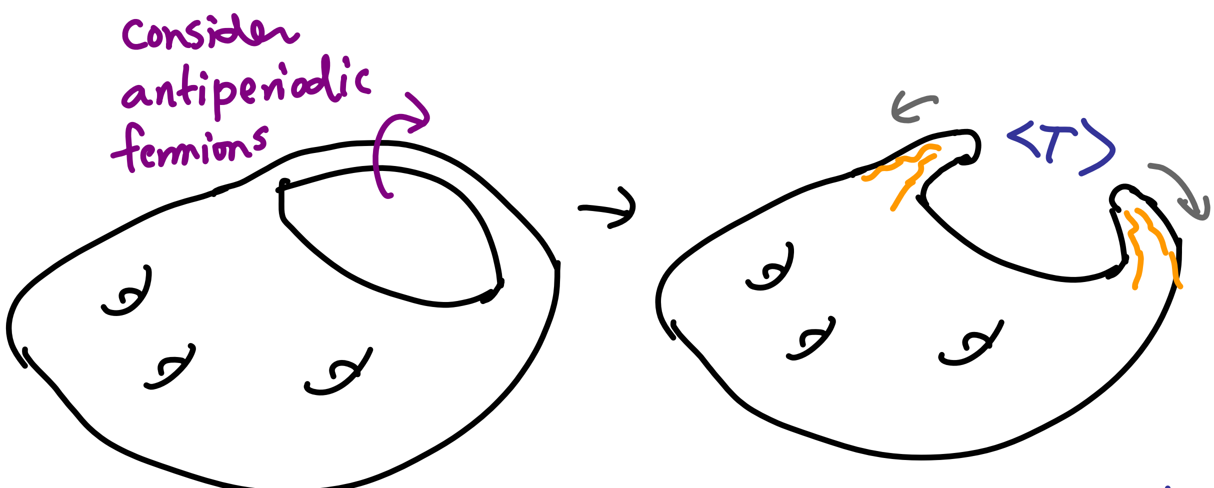


Yeats '19
ALMSS '05

- Having learned the basic effect of $\langle T \rangle$, we can study examples more interesting for spacetime singularity resolution.

- Topology change: Adams Liu McGreevy Saltman ES '05

Consider target space containing a Riemann surface (solving field eqns with time evolution, and/or metastabilize with other sources)

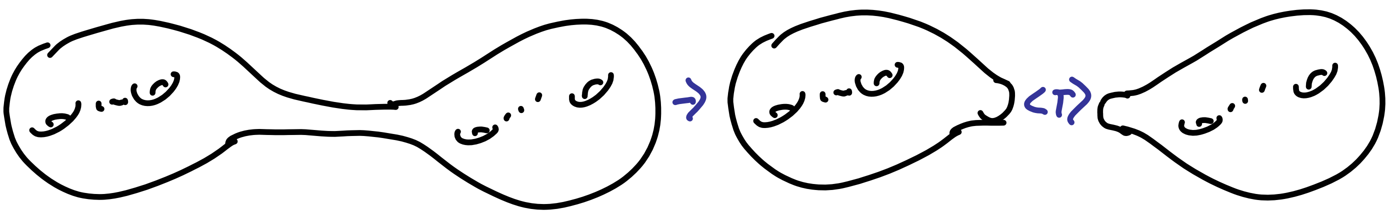


$\Rightarrow$ In String theory, $\exists$ transitions changing b_1

e.g. torus $\longleftrightarrow$ sphere

Things Fall Apart

- A consequence of this is that there also exist transitions changing b_0 (# of connected components):



→ Interesting questions about unitarity etc.*

Philosophy-term-dominated discussion may proceed at end...

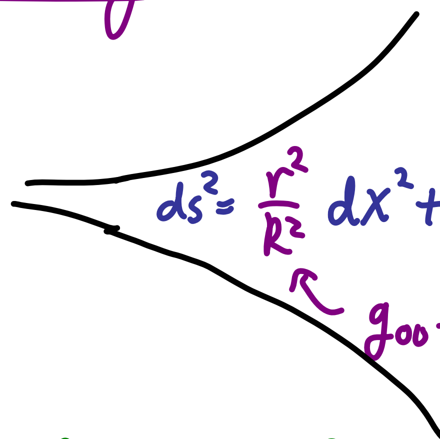
- <u>Quasilocal tachyons</u>

G. Horowitz '05
Horowitz ES '05

$\langle T \rangle$ can occur over an extended finite spacelike region.

1) AdS/CFT : Gauge/gravity duality extends to <u>confining</u> theories, with a mass gap.

Conformal case :

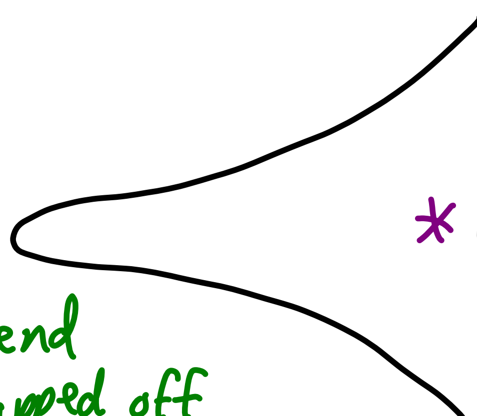


$ds^2 = \frac{r^2}{R^2} dx^2 + \frac{R^2 dr^2}{r^2} + ds_\perp^2$

$g_{00} \to 0$ at horizon

← QFT IR QFT UV →

gapped case :



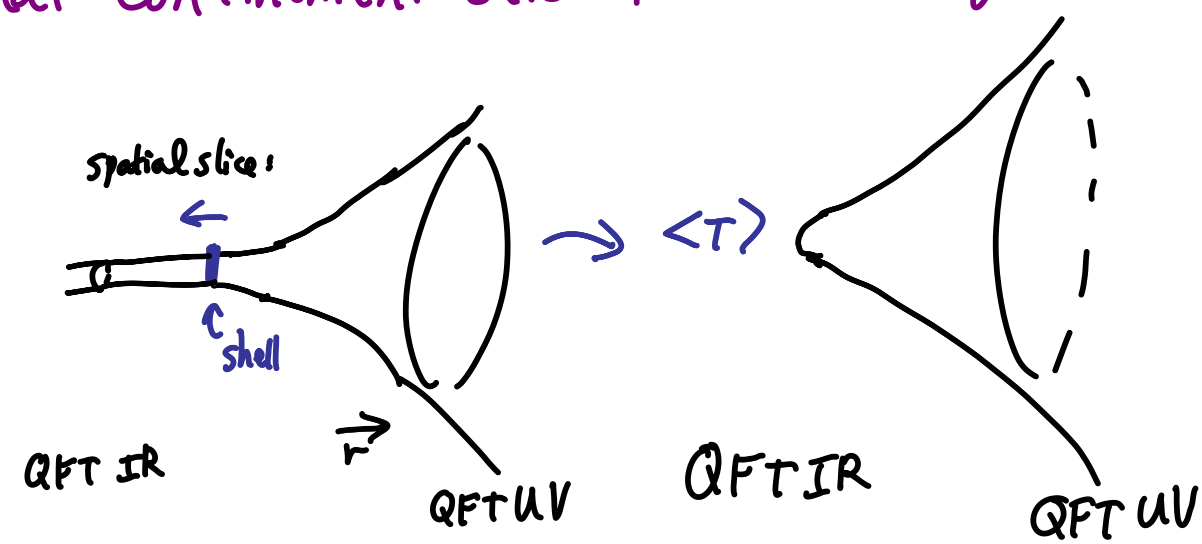
Polchinski-Strassler
Klebanov-Strassler
* Horowitz-Myers
...

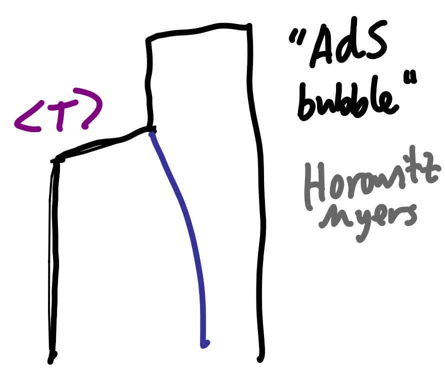
QFT IR end smoothly capped off

These can be connected via a tachyon condensate excising the IR region :

Consider the setup corresponding to $\mathcal{N}=4$ SYM on a circle with antiperiodic Fermion b.c.

Start in Coulomb phase $U(1)^N$ induced by scalar VEVs $\langle \phi \rangle$. Roll $\langle \phi \rangle \to 0$ so that confinement sets in at low energies.



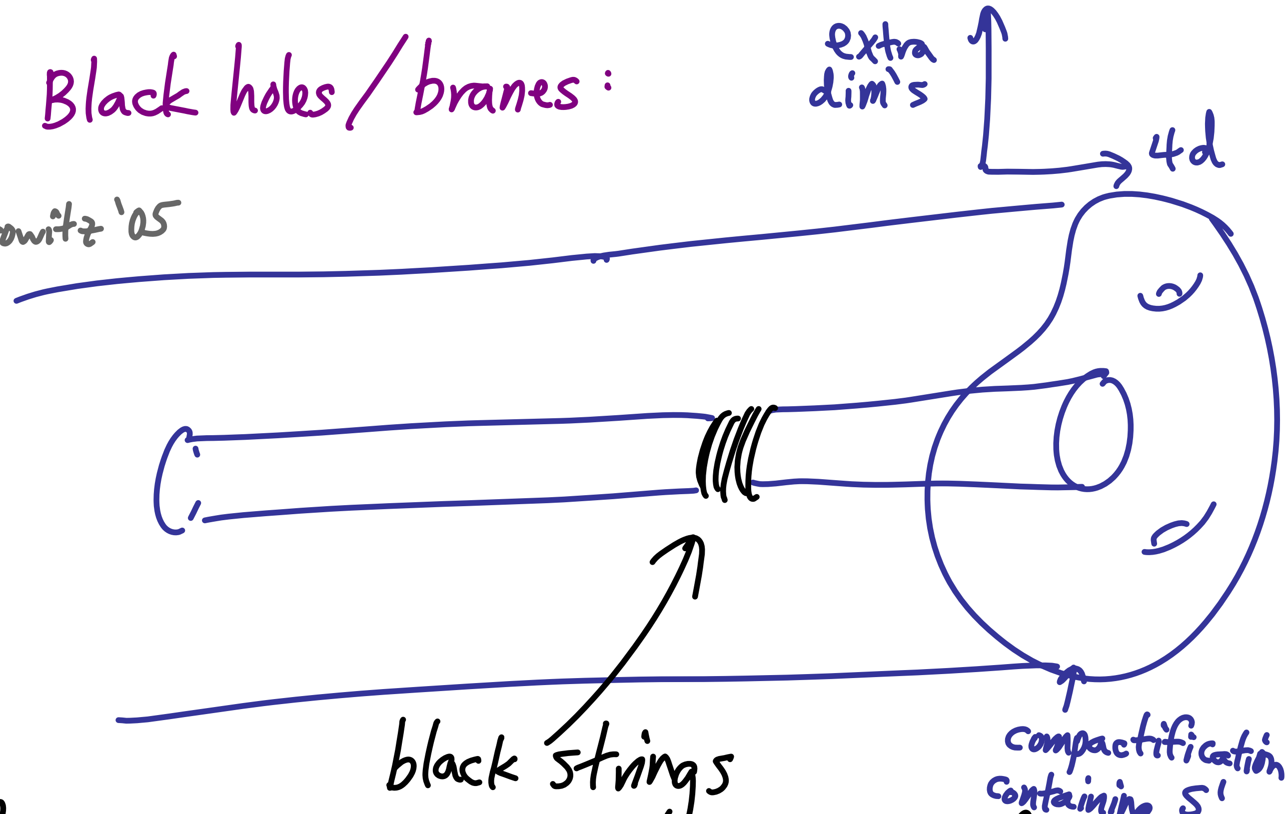


The fact that $\langle T \rangle$ excises spacetime fits neatly with dual QFT dynamics.

cf Nishioka, Takayanagi '06 entropy checks

2) Black holes / branes:

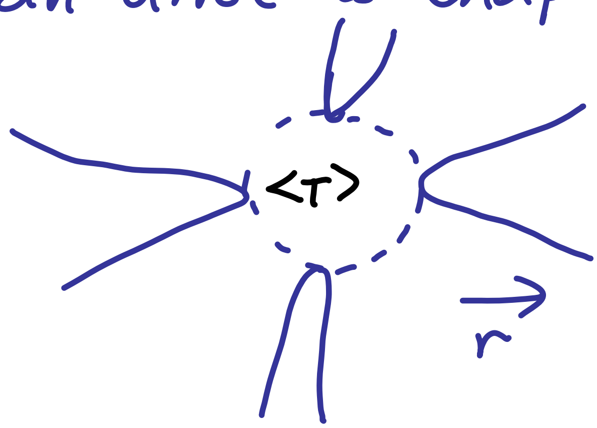
Horowitz '05



locally deform S^1 to shrink below l_s for sufficiently small M/Q. Can occur either outside or inside the horizon

outside: $\Rightarrow$ bubble of nothing*

can arise as endpoint of Hawking process!



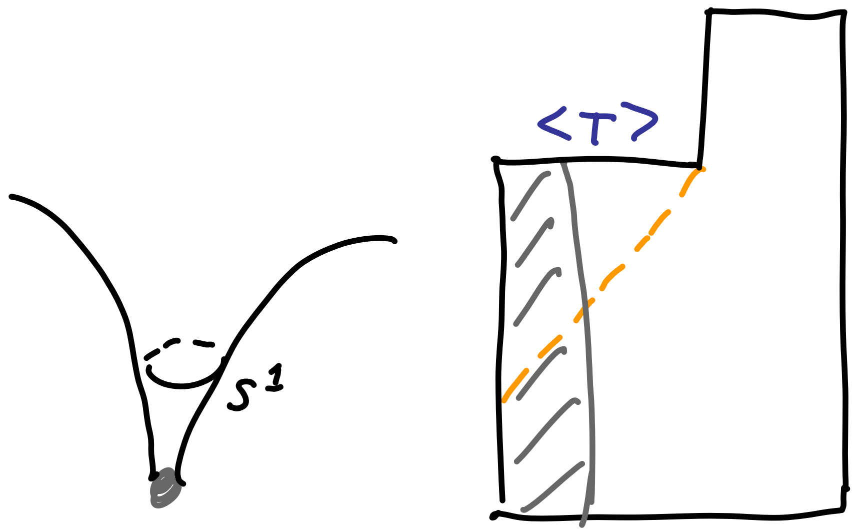
Catalyzed classically by $\langle T \rangle$

cf Witten, Brill/Horowitz, ..

inside: $\langle T \rangle$ replaces spacelike BH singularity

Horowitz / ES '06

- Also true for 3d BTZ uncharged BH $\subset AdS_3$ (this is the (2+1) dim'l analogue of Schwarzchild).

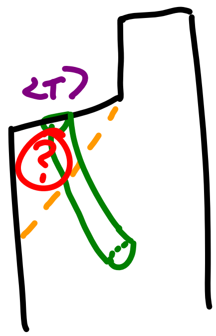


$l_s << r << R_{AdS}$

$\Rightarrow$ slowly shrinking cylinder with antiperiodic Fermion b.c.

$\hookrightarrow$ can we say anything about unitarity?

– requires understanding allowed state(s) in the $\langle T \rangle$ phase.



✗ What about other states? What happens to a particle/string sent into the <T> phase?

A priori, challenge for bulk spacetime unitarity

- Worldsheet path integral has saddle point classical solution with single string stuck in <T> phase
- Analogue QFT with $m^2(X^0,\vec{X}) = f(\vec{X})\,\mu\, e^{2\kappa X^0}$ has the property that particles get stuck and wavepackets stop expanding in massive region
- <T> $\rightarrow$ all modes massing up, including gravity multiplet, so back reaction caused by massive source is suppressed.

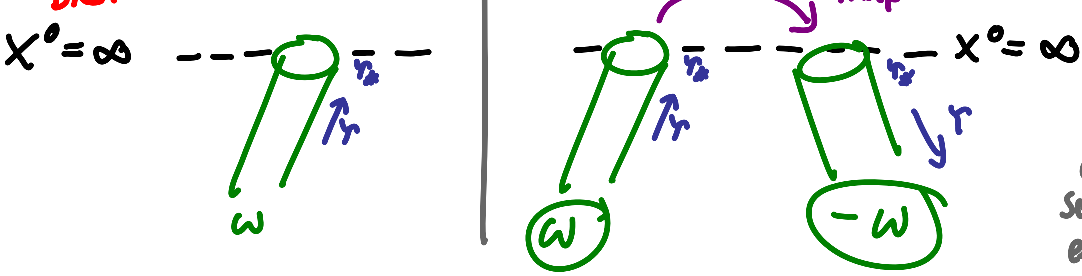
. However, these features do not survive in the full string theory, for 2 reasons:

① BRST invariance (decoupling of Q_B-trivial modes)

The strings impinge on $X^0=\infty$ at finite $\tau \equiv \tau_*$

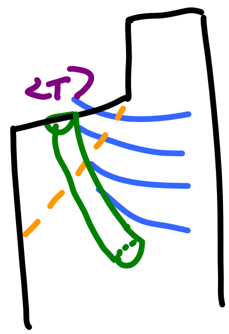
hole in worldsheet not BRST-invariant by itself

But $\mathcal{S}$ unitary map removes this BRST anomaly



cf Schomerus self-adjoint extension

② Dynamics :



Infalling string sources fields (e.g. gravitational field).

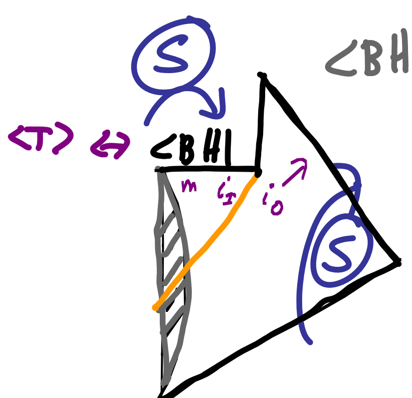
This field gets heavy in the <T> phase as well. In QFT analogue $m^2(x^0,\vec{x}) = f(\vec{x})M^2(x^0) + m_0^2$

$$E = m_0^2 \underline{\lambda^2 M(x^0)} \cos^2\left(\int^{x^0} M(t')dt'\right) \times \int d^{d-1}x\, f(|\vec{x}|)\left(\int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{e^{i\vec{k}\cdot\vec{x}}}{\omega_{\vec{k}}^{3/2}}\right)^2$$

→ forces out configurations sourcing the heavy fields

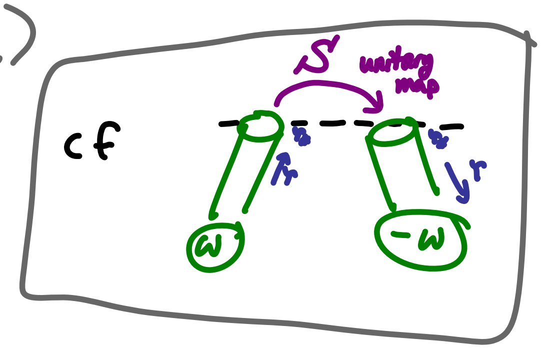
e.g. gravitational fld ⇒ $\Sigma\omega = 0$

This suggests microphysical realization of the "Black Hole Final State" Horowitz/Maldacena



$$\langle BH|\Psi\rangle = \underbrace{|i\rangle_0 S^{im}}_{\text{unitary}} \langle m|\Psi_m\rangle$$

$$|\Psi\rangle = |\Psi\rangle_m \otimes \Sigma |i\rangle_I \otimes |i\rangle_0$$



At linearized level, $\langle BH| = S^{mi} \langle m| \otimes \langle i|_I$

↗ unitary matrix ↗ matter ↖ inner Hawking

Relations to other approaches?

- BKL etc. Berger Garfinkle ...

  Velocity dominance → tachyon dominance in these examples?

- Other perturbative attempts Liu Moore Seiberg Berkooz Rozali ..

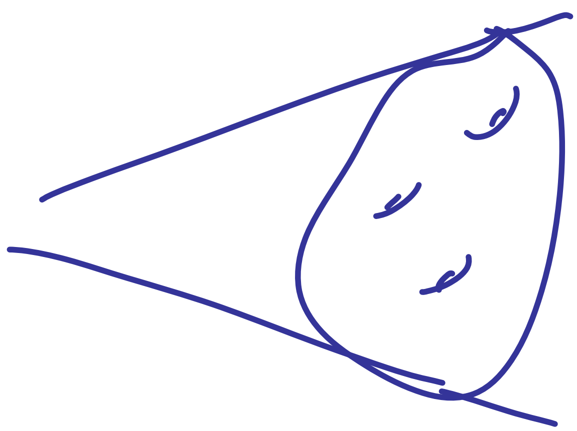
  For periodic F boundary conditions → no tachyon, but winding modes created by time-dependent mass due to shrinking circle. (Schwarzschild has both effects...)

- AdS/CFT correlators Shenker et al Hubeny Rangamani Liu ..

  See effect of $\langle T \rangle$ in analytic structure of correlators?

II. New Dimensions from Wound Strings and D-duality

- ES '05
- McGreevy ES Starr '06
- Green Lawrence McGreevy Morrison ES in progress



most n-manifolds have a fundamental group of exponential growth $\Rightarrow$

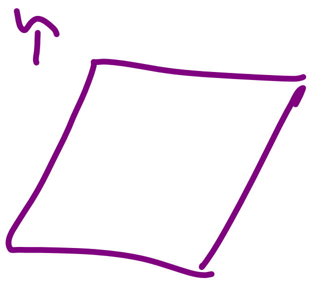
$$\rho(\ell) = e^{\ell/\ell_0} = e^{(\text{mass})\alpha'/\ell_0}$$

# of closed geodesics of length ℓ

(True of all negatively curved spaces Milnor Margulis Selberg ...)

$\Rightarrow$ Winding strings in themselves contribute a Hagedorn density of single-string states

Read off UV density of states from the single-string partition ftn:



$$= \mathrm{Tr}\int \frac{d^2\tau}{4\tau_2}(-1)^F q^{L_0}\bar{q}^{\tilde{L}_0}$$

where $q = e^{2\pi i\tau_1}e^{-2\pi\tau_2}$, $L_0+\tilde{L}_0$ = worldsheet Hamiltonian

$$\sim \int \frac{d\tau_2}{\tau_2}(-1)^F \sum_m \rho(m)\, e^{-\pi\tau_2 \alpha' m^2}$$

$\tau_2 \to \infty$ IR
$\tau_2 \to 0$ UV

related by modular invariance

$$\sim \int \frac{d\tau_2}{\tau_2}\, e^{\frac{\pi c_{eff}}{6\tau_2}}$$

"effective central charge" = dimension D in GR limits

cf Cardy, Kutasov/Seiberg.

For constant negative curvature solutions (in vacuum, expand with scale factor $a(t) \propto t$), one calculates

$$C_{\text{eff}} = C_{\text{eff}}^{\text{critical}} + \frac{3\alpha'(n-1)^2}{2t^2}$$

which agrees precisely with the deep IR behavior predicted by modular invariance. → New effective* dimensions emerge from topology!

* → Is there a dual description in terms of supercritical string theory?

Note: String theory a priori admits solutions in diverse dimensions:

$$S_D = \int d^D x\, e^{-2\Phi} \left(M_p^{D-2} R + M_p^{D-2} (\partial\Phi)^2 - \frac{D - D_{crit}}{\alpha'} + \cdots \right)$$

In the critical dimension, there are classical <u>exactly</u> flat solutions. Generically, there is a tree-level moduli potential. Both are consistent with realistic compactifation down to <u>four</u> large dimensions.

Supercritical case:

$$\underset{\substack{\text{Einstein}\\\text{frame}}}{U_{4d}}(\Phi) \sim e^{2\Phi} \frac{\left(c_{eff} - c_{eff}^{crit}\right)}{\alpha'} \times \left\{ \begin{matrix}\text{function}\\ \text{of other}\\ \text{moduli}\end{matrix} \right\}$$

D-duality

Before addressing our case, recall T-duality for strings on a T^n

momentum modes $\quad p \sim \frac{n}{R}$

winding modes $\quad w \sim n\frac{R}{\alpha'}$

$R \to \frac{1}{R}$

small circle $\cong$ large circle

- Path integral derivation: Buscher, Rocek/Verlinde ...

$$S = \int d^2\sigma \left\{ R^2 (\partial\theta - A)^2 + \hat{\theta} F \right\}$$

- Integrate $\hat{\theta}$ out $\Rightarrow F = 0 \Rightarrow A$ pure gauge

$$\Rightarrow \quad S \to \int d^2\sigma \frac{R^2}{\alpha'} (\partial\theta)^2 \qquad S^1_R$$

- Integrate A out $\Rightarrow$

$$S \to \int d^2\sigma \frac{\alpha'}{R^2} (\partial\hat{\theta})^2 \qquad S^1_{\alpha'/R}$$

- D-brane probes see the dual torus as moduli space of Wilson lines

Consider e.g. a Riemann surface $\mathcal{M}_2$ of genus h. There are $2h$ conserved winding charges, but no conserved momentum. (cf ALE / NS5 duality Ooguri Vafa & Tong, Strominger Yau Zaslow mirror symmetry)

- Wrapped D-brane has a T^{2h} moduli space : the Jacobian torus

$$X \equiv X + \underbrace{\underbrace{\oint_{z_0}^{z}}_{\text{closed basis loop }\gamma} \underbrace{\omega}_{\text{holomorphic 1-forms on RS}}}_{h \text{ complex } h\text{-vectors}}$$

→ Is there a dual description in terms of T^{2h} ?

T^{2h} by itself is manifestly different from a large-radius Riemann surface $\mathcal{M}_2$

* But T^{2h} + a tachyon condensate $T(X)$ would be consistent, where $T(X)$ provides a potential energy restricting the string to $\mathcal{M}_2$ at late times.

* $T(X)$ breaks the momentum symmetry but leaves the $2h$ winding charges unbroken ✓

worldsheet path integral:

$$\int Dz\, e^{-\int d^2\sigma \left\{ \underbrace{\omega^a \partial_\gamma z\, \mathrm{Im}\,\Omega^{-1}_{ab}\, \omega^b \partial^\gamma \bar{z}}_{\substack{\text{pullback of flat} \\ \text{metric on Jacobian } J \\ \text{for Riemann surface}}} \right\}} \prod_{j=1}^{n} O_j[z]$$

embedded in J: $\int^z \omega^a = X^a$, $a = 1 \cdots h$

evolves slowly via $\Omega_{ab}(X^0)$ [take supercritical limit to get RG evolution]

gauge the axial symmetry →

$$S = \int d^2\sigma \left\{ (\omega \partial z - *A)_\gamma\, \mathrm{Im}\,\Omega^{-1} (\bar{\omega} \partial \bar{z} - *\bar{A})^\gamma + X\, \mathrm{Im}\,\Omega^{-1} \bar{F} + \bar{X}\, \mathrm{Im}\,\Omega^{-1} F + \frac{F\bar{F}}{e^2} \right\}$$

$\delta_X S = 0 \Rightarrow F = 0$, A pure gauge

$\Rightarrow$ recover original model

$$S = \int d^2\sigma \Big\{ \left(\omega^a \partial z - *A\right)_\gamma \mathrm{Im}\,\Omega^{-1}_{ab} \left(\bar{\omega}^b \partial \bar{z} - *\bar{A}\right)^\gamma$$

$$+ X\,\mathrm{Im}\,\Omega^{-1} \bar{F} + \bar{X}\,\mathrm{Im}\,\Omega^{-1} F$$

$$+ \frac{F\bar{F}}{e^2} \Big\}$$

$\delta_A S = 0$ classically $\longrightarrow$

$$S \to \int d^2\sigma \Big\{ \partial X^a\, \mathrm{Im}\,\Omega_{ab}\, \partial \bar{X}^b$$

$$+ X\, \partial_\gamma \left(\omega \partial^\gamma z\right) + \bar{X} \partial_\gamma \left(\bar{\omega} \partial^\gamma \bar{z}\right) \Big\}$$

$$\delta_z S = 0 \Rightarrow \bar{\omega}^a\, \mathrm{Im}\,\Omega^{-1}_{ab}\, \Box X^b = 0$$

Quantum mechanically, expect potential and other effects to be generated. cf Tong

* Rank $2h$ kinetic term arises from $2h$ winding symmetries

Simpler candidate formal derivation

$$A_n = \int Dz\, e^{-\int d^2\sigma \left\{ \omega^a \partial_\tau z \,\mathrm{Im}\,\Omega^{-1} \omega^b \partial^\tau \bar{z} \right\}} \prod_{j=1}^{n} \mathcal{O}_j[z]$$

$$= N \int Dz\, DX\, e^{-\int d^2\sigma \left\{ (\omega \partial z)^2 + (\omega\partial z - \partial X)^2 + U\left(X - \int_\omega^z\right) \right\}} \prod_{j=1}^{n} \mathcal{O}[z]$$

where $N = \int DK\, e^{-\int (\partial K)^2 + U(K)}$

since integrating out X leads to the original expression.

Integrate out $z \rightarrow$

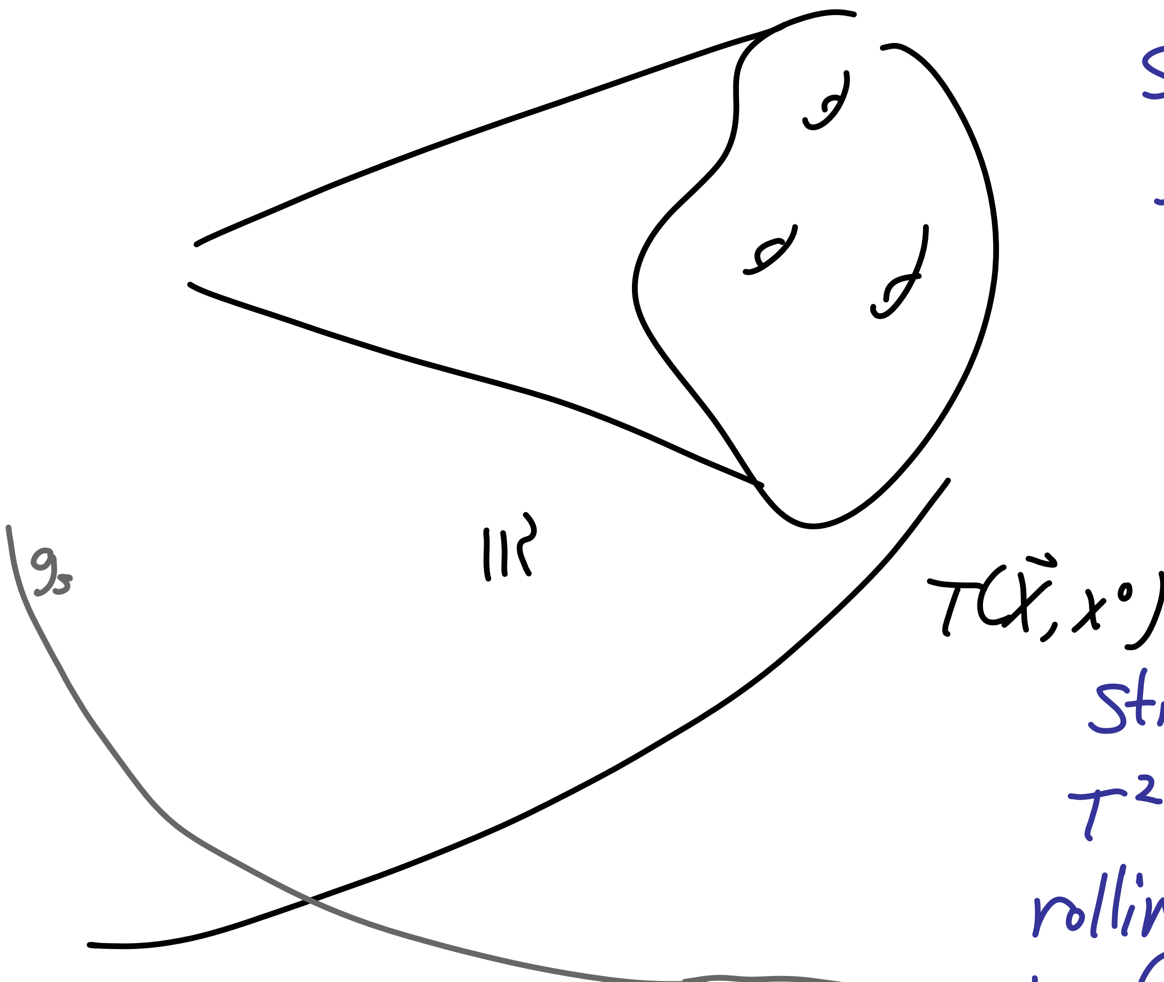
$$A_n = \int DX'\, e^{-S[X']}\, \mathcal{O}[X']$$

where $S[X']$ has rank-$2h$ kinetic term, plus a potential term which vanishes on $\mathcal{M}_2 \subset T^{2h}$ ✓

Preserving the $U(1)^{2h}$ winding symmetry,*
we expect the minimal c_{eff}
attained by $\mathcal{M}_2$ as it shrinks
in the far past to be $2h$, with
the tachyon turning off, leaving
the system in a T^{2h} + dilaton

- Other solutions are possible with additional degrees of freedom
- *Winding tachyons may come along and break the symmetry for some configurations.

D-duality (conjecture, maybe derivable from path integral + symmetries)

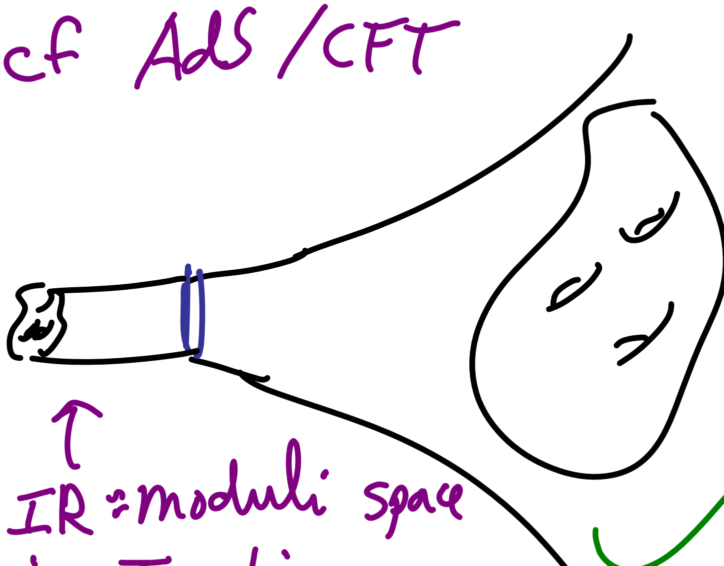


strings on $\mathcal{M}_2$

strings on T^{2h} with rolling closed string tachyon

- Upshot: in (generic) spaces, new dimensions emerge from topology. $\Rightarrow$ The resolution of the corresponding spacelike singularity in the framework of string theory will involve these degrees of freedom.

cf AdS/CFT



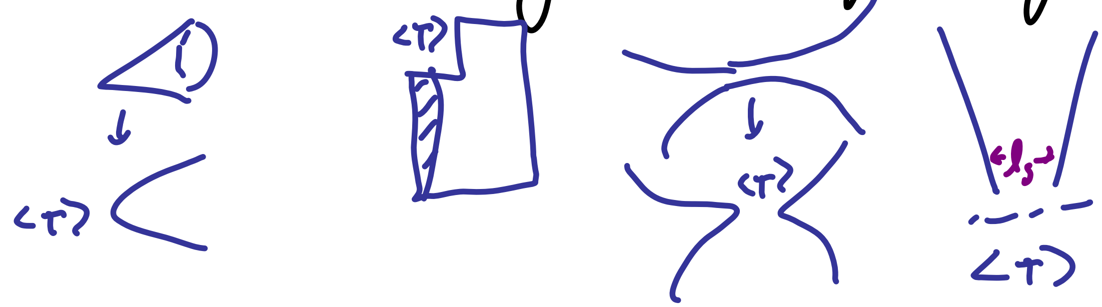
CFT on $\mathcal{M}_3 \times$ time

May provide non-perturbative formulation...

IR: moduli space is Jacobian

Summary   2 Cases

I. Winding tachyon condensate dominates, evading GR singularity



II. Winding mode spectrum builds up new effective dimensions