Queuing in the asymmetric exclusion process, faceting in paper combustion, and polymer localization

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and Marcel den Nijs

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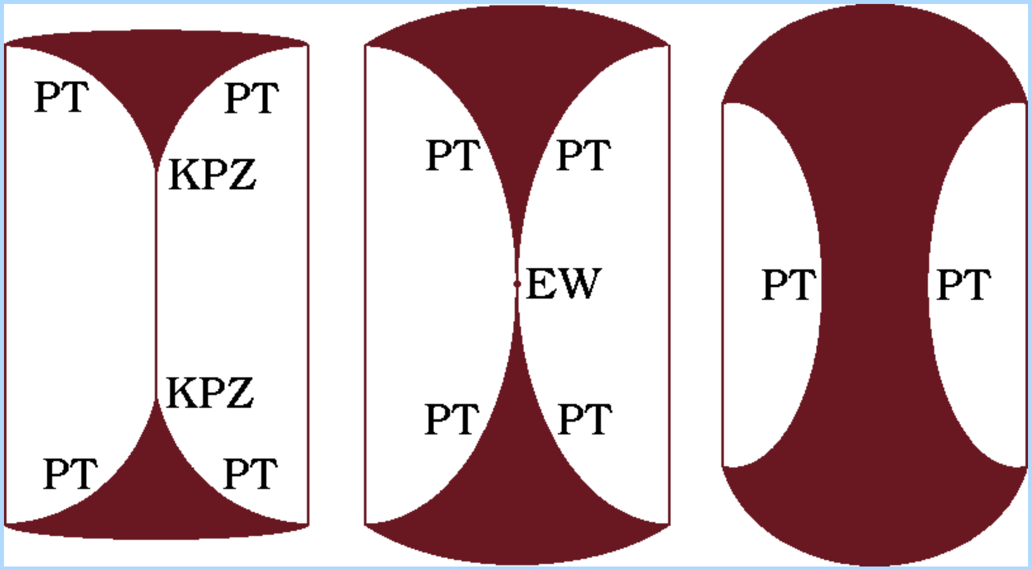
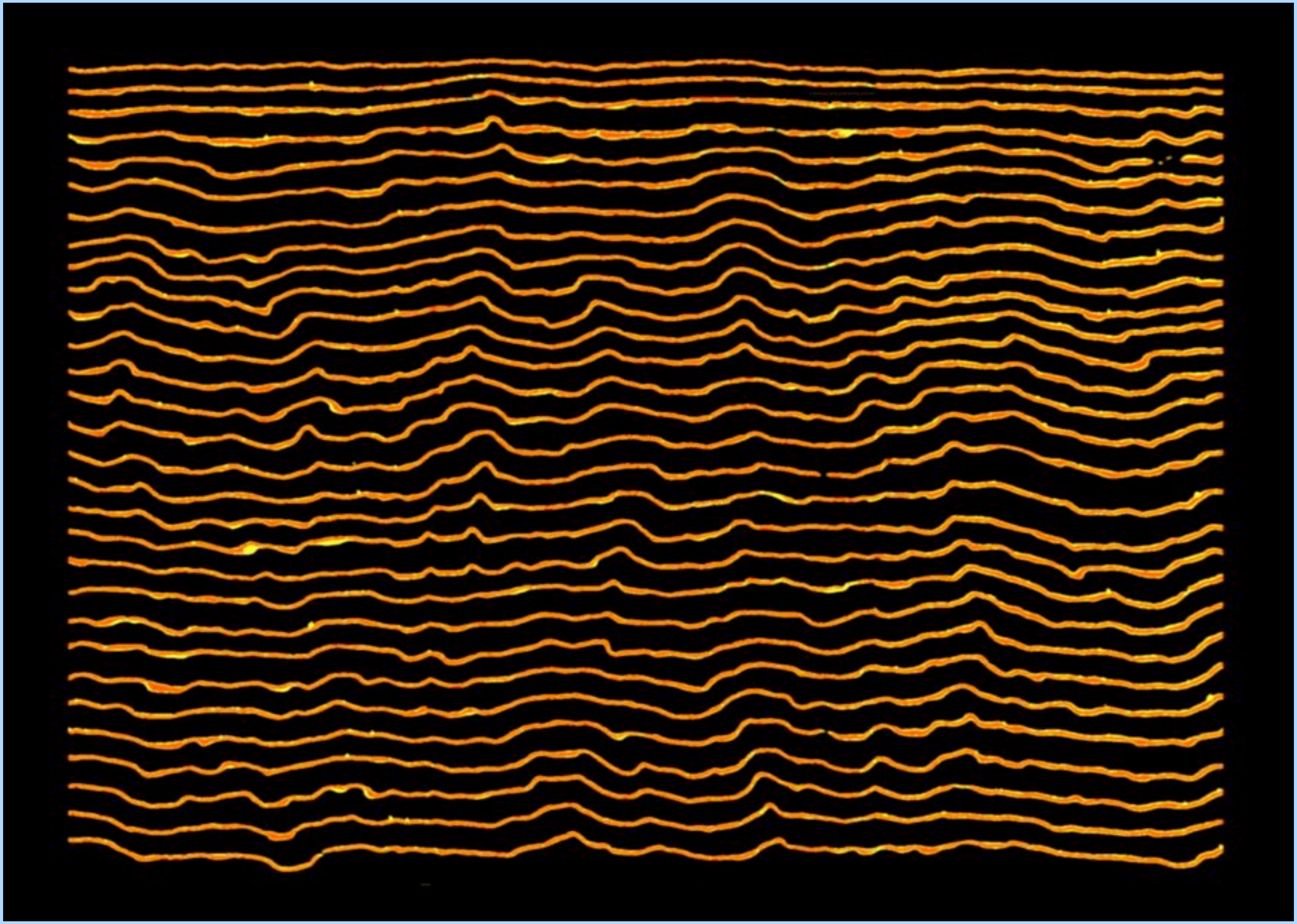
driven stochastic flow through1D channels

1+1D KPZ surface growth

2D polymer localization

facet ridge end points in 3D equilibrium crystals





KPZ Langevin equation

If the local geometry of the interface is the only relevant degree of freedom, then the large scale properties of its evolution must be governed by (universality):

$$\frac{d}{dt}h(\vec{r},t) = v_0 + \nu\nabla^2 h + \frac{1}{2}\lambda(\nabla h)^2 + \eta$$

the KPZ equation with uncorrelated noise

$$\langle \eta_{r_2,t_2}\eta_{r_1,t_1}\rangle = 2\Gamma\delta_{r_1,r_2}\delta_{t_1 t_2}$$

The growth rate v_0 is modified by the local curvature of the surface (the ν-term) and its local slope (the λ-term), and random fluctuations in the paper (density, flocking, potassium nitrate concentration).

Surface roughness

The moments of the height distribution

$$W_n(L,t) = L^{-1} \sum_r \langle (h_r - h_{av})^n \rangle$$

scale as

$$W_n(t,L) = b^{n\alpha} W_n(b^{-z}t, b^{-1}L)$$

$$\alpha = \tfrac{1}{2} \qquad z = 2 - \alpha = \tfrac{3}{2}$$

SLOW FLAMELESS COMBUSTION OF PAPER

Alava, Ala-Nissila, Merikoski, and Timonen,
PRE 64 036101, 2001.

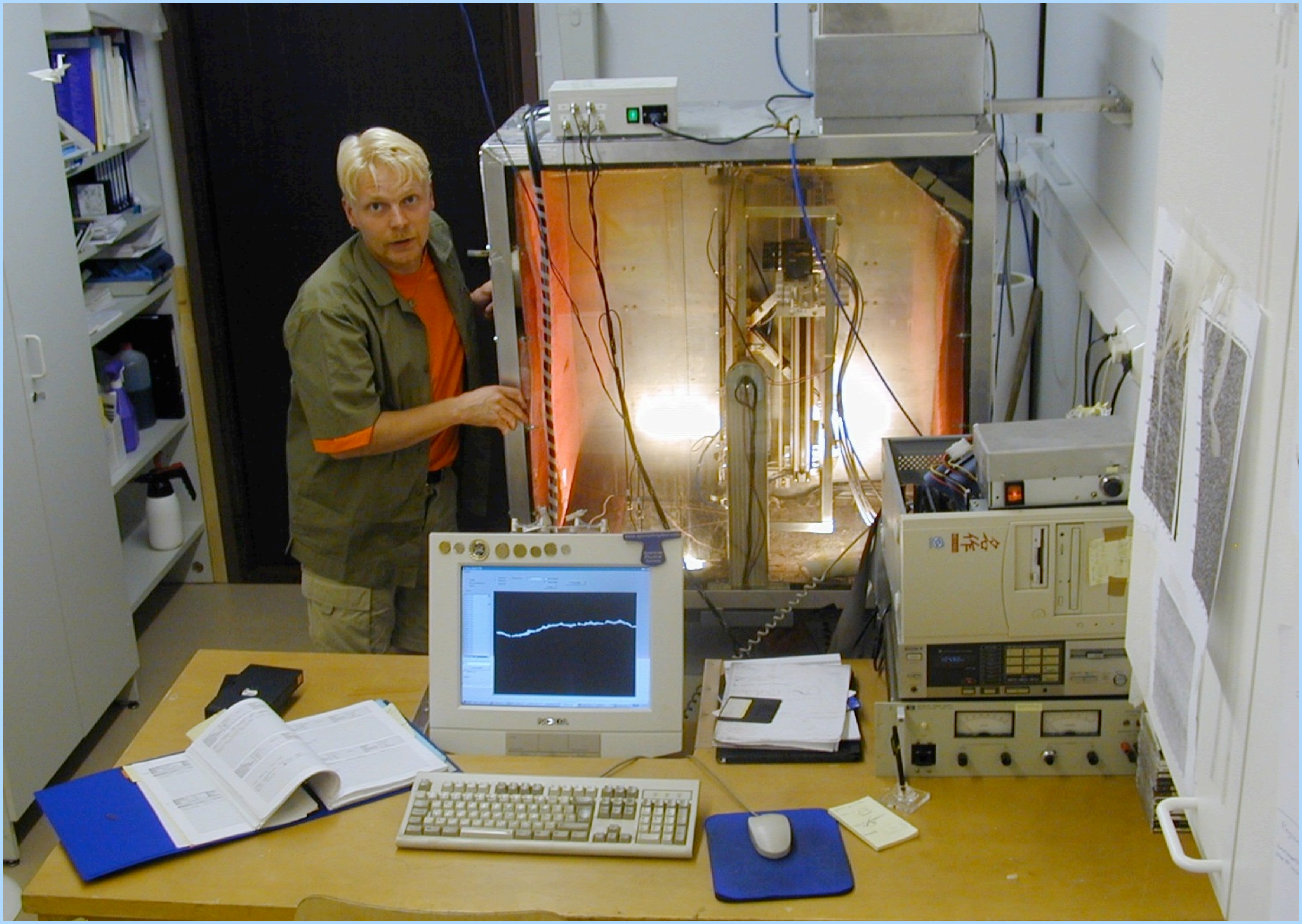
One of the few established realizations of KPZ growth.

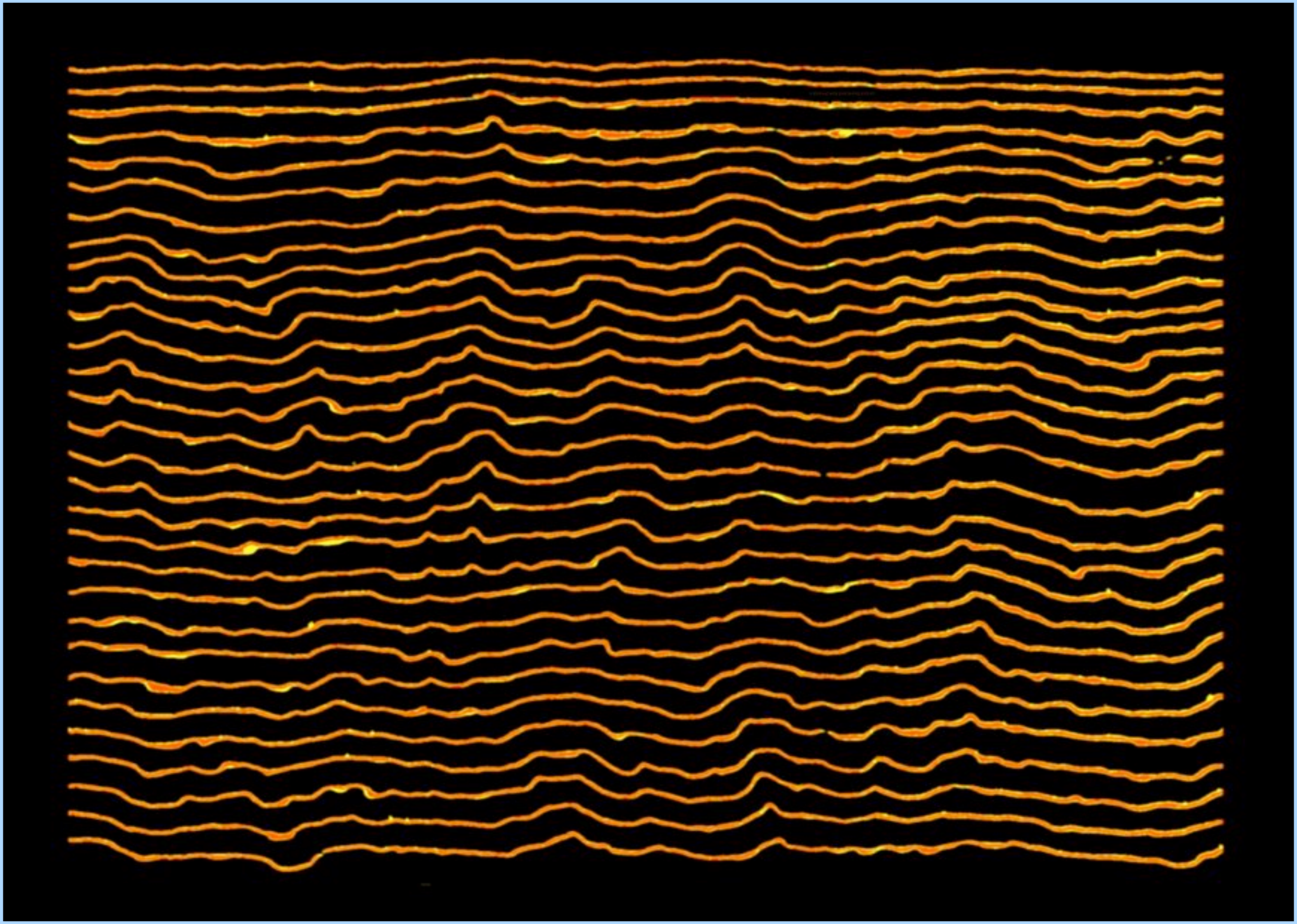
Only at length scales $l \geq 0.8$ cm (quenched impurities).

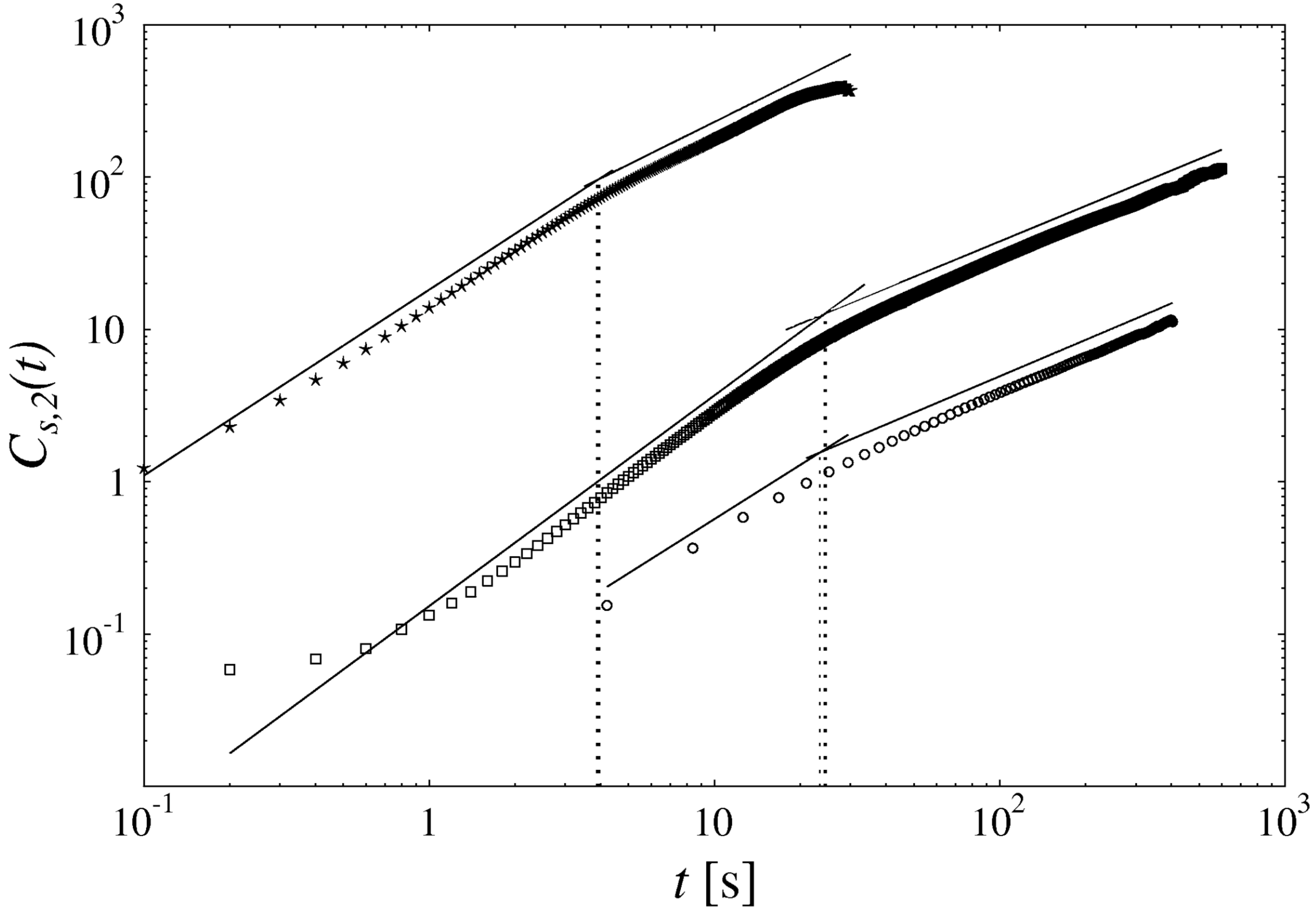
Impregnate paper with potassium nitrate.

Average over hundreds of independent burns.

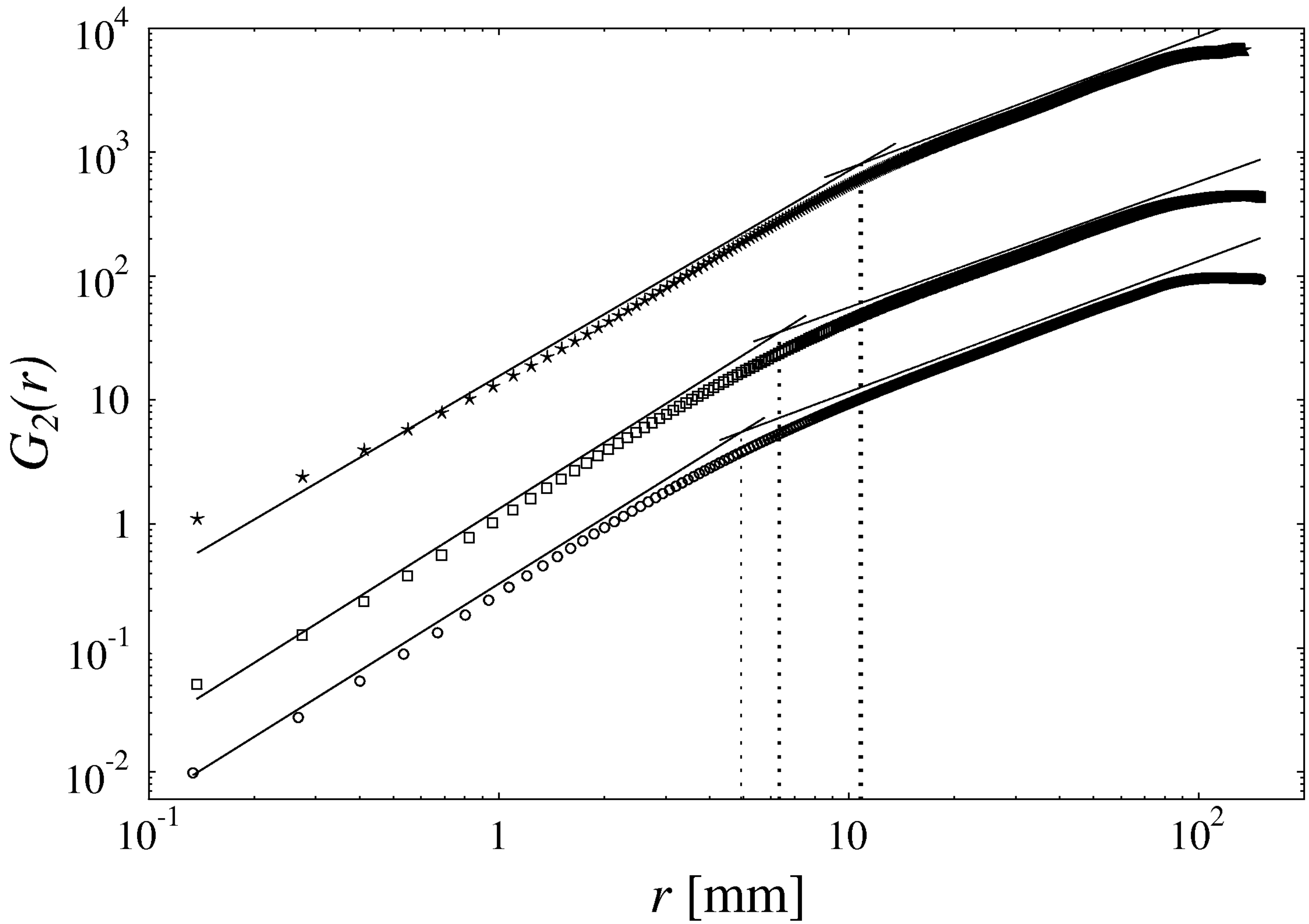
Paper grade	α	β	g^*	$r_{crossover}$	$t_{crossover}$
70 g/m^2	0.50(4)	0.36(3)	0.79(9)	4.7(4)	25(9)
80 g/m^2	0.47(4)	0.34(4)	0.76(8)	6.0(5)	27(5)
9.1 g/m^2	0.50(6)	0.43(6)	1.0(2)	11(2)	3.7(4)







$$C(t) = \langle \Big((h_x - \overline{h})_{t+t_0} - (h_x - \overline{h})_{t_0} \Big)^2 \rangle \sim t^{2\beta}$$

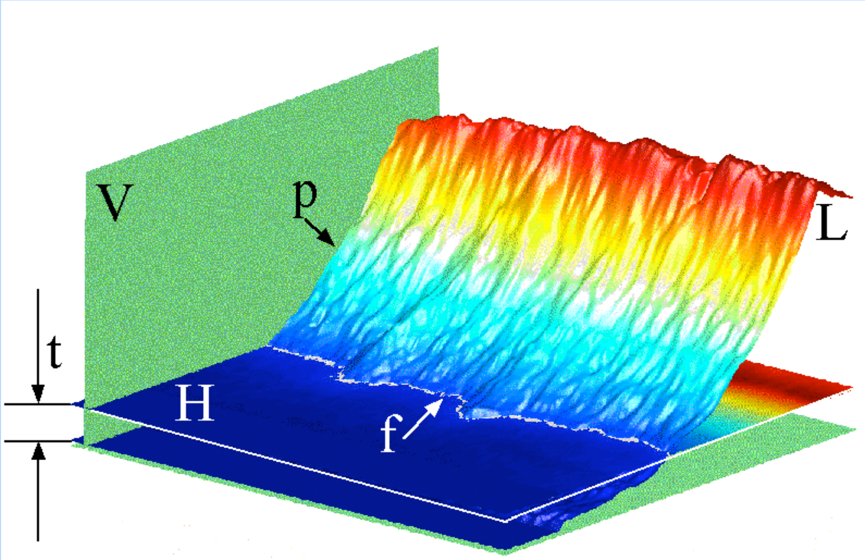


$$G_2(r,t) = \langle (h_{r+r_0} - h_{r_0})^2 \rangle_t \sim r^{2\alpha}$$

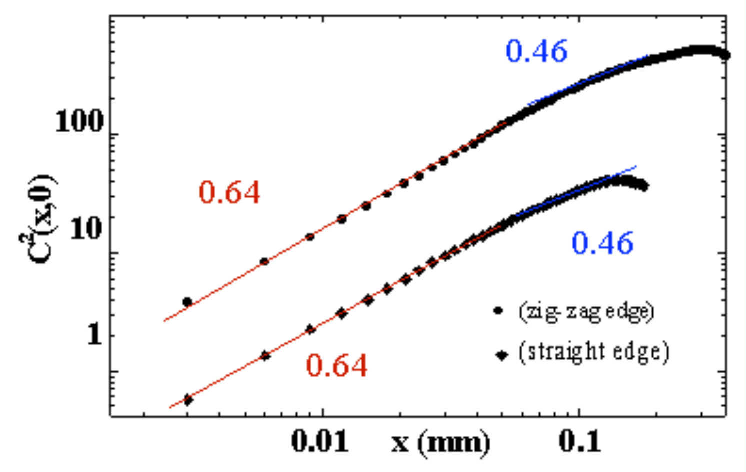
Another example of KPZ growth:

Flux front propagation in High Tc super conductors

Wijngaarden's group
at the VU in Amsterdam
PRL 83, 2064 ('99)

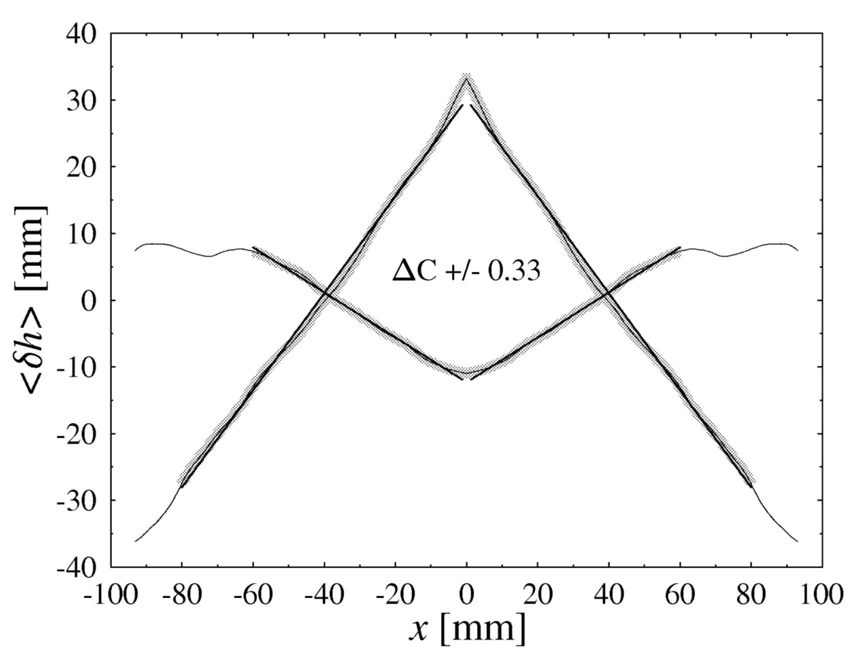


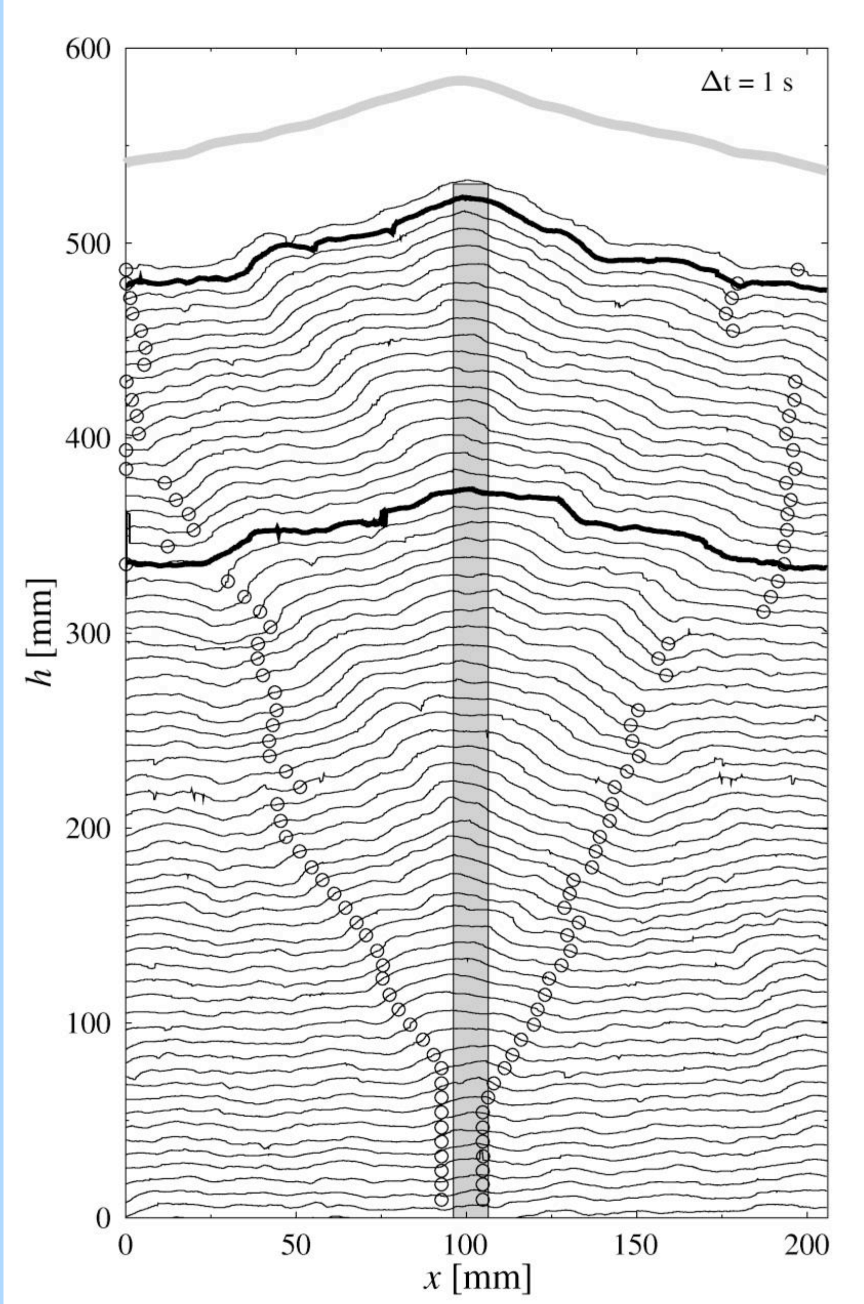
Spatial distribution of vortex density (plotted along the vertical axis) in a $YBa_2Cu_3O_7$ thin film in a field of 11 mT.



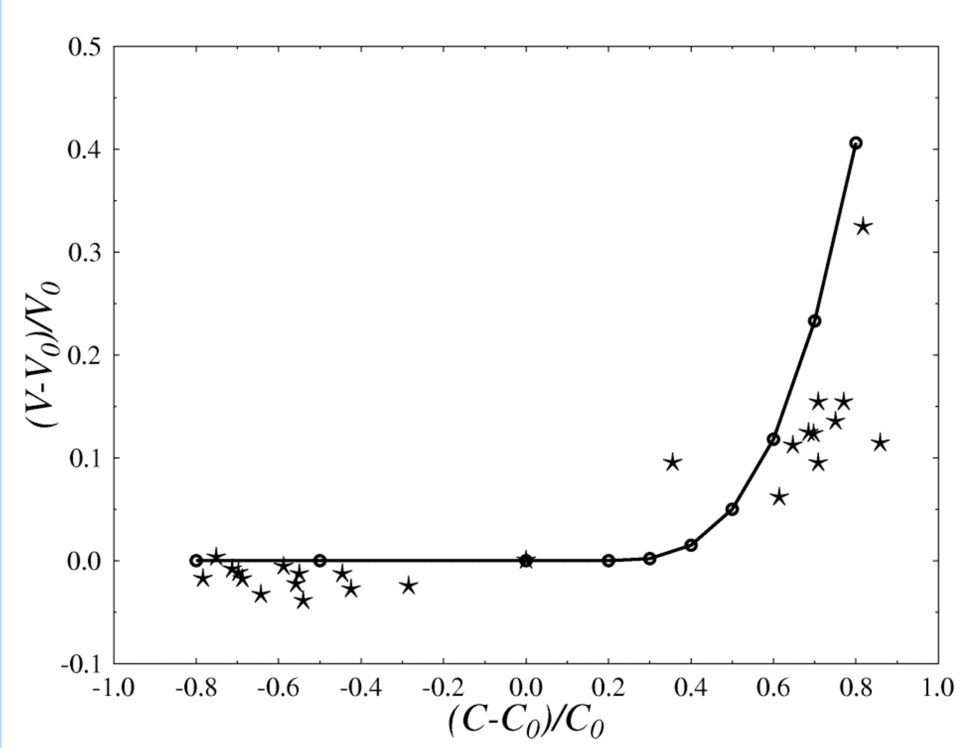
Defects induced faceting

Vary the potassium nitrate concentration in a narrow band to illustrate the presence of the non-linear term.





mean field theory predicts faceting for enhanced concentration and a logarithmic non-faceted profile for reduced concentration (if $\lambda > 0$).



Faceted surfaces grow faster:

$\star$ from experiment; drawn line from ASEP (numerical).

What is the real profile? Is there a faceting transition?

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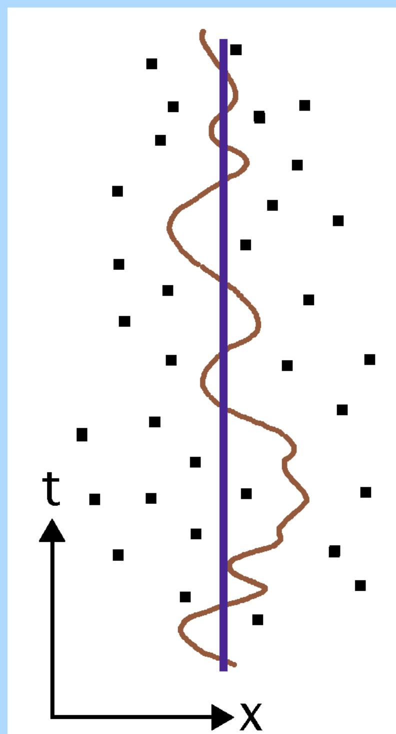


directed polymer localization

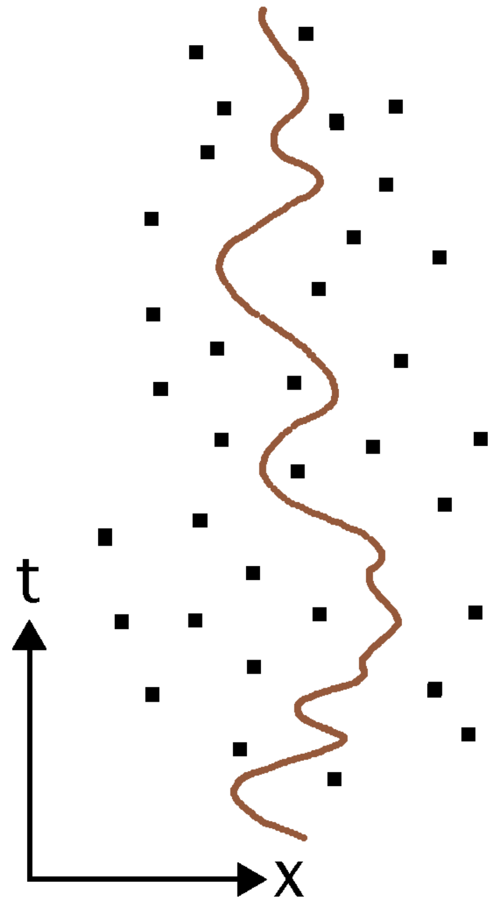
The directed polymer community was focused on the slow bond issue in the mid 1990-ties (e.g., Tang, Balents, Kinzelbach, Hwa, Straley, Lassig). The driving force behind these studies are applications of such directed polymers in terms of flux tubes in type-II dirty superconductors.

KPZ type growth is equivalent to a directed polymer in 2 dimensions subject to a random potential, by the Hopf-Cole transformation, $W = \exp(\frac{\lambda}{2\nu} h)$,

$$\frac{dh}{dt} = \nu \nabla^2 h - \lambda (\nabla h)^2 + \eta \;\rightarrow\; \frac{dW}{dt} = \nu \nabla^2 W - \eta W$$

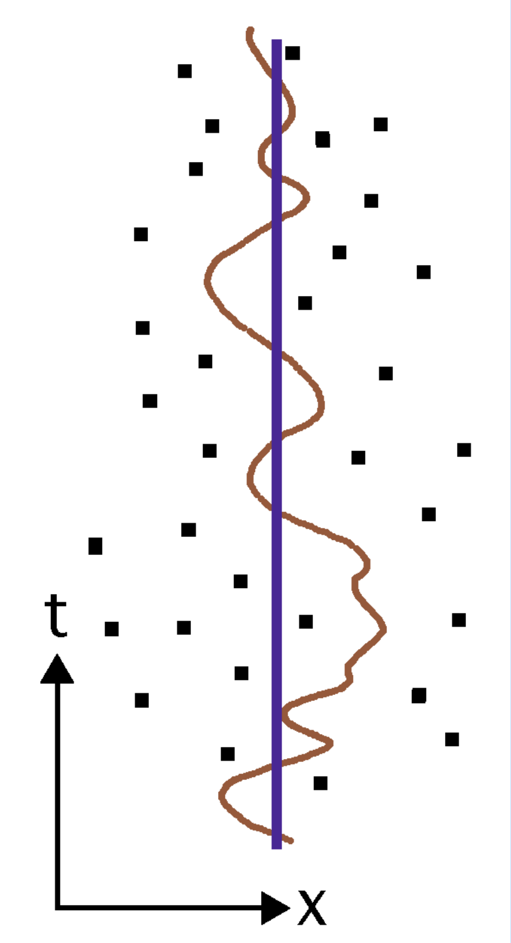
This is the master equation for a single random walker in a time dependent quenched random medium.

The walker spreads in time as $x \sim t^{1/z}$ with $z = z_{kpz} = 3/2$ instead of the free RW value $z = 2$.



The walker is also the path integral partition function of a 2D non-back bending polymer in a random potential.

The slow bond becomes a columnar defect with short ranged attraction.
Queuing translates to how this potential localizes the polymer.



Above a critical dimension D_c it should be localized for all $r < 1$; Power counting in the KPZ equation and associated field-theoretical renormalization studies suggest $D_c = 1$.

Our results do not contradict these field-theoretical results. The structure is more complex than anticipated.

Our queued phase represents the strongly localized state. It exists only beyond a critical defect strength $r_c < 1$.

The power-law shaped profile that remains for weaker slow bonds, represents a form of weaker localization, a stretched exponential,

$$\langle W \rangle \sim e^{\frac{\lambda}{2\nu}\langle h \rangle} \sim e^{-Cx^{1-\nu}} \quad \text{with} \quad \nu = 1/3.$$

Numerical studies in the directed polymer representation confirmed localization in $D = D_c = 1$ for all $r < 1$, but were insensitive to such details.

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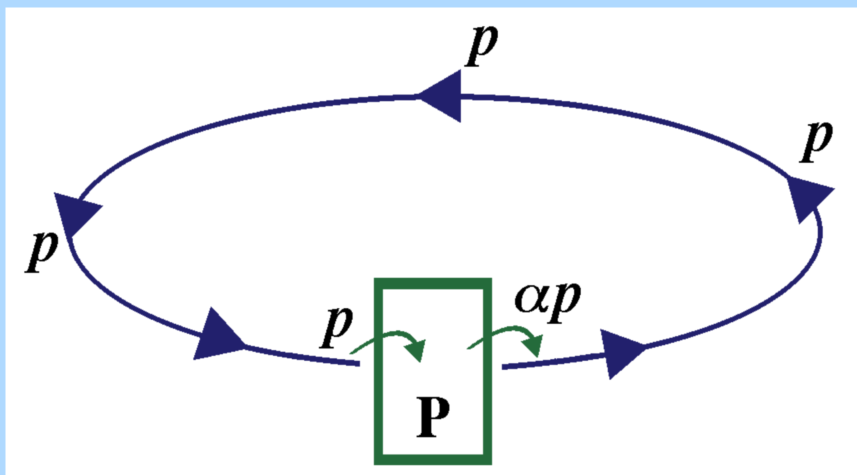

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the KPZ equation with uncorrelated noise

$$\langle \eta_{r_2,t_2}\eta_{r_1,t_1}\rangle = 2\Gamma\delta_{r_1,r_2}\delta_{t_1 t_2}$$

The growth rate v_0 is modified by the local curvature of the surface (the ν-term) and its local slope (the λ-term), and random fluctuations in the paper (density, flocking, potassium nitrate concentration).

the Burgers equation

For randomly stirred vortex free (curl free) fluids is equivalent to the KPZ equation (at $\lambda \equiv 1$).

$$\frac{d\vec{v}}{dt} + \lambda \vec{v} \cdot \vec{\nabla} \vec{v} = \nu \nabla^2 \vec{v} + \vec{f}(\vec{r}, t)$$

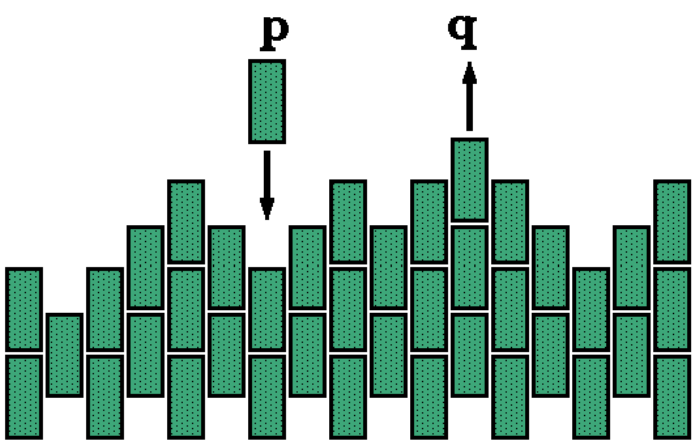
with velocity $\vec{v}(r) = -\vec{\nabla} h(r)$, viscosity ν, and random force $\vec{f} = -\vec{\nabla}\eta$.

The non-linear λ term arises here logically as part of the total derivative of the velocity.

In 1+1D, velocity is a scalar and we can reinterpret it as a particle density. The Burgers equation then describes a stochastic driven flow. The asymmetric exclusion process is the canonical example of this.

BCSOS (brick laying) growth

- Rectangular building blocks (brick wall on its side).
– Nearest neighbour heights differ by only $\Delta h = S_n^z = \pm 1$.



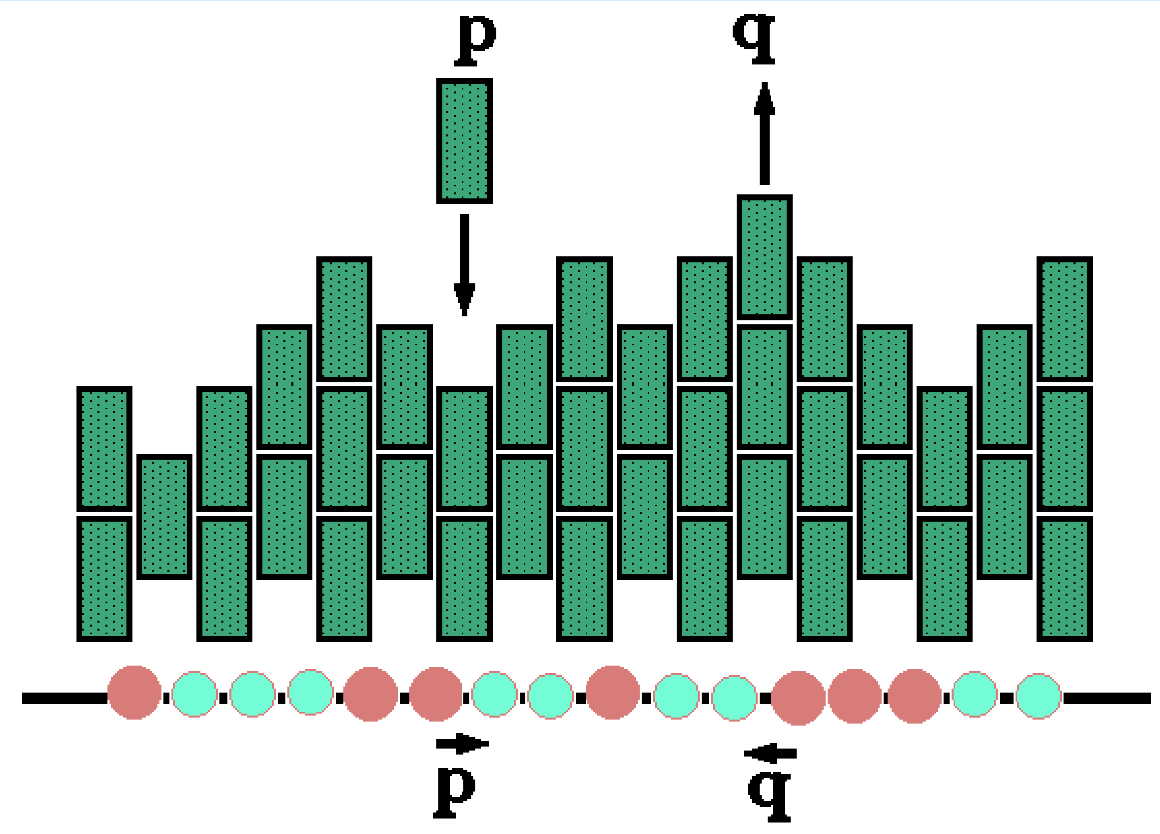
Growth rule: Select at random one of the columns. If this column is the bottom (top) of a local valley (hill top), a particle adsorbs (disorbs) with probability p (q) Local slopes are inactive ($\rightarrow \lambda < 0$).

– Early numerical studies: Meakin, Family, $\cdots$.
– In 1D, surface fully characterized by spin-$\frac{1}{2}$ type step variables $\rightarrow$ Master equation: XXZ quantum spin chain.
– Bethe Ansatz exact solution in 1D: Dhar, Gwa/Spohn.

asymmetric exclusion process (ASEP)

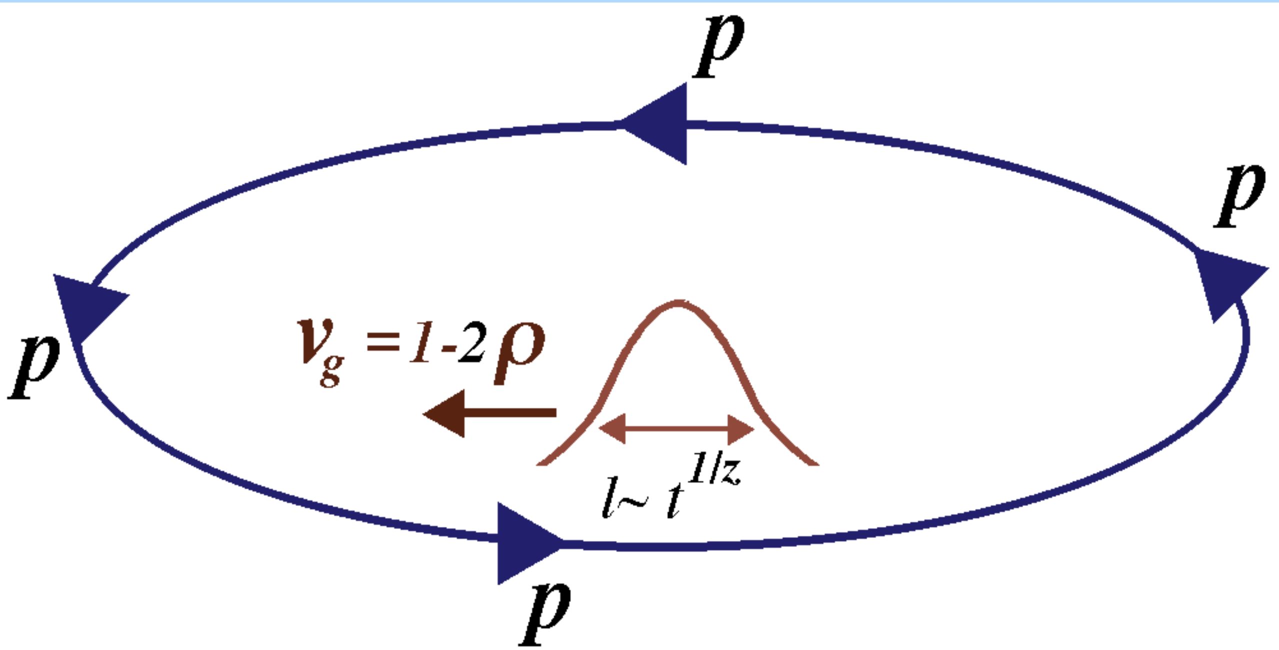
The BCSOS interface model (KPZ growth) is equivalent to a driven flow of particles with hard core repulsive interactions:

Interpret the $S^z_n = -1$ down-steps as particles and the $S^z_n = +1$ up-steps as empty sites



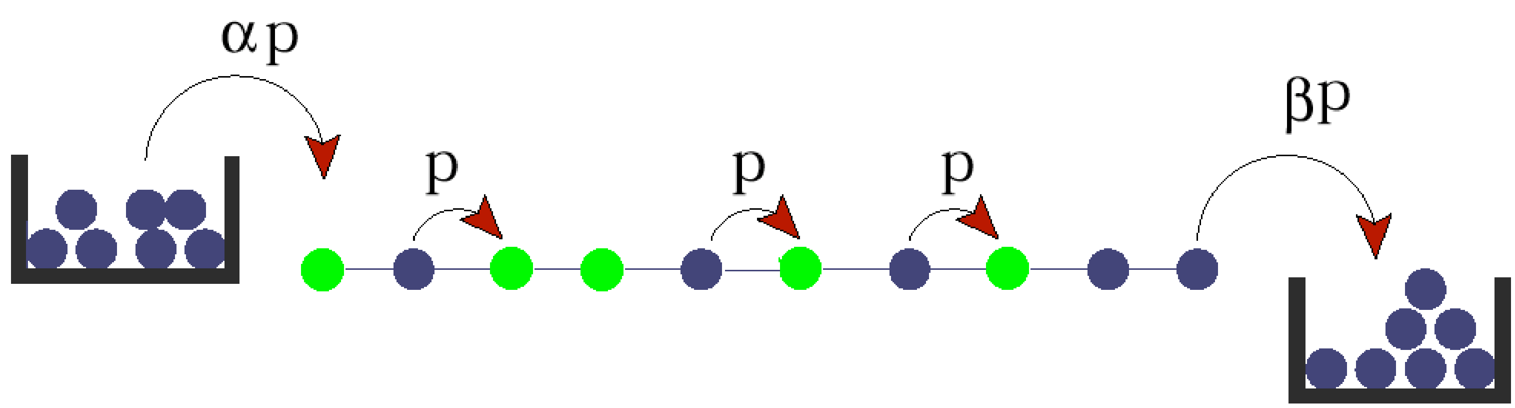
stationary state, fluctuations, and group velocity

The stationary ASEP state for periodic boundary conditions is disordered, random, without any correlations, but fluctuations scale in time as $l \sim t^{1/z}$ with the KPZ dynamic exponent $z = \frac{3}{2}$, and move with group velocity $v_g = 1 - 2\rho$ (tilt of KPZ surface).



boundary induced phase transitions

Phase transitions take place in open road set-ups with reservoirs on both ends; (exact matrix formulation results of the stationary state by, e.g., Derrida *et.al.*)

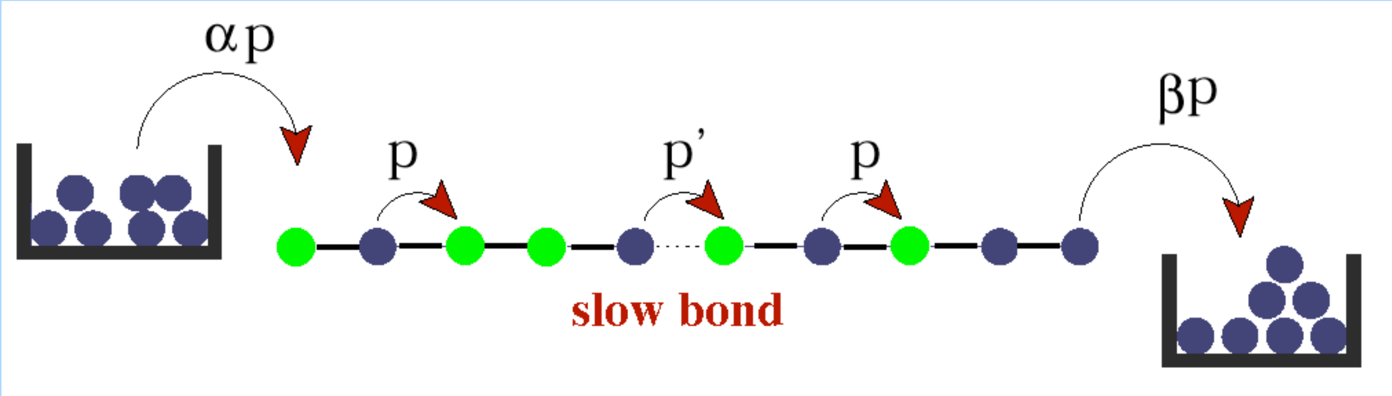


In the maximum current (MC) phase the road controls the density, but in the low (high) density phase the input (output) reservoir (α or β) controls the bulk density.

queuing due to slow bonds

Studies of slow bonds in the ASEP goes back at least 10 years. The hopping rate through the slow bond is reduced to $p' = rp$.

Behind the slow bond a traffic jam develops. The issue us whether the queue is finite or infinite in length (does it scale with the system size in the thermodynamic limit, like in bose condensation); and also the detailed shape of the density profile.



earlier work

Mean field theory predicts an infinite queue for all $r < 1$, and no queue for fast bonds, $r > 1$ (Wolf and Tang, '90).

Kandel and Mukamel (1992) suggested (for a slightly different model) the presence of a queuing transition at a $r_c < 1$ but their simulation data were inconclusive.

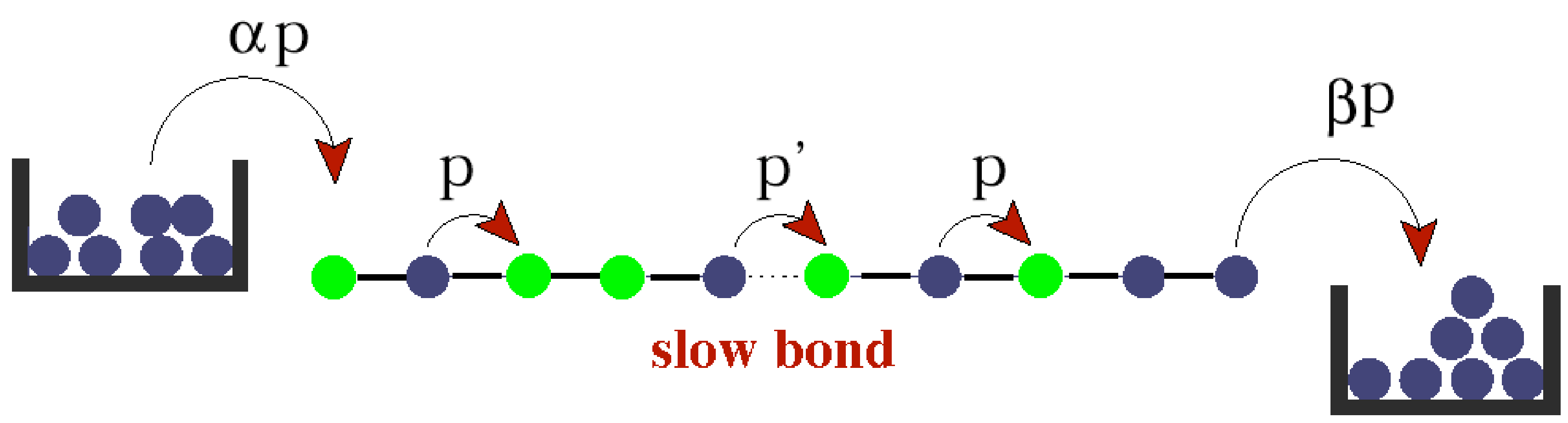
Janowsky and Lebowitz (1992-94) draw pictures as if $r_c = 1$, but their focus was on the shock wave fluctuations in the faceted phase far away from the slow bond.

Schütz ('93) determined the exact stationary state for periodic boundary conditions and parallel updating (the matrix method) and found $r_c = 1$. This does not contradict our results, because in parallel updating stochastic noise is weaker than in random sequential updating.

open boundary conditions

Choose the fully asymmetric ASEP ($q = 0$) with $p = \frac{1}{2}$.

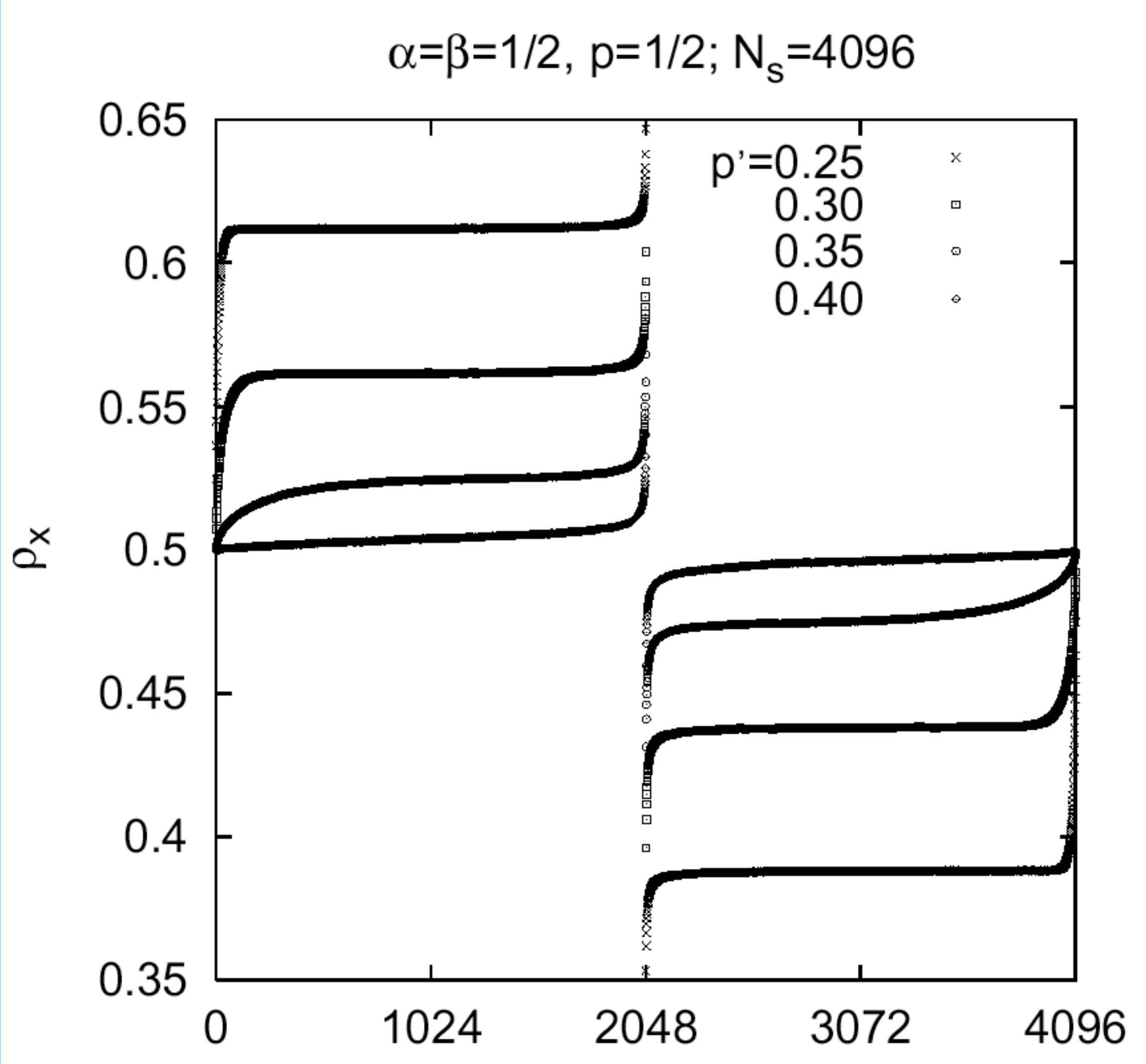
Use open boundary conditions and $\alpha = \beta = \frac{1}{2}$, such that at $r = 1$ (no slow bond; $p' = rp = p$) the density profile is featureless with all $\rho(x) = \frac{1}{2}$ the same, including near the edges.



faceted density profiles

open edges,
slow bond
in the middle

particle-hole
symmetry



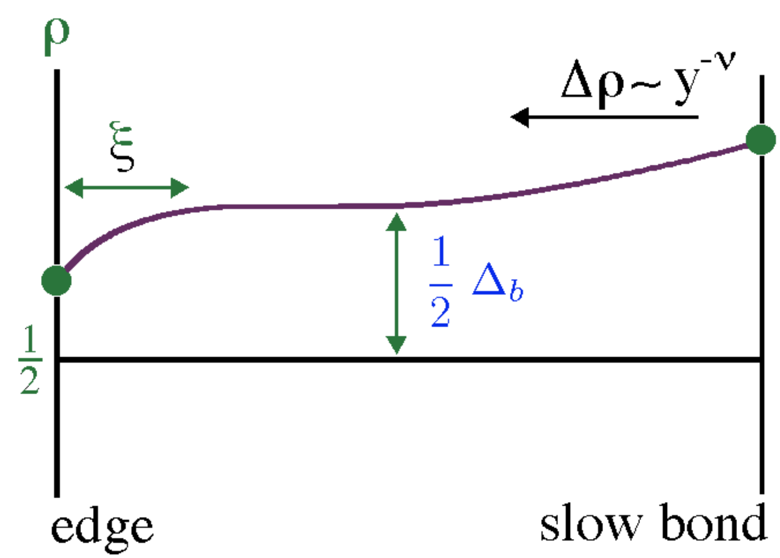
order parameter

Various aspects of the profile are linked to the current:

$$j = \alpha p\langle(1-\rho_1)\rangle = p\langle\rho_x(1-\rho_{x+1})\rangle = p\rho_b(1-\rho_b)$$

In the bulk (flat part) the stationary is uncorrelated.

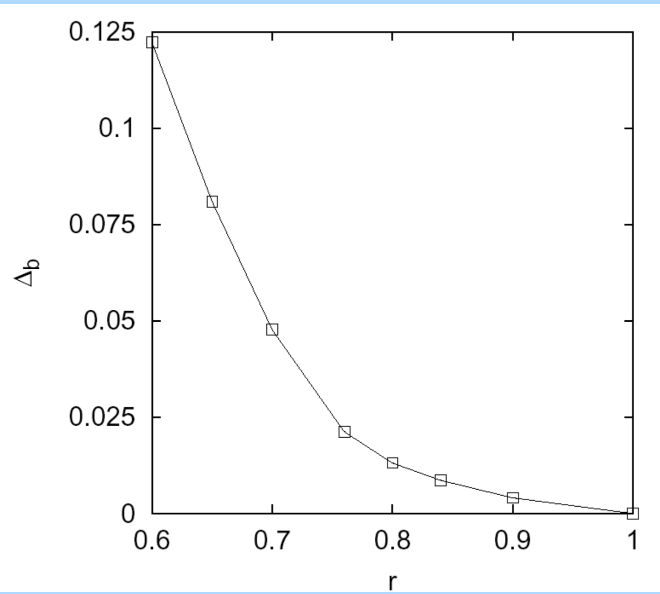
The profile at the edges is exponential (as in open bc case without slow bond)

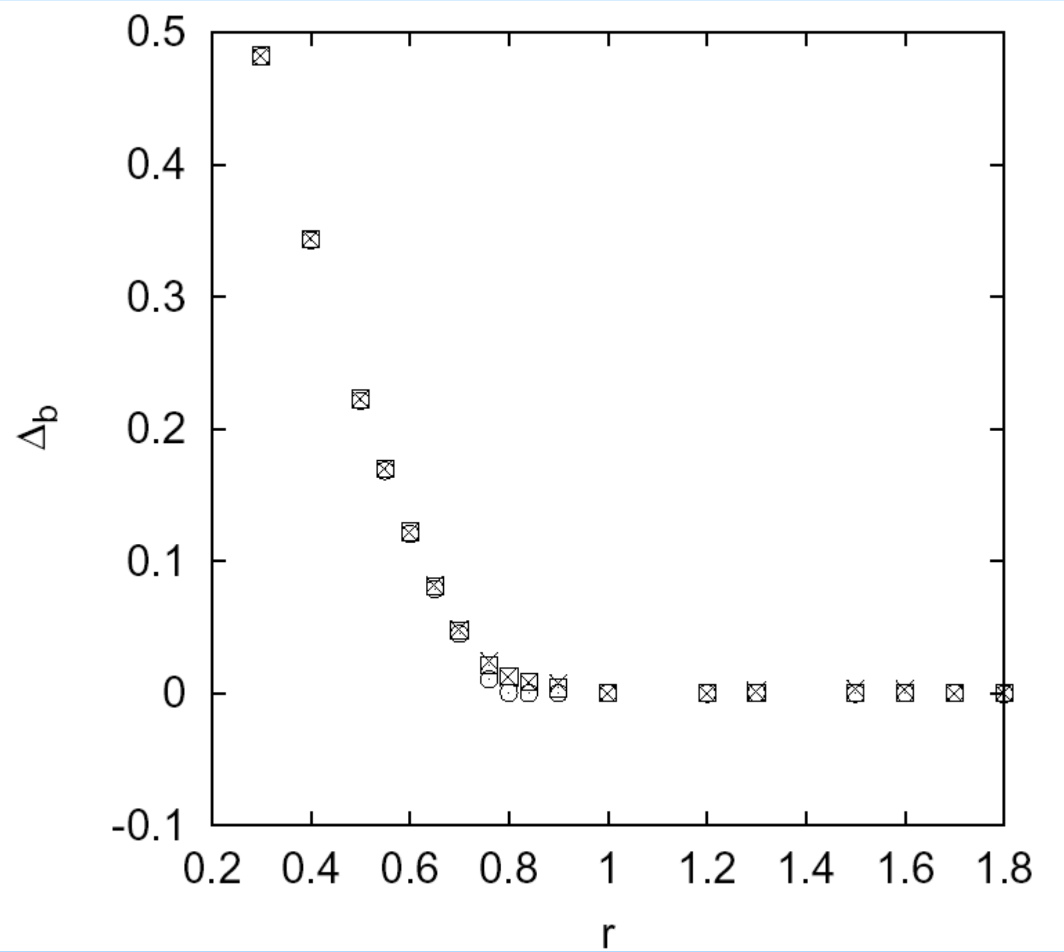


$$\rho(x) \sim e^{-x/\xi} \;\; ; \;\; \xi \sim \Delta_b^{-\frac{1}{2}} \;\; ; \;\; \rho_b = \frac{1}{2}(1+\Delta_b)$$

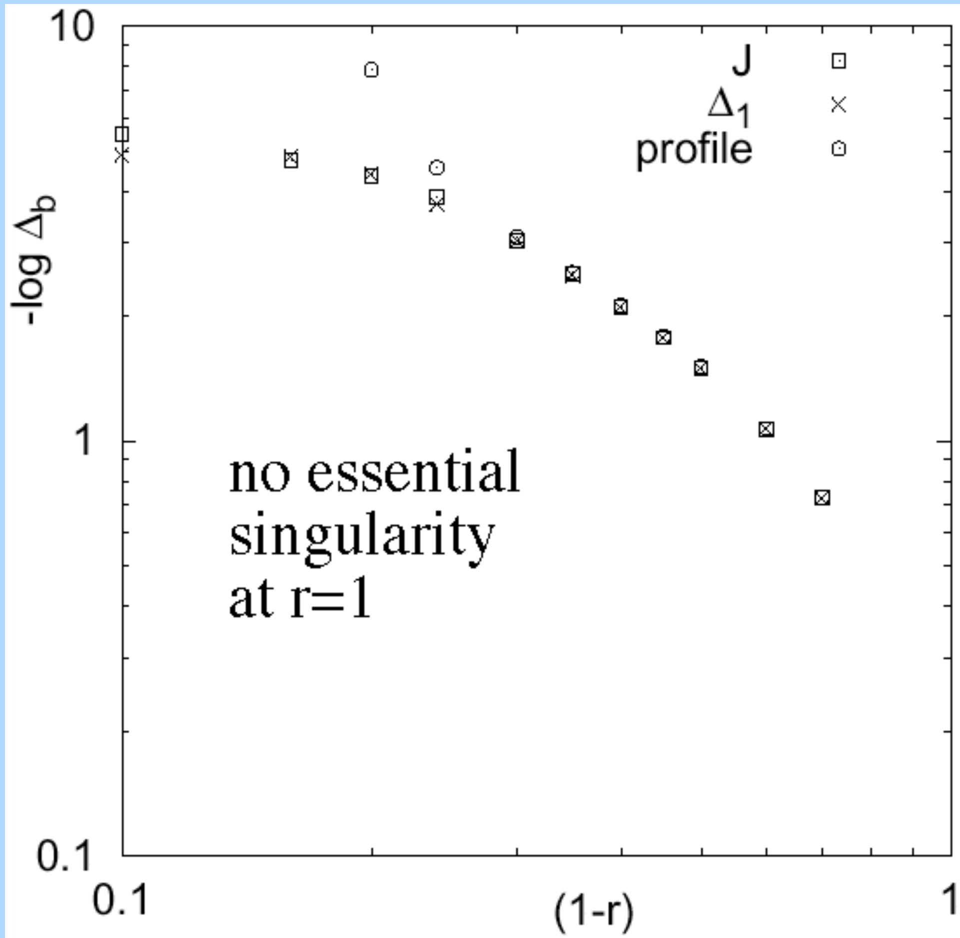
the order parameter

The numerical data for the current, the first site density, and the plateau value Δ_b (faceting angle), agree very well.





An exponential essential-singularity type infinite-order transition with $r_c = 1$, as suggested by the directed polymer renormalization studies does not fit our numerical data.

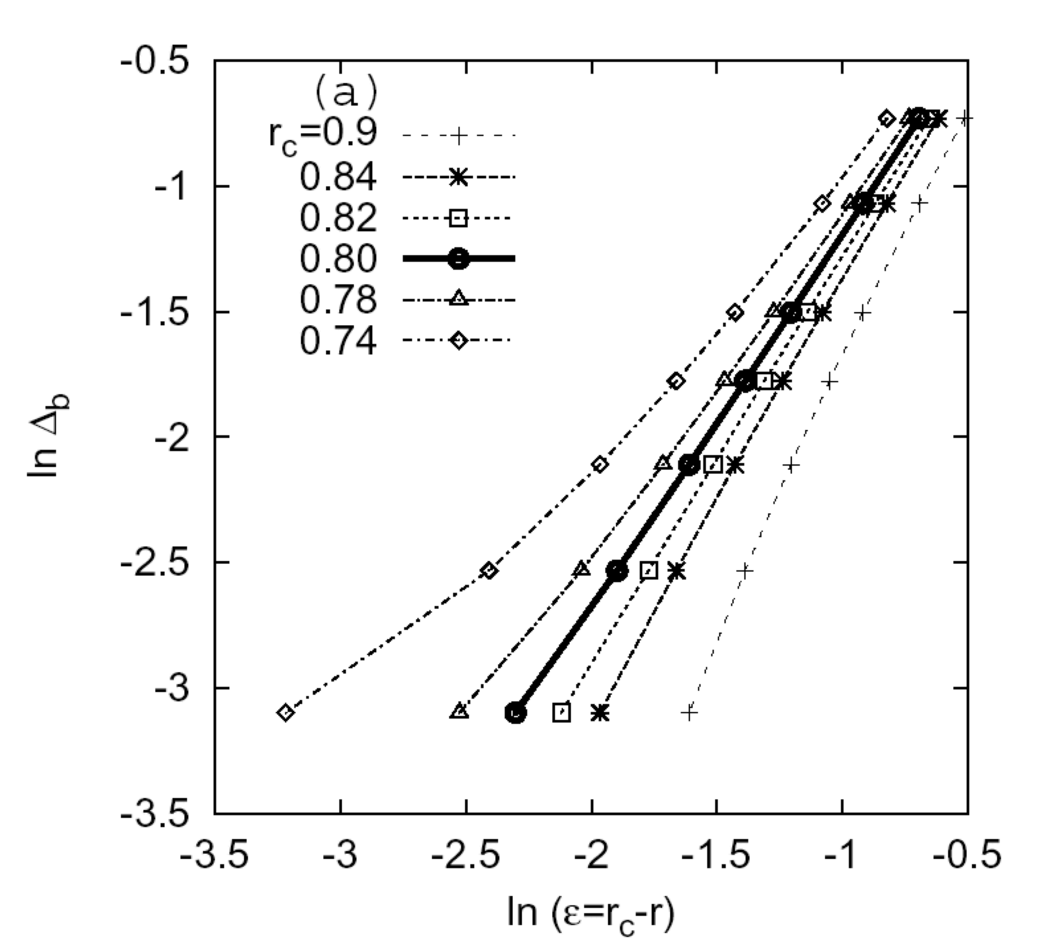


the critical point

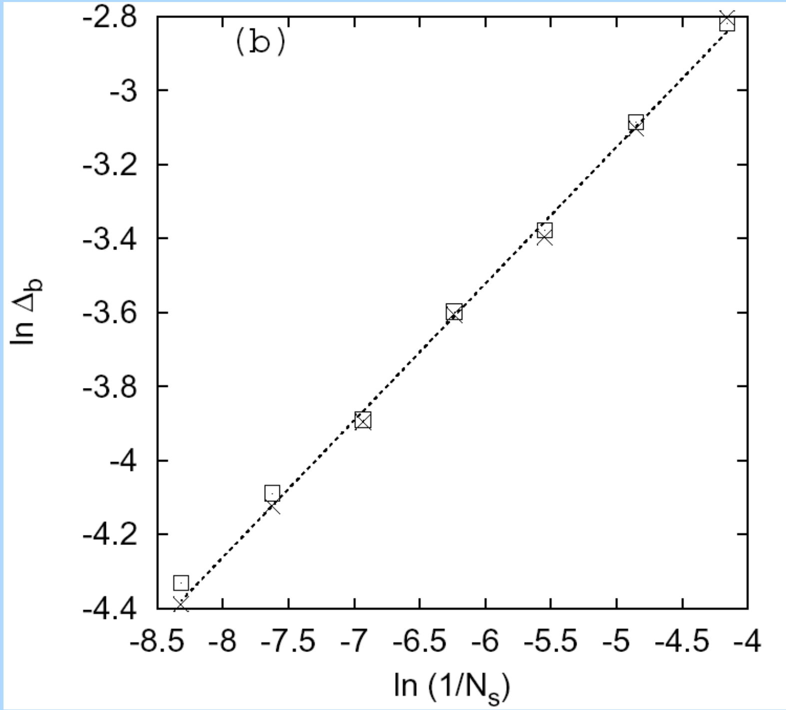
Assume the order parameter vanishes as a powerlaw $\Delta_c \sim |r - r_c|^{\beta}$.

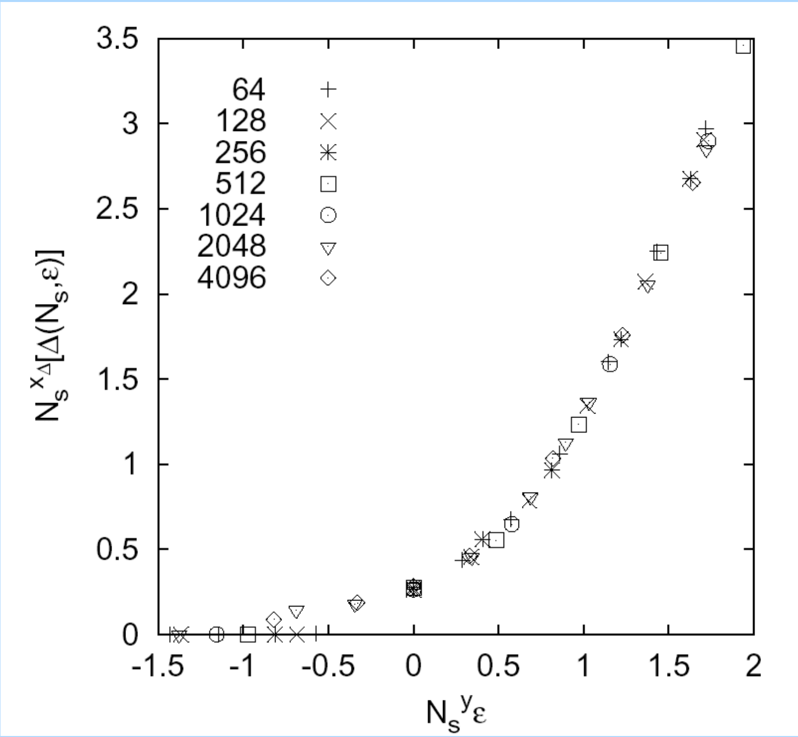
From straight line fits in log-log plot:

$r_c = 0.80 \pm 0.02$
$\beta = 1.5 \pm 0.01$



Finite size scaling of $\Delta_b \sim N_s^{-x_\Delta}$ at $r = 0.80$.
$x_\Delta = 0.370(5)$





Data collapse of the FSS scaling function

$\Delta_b(N_s, \epsilon) = N^{-x_\Delta} \mathcal{S}(N_s^y \epsilon)$

with $x_\Delta = \beta y$

our results

We show that $r_c = 0.80 \pm 0.02$.

This is implies that stochastic fluctuations destroy the (macroscopic) traffic jam behind weak obstacles.

Moreover, the density profile has always a power law tail near the slow bond:

$$\rho(y) = \frac{1}{2} + \Delta_b + Ay^{-\nu}$$

with:

$\nu = \frac{1}{2}$ in the faceted phase (when $\Delta_b \neq 0$);

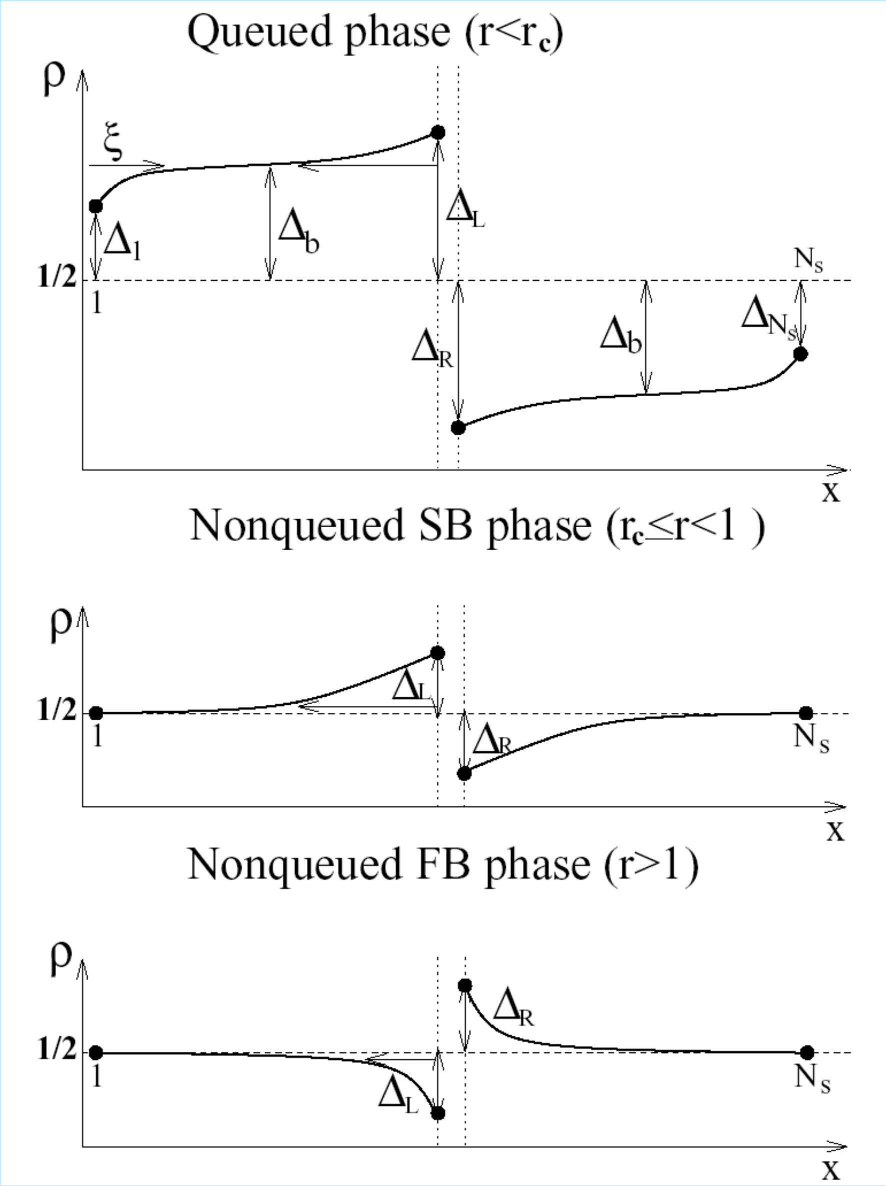
$\nu = \frac{1}{3}$ for weak slow bonds ($r_c < r < 1$), and

$\nu = \frac{2}{3}$ for fast bonds ($r > 1$; never facets)

Density profile power law tails:

In all three phases, the density profile has a powerlaw shape near the slow bond.

The details depend on if or how information travels through the slow bond, how information spreads along the chain (the KPZ dynamic exponent $z = \frac{3}{2}$), and the group velocity of such fluctuations.

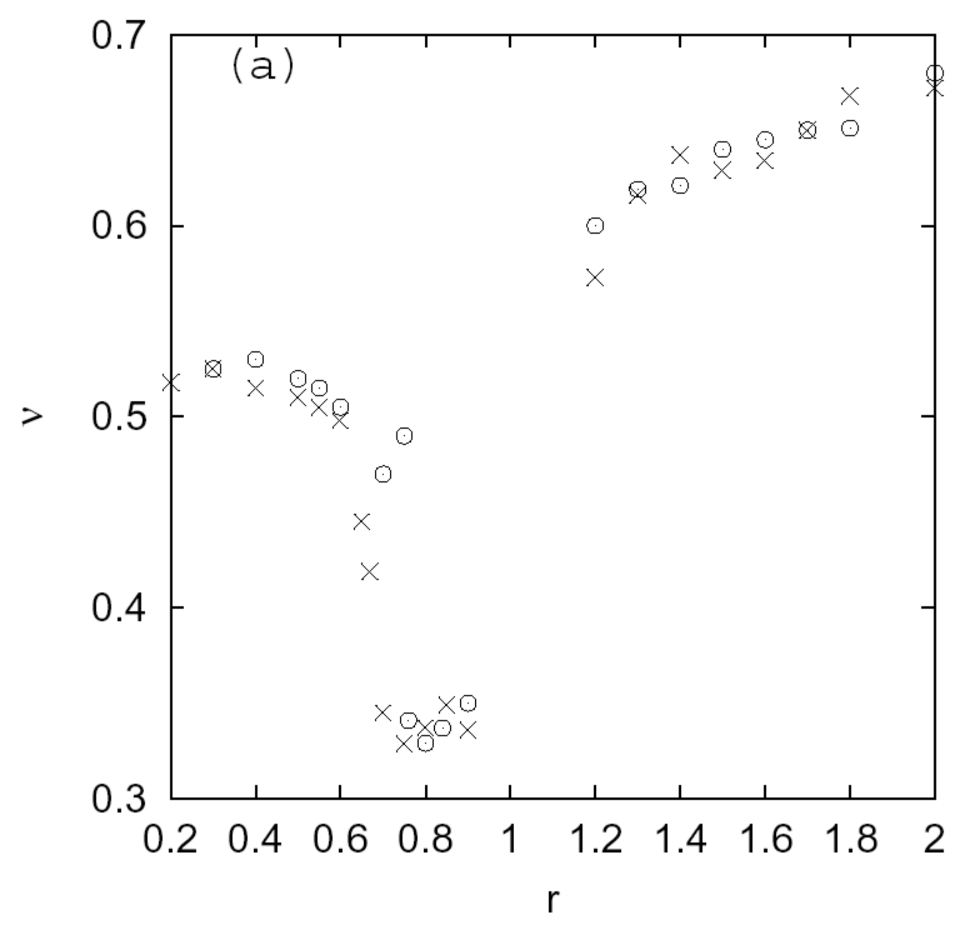


<u>density profile exponent ν</u>

Power law fits to
the density profile

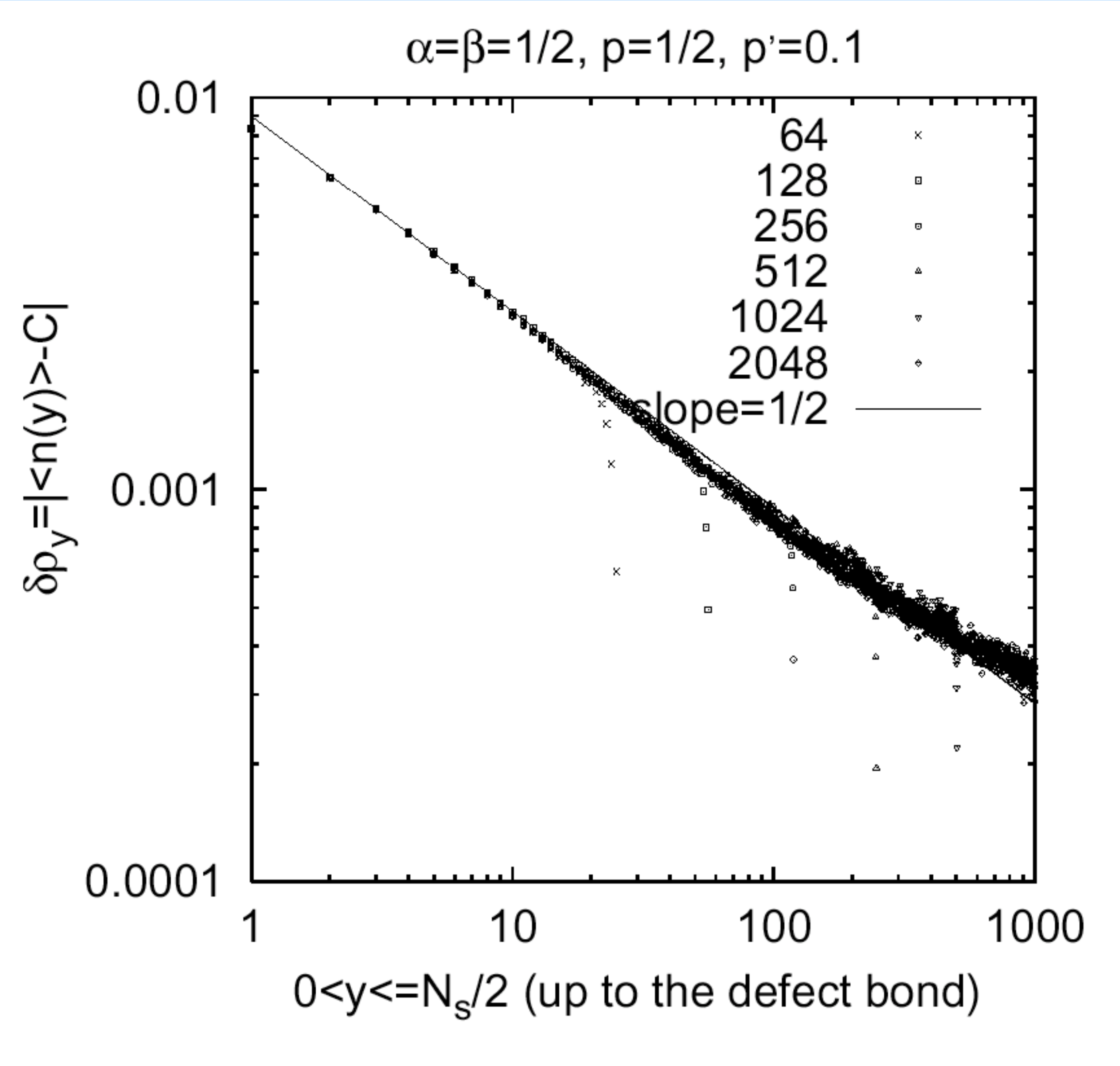
$$\Delta(y) \simeq \Delta_b + Ay^{-\nu}$$

ν as function of r
shows the three
distinct phases.



faceted
powerlaw
profile

The log-log plot
of the profile
$\Delta(y) \simeq \Delta_b + Ay^{-\nu}$
yields $\nu \simeq \frac{1}{2}$



explanation: an uncorrelated passage process

The powerlaw density tail, $\rho \sim y^{-\nu}$, with exponent $\nu = 1/2$, reflects that the passages of particles through the slow bond (SB) are stochastic uncorrelated events.

The group velocity v_g of fluctuations points away from both sides of the SB. The number of passage fluctuations in the system is therefore proportional to $t_{\text{flight}} \sim N_s$.

The passage process is biased. It favors vacancies. The passing probability of particles is reduced by a factor r.

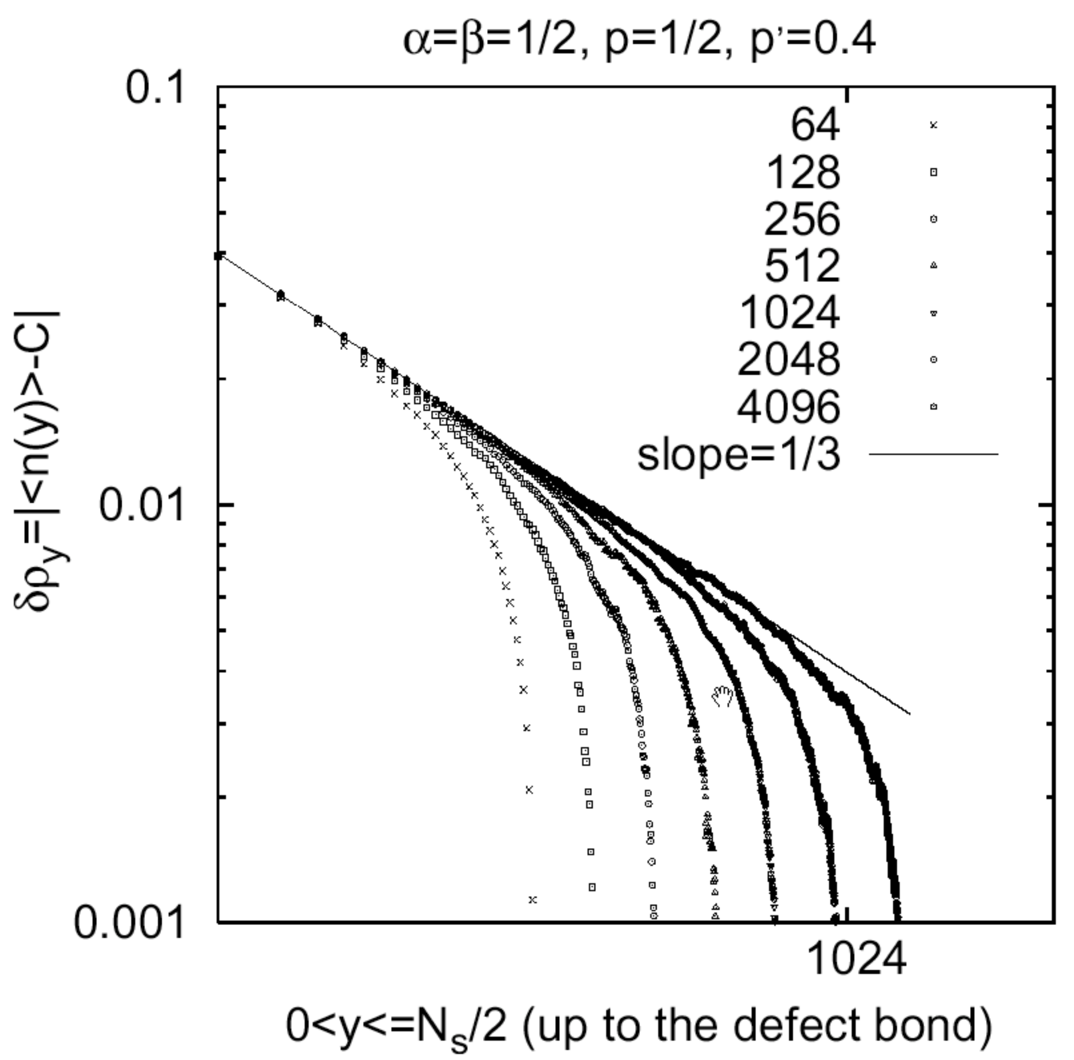
This explains why an excess of particles (vacancies) builds up in front (behind) the SB. The total mass of the power law tail $\rho \sim y^{-1/2}$, scales in the macroscopic queued phase as in a random walk, $\delta N \sim N^{1/2}$.

density profile at and above r_c

Above and at r_c the slow bond does not create a macroscopic queue, but only a power law shaped density correction

$$\rho(y) \simeq \frac{1}{2} + Ay^{-\nu}$$

$$\nu \simeq \frac{1}{3}$$



explanation: self consistent uncorrelated SB passage

The group velocity is equal to $v_g \sim \frac{1}{2} - \rho \sim y^{-\nu}$. The time of flight of a fluctuation, from the slow bond (SB) to the reservoir, scales as $t_{\text{flight}} \sim \int dy\, y^{\nu} \sim N_s^{\nu+1}$.

The SB processes vacancies more efficiently than particles. Fluctuations still detach from the SB and passage through it remains uncorrelated, if $\nu < \frac{1}{2}$.

The excess number of cars in the queue represents again t_{flight} uncorrelated events, $\delta N \sim t_{\text{flight}}^{1/2}$. Assume this is distributed as a power law. Self consistency implies:

$$\delta N \sim t_{\text{flight}}^{1/2} \sim N_s^{\frac{\nu+1}{2}} \to (\nu+1)/2 = -\nu+1 \to \nu = \frac{1}{3}.$$

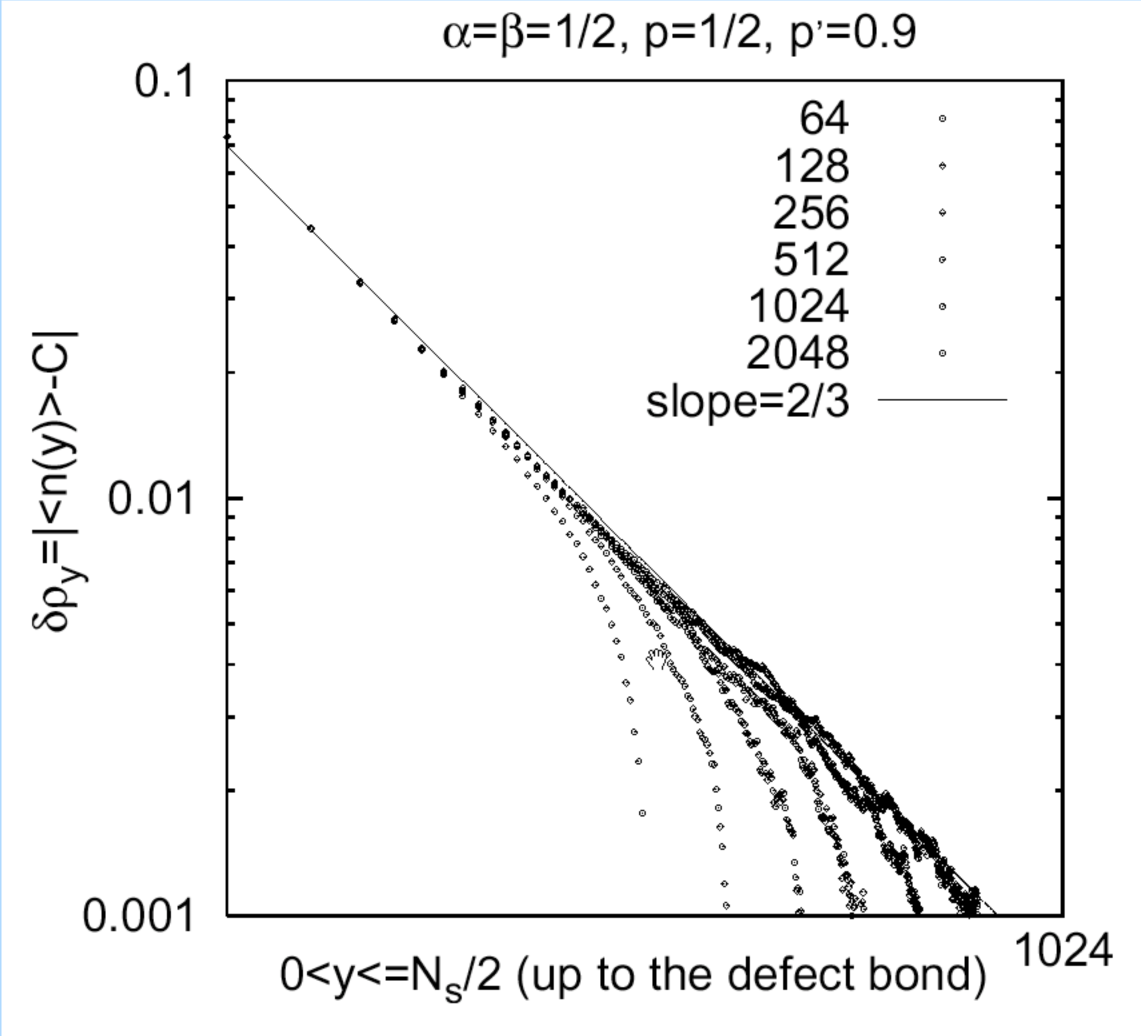
No KPZ properties are used. This result is very general.

density profile for fast bonds

A fast bond. $r > 1$, never creates a macroscopic queue; only a power law shaped density correction

$$\rho(y) \simeq \frac{1}{2} - A y^{-\nu}$$

$$\nu \simeq \frac{2}{3}$$



weak explanation: $\nu = 1/z$ (KPZ dynamic exponent)

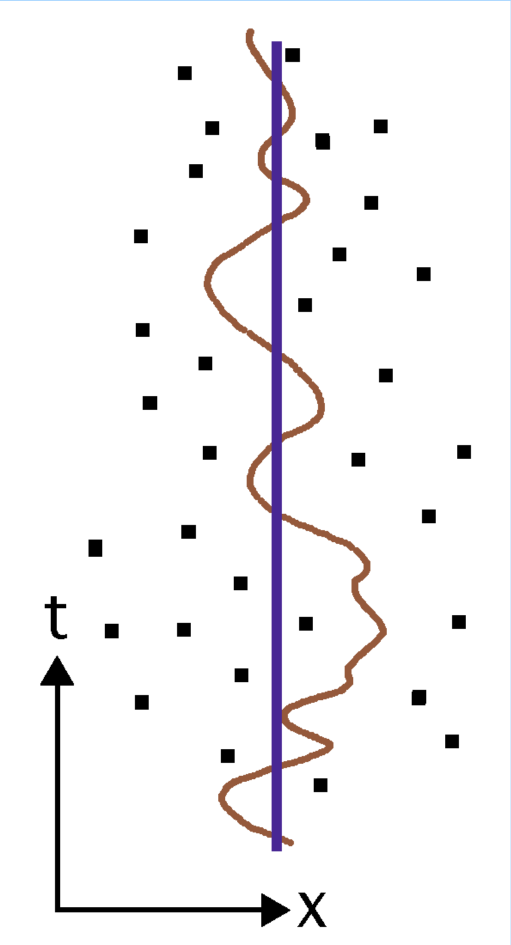
The fast bond (FB) depletation power law is caused by fluctuations traveling towards the FB (instead of away) and particles passing more efficiently than vacancies.

For $\nu > z - 1 = \frac{1}{2}$, the time of flight of the center of mass of a fluctuation $t_{\text{flight}} \sim N_s^{\nu+1}$ is longer than the time it takes that same fluctuation to spread over the entire system $t \sim N_s^z$. The exponent ν can not be insensitive anymore to the value of z.

A fluctuation created at distance y arrives at the FB after $t \sim y^z$, with reduced amplitude (spreading) $A \sim y^{-1/z}$.

If superposition concepts apply, the density deficit at the SB scales as $\delta N \sim \int dy\, y^{-1/z} \sim N_s^{1-1/z} \sim N_s^{-\nu+1}$.

The slow bond becomes a columnar defect with short ranged attraction.
Queuing translates to how this potential localizes the polymer.



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The power-law shaped profile that remains for weaker slow bonds, represents a form of weaker localization, a stretched exponential,

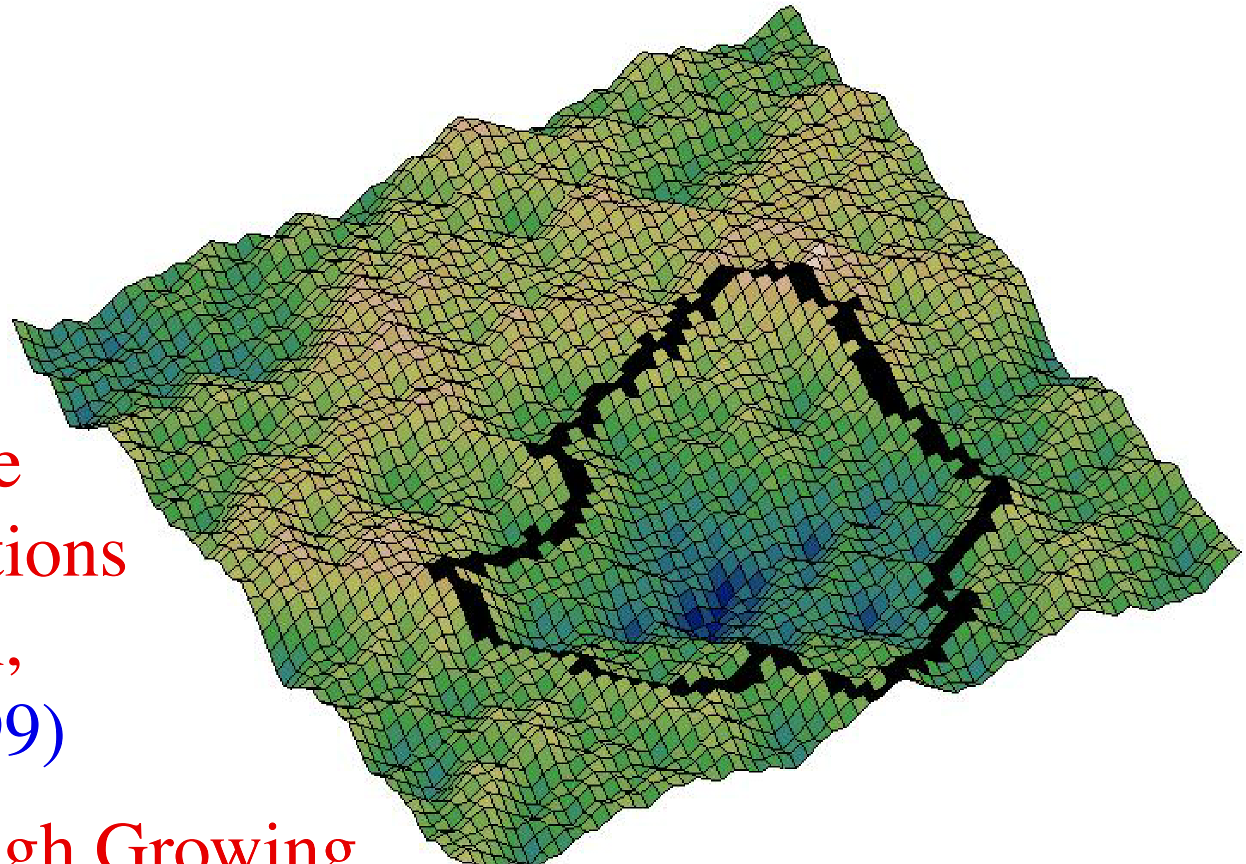
$$\langle W \rangle \sim e^{\frac{\lambda}{2\nu}\langle h \rangle} \sim e^{-Cx^{1-\nu}} \quad \text{with} \quad \nu = 1/3.$$

Numerical studies in the directed polymer representation confirmed localization in $D = D_c = 1$ for all $r < 1$, but were insensitive to such details.

KPZ GROWTH DURING SURFACE RECONSTRUCTION

Chen-Shan Chin

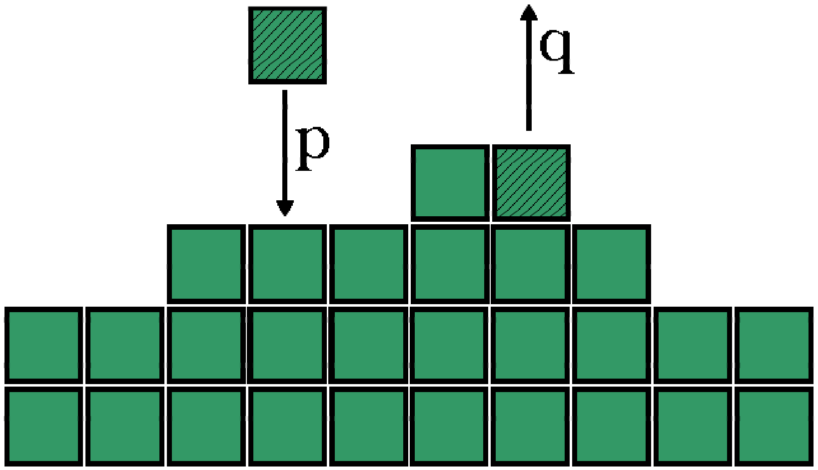
Marcel den Nijs



A: Stationary State
Skewness and finite
size scaling corrections
in 2D KPZ Growth,
PRE 59, 2633 (1999)

B: Reconstructed Rough Growing
Interfaces, Ridge Line Trapping
of Domain Walls, PRE 64, 031606 (2001)

RSOS (Kim-Kosterlitz) model



– discrete heights on a lattice
– cubic building blocks
– nearest neighbour heights differ by $\Delta h = S_n^z = 0, \pm 1$. (spin-1 type step variables).

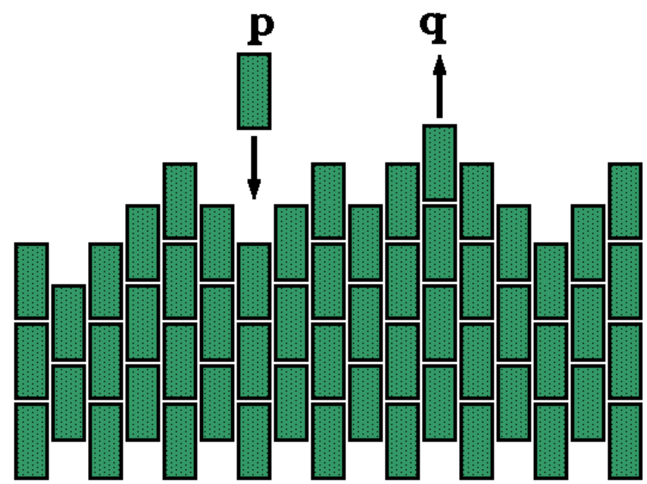
Growth rule: Select at random one of the columns. Attempt an absorption or desorption event. Absorb (disorb) a particle with probability p (q) if the $\Delta h = 0, \pm 1$ constraints remain satisfied.

– sloped areas are less active $\rightarrow \lambda < 0$
– the stationary state is skewed even in 1D.

BCSOS (brick laying) growth

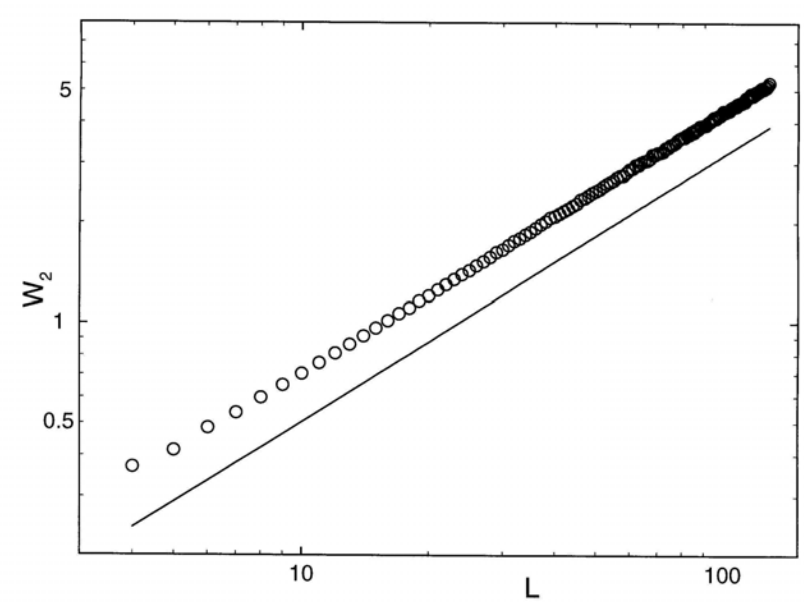
- Rectangular building blocks (brick wall on its side).

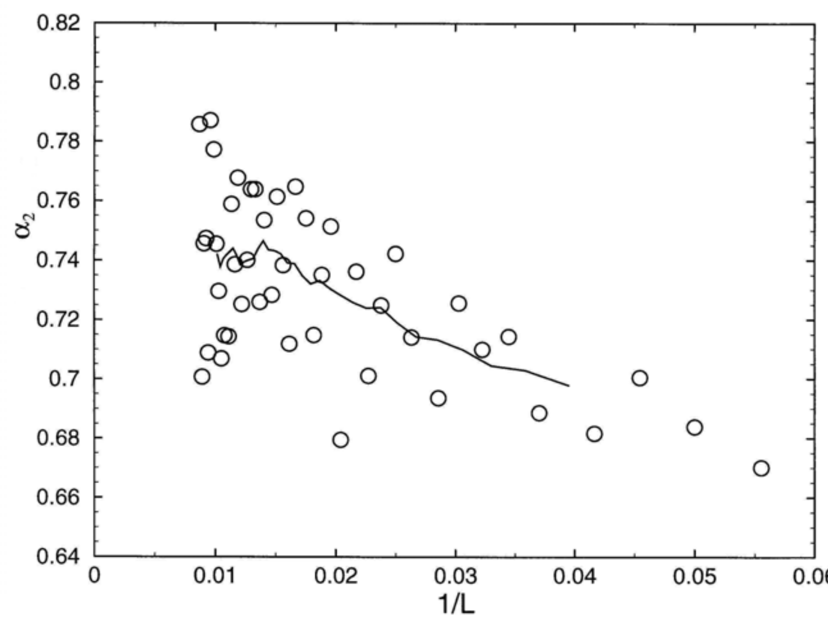
– Nearest neighbour heights differ by only $\Delta h = S_n^z = \pm 1$.



Growth rule: Select at random one of the columns. If this column is the bottom (top) of a local valley (hill top), a particle adsorbs (disorbs) with probability p (q) Local slopes are inactive ($\rightarrow \lambda < 0$).

A. FINITE SIZE CORRECTIONS TO SCALING AND STATIONARY STATE SKEWNESS IN 2D KPZ GROWTH





PRE 59, 2633 (1999)

1. Early numerical results for 2D dynamic roughness

Until recently, most numerical studies pretended to be very accurate, with very small quoted error bars, but the values of α between models were mutually inconsistent.

For the 2D BCSOS model, they were in the $\alpha \simeq 0.38$ range while for the 2D Kim-Kosterlitz model $\alpha \simeq 0.40$.

Indifference to this inconsistency illustrates how much universality is taken for granted nowadays.

Such a spread would have been a dead blow for the development of scaling theory during the 1960/70-ties in the context of equilibrium critical phenomena.

How universal are the KPZ exponents in $D > 1$?

2. Corrections to scaling

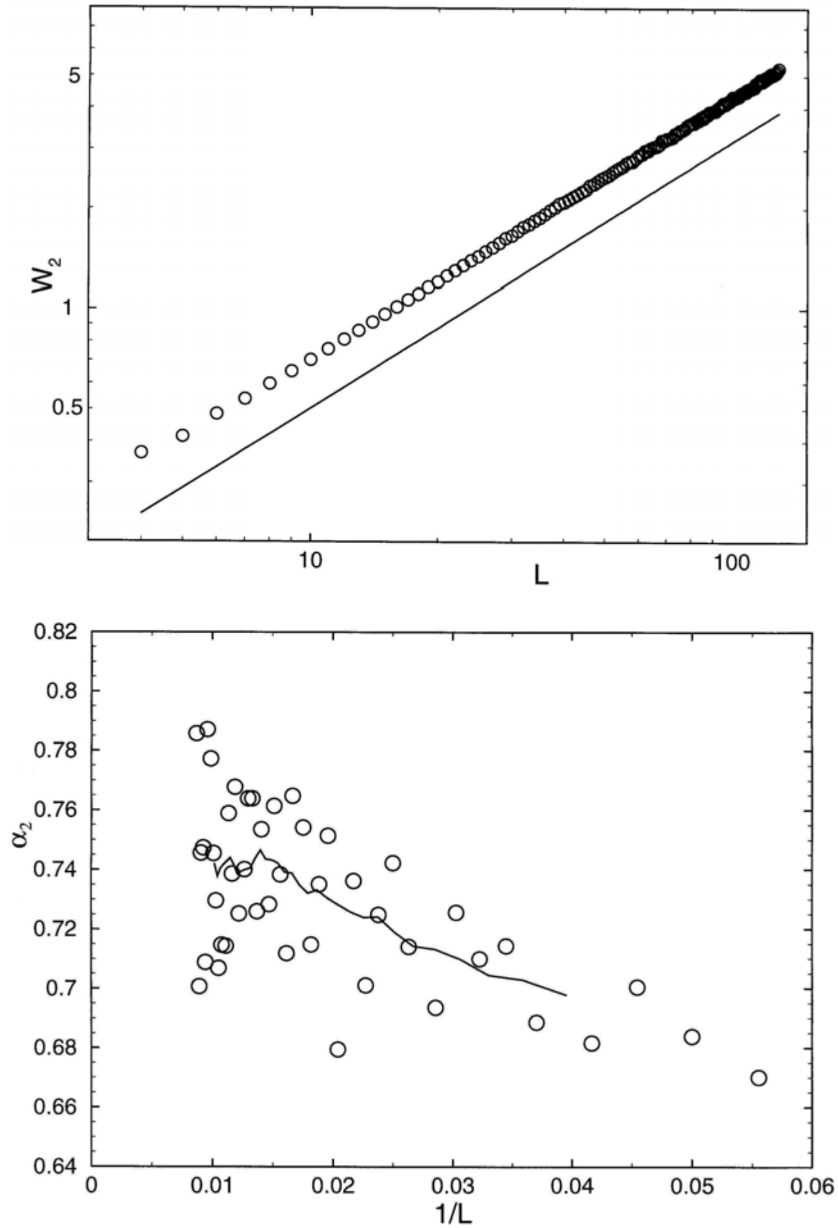
"Thou shall NEVER deduce the values of critical exponents from straight line fits to log-log plots."

Simple power laws without corrections to scaling are rare.

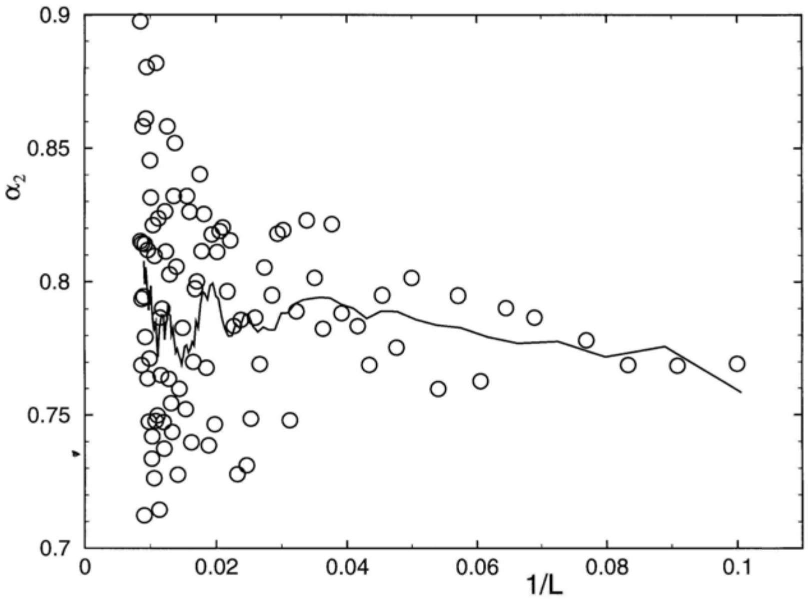
$$W_2 \sim N^{2\alpha}(A + BN^{y_{ir}} + \cdots)$$

with $y_{ir} < 0$ is the norm.

Plot the local slopes of the log-log plot as function of N.

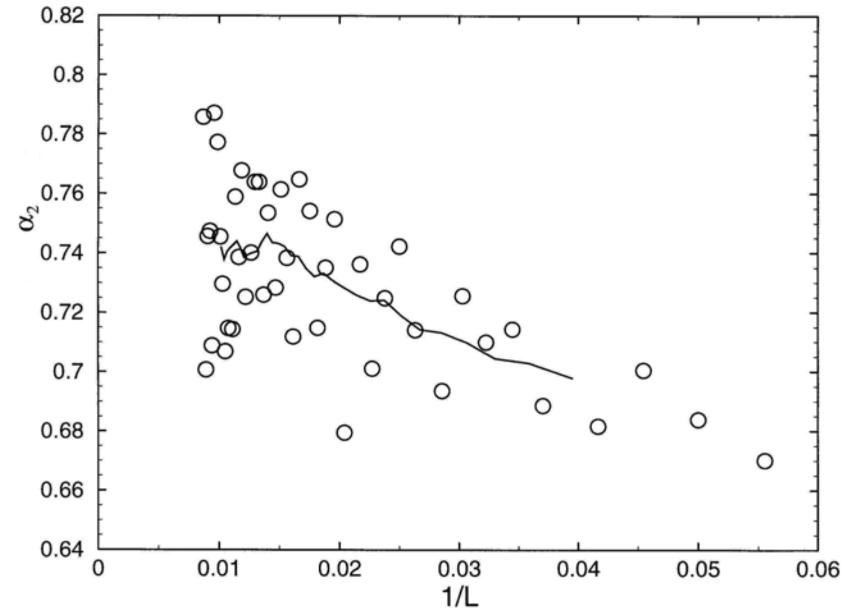


3. Corrections to scaling exponent for the 2-nd moment



The corrections to scaling are small in the KK model but large in BCSOS growth (farther from fixed point).

RSOS $\uparrow$ BCSOS $\longrightarrow$

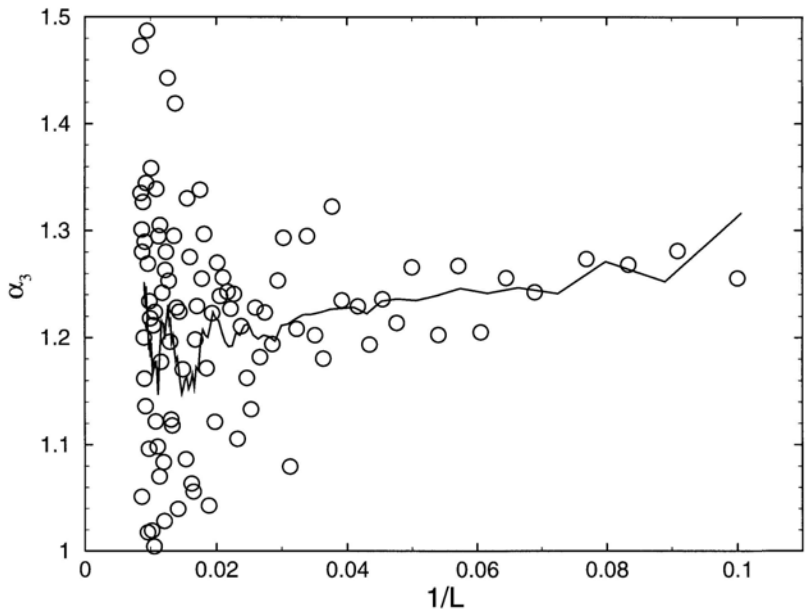


Following the Kim Kosterlitz and Lässig type conjectures W_2 scales with

$\alpha_2 = 2 \times \alpha = 0.80$

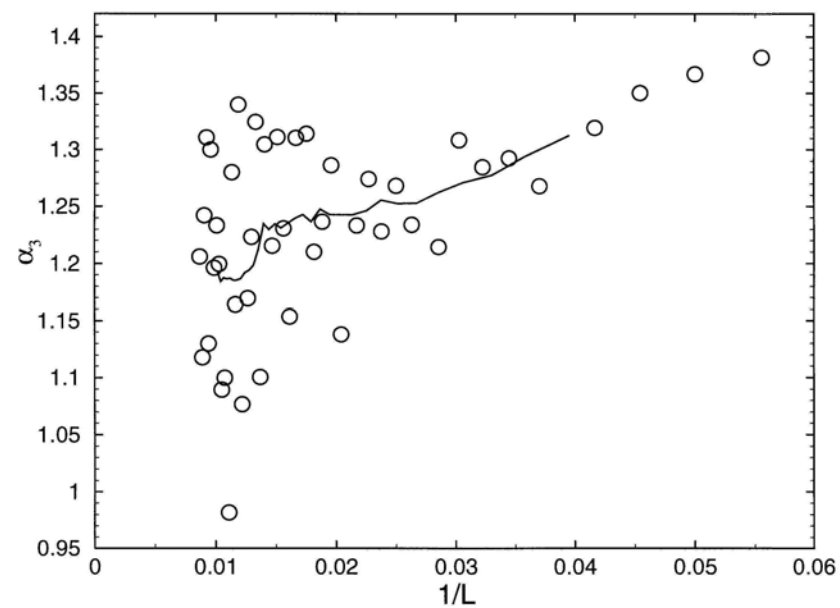
numerics requires: $y_{ir} = -0.6\ (2)$

4. Corrections to scaling exponent for the third moment



The leading corrections to scaling exponents for the odd and the even moments are different.

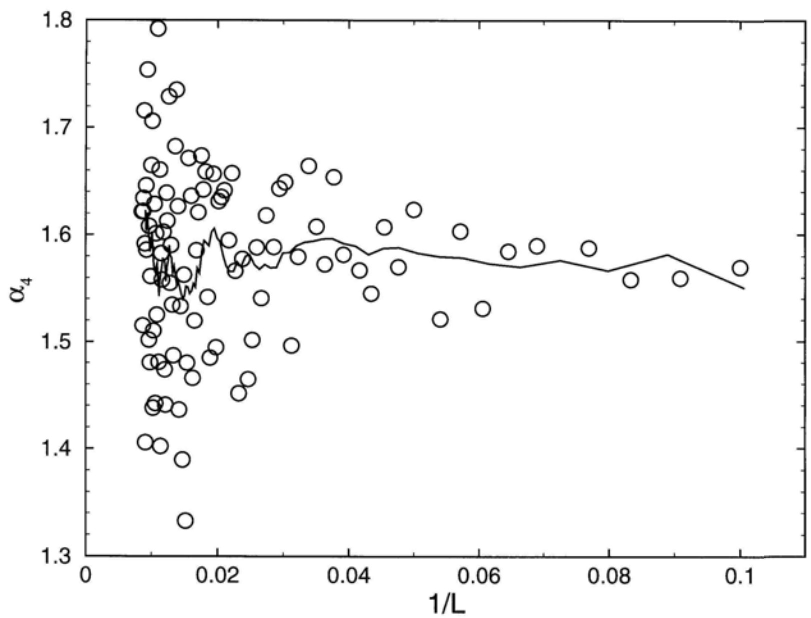
RSOS ↑ BCSOS ⟶



According to the conjectures the 3-rd moment scales as $\alpha_3 = 3 \times \alpha = 1.20$

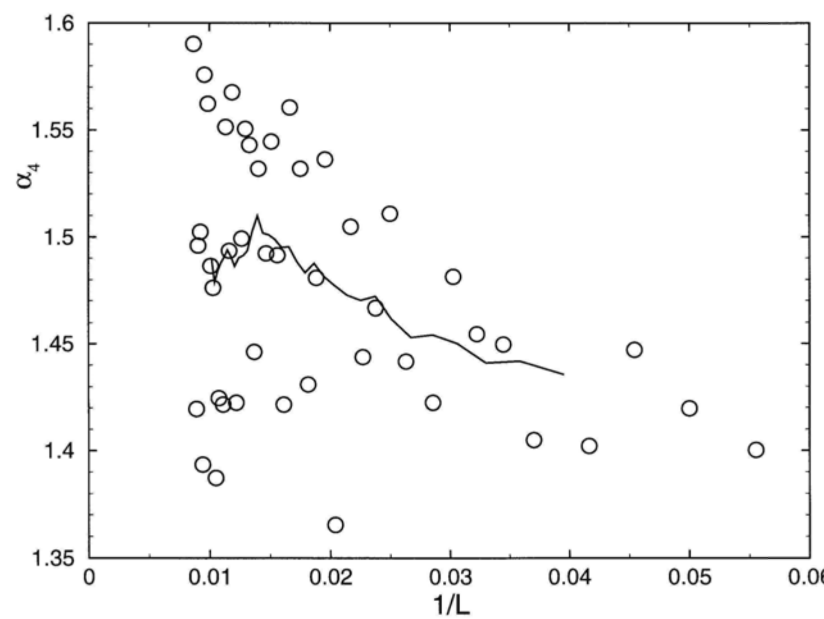
numerics requires: $y_{ir} = -1.7\ (3)$

5. Corrections to scaling exponent for the fourth moment



The even moments have the same leading corrections to scaling exponent

RSOS $\uparrow$ BCSOS $\longrightarrow$



According to the conjectures the 4-th moment scales as $\alpha_4 = 4 \times \alpha = 1.60$

numerics requires: y_{ir}= -0.6 (2)

6. Identifying the corrections to scaling operators

Lässig conjecture presumes that the KPZ operator algebra contains only a few independent elements, and thus that the scaling exponents of the higher-order operators have power counting values.

Our results, $y_{ir}= -0.6\ (2)$ (even) and $y_{ir}= -1.7\ (3)$ (odd) are close to what we could expect:

The curvature operator $\partial^2 h/\partial x^2$ is irrelevant, but not by much. Power counting suggests $y_\nu = -\alpha$.

$\partial^2 h/\partial x^2$ does not affect odd moments.

A leading candidate for the odd moments corrections to scaling is $(\partial^2 h/\partial x^2)^2$, with $y_{sk} = -2$.

7. Universal KPZ stationary state distribution function

KPZ growth is by probably the best understood system with driven non-equilibrium critical behaviour.

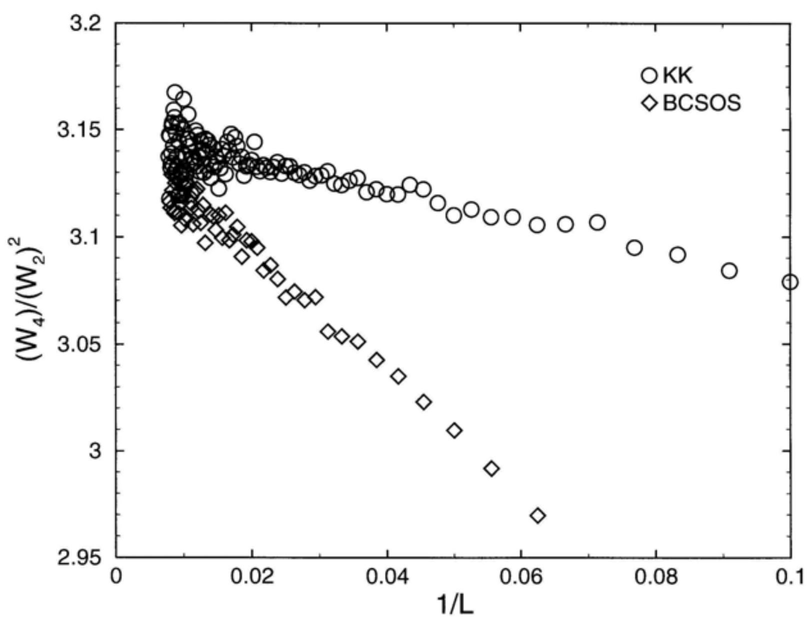
In 1D, the stationary state remains the trivial Gaussian distribution (uncorrelated up/down steps).

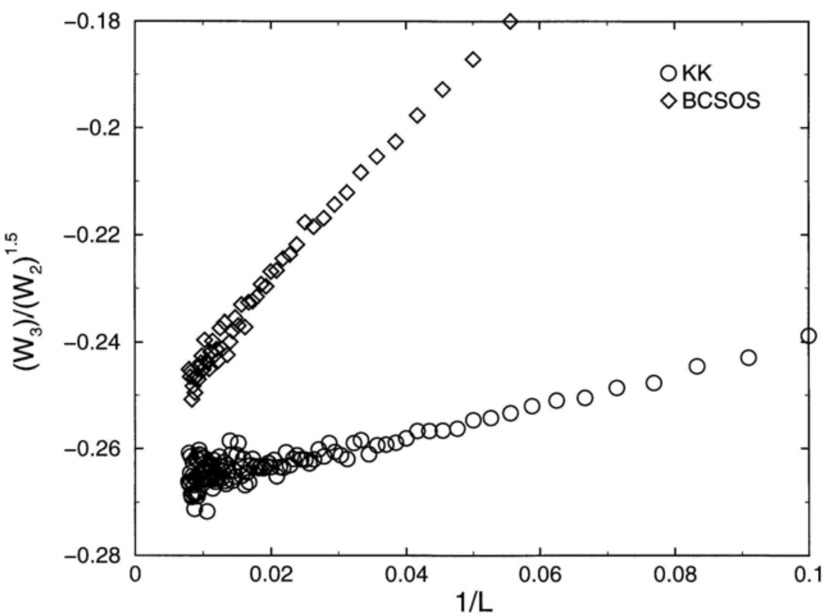
In 2D the stationary state is a novel non-Gaussian distribution, which needs still further investigation.

Parisi's group obtained more accurate MC data, after our study. They claimed to see deviations from $\alpha = 0.4$ at the 1% level. Unfortunately they introduced a systematic error in their FSS analysis by forcing $y_\nu = y_{sk}$. The leading corrections to scaling in even and odd moments are clearly not equal in our analysis.

8. Universal moment amplitude ratio's in 2D

The amplitude ratio's $R_n = W_n/(W_2)^{\frac{n}{2}}$ converge smoothly to $R_3 = -0.27(1)$ and $R_4 = +3.15(2)$.





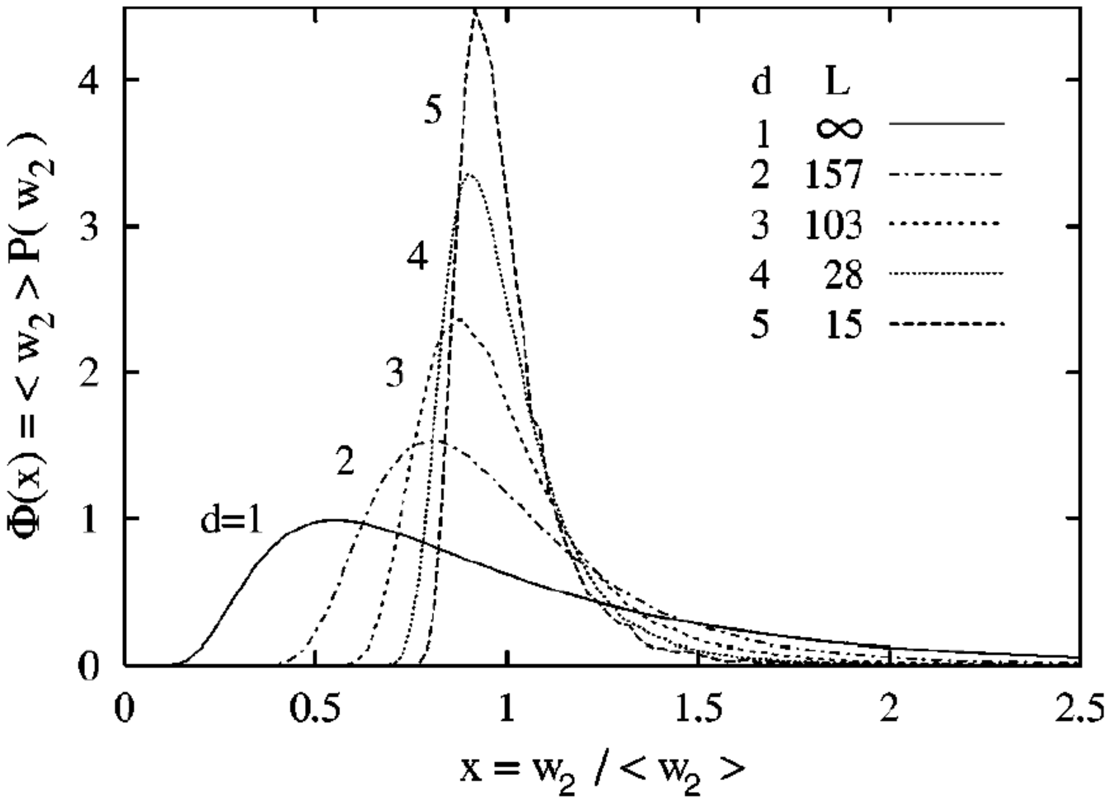
Skewness is negative, $R_3 < 0$.

Maybe $R_4 = \pi$. ($R_4 = 3$ in Gaussian distributions.)

9. Width distribution functions

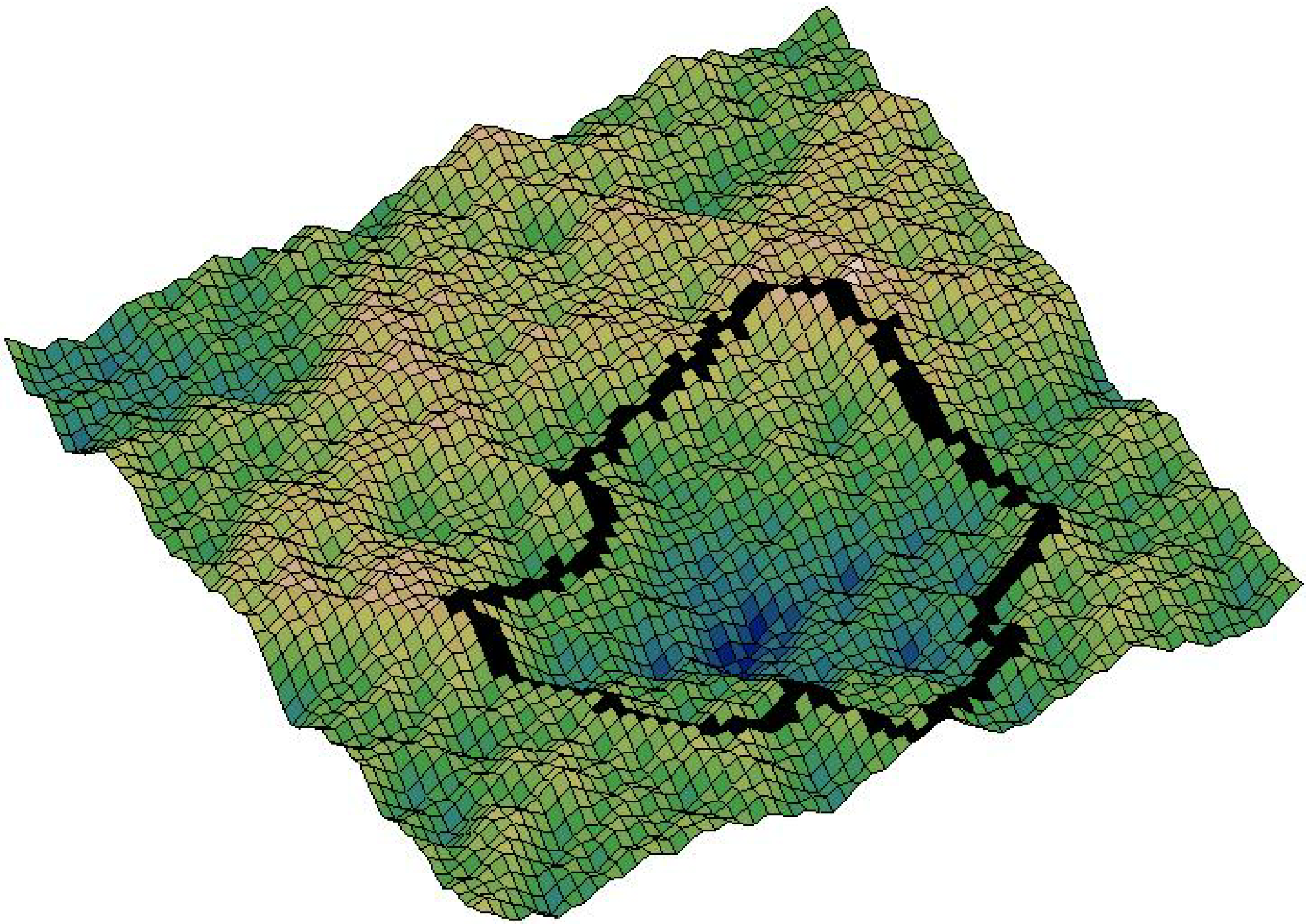
$$P(w_2) \sim \frac{1}{w_2(L)} \Phi\left(\frac{w_2}{w_2(L)}\right) \; ; \; w_2^2 = \frac{1}{L^D} \sum_r (h_r - h)^2$$

$\langle w_2 \rangle = w_2(L)$ is the ensemble averaged width; w_2 is the width in one specific sample. Φ is the scaling function



Marinari *et al*, PRE 65, 026136 (2002)

B. RECONSTRUCTED ROUGH GROWING INTERFACES, RIDGELINE TRAPPING OF DOMAIN WALLS



PRE 64, 031606 (2001)

1. Surface reconstruction during growth

Question: Do surface reconstruction order-disorder phase transitions exist in stationary states of growing surfaces?

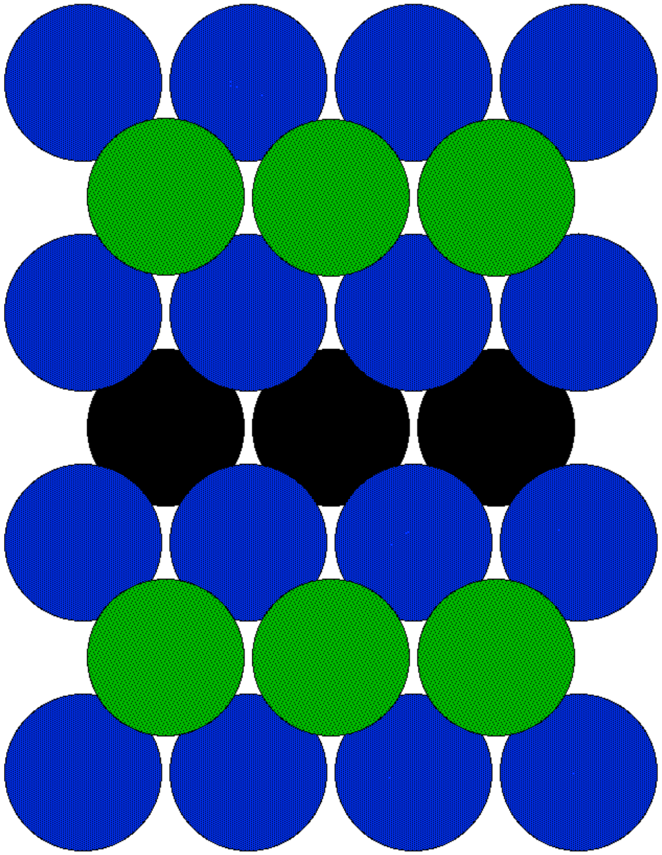
– Growing surfaces are always rough (although they might seem, e.g., to grow layer-by-layer at most experimental length scales for $T < T_R$).

– Reconstructed rough phases, and equilibrium deconstruction transitions exist while the surface is rough.

Question: Can reconstructed rough order exist in stationary states of growing surfaces like they do in equilibrium?

2. Missing row reconstruction in Au and Pt(110)

In equilibrium SC(110) surfaces the MR reconstruction order and roughness are topologically compatible, but not so in MR FCC(110) facets.



In cases where steps excitations are topologically compatible with the Ising type reconstruction order → Reconstructed rough phase.

MdN PRB 46, 10386 (1992)

3. 2D RSOS model with negative step energies

The RSOS model contains a RR phase in equilibrium.

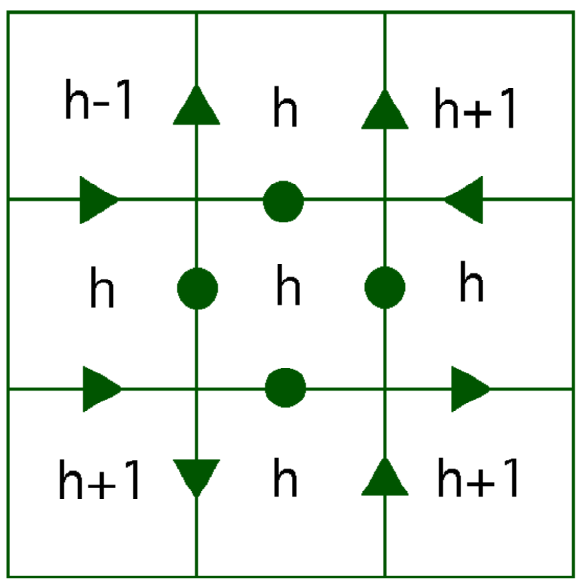
Heights at nearest neighbour sites differ by $dh = 0, \pm 1$

Nearest neighbour interactions $E = \sum_{\langle i,j \rangle} K(h_i - h_j)^2$

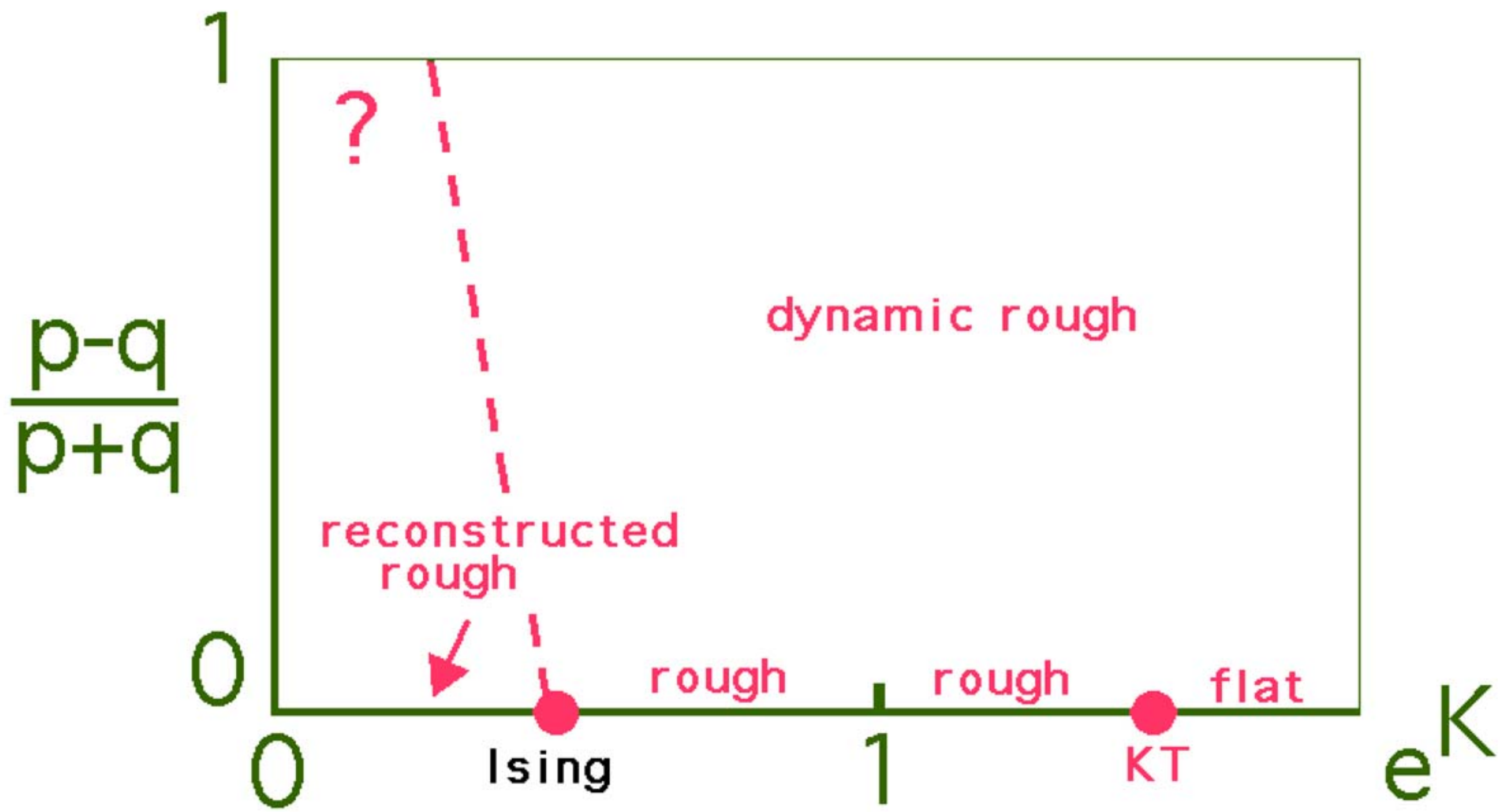
In limit $K \to -\infty$ the $dh = 0$ states are frozen out (BCSOS model)

checkerboard type RR order.

Ising type equilibrium deconstruction transition inside the rough phase (MdN 1985)



4. Ising field coupled to KPZ



The AF RSOS model provides a compact way to couple an Ising field to KPZ.

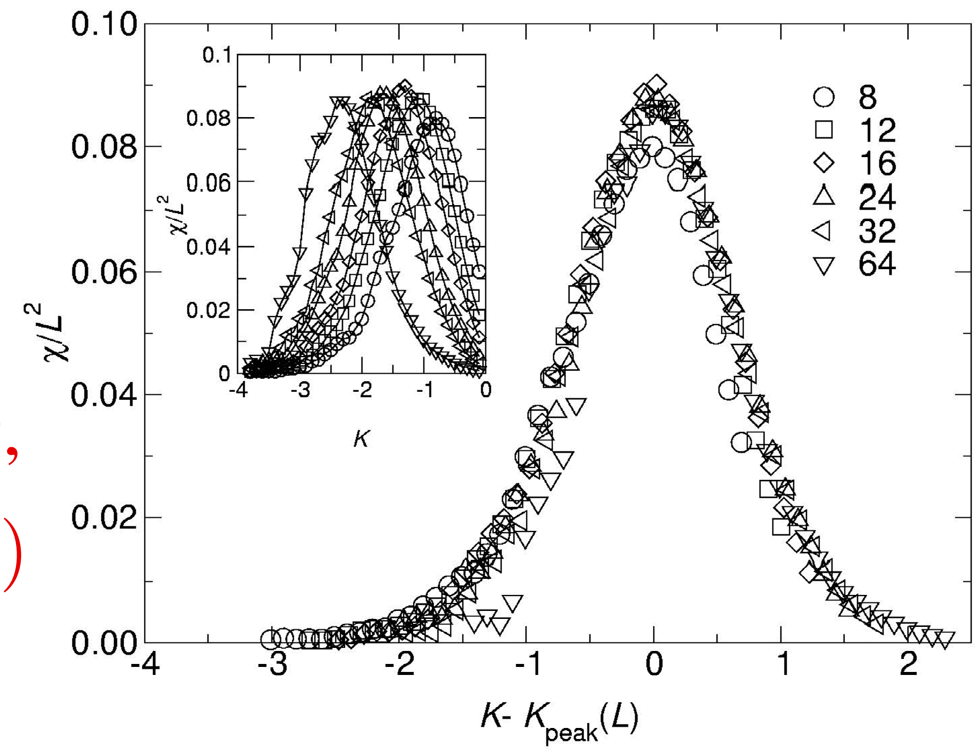
5. MC results: $K < 0$ susceptibility

The RR checkerboard type order is characterized by $m = \langle (-1)^{x+y}\, S_r \rangle$ with $S_r = \exp(i\pi h_r) = \pm 1$.

The susceptibility peak, $\chi = L^2(\langle m^2 \rangle - \langle |m| \rangle^2)$ suggests a transition.

But the peak scales with system size only as $\chi \sim L^2$ and keeps moving,

$K_{\mathrm{peak}}(L) \simeq -A \ln(L/L_0)$



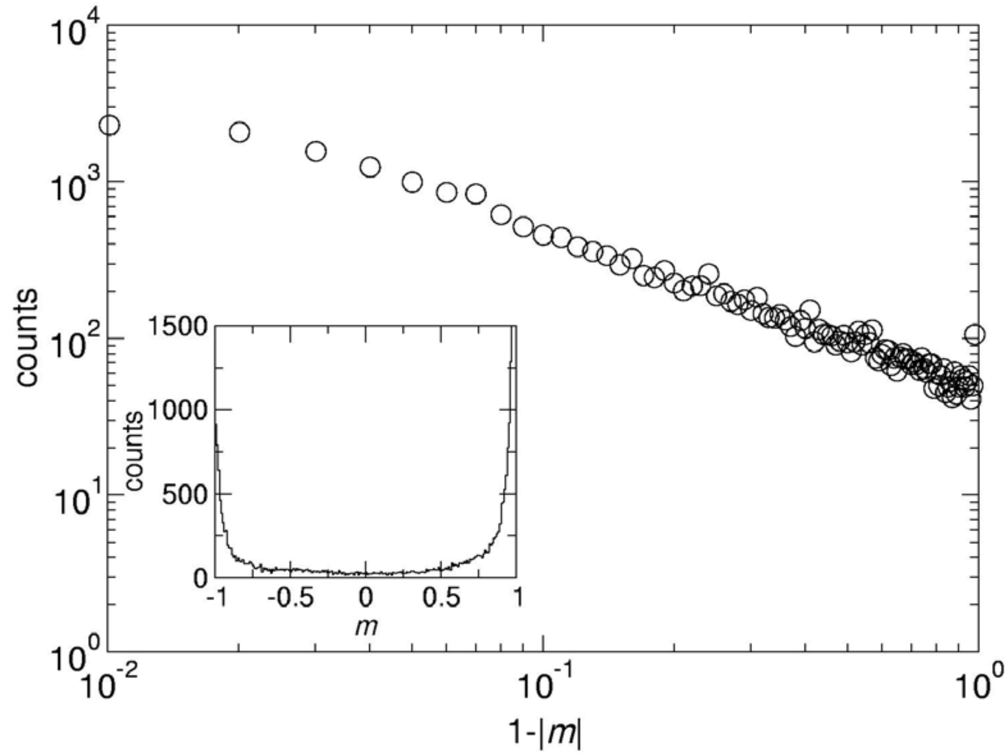
$A = 0.77(5)$, $L_0 = 2.2(2)$

6. MC results: quasi-critical reconstruction fluctuations

Power law tails in the time series histogram of the staggered magnetization suggest critical fluctuations in the RR order for $K < K_{\text{peak}}(L)$.

Quasi critical order exists within a length scale $l_{rec}(K)$ for every $K < 0$.

The susceptibility peak signals the K where $l_{rec}(K) \simeq L$.



7. The life cycle of loops; ridge line trapping

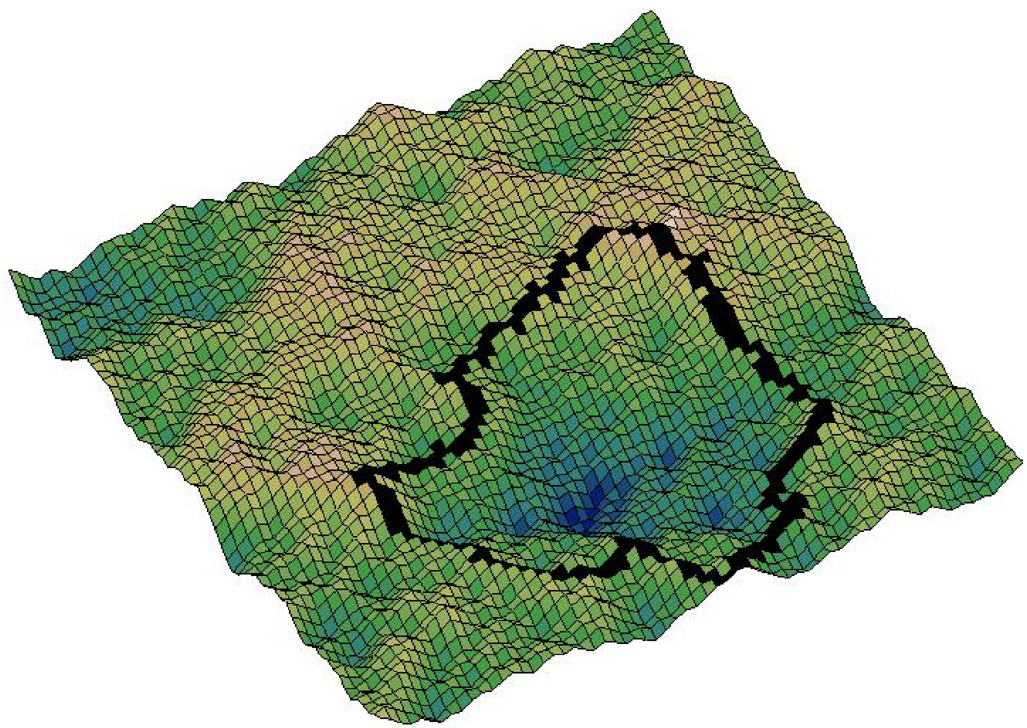
Reconstruction domain wall loops ($dh = 0$ contours) are nucleated in valley bottoms.

Loops grow, subject to an up-hill bias until they coincide with a ridge line.

A Loop remains strapped to the ridge lines until:

$\rightarrow$ another loop nucleation event annihilates it.

$\rightarrow$ KPZ fluctuations to which it is slaved fill-up the enclosed valley.



8. The nucleation of new loops

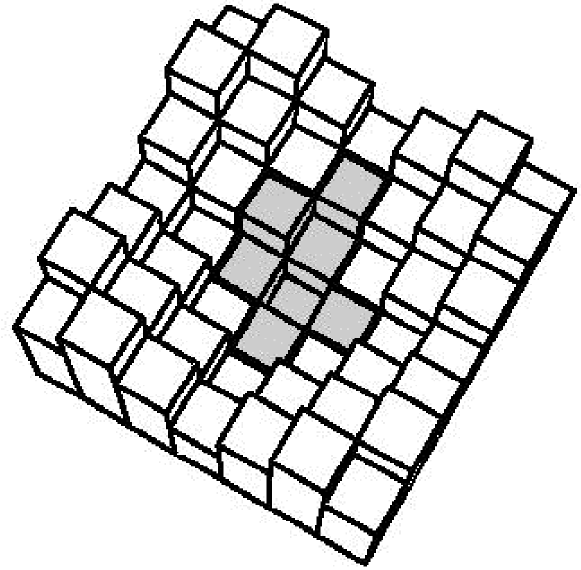
Numerically:

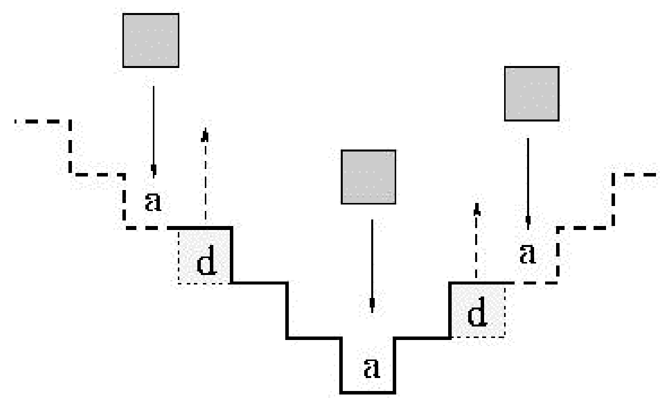
$\tau_n \sim \exp(-aK)$ with $a = 3.0 \pm 0.1$ (in BCSOS time units).

Prepare surface in the BCSOS KPZ stationary state at $K \ll K_{\mathrm{c}}$; record the intervals between macroscopic loop events.

Qualitatively:

Depositions remain indistinguishable from KPZ growth events until $l_c^2 \simeq 6$ $\rightarrow \tau_n \sim L^{-2} e^{-4K}$.

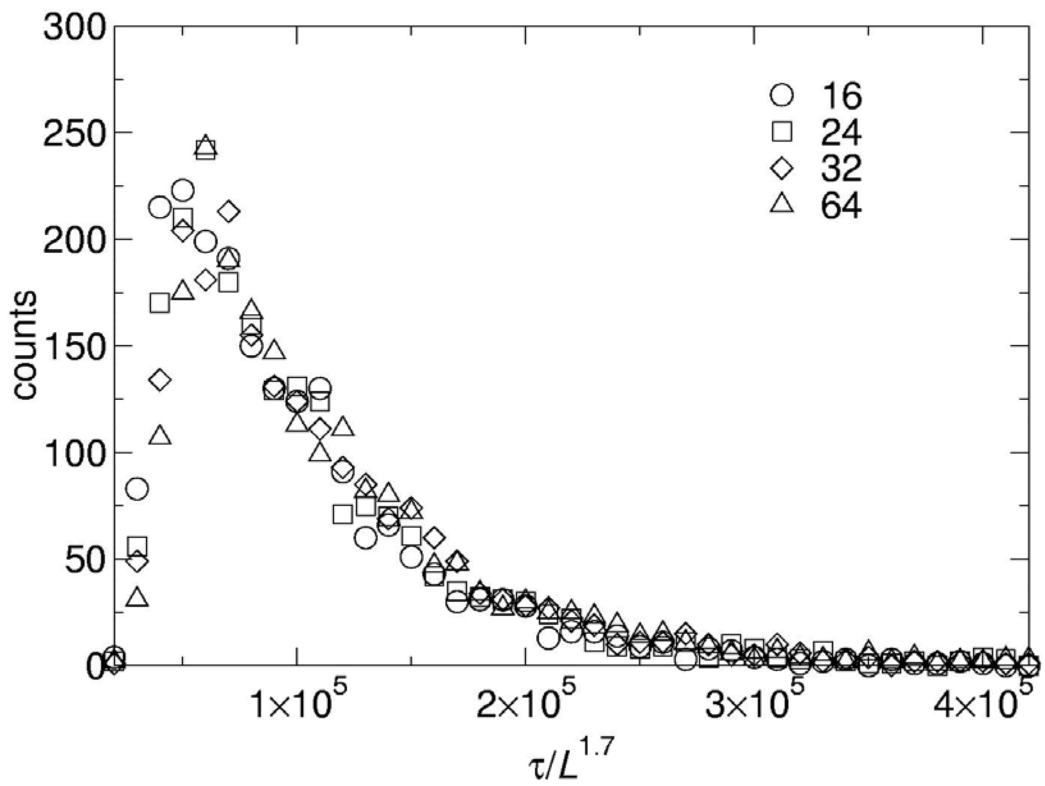




9. The KPZ trapping time scale

Measure the life time of a marked ridge line of diameter l on the KPZ growing surface.

The valley encompassed by the ridge line (it forms the watershed) vanishes by KPZ growth at a time scale $\tau_z \sim l^z$, with $z = \frac{8}{5}$.



The distribution of life times scales numerically with $z \simeq 1.7 \pm 0.1$.

10. 1D Gaussian interface width distribution !?

PHYSICAL REVIEW E VOLUME 50, NUMBER 2 AUGUST 1994

Width distribution for random-walk interfaces

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(Received 18 May 1994)

Roughening of a one-dimensional interface is studied under the assumption that the interface configurations are continuous, periodic random walks. The distribution of the square of the width of interface, w^2, is found to scale as $P(w^2)=\langle w^2\rangle^{-1}\Phi(w^2/\langle w^2\rangle)$ where $\langle w^2\rangle$ is the average of w^2. We calculate the scaling function $\Phi(x)$ exactly and compare it both to exact enumerations for a discrete-slope surface evolution model and to Φ's obtained in Monte Carlo simulations of equilibrium and driven interfaces of chemically reacting systems.

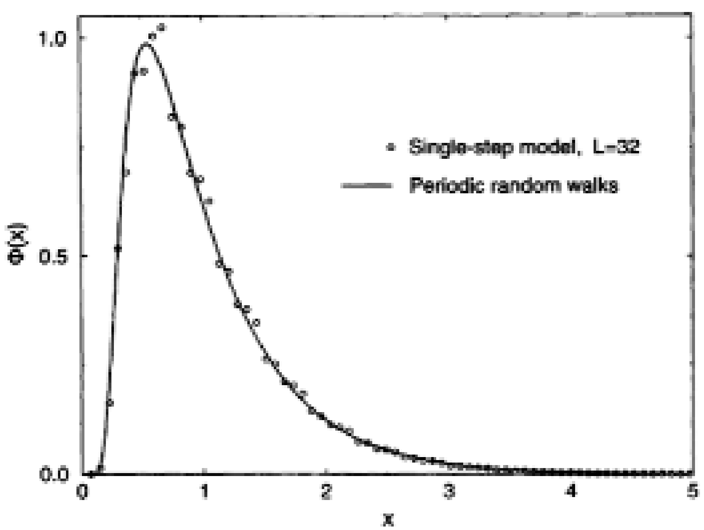


FIG. 2. Scaling function for the finite ($L=32$) single-step model as compared with the result for the periodic random-walk model.

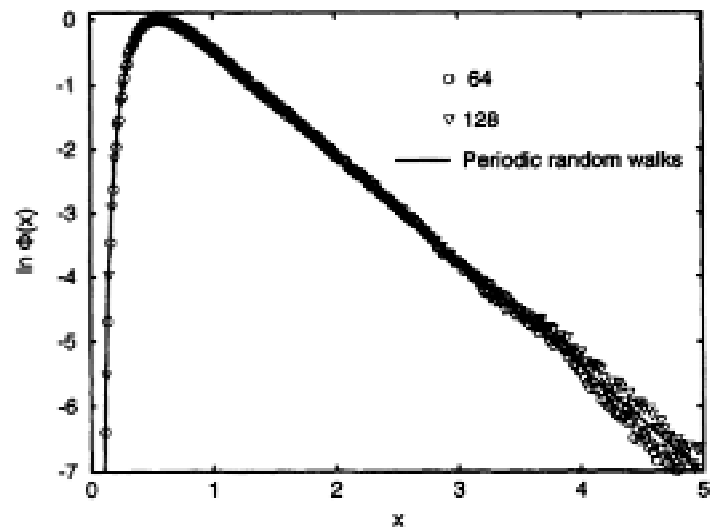
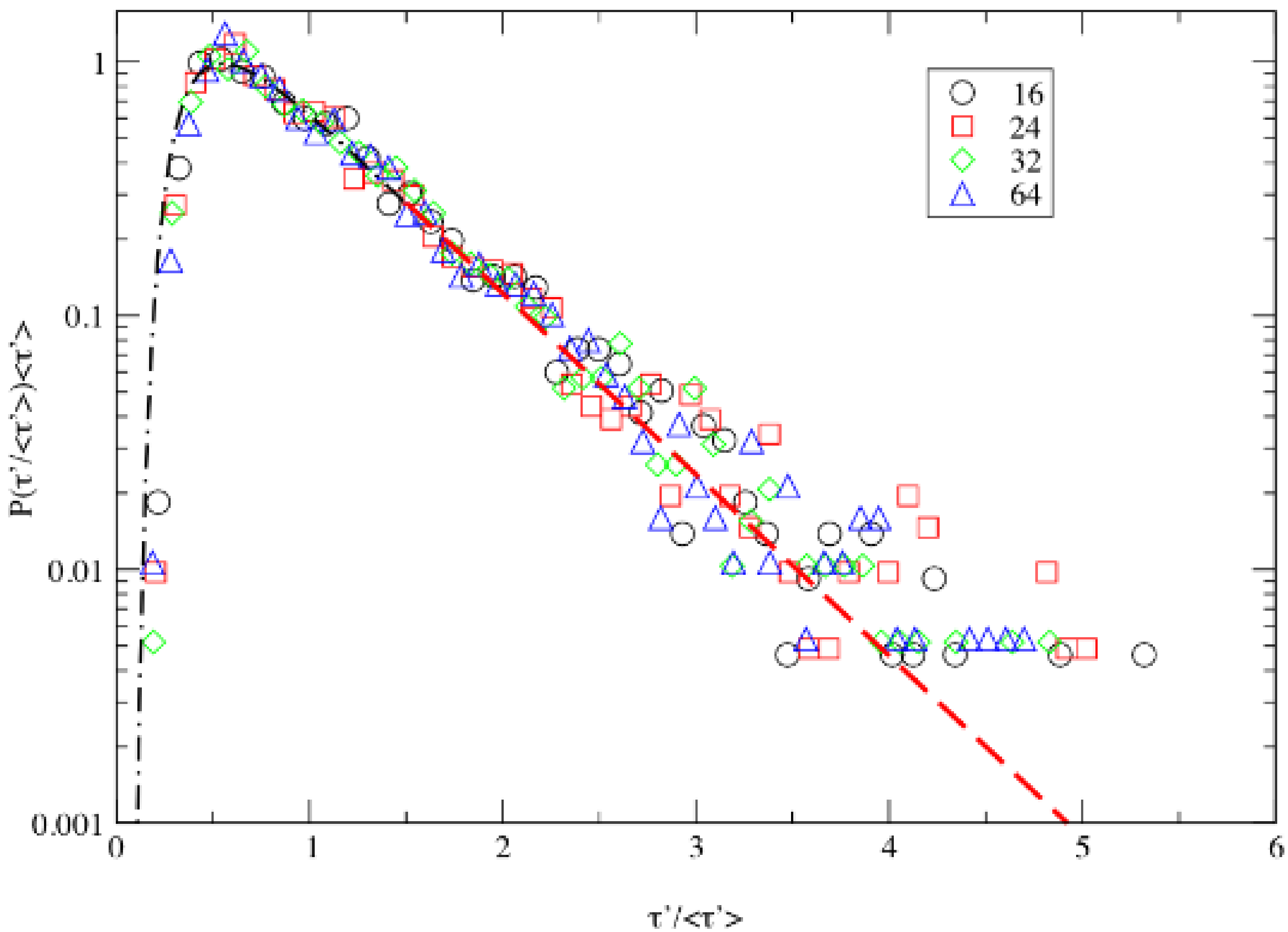


FIG. 3. Comparison of scaling functions for the catalytic reaction model ($p=\frac{1}{2}$, standing interface) and the periodic random-walk model.

11. Gaussian fluctuations!?

The life time distribution is well fitted by the width distribution of 1D Gaussian interfaces



12. The crossover reconstruction length scale

KPZ critical fluctuations show up below a characteristic length scale $l_{\rm rec}$, where the nucleation time scale $\tau_{\rm n} \sim \exp(-aK)$ is larger than the KPZ time scale, $\tau_z \sim L^z$.

$$\tau_z \simeq \tau_n \quad \rightarrow \quad l_{rec} \sim \exp(\frac{a}{z}K)$$

The susceptibility peak shifts logarithmically, consistent with $\tau_n \simeq \tau_z \rightarrow K_{\rm c} = -\frac{z}{a}\ln(L/L_0)$.

The amplitude of the shift is numerical about 30% larger than z/α; the argument ignores, e.g., the self similarity of the rough surface and its ridge line network. Trapped loops can jump over adjacent subvalleys by nucleation events in those subvalleys. Such events renormalize τ_z.

13. Conclusions

- During surface growth, the stationary states lack true macroscopic reconstructed rough order.
- Trapping of domain wall loops at ridge lines in the growing surface (caused by an upwards drift) is the fundamental mechanism.
- The competition between domain loop nucleation and ridge line KPZ fluctuations sets a temperature dependent crossover length scale $l_{rec}(K)$.
- Within l_{rec}, the surface appears as reconstructed rough, with quasi-critical fluctuations (power law shape diffraction peaks) .