Universal Low Temperature Physics and Pseudogaps in Coulomb and Spin Glasses

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Markus Müller

S. Pankov (Tallahassee)
L. B. Ioffe (Rutgers)

HARVARD

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Road map

- Introduction to spin/electron glasses with long range interactions (Coulomb):
    - Pseudogaps and glassy behavior
- Theoretical mean field approach to electron glasses
    - Physics of the glass transition and replica symmetry breaking
- Solution and low temperature
    - Temporal ‘RG’ flow, fixed points, and universality
- Connection with Experiments

Introduction

Glasses with quenched disorder

- Interactions + disorder $\rightarrow$ Frustration and glassy behavior

- No simple order, but randomly patterned "spin glass order" in many different pure states

- Absence of order $\rightarrow$ no hard gaps, but soft pseudogaps

- Multitude of metastable configurations leads to out of equilibrium behavior and history dependence

Coulomb glasses

Anderson insulators with strong electron-electron interactions

M. Pollak (1970)
A. Efros, B. Shklovskii (1975)
J.H. Davies, P.A. Lee, T.M. Rice (1982,84)

Efros-Shklovskii model

$$H = \frac{1}{2}\sum_{i \neq j}\left(n_i - \nu\right)\frac{e^2}{r_{ij}}\left(n_j - \nu\right) + \sum_i n_i \varepsilon_i$$

$n_i = 0,1$: Occupation of sites on a given lattice

Unscreened Coulomb interactions

Neutralizing background charge

Disorder

$$P\left(\varepsilon_i\right) = \frac{\exp\left[-\left(\varepsilon_i / W\right)^2 / 2\right]}{\sqrt{2\pi W^2}}$$

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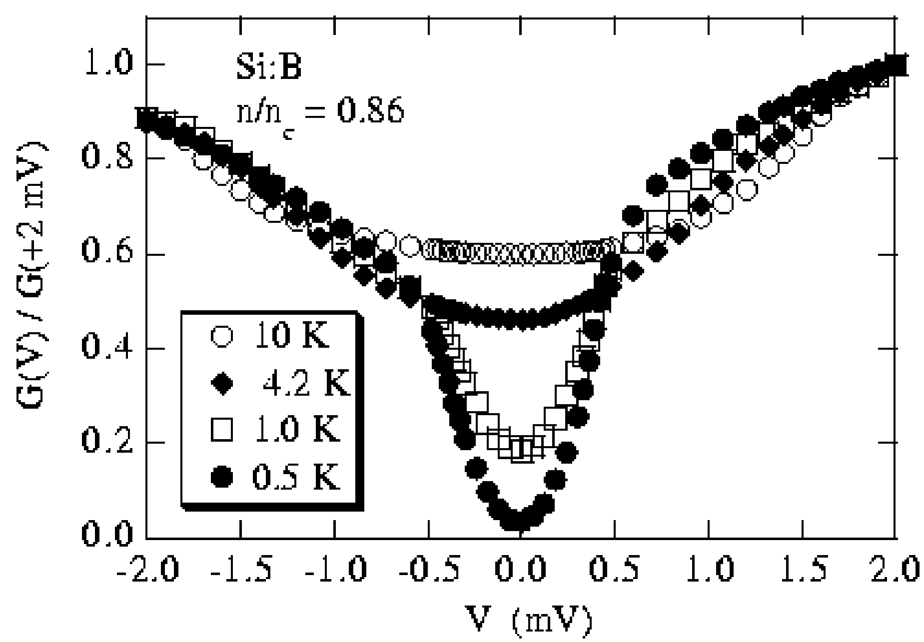
Strongly localized electrons → Classical problem with strong frustration

$\nu = 1/2 \quad \rightarrow \quad s_i \equiv n_i - 1/2$ ↔ Long range antiferromagnetic spin glass

I. Pseudogaps

Coulomb gap : Tunneling DOS



*J. G. Massey and M. Lee, PRL **75**, 4266 (1995)*

Boron-doped silicon matrix

$$n = 4.0 \cdot 10^{18}\, cm^{-3}$$

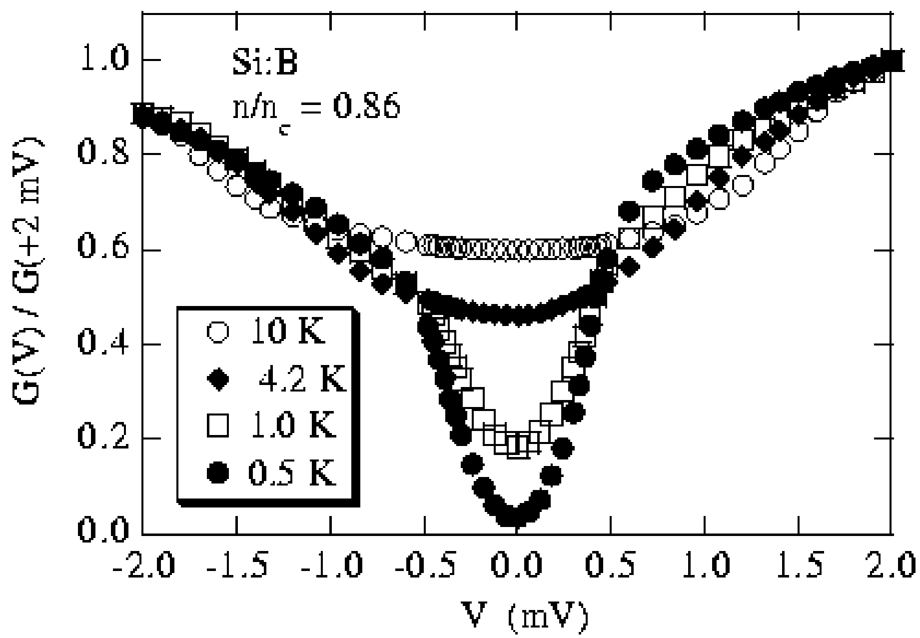
$$n/n_c = 86\%$$

Soft "Coulomb gap" in the density of states in the classical limit

Local fields: $E_i = \sum_{j \neq i} \frac{e^2}{\kappa r_{ij}} n_j + (\varepsilon_i - \mu)$ $\quad \rho(E) = \frac{1}{N} \sum_{i=1}^{N} \delta(E - E_i)$

Efros-Shklovskii: $\rho(E) = C(\kappa/e^2)^3 E^2$ $\quad \sigma(T) \propto \exp\left[-(T_{ES}/T)^{1/2}\right]$

Coulomb gap : Tunneling DOS



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↔ Mott insulator (charge ordered state): Hard gap

Long range spin glasses (SK-model)

SK model (N spins) + random fields

Sherrington and Kirkpatrick (1975)

G. Parisi (1979)

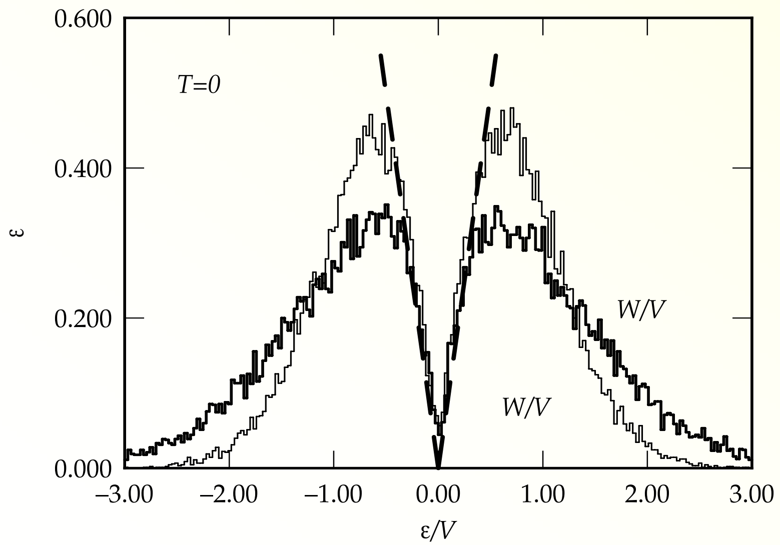
$$H = \frac{1}{2}\sum_{i \neq j} s_i J_{ij} s_j + \sum_i s_i h_i$$

Random exchange

$$P\left(J_{ij}\right) = \exp\left[-\frac{J_{ij}^2}{2NV^2}\right] \Big/ \sqrt{2\pi NV^2}$$

Random fields

$$P\left(h_i\right) = \exp\left[-\frac{h_i^2}{2W^2}\right] \Big/ \sqrt{2\pi W^2}$$



Linear 'Coulomb' gap!

Thouless, Anderson and Palmer, (1977)
Palmer and Pond (1979)
Bray, Moore (1980)
Sommers and Dupont (1984)

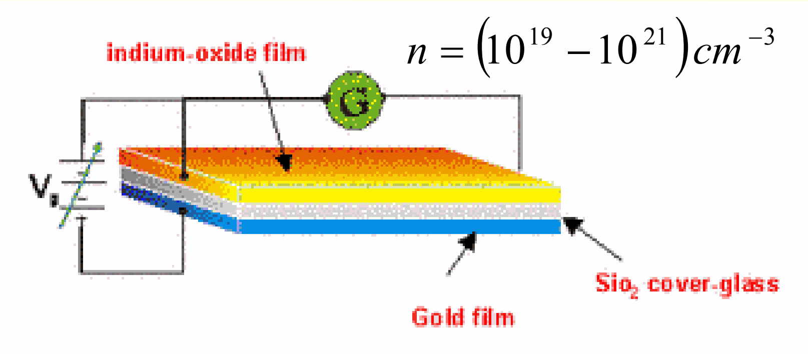
Dobrosavljevic, Pastor (1999)

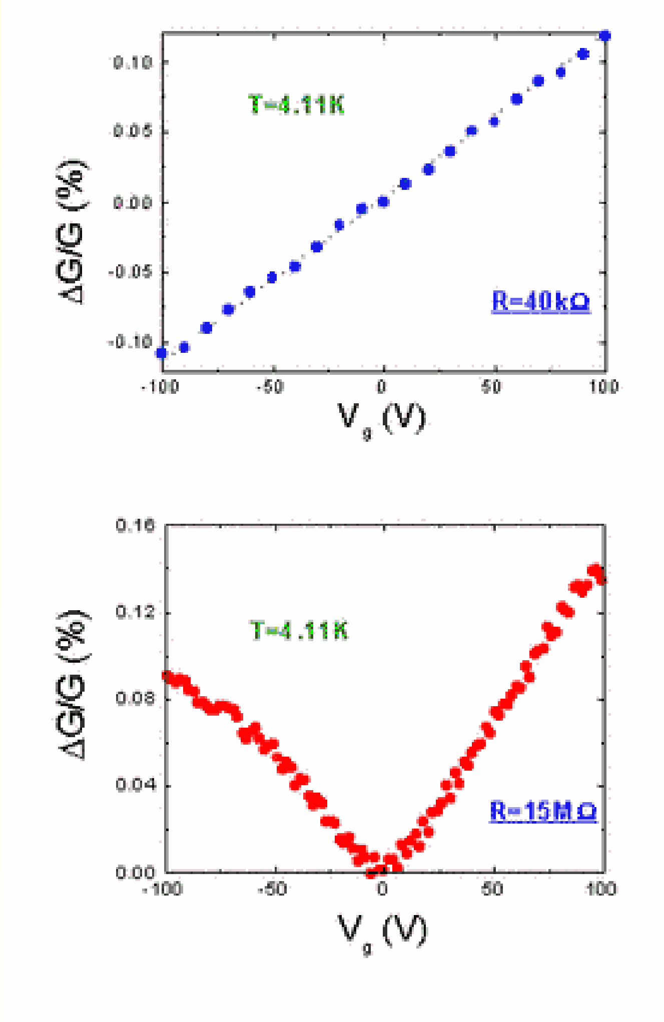
II. Glassy behavior in electronic systems

Electron glasses: Anomalous field effect

Indium-oxides In_2O_{3-x}

Z. Ovadyahu et al.





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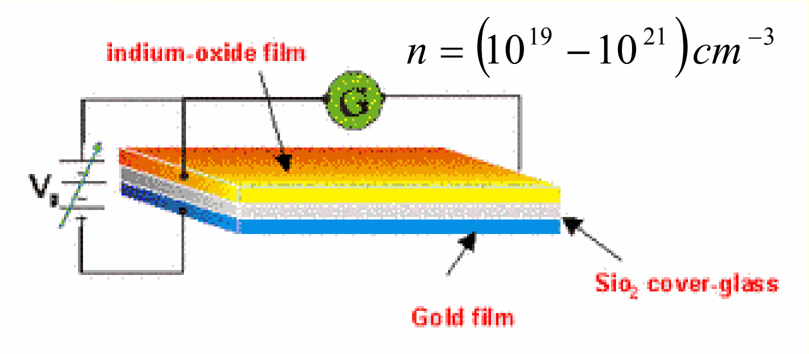
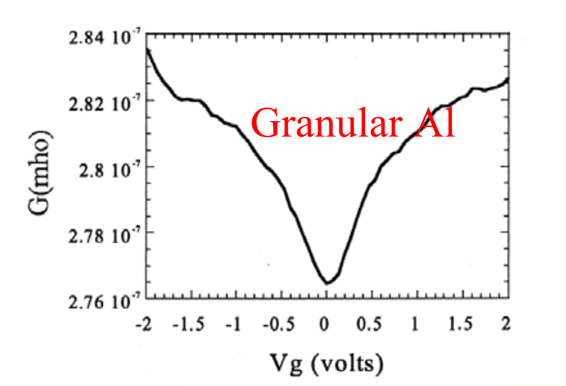
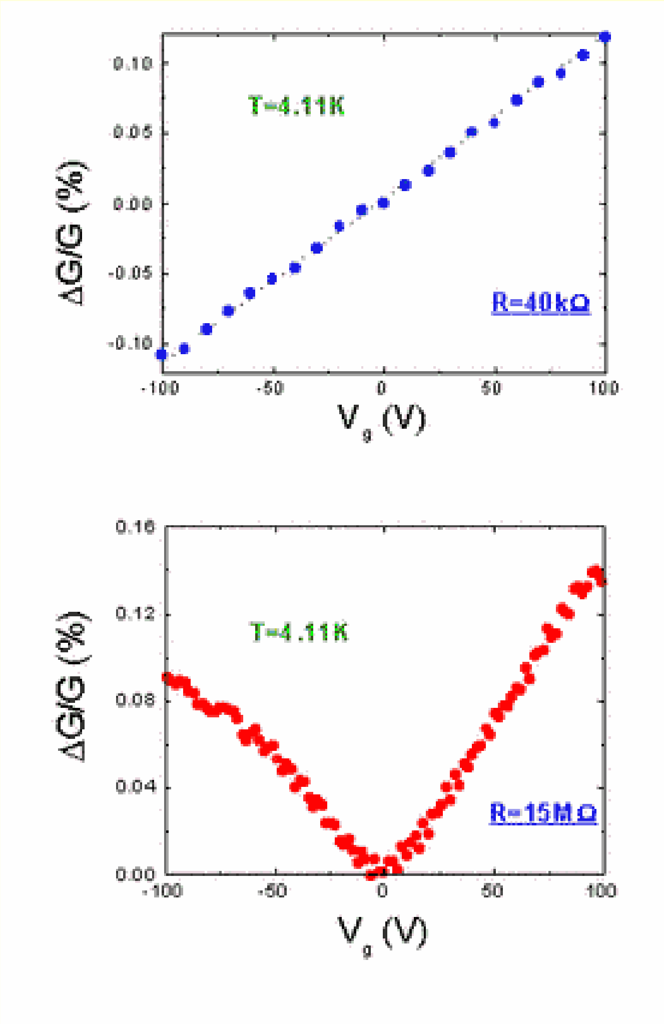




*•T. Grenet, EPJ B **32**, 275 (2003)*



+
- Slow relaxation
- Aging
- Memory

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Questions

- Why is the Coulomb gap so universal?
- How is the pseudogap related to glassiness?
- Low temperature description?
- Experimental consequences of the glass?

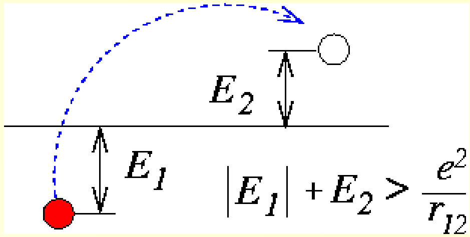
? ?

Review: The Coulomb gap

A. Efros, B. Shklovskii (1975)

Stability of ground state with respect to one particle hop:

The density of states at the Fermi level must vanish at T = 0.



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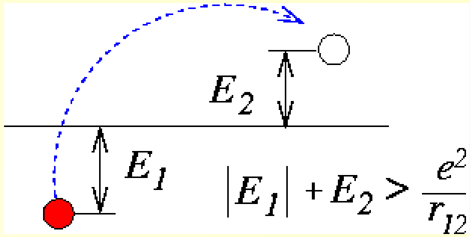

Self-consistent argument:

$$R_E = \frac{e^2}{E} \; ; \; R_E^D \cdot \int_0^E \rho(E)\, dE \leq 1$$

→ Parabolic pseudogap in D = 3.

$$\rho(E) = cst.\left(\kappa / e^2\right)^D E^{D-1}$$

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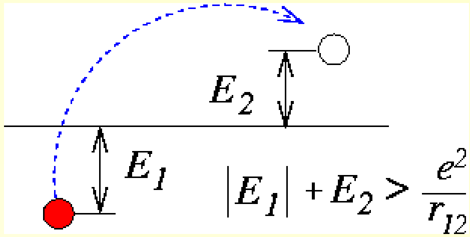

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?

Why is this upper bound saturated?
Why is the gap so universal?

?

Locator approximation for the Coulomb glass

MM and L.B. Ioffe, PRL 2004
S. Pankov and V. Dobrosavljevic, PRL 2005
MM and S. Pankov, condmat - 0611021

Locator approximation based on a systematic diagrammatic technique.

- Glass transition due to critical fluctuations in the screening
- Marginal stability and its relation to the saturated Efros-Shklovskii Coulomb gap.
- Low temperature universality

High T expansion

S. R. Johnson, D.E. Khmel'nitskii (1996)

Hamiltonian (Coulomb glass)

$$H = \frac{1}{2}\sum_{i\neq j}(n_i - \nu)\frac{e^2}{r_{ij}}(n_j - \nu) + \sum_i n_i \varepsilon_i$$

Particle hole symmetric case

$$\nu = 1/2 \qquad s_i \equiv n_i - 1/2$$

$$J_{ij} \equiv e^2/r_{ij}$$

$$\rightarrow \quad H = \frac{1}{2}\sum_{i\neq j} s_i J_{ij} s_j + \sum_i s_i \varepsilon_i$$

Partition function

$$Z = \sum_{\{s_i\}} \exp\left\{-\frac{1}{2}\sum_{i\neq j} s_i (\beta J)_{ij} s_j + \sum_i \beta\varepsilon_i s_i\right\}$$

$$= \int \prod_i d\varphi_i \sum_{\{s_i\}} \exp\left\{-\frac{1}{2}\sum_{i\neq j} \varphi_i (\beta J)^{-1}_{ij} \varphi_j + \sum_i (\beta\varepsilon_i + i\varphi_i) s_i\right\}$$

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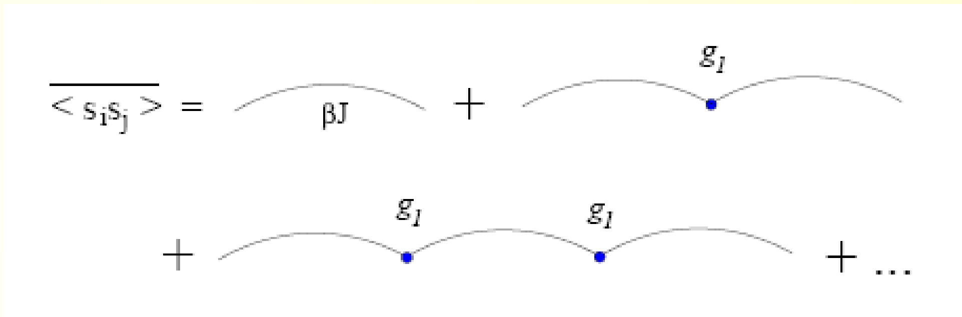
Replica trick

$$-\beta\overline{F} \equiv \overline{\ln[Z]} = \lim_{n\to 0}\frac{\overline{Z^n - 1}}{n}$$

Glass transition I

Disorder-averaged correlations

$$\overline{\langle s_i s_j \rangle_c} = C\,\frac{e^{-r/\xi_1}}{r}\,, \qquad \xi_1 \propto a\sqrt{W/E_{Cb}}$$



$$g_1 \propto \left\langle \frac{\beta}{\cosh^2(\beta\varepsilon)} \right\rangle_\varepsilon \propto \frac{1}{W}$$

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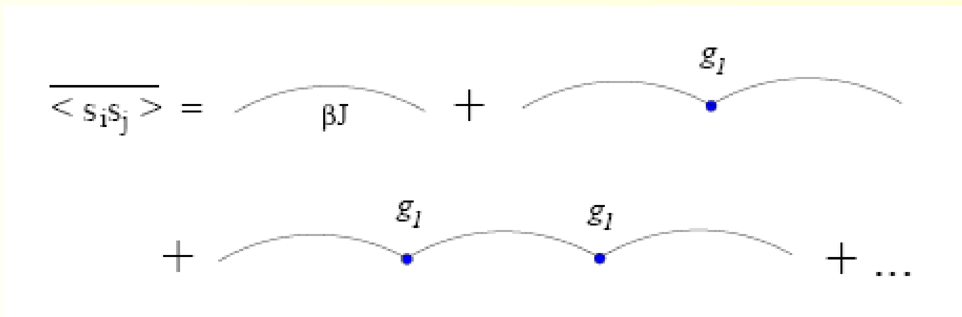

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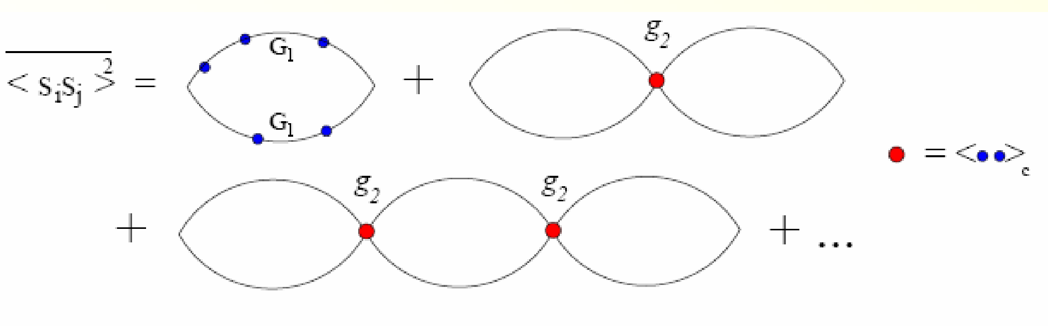

$$g_2 \propto \left\langle \frac{\beta^2}{\cosh^4(\beta\varepsilon)} \right\rangle_\varepsilon - g_1^2$$

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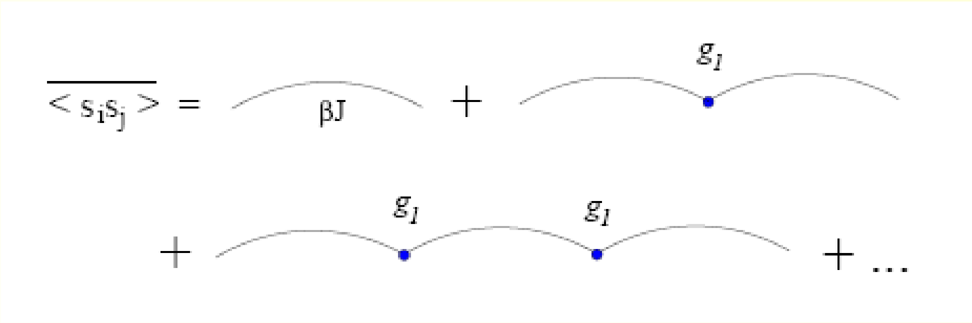

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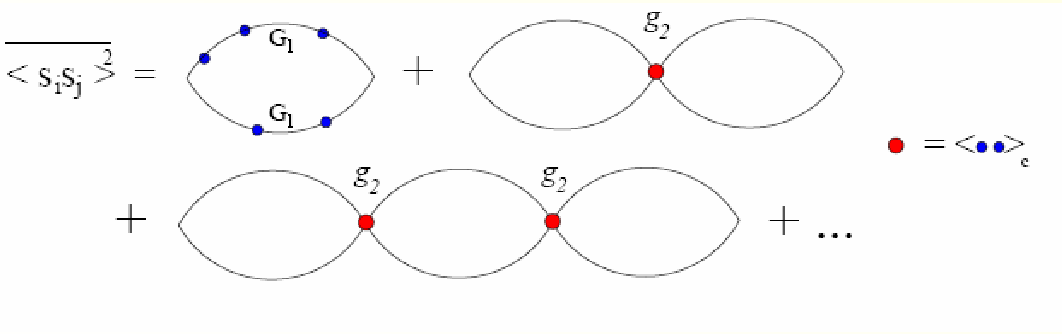
$\xi_2 \to \infty$ for $T \to T_c$



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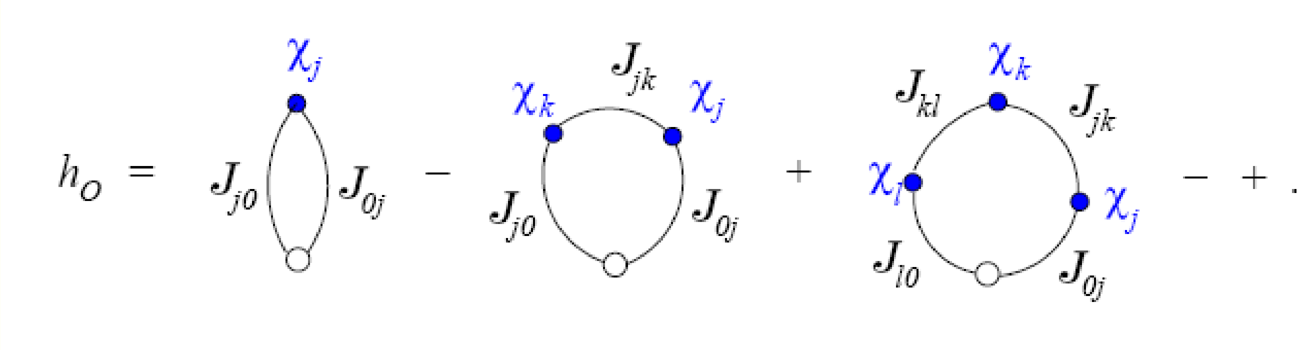
Glass transition II/III

Alternative views of T_c: II) Onsager back reaction



Back reaction of environment ~ T

$$h_O \approx \int_0^{\xi_1} d^3r \frac{J^2(r)}{W} \approx T_c$$

→ Transition to collective, correlated state

Glass transition II/III

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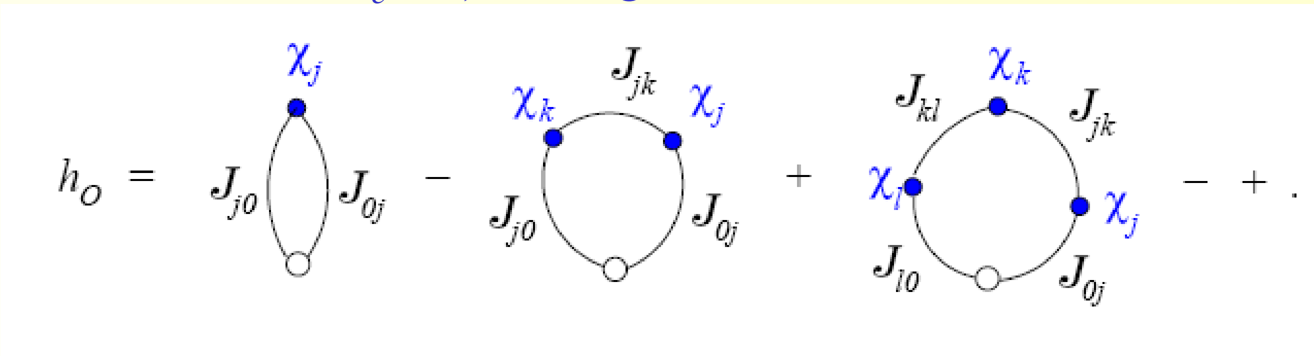

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$$T_c = \frac{e^2/a}{6(2/\pi)^{1/4}} \sqrt{e^2/aW}$$

Width of Efros-Shklovskii gap!

Glass transition II/III

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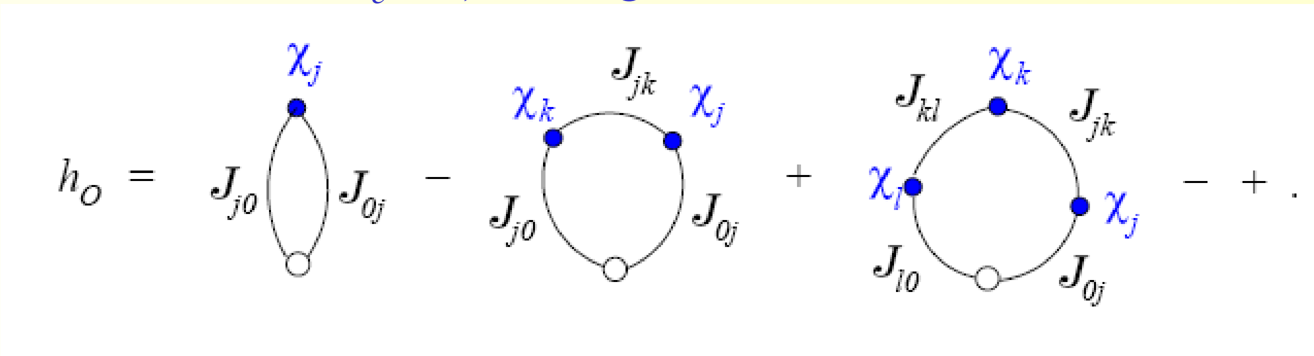

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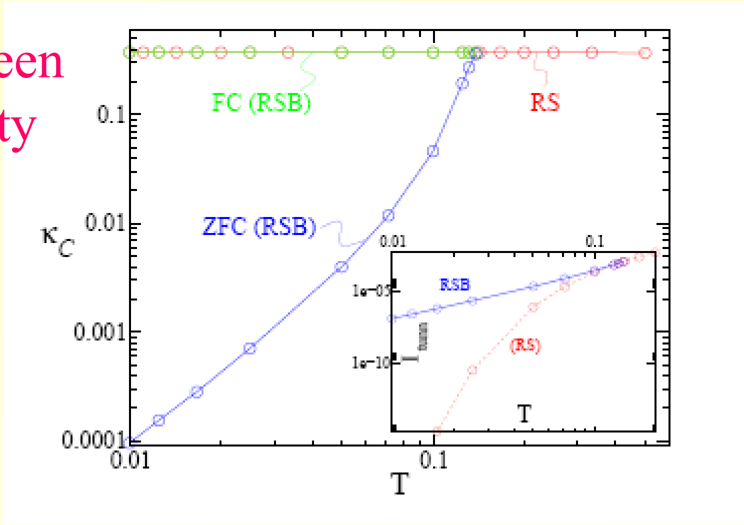
Width of Efros-Shklovskii gap!

III) Local approximation (MF theory): Instability of the high T (replica symmetric) phase

→ Continuous glass transition, same universality as the RF-SK model.

Properties of the glass phase

- Large number of pure states → Difference between field cooled, and zero field cooled compressibility



- Broken ergodicity

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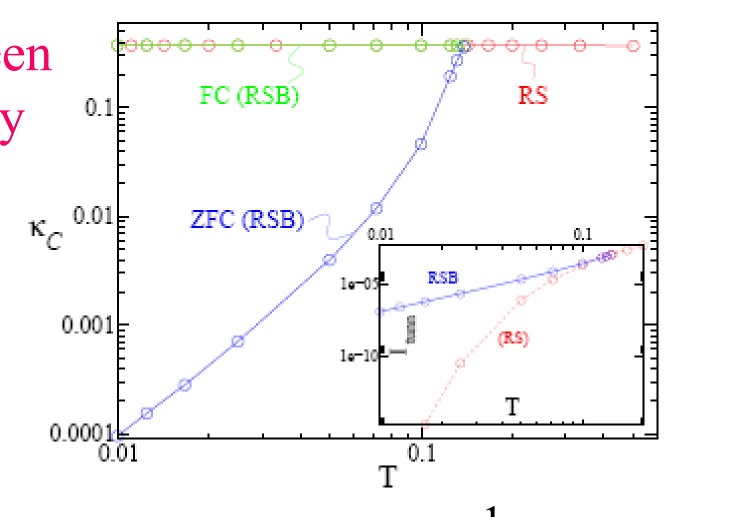


- Broken ergodicity

- Marginal stability

→ Widely spread charge response (screening) $\langle n_j | n_i = n \rangle - \langle n_j \rangle \propto \frac{1}{r^{\alpha}}$

→ Detect glass phase by non-local charge response!

→ The system is permanently in a critical (almost unstable) state with excitations down to zero energy.

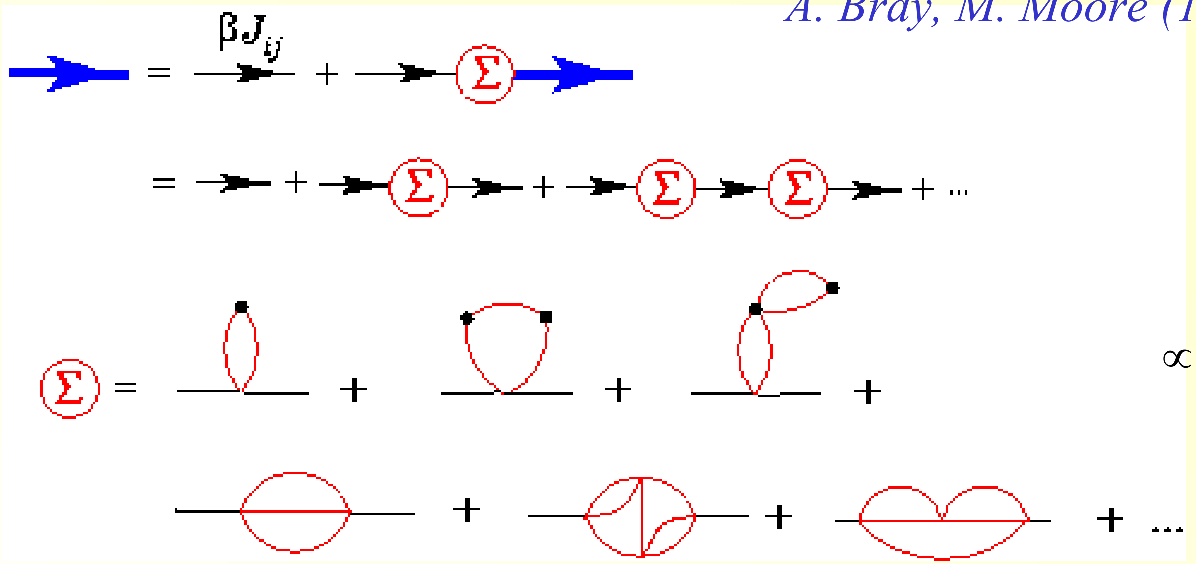
→ Soft collective modes and slow dynamics.

→ Expect effects of these modes on activated transport (hopping).

Below T_c: Locator approximation

M. Feigel'man, A. Tsvelik (1979)
A. Bray, M. Moore (1979)

$$\langle \varphi_i \varphi_j \rangle \equiv$$



$$\propto \left(\frac{\beta E_{Cb}^2}{W} \right)^{1/2} \frac{1}{(\beta W)^3}$$

$$\propto \left(\frac{E_{Cb}}{W} \right)^{3} \frac{1}{(\beta W)^3}$$

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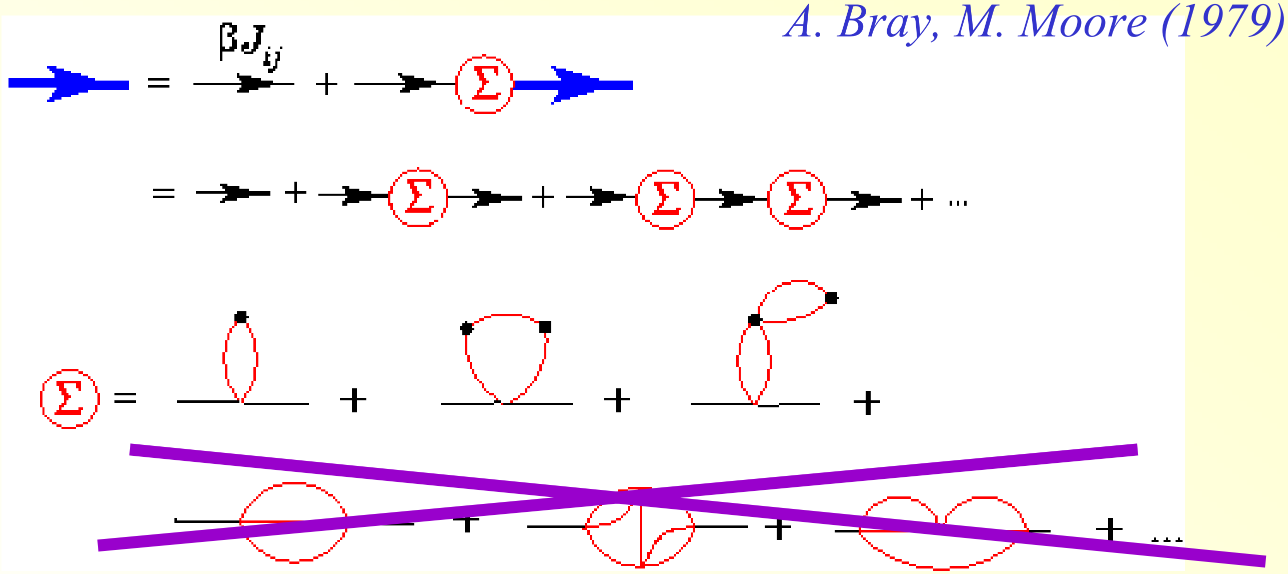

Local self-energy with non-trivial replica structure

$$\Sigma_{ab}(k) \approx \Sigma_{ab}$$

Map to an effective single-site model with a selfconsistent self-energy Σ ("local field").

Mapping to a single-site model

$$\beta H\left(\{s_i\}\right) = \beta\left(\frac{1}{2}\sum_{i\neq j} s_i J_{ij} s_j + \sum_i s_i \varepsilon_i\right)$$

$$\longrightarrow \quad \beta H_0\left(\{s^a\}\right) = -\frac{\beta^2}{2}\sum_{a,b} s^a\left(\Lambda_{ab} + W^2\right)s^b$$

→ Resummation of all diagrams with local self-energies.

Self-consistency of the coupling Λ_{ab}

$$Q_{ab} \equiv \frac{1}{N}\sum_i \left\langle s_a^i s_b^i \right\rangle_H = \left\langle s_a s_b \right\rangle_{H_0}$$

→ Exact for SK spin glass, controlled approximation for Cb-glass.

Replica symmetry breaking (RSB)

G. Parisi (1979)

Effective single site problem: How to break replica symmetry?

$$\beta H_0\left(\left\{s^a\right\}\right) = -\frac{\beta^2}{2}\sum_{a,b} s^a\left(\Lambda_{ab} + W^2\right)s^b$$

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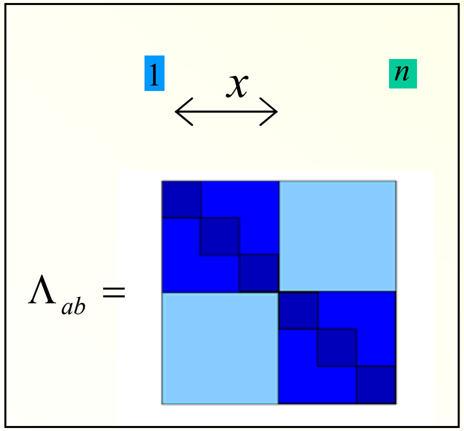
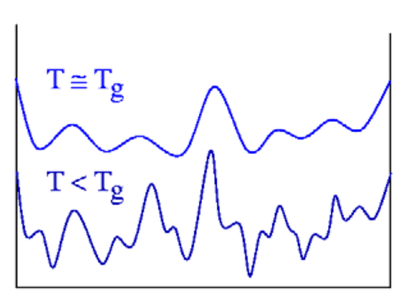


Ultrametric hierarchy of replica clusters ↔ Valley structure in energy landscape. Exponential distribution of energies

$$P(F_k) \propto \exp\left[+x_k \beta F_k\right]$$



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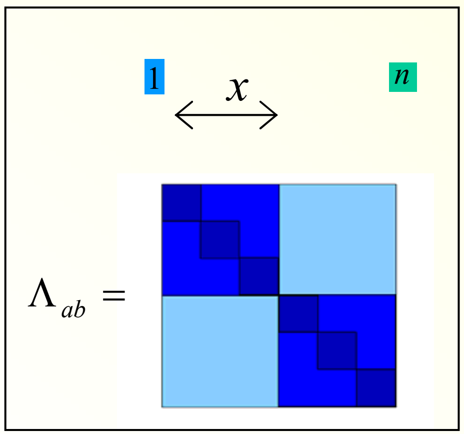

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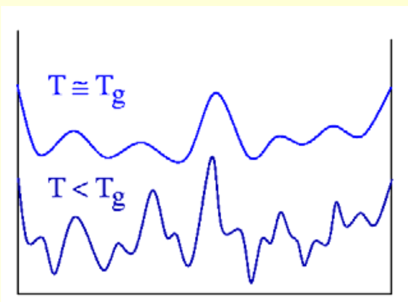

$n \to 0$ Continuous RSB: $\Lambda_{ab} \to \Lambda(x)$

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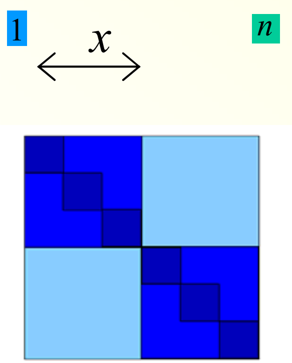
$\Lambda_{ab} =$



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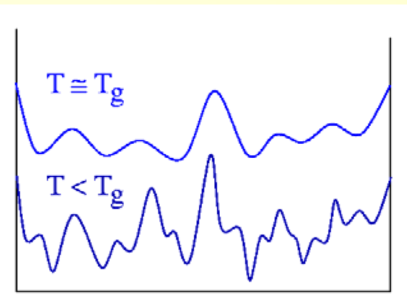

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Dynamical interpretation: Hierarchy of time scales

H. Sompolinsky, A. Zippelius (1981)

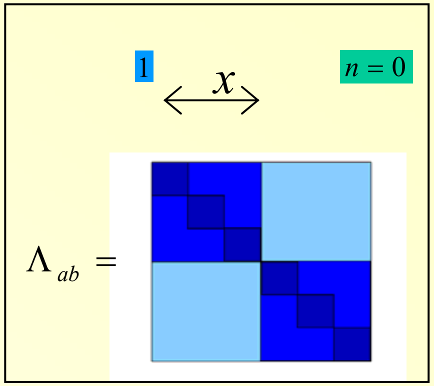
$$t_{\text{micro}} \ll t_k \ll t_{k-1} \ll \ldots \ll t_2 \ll t_1 \ll t_{\max}$$

$$1 > x_k > x_{k-1} > \ldots > x_2 > x_1 > 0$$

$k \to \infty$

How to solve the single site problem

$$\beta H_0\left(\left\{ s^a \right\}\right) = -\frac{\beta^2}{2} \sum_{a,b} s^a \left(\Lambda_{ab} + W^2\right) s^b$$



Hierarchy of time scales

$$t_{loc} \ll t_k \ll t_{k-1} \ll \ldots \ll t_2 \ll t_1 \ll t_{\max}$$
$$1 > x_k > x_{k-1} > \ldots > x_2 > x_1 > 0$$

Parisi (1979)
Duplantier (1981)
Sommers, Dupont (1984)

Average magnetization of a spin on time scale x in presence of a frozen field y:

$$m(x = x(t), y)$$

Distribution of frozen fields on times scale t_x (= Density of states at x=1!)

$$P(x = x(t), y)$$

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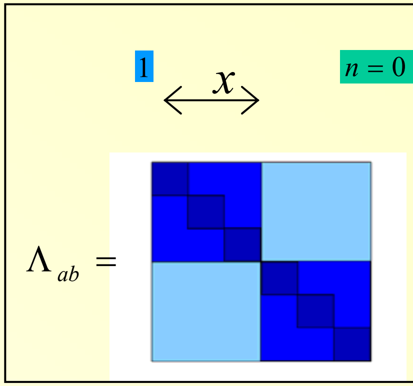

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Temporal flow equations

$$\dot{P}(x,y) = \frac{\dot{\Lambda}(x)}{2}\left[P'' - 2x\beta\left(P'm + Pm'\right)\right]$$

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← Continuous $\Lambda(x)$

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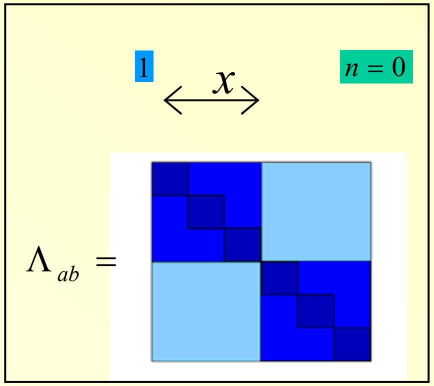

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Selfconsistency

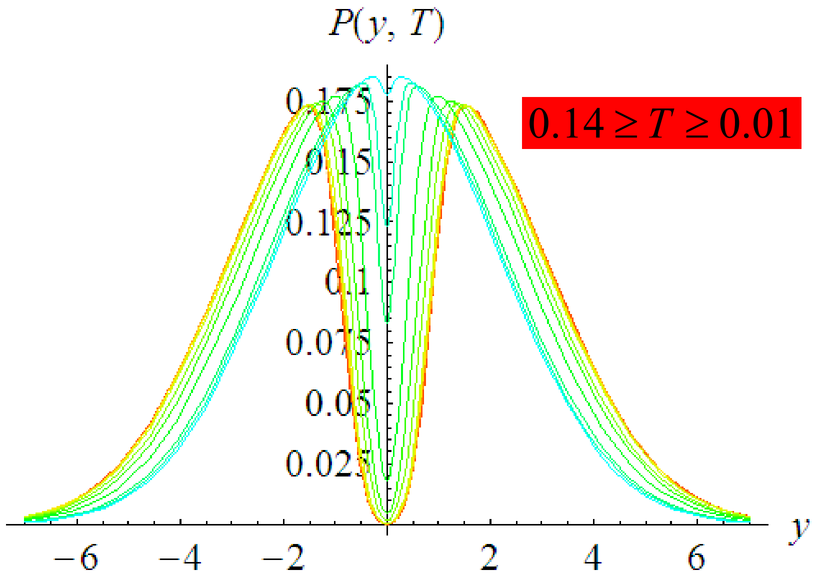
$$Q(x) = \int_{-\infty}^{\infty} dy P(x,y) m^2(x,y)$$

$$\Lambda(x) = \Lambda\{Q(x')\}$$

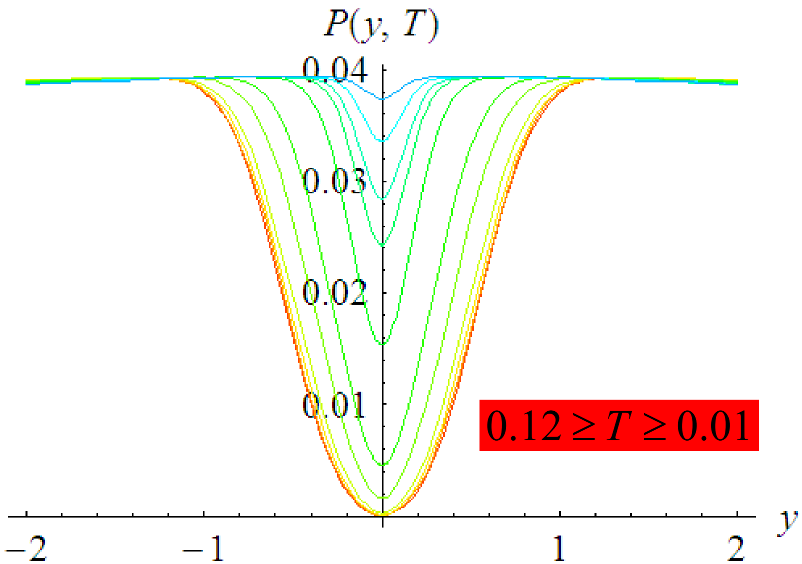
$D, J(r)$

Results: Temperature Evolution of the Coulomb gap

$W = 2e^2/2a$ $T_c = 0.140\,e^2/2a$



$W = 10\,e^2/2a$ $T_c = 0.123\,e^2/2a$



Results: Low T scaling

Continuous replica symmetry breaking

⟷ Marginal stability

Excitation spectrum around local minima extends down to zero.

Results: Low T scaling

Continuous replica symmetry breaking

↔ Marginal stability

Excitation spectrum around local minima extends down to zero.

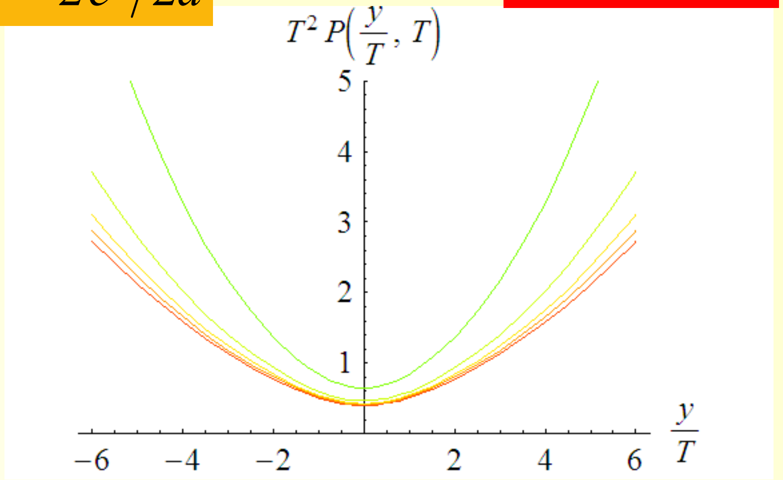
→ Universal Coulomb gap at low T

$$P(y) \xrightarrow{T \to 0} T^2 \Psi(y/T)$$

$$P(y) \propto y^2 \quad \text{for} \quad y > T$$

$W = 2e^2/2a$

$0.05 \geq T \geq 0.01$



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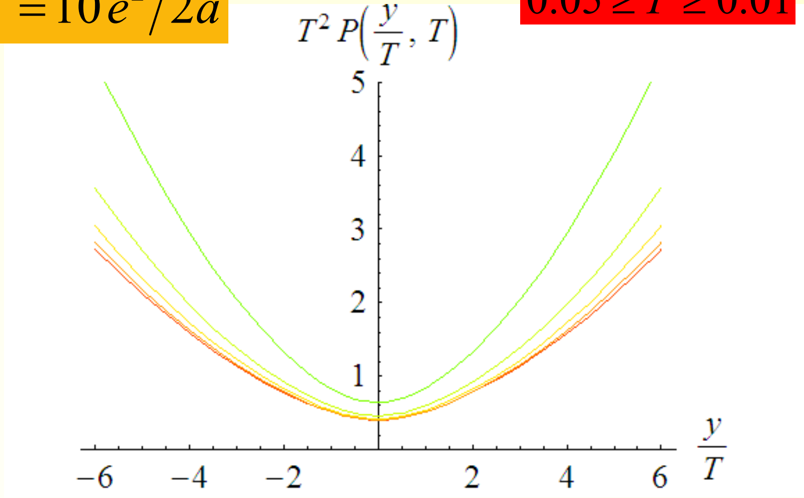

Results: Low T scaling

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$\longleftrightarrow$ Marginal stability

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$\longrightarrow$ Universal Coulomb gap at low T

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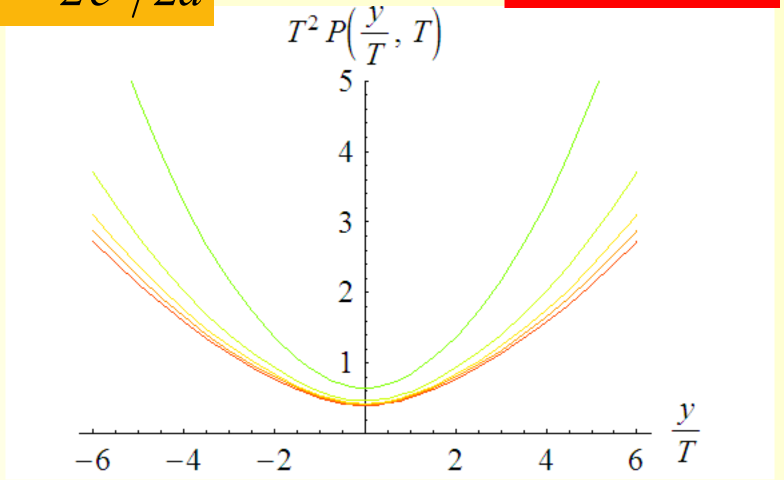
General interactions:

$J(r) \propto 1/r^{\alpha}$

D dimensions

$2 \rightarrow D/\alpha - 1$

$W = 2e^2/2a$ $\quad 0.05 \geq T \geq 0.01$



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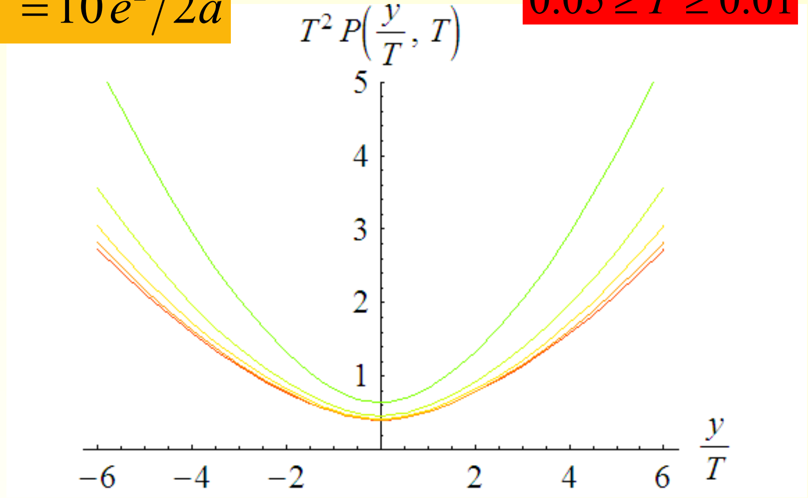


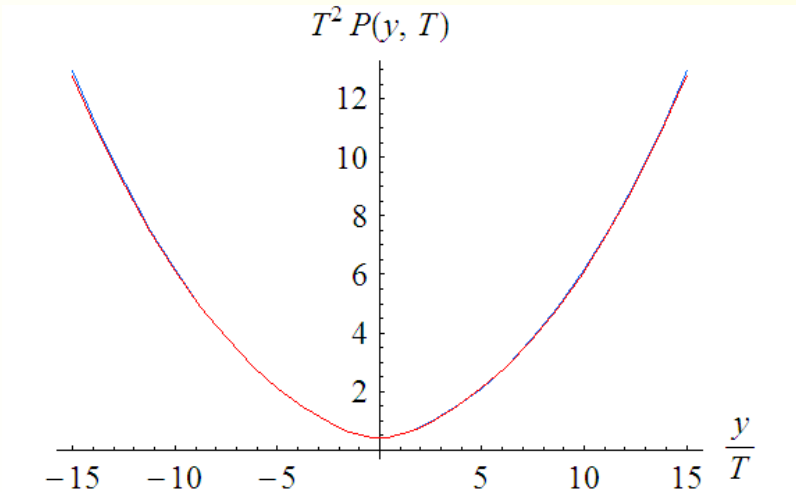
Results: Independence of disorder

3D Coulomb glass

Independence of disorder at low T!

$W = 2, 10$

$T = 0.01\, e^2/2a$



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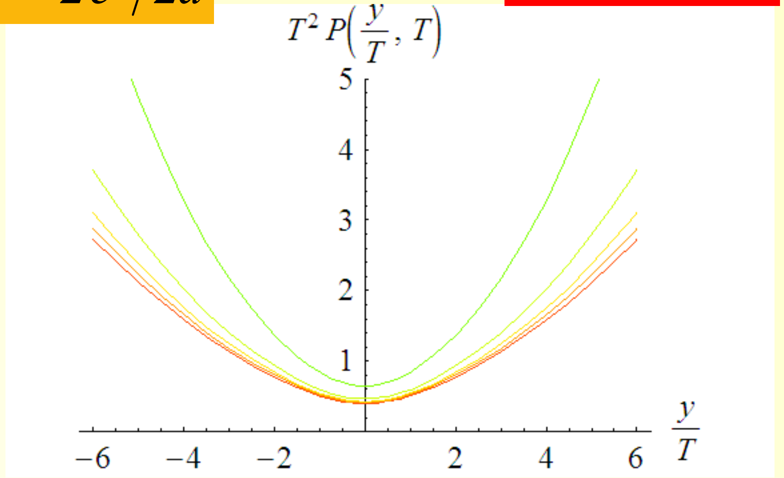

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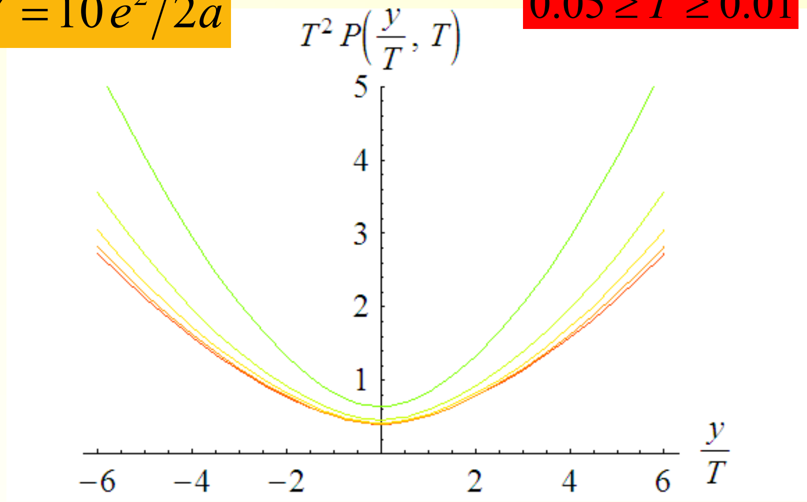

Why is the low T behavior so universal?

Fixed point in flow equations: Selfsimilarity in dynamics

S. Pankov (2006)

Rewrite temporal flow equations in natural variables

$$x \to a \equiv \beta x \equiv 1/T_{eff} \quad \text{(Sompolinsky time or effective } T\text{)}$$

$$y \to z \equiv \beta x y = y/T_{eff} \quad \text{(Local field)}$$

$$\tilde{p}(a,z) \equiv (\beta x)^2 P(x, y = z/\beta x)$$
$$\tilde{m}(a,z) \equiv m(x, y = z/\beta x)$$

→

$$a\partial_a \tilde{m}(a,z) = -z\tilde{m}' - \frac{a^3\dot{\Lambda}(a)}{2}\left[\tilde{m}'' + 2\tilde{m}\tilde{m}'\right]$$

$$a\partial_a \tilde{p}(a,z) = 2\tilde{p} - z\tilde{p}' + \frac{a^3\dot{\Lambda}(a)}{2}\left[\tilde{p}'' - 2(\tilde{p}'\tilde{m} + \tilde{p}\tilde{m}')\right]$$

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$$a^3\dot{\Lambda}(a)/2 \to c;$$

Like RG in time a!

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$$a^3\dot{\Lambda}(a)/2 \to c;$$

$$\tilde{m}(a,z) \to m^*(z)$$

$$\tilde{p}(a,z) \to p^*(z)$$

$$\beta >> a \equiv \beta_{eff} >> \beta_c$$

Like RG in time *a*!

Fixed point in flow equations: Selfsimilarity in dynamics

MM, S. Pankov (2006)

$$\tilde{m}(a,z) \to m^*(z)$$
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Consequences

- Disorder independence: Fixed point (short times) independent of W

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- Selfsimilarity of time averaged magnetizations

$$\rho(m,x) \equiv \int dy\, \delta(m - m(x,y))\, P(x,y) = \frac{1}{x^2}\rho^*(m)$$

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- Generalization of the fluctuation-dissipation relation: Exact for every x *(Sompolinsky).*

$$R(t,t') = \beta \frac{\partial C(t,t')}{\partial t'} \Rightarrow R(t,t') = \beta x(C) \frac{\partial C(t,t')}{\partial t'}$$

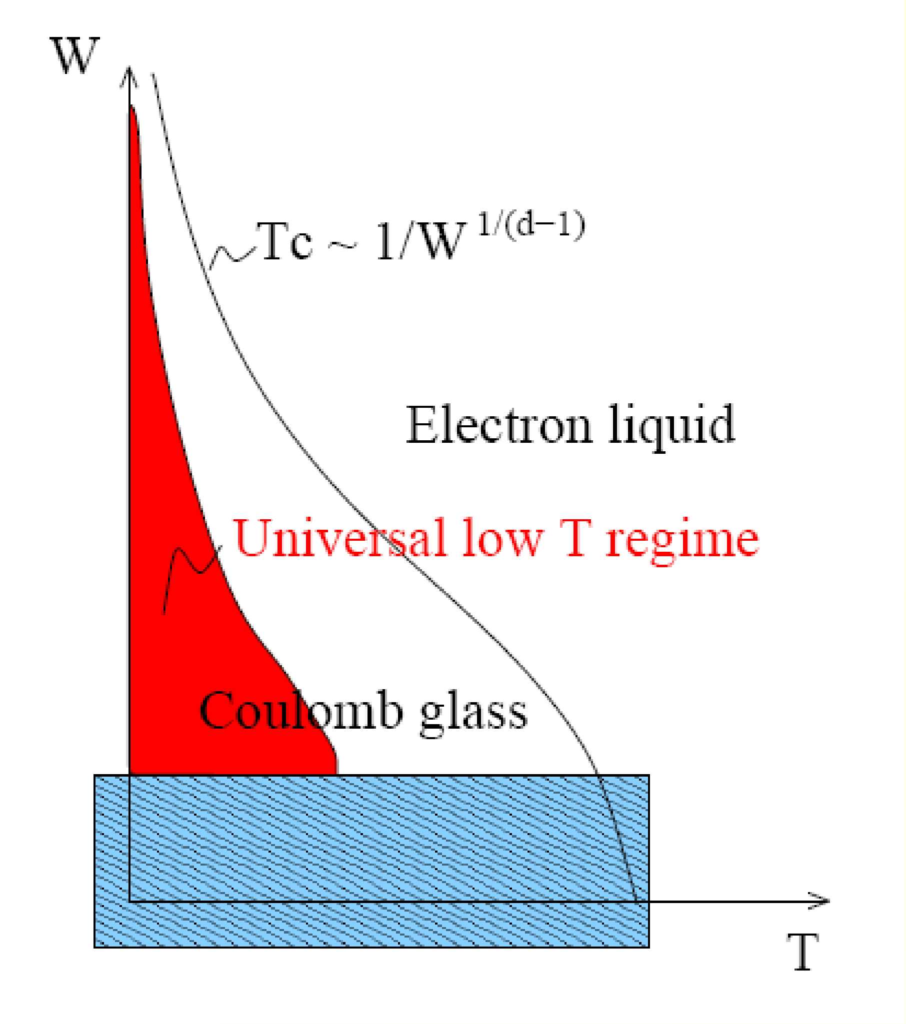
$$\beta \Rightarrow \beta_{eff} = x\beta$$

- Local meaning of T_{eff}

Time-averaged magnetization: Function only of y/T_{eff}

$$m(x,y) \approx m^*\left(y/T_{eff}(x)\right)$$

Summary of theoretical results



Connection with Experiments

—

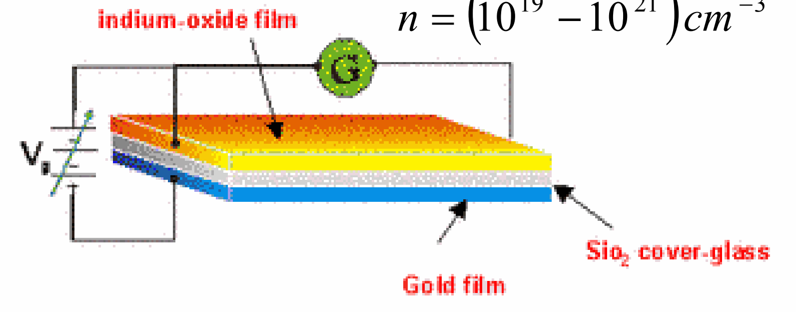
Aging

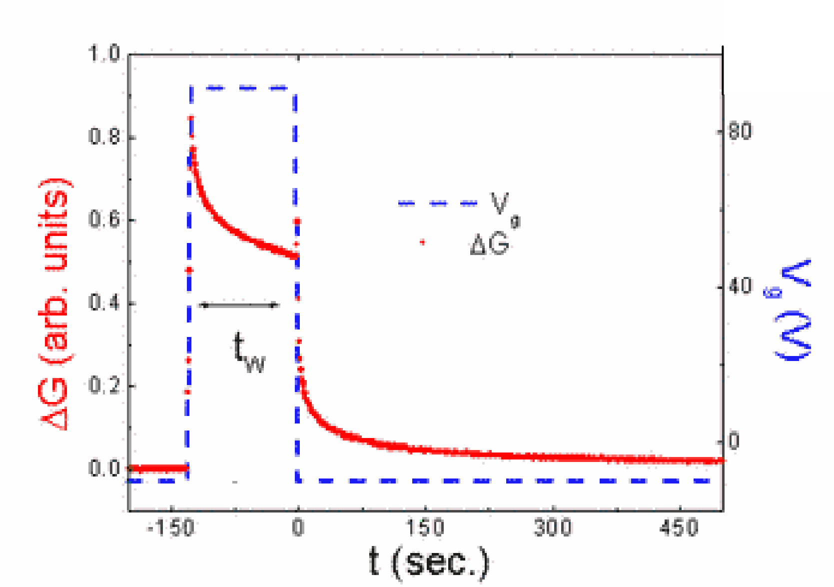
Electron glasses: Relaxation and aging

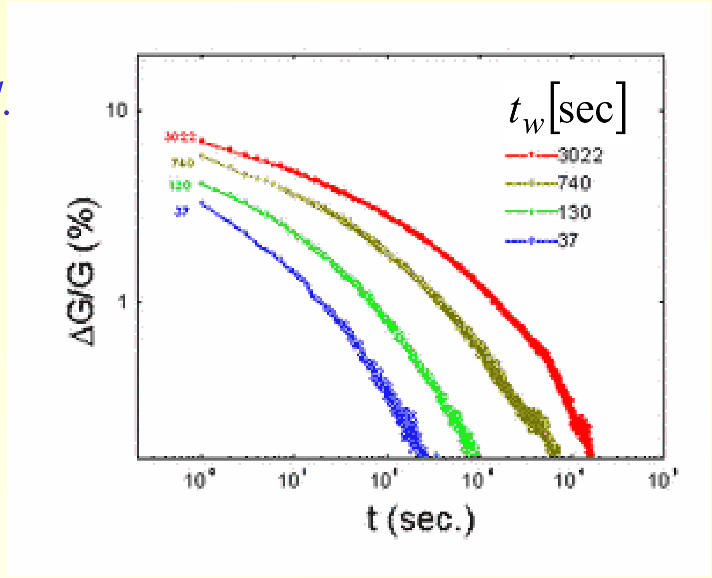
Indium-oxides In_2O_{3-x}

Z. Ovadyahu et al.

$$n = \left(10^{19} - 10^{21}\right) cm^{-3}$$







*A. Vaknin et al., PRL **84**, 3402 (2000)*

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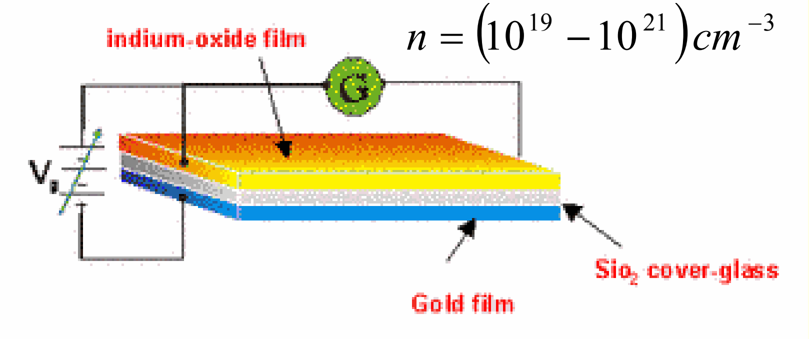

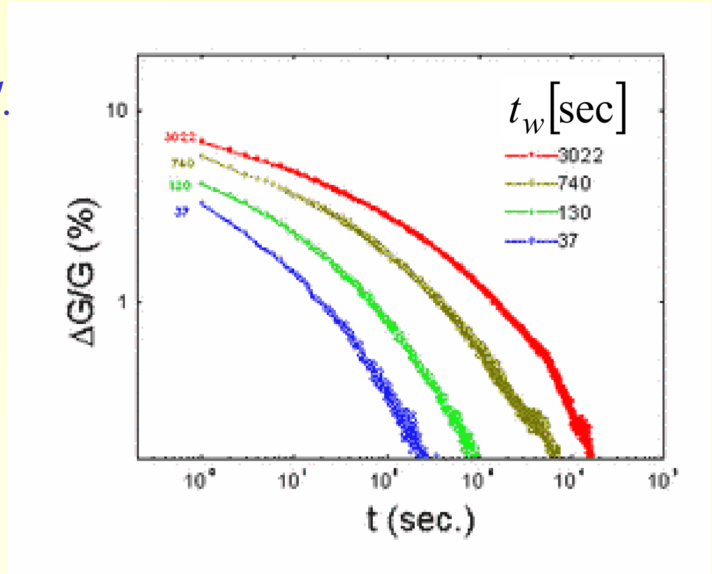

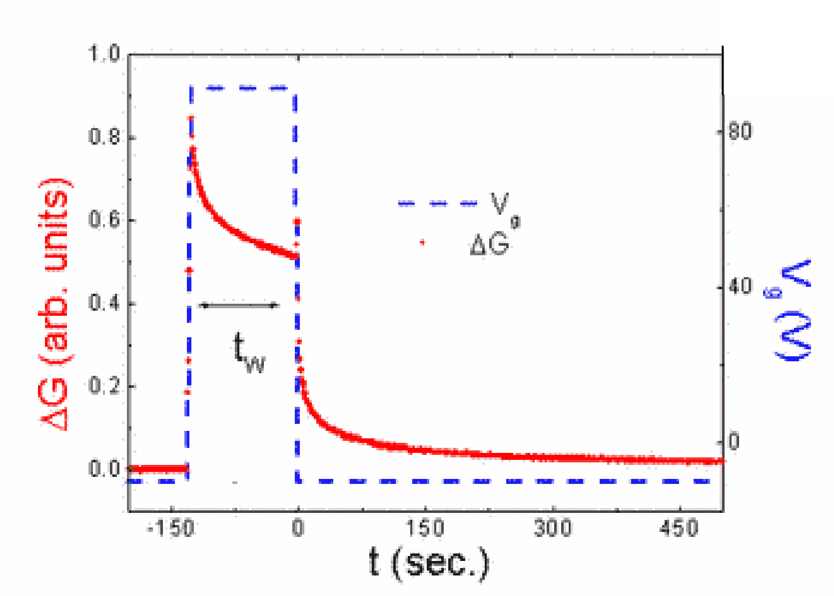

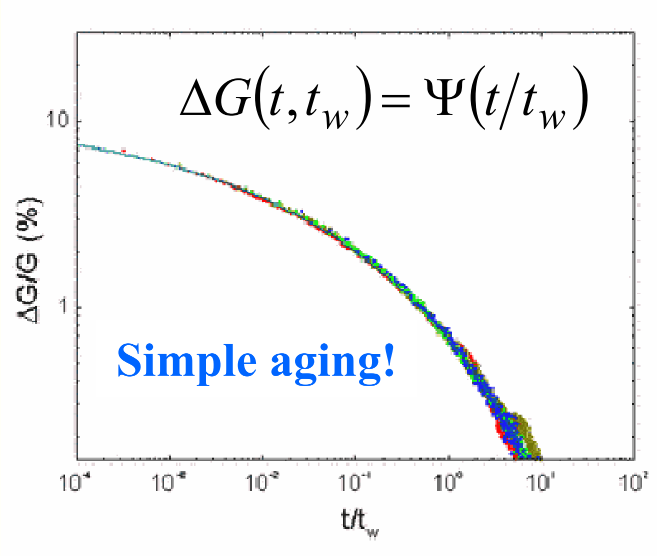

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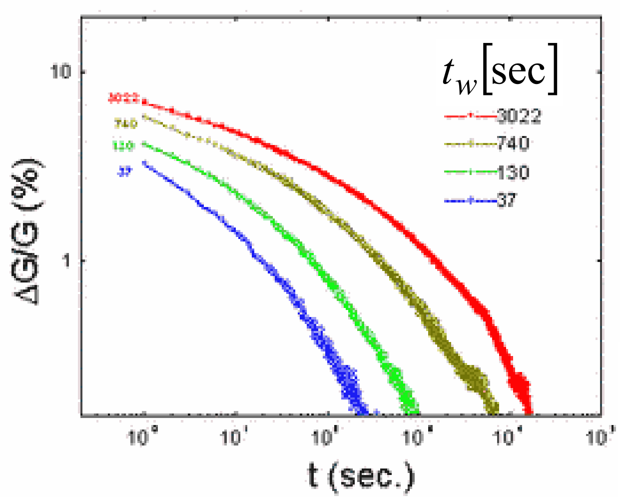
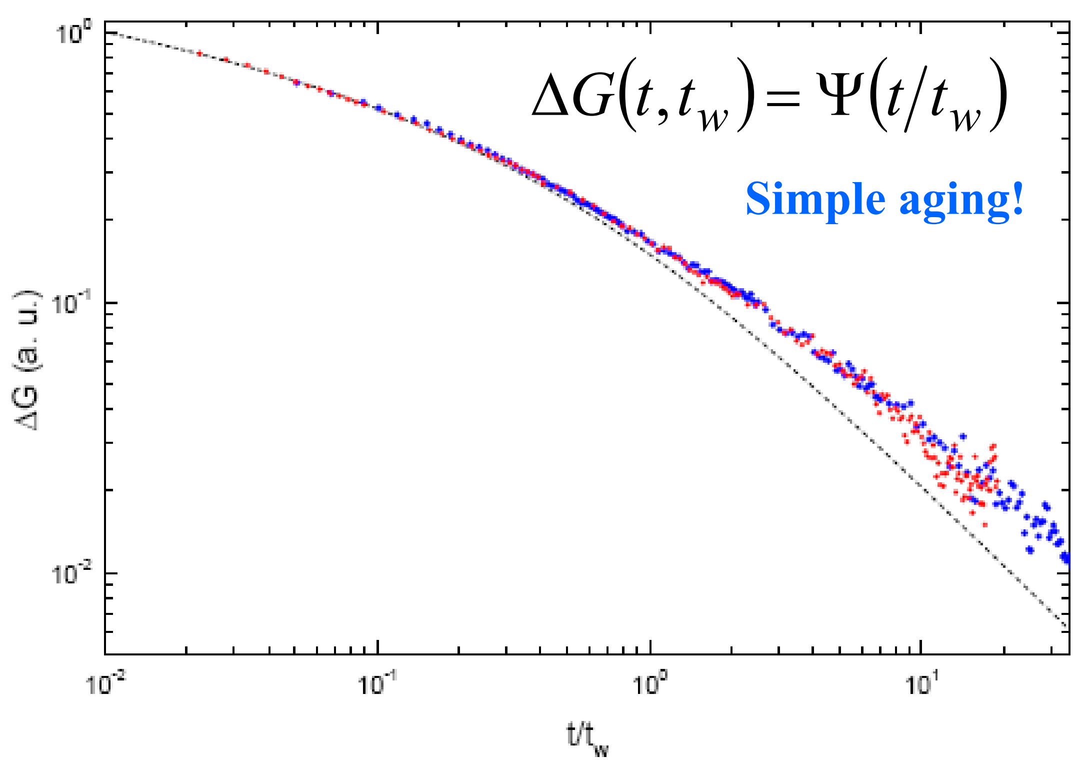
Connections with experiment?

→ • Aging: Properties of slow relaxation and simple aging



Z. Ovadyahu (2006)

Indium-oxide



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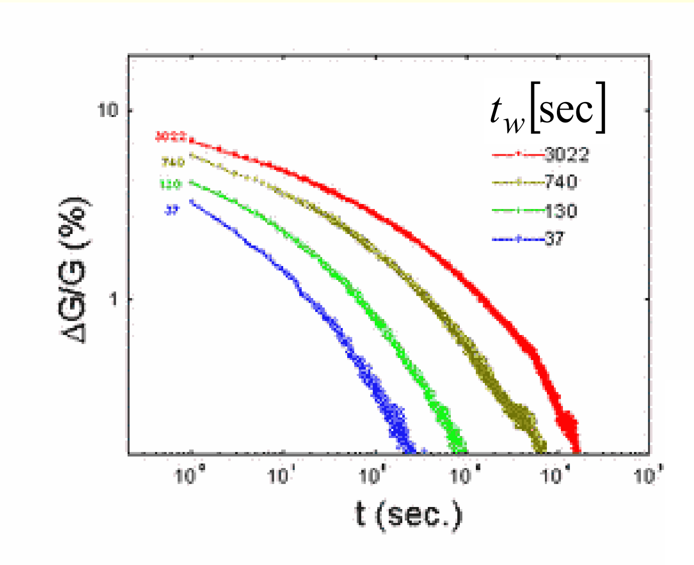

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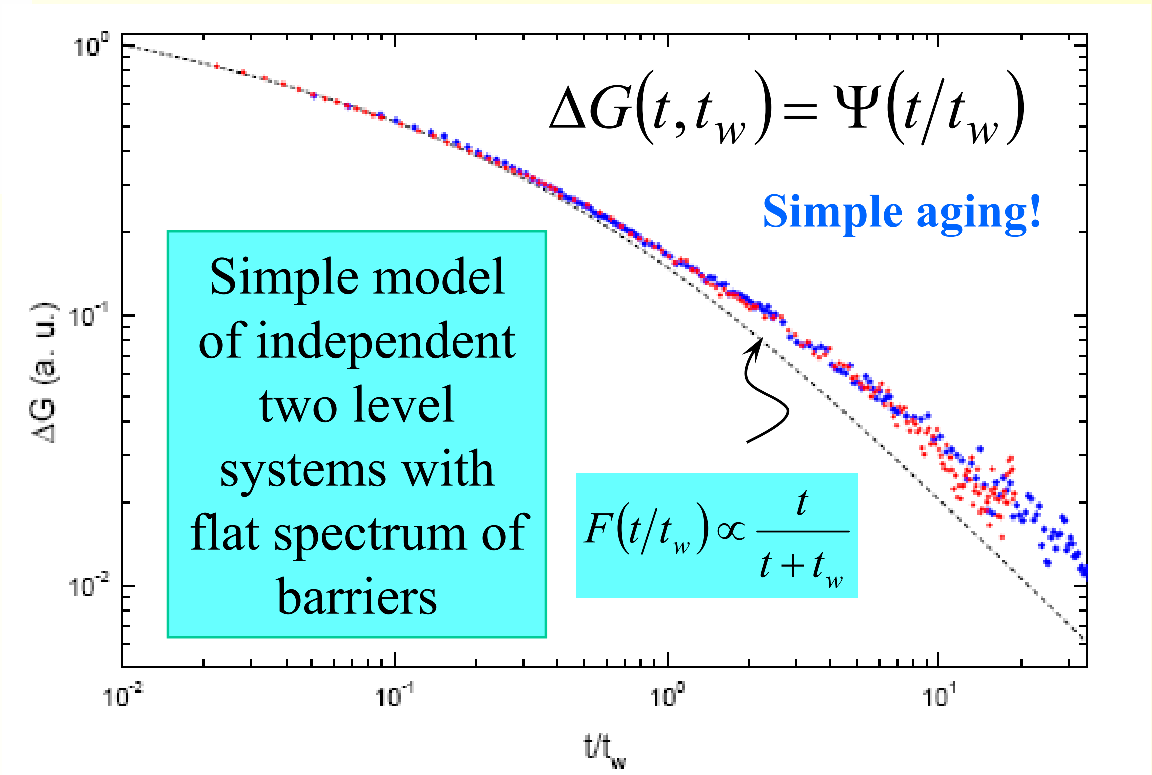

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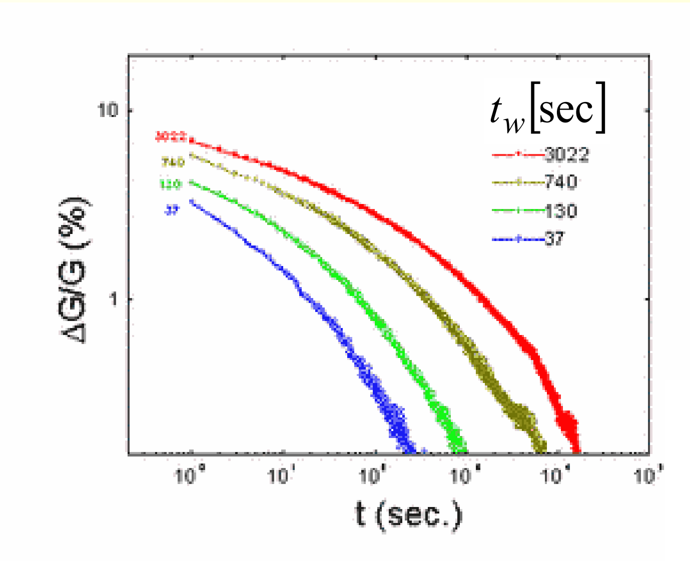
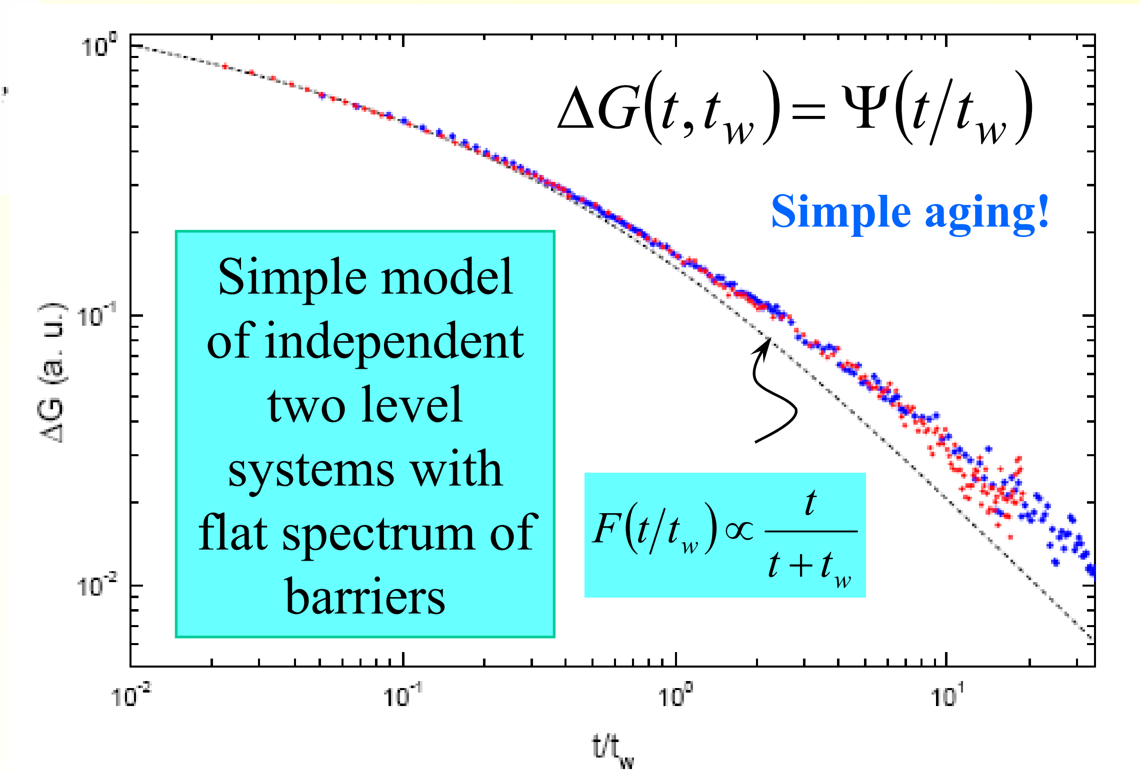


Z. Ovadyahu (2006)

Indium-oxide

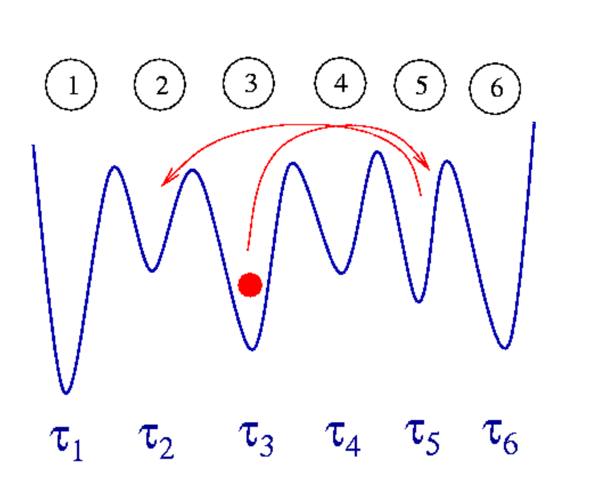
There is more structure to the glass!



Aging on a Parisi tree

J.P. Bouchaud,
D. Dean

Trap model



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$$\tau_i = \exp(\beta F_i)$$

$$\rightarrow \tilde{P}(\tau) \propto \frac{1}{\tau^{1+x}}$$

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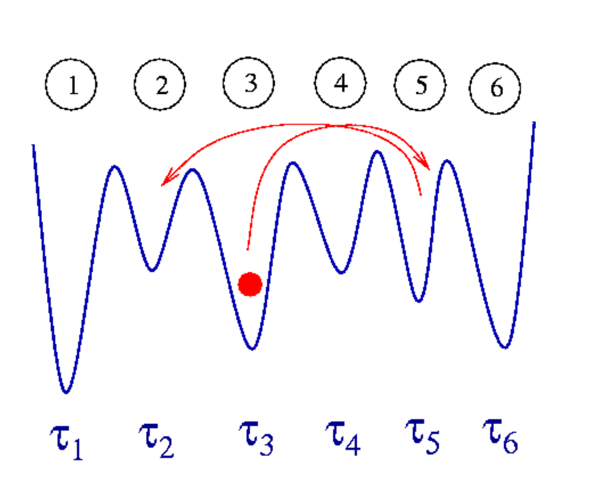

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$\rightarrow$ $$\Delta G \propto \left(t_w / t + t_w\right)^x \quad t > t_w$$

$$x = ?$$

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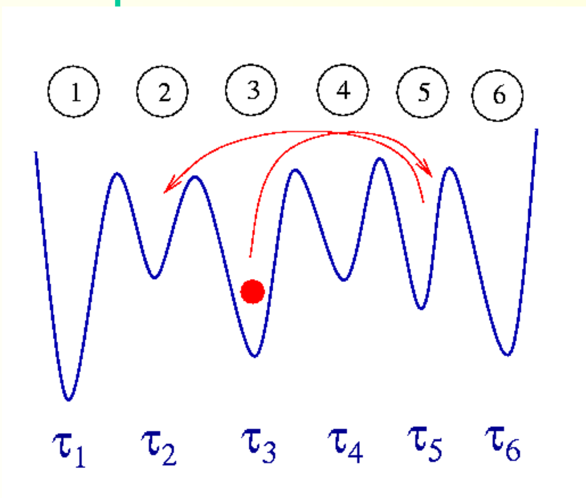

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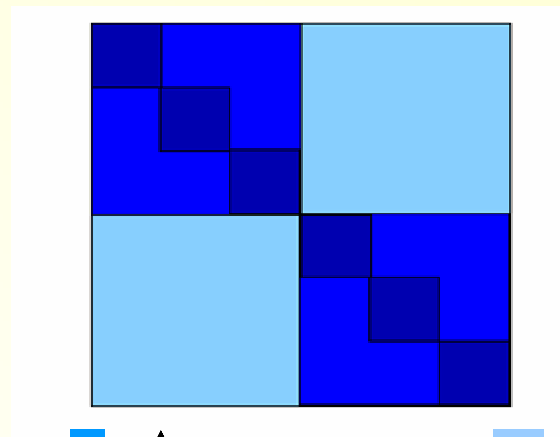

1 ↑ 0

$$x_{\max} \approx 0.8$$

(3D Coulomb)

Aging on a Parisi tree

J.P. Bouchaud, D. Dean

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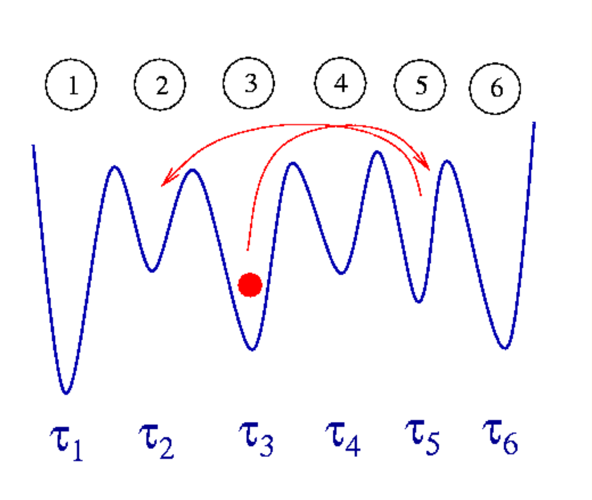
Trap model



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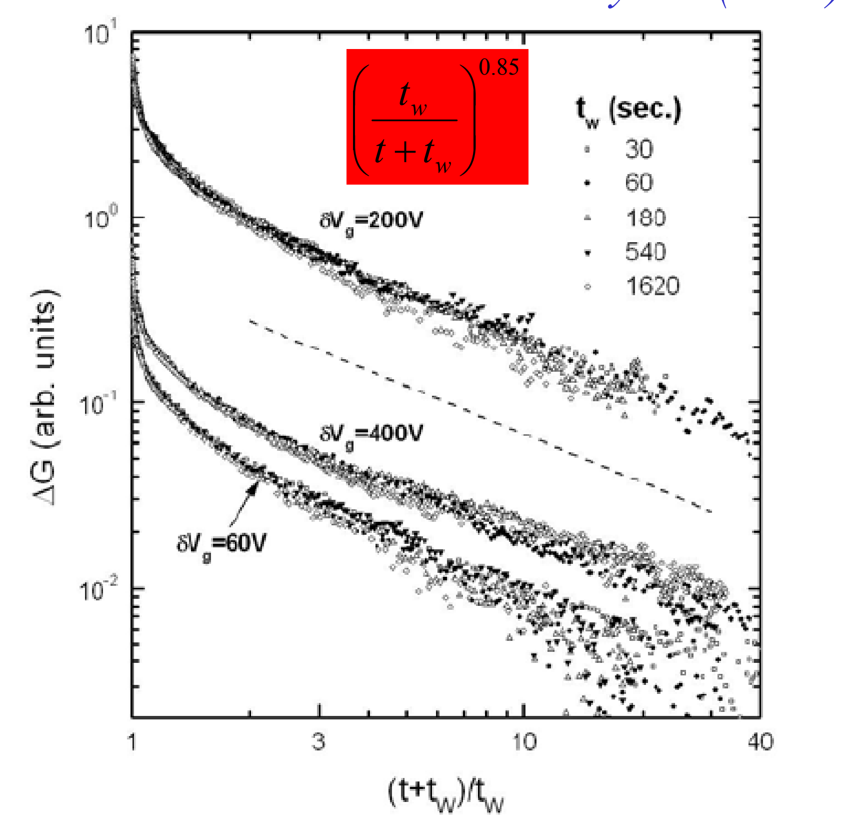

Conclusions

Low T analysis of the Coulomb glass phase:

→ Marginal stability → prediction of collective soft modes

→ Saturation and universality of the Coulomb gap

→ Selfsimilarity in temporal evolution

→ Relation with functional RG?

→ Prediction for aging.