Field Theory for Distribution Functions in Disordered Conductors

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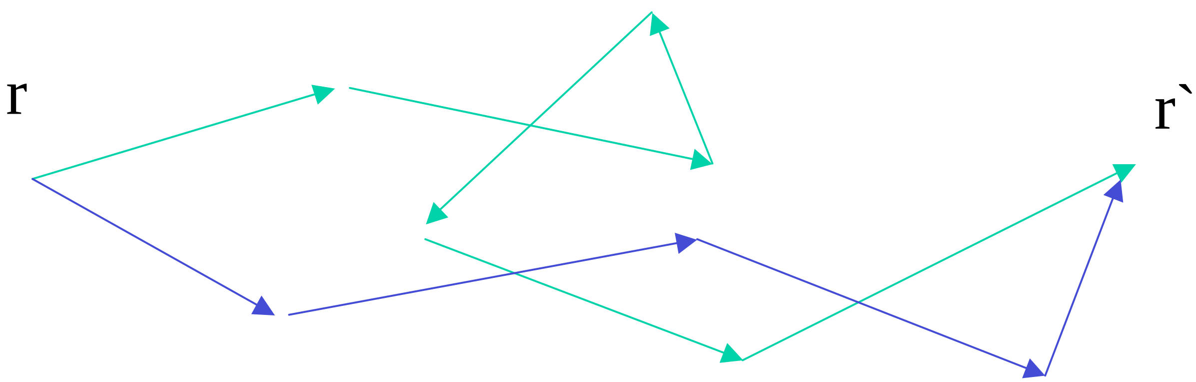
Electrons (waves, …) in a random potential

$$[E - H_0 - U(r) \pm i\delta/2]G^{R(A)}(\mathrm{r};\,\mathrm{r}`;\,\mathrm{E}) = \delta(\mathrm{r} - \mathrm{r}`)$$

$$H_0 = -\frac{\nabla^2}{2m} \qquad < U(r)U(r') = \gamma\delta(r - r') \qquad \frac{1}{\tau} = 2\pi\bar{\nu}(E_F)\gamma$$

$G^{R(A)}$ (r; r`; E) – amplitude of electron propagation:



mean free path

$$l = v_F\tau << \lambda$$

Exact eigenstates:

$$[H_0 + U(r)]\phi_m(r) = E_m\phi_m(r)$$

Spectral representation:

$$G^{R(A)}(r,r';E) = \sum_m \frac{\phi_m(r)\phi_m^*(r')}{E - E_m \pm i\delta}$$

Physical quantities:

Global density of states (DOS):

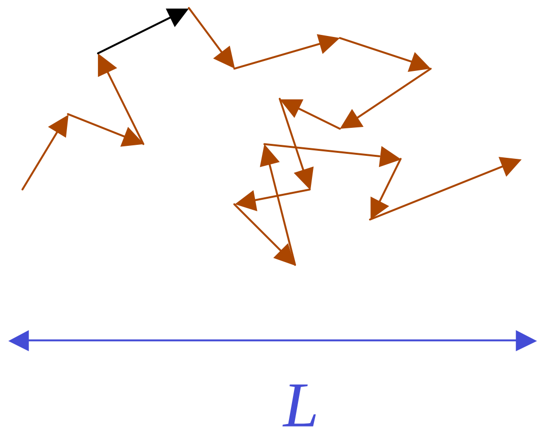
$$\nu(E) = L^{-d} \sum_m \delta(E - E_m) = i/(2\pi L^d)\mathrm{Tr}\{[G^R(E) - G^A(E)]\}$$

Conductance: $G = I/V = \sigma L^{d-2} = (e^2/h)g$

$$g = g_{xx} = -1/(2L^2)\mathrm{Tr}\{v_x[G^R - G^A]v_x[G^R - G^A]\};$$

$$(E = E_F = k^2_F/2m)$$

Diffusion propagation



Fast relaxation of momentum: $l = v_F \tau << L$

Diffusion coefficient $D = \frac{l^2}{\tau d}$ “Diffusion time” $\frac{L^2}{D}$

“Thouless energy” $E_c = \frac{D}{L^2}$

Conductance: $\bar{g} = 2\pi E_c / \Delta >> 1$ (Δ - mean level spacing)

Some physical results

Averaged DOS - no change: $<\nu(E)>=\overline{\nu}_d(E)$

$(\overline{\nu}_{d=2}(E)=m/2\pi)$

Averaged conductance: $<g>=\overline{g}+\delta\overline{g}$

$$\frac{\delta\overline{g}}{\overline{g}}=[d-2-\frac{a}{\overline{g}}+\ldots]\log\frac{L}{l}$$

Scaling hypothesis: Abrahams, *et al*. (1979)

One-parameter scaling hypothesis

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Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

E. Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854

and

P. W. Anderson,[a] D. C. Licciardello, and T. V. Ramakrishnan[b]
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540
(Received 7 December 1978)

$$dg/d\ln L = g(d-2-a/g+\ldots). \quad (9)$$

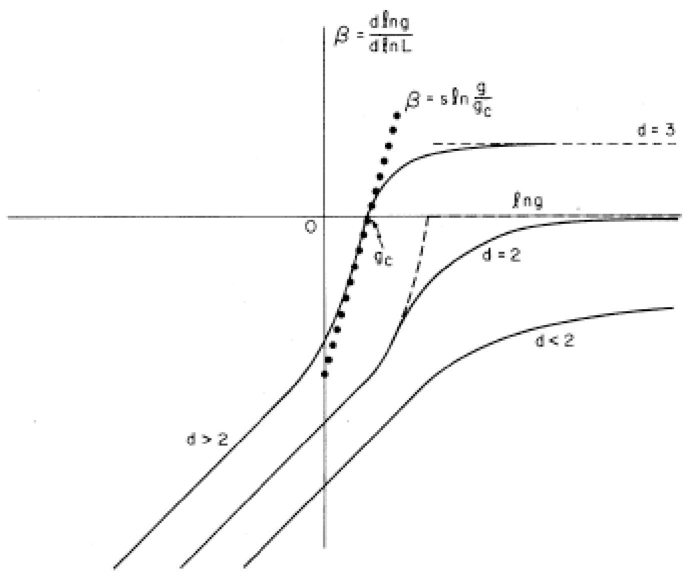


FIG. 1. Plot of $\beta(g)$ vs $\ln g$ for $d>2$, $d=2$, $d<2$. $g(L)$ is the normalized "local conductance." The approximation $\beta = s\ln(g/g_c)$ is shown for $g>2$ as the solid-circled line; this unphysical behavior necessary for a conductance jump in $d=2$ is shown dashed.

Mesoscopics

Sample to sample variations

Conductance: $< (\delta g)^2 > \sim 1$ in units of $\frac{e^2}{\hbar}$

"Universal Conductance Fluctuations"

[Altshuler (1985); Lee & Stone (1985)]

Density of states variations (d=2): $< (\delta \nu / \overline{\nu})^2 > \sim 1/\overline{g}^2$

[Altshuler & Shklovskii (1986)]

Beyond diagrams

Averaging over disorder – field-theoretical problem

Powerful tool for calculation of *averaged* quantities – nonlinear sigma-model (reduction to slow degrees of freedom)

Mesoscopic fluctuations - a need for *distribution functions* of physical quantities

Field theory for distribution functions?

“Gang of 4”: Scaling hypothesis

Gor’kov, Larkin, Khmelnitskii (renormalizability)

Wegner: (bosonic) nonlinear σ-model

no interaction

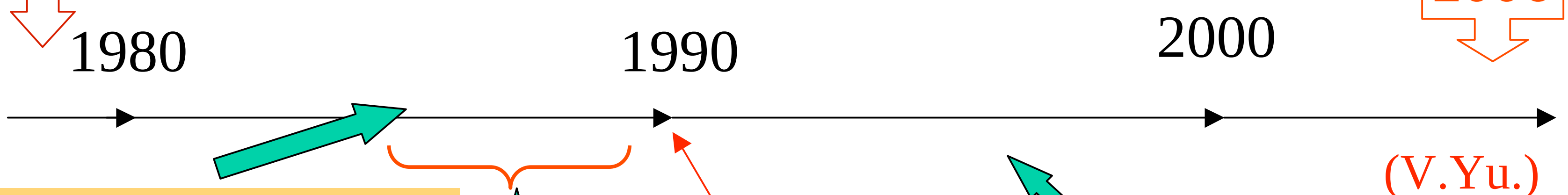
$2 \le d \le 4$

1979

Efetov, Larkin, Khmelnitskii: fermionic σ-model

Efetov: supersymmetric σ-model (for $< g >$)

2006



Altshuler; Stone & Lee (mesoscopics)

Wegner (n-vector)

Muzykantskii & Khmelnitskii (secondary saddle-point; tails of $g(t)$; ballistic effects)

Kravtsov, Lerner, Yudson: anomalous dimensions of vertices in extended σ-model

Altshuler, Kravtsov, Lerner: log-normal tails of distribution functions $P(g)$, $P(\nu)$, $g(t)$ ←→ $P(t_\phi)$

Content

- Averaged quantities and nonlinear σ-model
- [Instabilities in σ-model, averaged moments, and log-normal tails of P(g), P(v), and $P(t_\phi)$ – sketch]
- [Secondary saddle-point approach and log-normal tails of $P(t_\phi)$ - sketch]
- Field theory for P(v)

Field theory and nonlinear σ-model

Basic object: Green's function

Primary representation:

$$G^{R,A}(\mathbf{r},\mathbf{r}') \propto \int d\Psi d\Psi^* \ \Psi(\mathbf{r})\Psi^*(\mathbf{r}') \exp\left[\pm i\Psi^*(E - H_0 - U \pm i\delta/2)\Psi\right]$$

Primary variable: local field $\Psi(r)$

Averaging over the Gaussian disorder is elementary!

Price for the absence of the denominator:

(Replica trick OR Shwinger-Keldysh contour, OR …)

"Supersymmetry" (Efetov): $\Psi = (S, \eta)$; η - Grassmann variables:

$$\int dS dS^* \exp[\pm i S^* (M \pm i\delta) S] \propto [\det(M \pm i\delta)]^{-1}$$

$$\int d\eta d\eta^* \exp[\pm i \eta^* (M \pm i\delta) \eta] \propto \det(M \pm i\delta)$$

$$\boxed{\int d\Psi d\Psi^* \exp[\pm i \Psi^* (M \pm i\delta) \Psi] = 1}$$

$$G^{R(A)}(r, r'; E) \propto \int D[\Psi, \Psi^*] \Psi(r) \Psi^*(r') \exp[\pm i \Psi^* (E - H_0 - U \pm i\delta) \Psi]$$

$$\Psi^a \ (a = R, A) \ ; \ \Psi^* \to \bar{\Psi} = K \Psi^* \ ; \ K = (\Lambda, I); \ \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \Lambda_z$$

$$g(\omega) \propto \int D\Psi D\overline{\Psi}[...]\exp[i\overline{\Psi}(E - H_0 - U + \frac{\omega + i\delta}{2}\Lambda_z)\Psi]$$

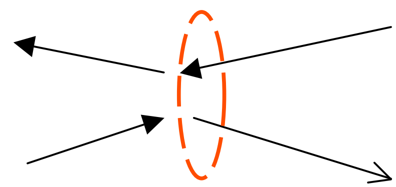
Averaging over disorder

$<U(r)U(r') = \gamma\, \delta(r - r')$ $\quad$ $1/\tau = 2\pi\nu_d\gamma$

$$< \exp[-i\bar{\Psi}(r)U(r)\Psi(r)] >= \exp[-\frac{\gamma}{2}(\bar{\Psi}(r)\Psi(r))^2]$$

Hubbard-Stratonovich decoupling

$$Q(r) \sim < \Psi(r) \otimes \overline{\Psi}(r) >$$



$$< g(\omega) > \propto \int DQD\overline{\Psi}D\Psi \exp\{i\overline{\Psi}[E + (\omega + i\delta)\Lambda_z / 2 - H_0 + iQ/(2\tau)]\Psi - \pi\nu/(8\tau)\mathrm{Str}Q^2\}$$

$$\mathrm{Str}M = \mathrm{Tr}M^{bb} - \mathrm{Tr}M^{ff}$$

"Primary" saddle-point: $Q^2 = I;$ $\quad$ $Q = V\Lambda_z V^{-1}$

$$< g(\omega) > \propto \int_{Q^2=I} \mathcal{D}Q(r)\ [\ldots]\exp[-\underline{\mathrm{S}}\mathrm{tr}\ln(E + \underbrace{(\omega + i\delta)\Lambda/2}_{\text{symmetry breaking term}} - H_0 + iQ/2\tau)]$$

symmetry breaking term

“Hydrodynamic” expansion ($\omega\tau$, $l/L << 1$) :

$$\mathrm{Str}\ln\{\ldots\} = -(\pi\nu_d D/4)\,\mathrm{Str}\{(\nabla Q)^2 + 2i\omega/D\,\Lambda_z Q \quad \leftarrow \text{usual } \sigma\text{-model}$$

$$+ c_1 l^2 (\nabla Q)^4 + c_2 l^2 (\nabla^2 Q)^2 + c_3\tau\,\omega^2/D\,(\Lambda_z Q)^2 + \ldots\} \quad \leftarrow$$

“extended”

$$< g(\omega) > = \int \ldots \exp\left[\frac{\pi\nu_d D}{4}\int \mathrm{Str}\{(\nabla Q)^2 + 2i\frac{\omega}{D}\Lambda_z Q\}dr\right] DQ(r)$$

$\updownarrow$

$$2\pi\nu_d D = 2\pi\frac{D}{\Delta L^2} = 2\pi\frac{E_c}{\Delta} = g >> 1$$

Moments $\langle g^n \rangle$, $\langle \nu^n \rangle$ (modifications: $\Psi \to \Psi_i$; $Q \to Q_{ij}$; $i,j = 1, \ldots, n$)

$$< g^n > \propto \int_{Q^2=I} \mathcal{D}Q(r)\ [\ldots] \exp\left[-\underline{\mathrm{S}}\mathrm{tr} \ln\left(E + (\omega + i\delta)\Lambda/2 - H_0 + iQ/2\tau\right)\right]$$

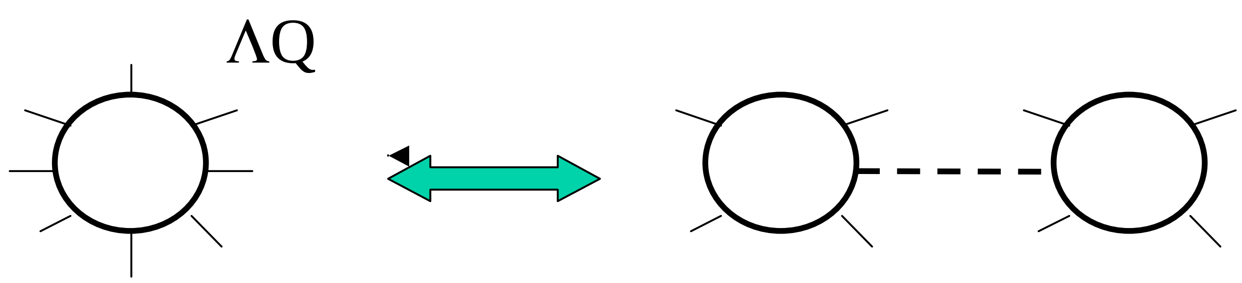
"Hydrodynamic" expansion ($\omega\tau$, $l/L \ll 1$) :

$\mathrm{Str}\ln\{\ldots\} = -(\pi\nu_d D/4)\,\mathrm{Str}\{ (\nabla Q)^2 + 2\omega/D\, \Lambda_z Q$ + ← usual σ-model

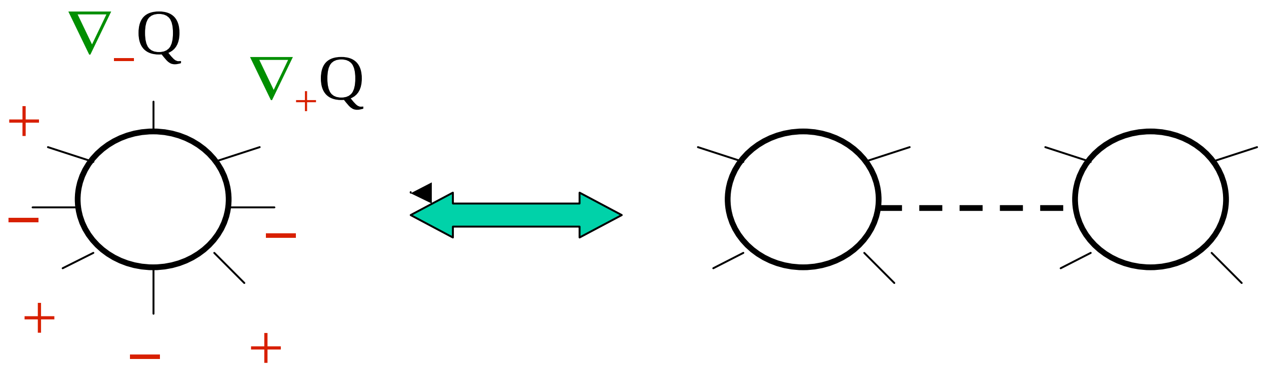
$+ c_1 l^2 (\nabla Q)^4 + c_2 l^2 (\nabla^2 Q)^2 + c_3 \tau\, \omega^2/D\, (\Lambda_z Q)^2 + \ldots \}$ ← "extended"

Anomalies (KLY)

RG transformation of additional scalar vertices



RG transformation of additional vector vertices (KLY 1988-89)



Conformal structure: $\nabla_{\pm} Q = \nabla_x Q \pm i \nabla_y Q$ � d = 2

Gradient vertex: $z_n \mathrm{Str}\{(\nabla_+ Q \nabla_- Q)^n\}$

Growth of charges: $z_n \sim z_n(0) \exp[u(n^2 - n)]$

$u = (1/\pi g) \ln(L/l) << 1$

AKL: Growth of cumulant moments:

$$<(\delta g/g)^n>_c \sim <(\delta\nu/\nu)^n>_c \sim g^{1-n} (l/L)^{2(n-1)} \exp[u(n^2 - n)],$$

$$n > n_0 \sim u^{-1} \ln (L/l) \sim g;$$

$$\sim g^{2-2n}, \quad n < n_0$$

Log-normal asymptotics of distribution functions:

$$P(x) \sim \exp[-(1/4u) \ln^2(x/\tau\Delta)], \quad x = \delta\nu/\nu > 0 \quad \text{OR} \quad -\delta g/g > 0$$

$\Delta = 1/(\nu_d L^d)$ - mean level spacing

$u = (1/\pi g) \ln(L/l)$

AKL: Long-time current relaxation - distribution of relaxation times

Anomalous contribution to $\langle g(\omega) \rangle$ $\rightarrow$

log-normal asymptotics of $\langle g(t) \rangle$

$$\langle g(t) \rangle - g_0 \exp[-t/\tau] \sim \exp[-(1/4u) \ln^2(t/\tau g)] \sim \;!\; \exp[-t/t_\phi] \, P(t_\phi) \, dt_\phi$$

Distribution of relaxation times: $P(t_\phi) \sim \exp[-(1/4u) \ln^2(t_\phi/\tau)]$

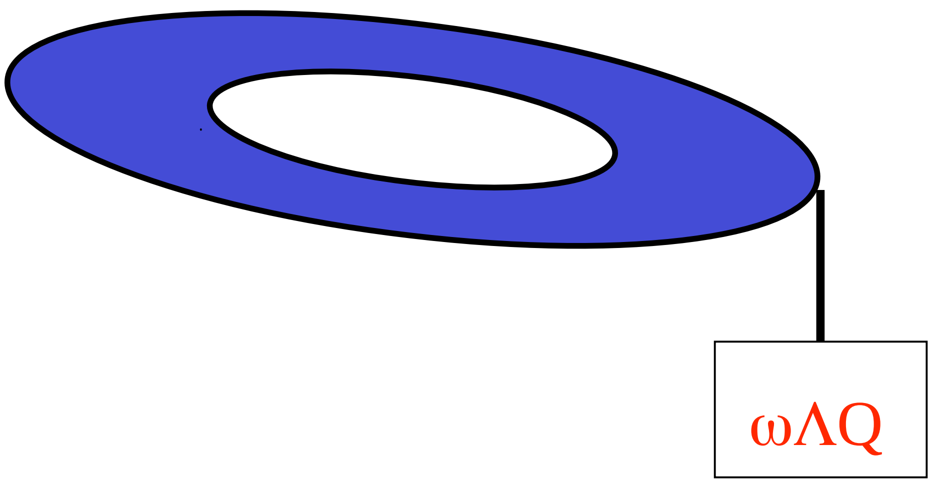
Muzykantskii & Khmelnitskii: Secondary saddle-point approach

$<g(t)> = g_0 \exp(- t/\tau) + ! \, d\omega/2\pi \exp(-i\omega t) \, ! \, DQ \, [\ldots] \exp(- S[Q])$,

$S[Q] = \pi\nu/4 \, ! \ dr \, \mathrm{Str}\{ D(\nabla Q)^2 + 2i\omega\Lambda Q \}$

$g(t) - g_0 \exp[-t/\tau] \sim \exp[- (1/4u) \ln^2(t/\tau g)]$ - ?

$Q^2 = I$



$<g(t)> = g_0 \exp(- t/\tau) + ! \; d\omega/2\pi \exp(-i\omega t) \; ! \; DQ \, [\ldots] \exp(- S[Q])$,

$S[Q] = \pi\nu/4 \; ! \; dr \, \mathrm{Str}\{ D(\nabla Q)^2 + 2i\omega\Lambda Q \}$

Saddle-point equation:

$$2D\nabla(Q\nabla Q) + i\omega[\Lambda, Q] = 0$$

Parametrization: $Q = VHV^{-1}$; $H = H(\theta, \theta_F)$

Saddle-point equation for $\theta(r)$: $\nabla^2\theta + \kappa^2 \sinh\theta = 0$; $\kappa^2 = i\omega/D$

Boundary conditions: $\theta|_{leads} = 0$; $\nabla_n\theta|_{insulator} = 0$

Integration over ω → self-consistency equation:

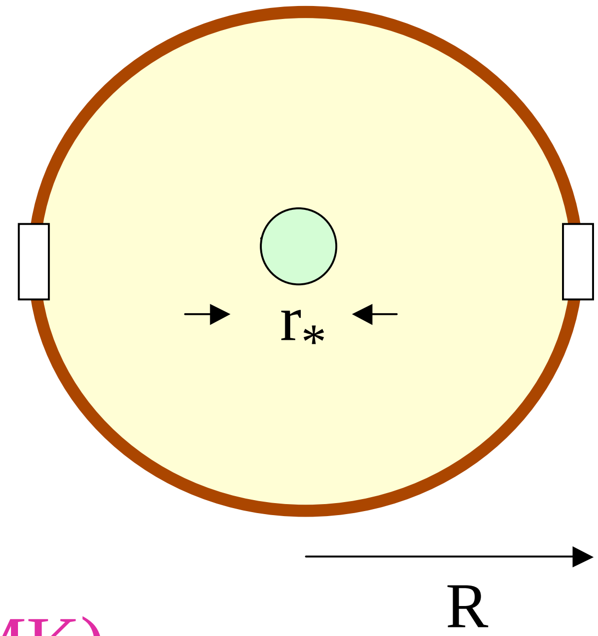
$! \; dr/L^d \, [\cosh\theta - 1] = t\Delta/\pi$ (Δ - mean level spacing) d=2 , disk geometry (Mirlin):

$\theta = \theta(r)$ - singular at $r \to 0$ at very large t
(breakdown of the diffusion approximation!)

Requirement: $|\theta'(r)| < 1/l$

Cutoff $r_* < r$: $\theta'(r_*) = 0$

$r_* \sim C\, l$



Solution: $(\beta = 1, 2, 4)$

$g(t) \sim (t\Delta)^{-2\pi\beta g}$, $1 \ll t\Delta \ll (R/l)^2$ (MK)

$g(t) \sim \exp[-\pi\beta g \ln^2(t/\tau g)/4\ln(R/l)]$, $t\Delta \gg (R/l)^2$

Coincides with the RG result of AKL

However, no alternative way for $P(\nu)$ and $P(g)$

(no field theory for $P(\nu)$ and $P(g)$!)

Wanted:

Field theory for distribution function

Requirements:

- disorder averaging
- slow functional description
- RG analysis
- non-perturbative solutions

Not every representation is suitable!

For instance:

$$P(\nu) = <\delta(\nu - \nu(E))>$$

$$\nu(E) = L^{-d} \sum_m \delta(E - E_m) = i/(2\pi L^d)\mathrm{Tr}\{[G^R(E) - G^A(E)]\}$$

$$G^{R,A}(\mathbf{r},\mathbf{r}') \propto \int d\Psi d\Psi^* \; \Psi(\mathbf{r})\Psi^*(\mathbf{r}')\exp\left[\pm i\Psi^*(E - H_0 - U \pm i\delta/2)\Psi\right]$$

Horrible!!!..

Primary representation for the characteristic function

$$< \exp[-s\nu/\bar{\nu}] > \equiv \mathcal{P}_\nu(s) = \int \mathcal{D}[\bar{\Psi}, \Psi] < e^{i[\bar{\Psi}\mathcal{M}\Psi - \sqrt{\tilde{s}}(\bar{\mathcal{B}}\Psi + \bar{\Psi}\mathcal{B})]} >$$

$\bar{\Psi}_{\mathbf{r}_1\mathbf{r}_2}$ and $\Psi_{\mathbf{r}_1\mathbf{r}_2}$ - bi-local primary superfields; $\tilde{s} = s\tau\Delta/(2\pi)$

$\Psi = (S^R, S^A; \xi^R, \xi^A)^t$ $\bar{\Psi} = \Psi^+\hat{K}$ K = diag{Λ, I}

Explicitly: $$\bar{\Psi}\mathcal{M}\Psi = \bar{\Psi}_{\mathbf{r}_2\mathbf{r}_1}\mathcal{M}_{\mathbf{r}_1\mathbf{r}_2;\mathbf{r}_3\mathbf{r}_4}\Psi_{\mathbf{r}_4\mathbf{r}_3}$$

$$\mathcal{M}_{\mathbf{r}_1\mathbf{r}_2;\mathbf{r}_3\mathbf{r}_4} = \delta_{\mathbf{r}_2\mathbf{r}_3}\tau[E - H + i\delta\Lambda_z/2]_{\mathbf{r}_1\mathbf{r}_4}$$

$$\mathcal{B}^\alpha_{\mathbf{r}_1\mathbf{r}_2} = B\delta_{\mathbf{r}_1\mathbf{r}_2}\delta_{\alpha b} \text{ (and similarly for } \bar{\mathcal{B}})$$

$$B \circ \bar{B} \equiv \Omega = \Lambda_z + i\Lambda_y$$

We choose $\bar{B} = \mathrm{diag}(1, 1)$ and $B = \mathrm{diag}(1, -1)^t$

$$\mathcal{P}_\nu(s) = \int \mathcal{D}[\bar{\Psi}, \Psi] < e^{i[\bar{\Psi}\mathcal{M}\Psi - \sqrt{\tilde{s}}(\bar{\mathcal{B}}\Psi + \bar{\Psi}\mathcal{B})]} >$$

Integrating over $\bar{\Psi}, \Psi$ Shift: $\Psi \to \Psi + \sqrt{\tilde{s}} M^{-1} B$

$$P_\nu(s) = < \exp[-\underbrace{\mathrm{Str}\{M\}}_{=0} - i\tilde{s}\bar{B}M^{-1}B] >$$

=0, due to supersymmetry

$$\bar{B}M^{-1}B = \mathrm{Tr}\{[\tau(E - H + i\delta\Lambda_z/2)]^{-1}(\Lambda_z + i\Lambda_y)\} = \tau^{-1}\mathrm{Tr}\{G^R(E) - G^A(E)\}$$

$$\nu(E) = i/(2\pi L^d)\mathrm{Tr}\{[G^R(E) - G^A(E)]\}$$

$$\tilde{s} = s\tau\Delta/(2\pi) \qquad \Delta = 1/(\bar{\nu}L^d)$$

$$i\tilde{s}\bar{B}M^{-1}B = \frac{s\tau}{\bar{\nu}}\frac{i}{2\pi L^d}\tau^{-1}\mathrm{Tr}\{G^R(E) - G^A(E)\} = \frac{s\nu(E)}{\bar{\nu}}$$

Finally: $\mathcal{P}_\nu(s) \equiv < \exp[-s\nu/\bar{\nu}] >$

"Physical sense" of the integration over bi-local variables $\bar{\Psi}, \Psi$

"Averaged" Ψ:

$$< \Psi_{r_1 r_2} > \equiv \int D[\overline{\Psi}, \Psi] \Psi_{r_1 r_2} \exp\{i[\overline{\Psi} M \Psi - \sqrt{\tilde{s}}\,(\overline{B}\Psi + \overline{\Psi} B)]\}$$

Shift: $\Psi \rightarrow \Psi + \sqrt{\tilde{s}}\,\mathrm{M}^{-1}\mathrm{B}$

$$< \Psi_{r_1 r_2} > \propto M^{-1} B \propto \begin{pmatrix} G^R{}_{r_1 r_2} & 0 \\ 0 & G^A{}_{r_1 r_2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} G^R{}_{r_1 r_2} \\ -G^A{}_{r_1 r_2} \end{pmatrix}$$

Resume:

Integration runs over *"Green's functions space";*

"Classical trajectory" is the *physical* Green's function

Using the Primary Representation:

$$\mathcal{P}_\nu(s) = \int \mathcal{D}[\bar{\Psi}, \Psi] < e^{i[\bar{\Psi}\mathcal{M}\Psi - \sqrt{\tilde{s}}(\bar{\mathcal{B}}\Psi + \bar{\Psi}\mathcal{B})]} >$$

$$\mathcal{M}_{\mathbf{r}_1\mathbf{r}_2;\mathbf{r}_3\mathbf{r}_4} = \delta_{\mathbf{r}_2\mathbf{r}_3}\tau[E - H + i\delta\Lambda_z/2]_{\mathbf{r}_1\mathbf{r}_4}$$

$$H = H_0 + U$$

Averaging over Gaussian disorder U and H-S decoupling of

$(\bar{\Psi}_{\mathbf{r}_1\mathbf{r}}\Psi_{\mathbf{r}\mathbf{r}_1})(\bar{\Psi}_{\mathbf{r}_2\mathbf{r}}\Psi_{\mathbf{r}\mathbf{r}_2})$ by a matrix field $Q_{\mathbf{r}_1\mathbf{r}_2}(\mathbf{r})$

As a result, $\mathcal{M} \to \mathcal{M}(\mathcal{Q}) = \mathcal{M}_0 + i\mathcal{Q}/2$

$$\mathcal{Q}_{\mathbf{r}_1\mathbf{r}_2;\mathbf{r}_3\mathbf{r}_4} \equiv \delta_{\mathbf{r}_1\mathbf{r}_4} Q_{\mathbf{r}_2\mathbf{r}_3}(\mathbf{r}_1)$$

"Primary" saddle-point approximation

Saddle-point: $$Q^2(\mathbf{r}) = I$$

Slow functional: $$\mathcal{P}_\nu(s) \; = \int \mathcal{D}[Q] \, \exp\left[F_0 + F_s\right]$$

$$F_0 = \frac{\pi\bar{\nu}}{4} \int d\mathbf{r} \, \mathrm{Str}\{D(\nabla Q(\mathbf{r}))^2 - 2\delta\Lambda_z Q(\mathbf{r})\}$$

$$F_s = -\tilde{s} \int d\mathbf{r} \, \mathrm{Str}\{O^{(\nu)}(\mathbf{r})\Lambda_z k Q(\mathbf{r})\}$$

$$\tilde{s} = s\tau\Delta/(2\pi)$$

k = diag{1, -1} in "superspace"

$$O^{(\nu)}_{\mathbf{r}_1\mathbf{r}_2}(\mathbf{r}) = C_{\mathbf{r}_2\mathbf{r}_1}\delta[\mathbf{r} - (\mathbf{r}_1 + \mathbf{r}_2)/2]$$

$$C = [I + 4\tau^2(H_0 - E_F)^2]^{-1}$$

Correspondence with the perturbation theory

$$\ln P(s) = -s + (s\Delta/2\pi)^2 \sum_{q\neq 0} (Dq^2)^{-2} + \ldots$$

$$\langle \delta\nu/\nu \rangle = 0 \;\; ; \;\; \langle (\delta\nu/\nu)^2 \rangle_c = \Delta^2/(2\pi^2) \sum q\neq 0 \, (Dq^2)^{-2}$$

Non-trivial "accidental" cancellation of
two-loop contributions to $\langle (\delta\nu/\nu)^3 \rangle_c$

Beyond the perturbation theory

Secondary saddle-point equation:

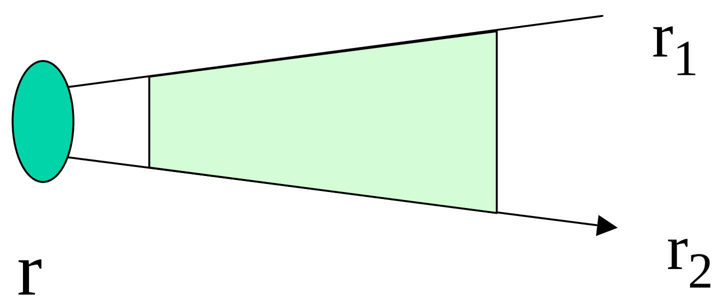
$$\nabla[Q(\mathbf{r})\nabla Q(\mathbf{r})] = \frac{\tilde{s}}{\pi\bar{\nu}D}[\Lambda_z k O^{(\nu)}(\mathbf{r}), Q(\mathbf{r})]$$

$\mathcal{E}_0$ - a sub-manifold of the saddle-point manifold $\mathcal{E}$

$Q_{\mathbf{r}_1\mathbf{r}_2}(\mathbf{r}) \in \mathcal{E}_0$ ⟺ smooth dependence on $(r_1 + r_2)/2$

small difference $|r_1 - r_2| \sim l$

$Q_{\mathbf{r}_1\mathbf{r}_2}(\mathbf{r})$



Wigner representation:

$$\mathbf{r}' = (\mathbf{r}_1 + \mathbf{r}_2)/2$$

$$A_{\mathbf{r}_1\mathbf{r}_2} = (1/L^d)\sum_{\mathbf{p}} A(\mathbf{r}';p)\exp[-ip(\mathbf{r}_1 - \mathbf{r}_2)]$$

$$F_0 = \frac{\pi\bar{\nu}}{4}\int d\mathbf{r}d\mathbf{r}'\frac{1}{L^d}\sum_{\mathbf{p}} \mathrm{str}\{D[\nabla^{(\mathbf{r})}Q(\mathbf{r};\mathbf{r}';\mathbf{p})]^2\}$$

$$F_s = -\tilde{s}\int d\mathbf{r}d\mathbf{r}'\frac{1}{L^d}\sum_{\mathbf{p}} C_p\,\mathrm{str}\{\Lambda_z Q(\mathbf{r};\mathbf{r};\mathbf{p})_{\mathrm{k}}\}$$

$$C_p = [1 + (p - p_F)l^2]^{-1}$$

In the considered hydrodynamical approximation (non-extended σ-model):

$$Q(\mathbf{r};\mathbf{r}';\mathbf{p})^2 = I$$

Long-tail asymptotics of $\mathcal{P}(\nu)$

Inverse transformation:

$$\mathcal{P}(\nu/\bar{\nu}) = \int_{-i\infty}^{i\infty} ds/(2\pi i)\exp[s\nu/\bar{\nu}]\mathcal{P}_\nu(s)$$

Parametrization of the bosonic sector:

$$Q^{bosonic}(r,r';p) = \begin{pmatrix} \cosh\theta & -\sinh\theta \\ \sinh\theta & -\cosh\theta \end{pmatrix}$$

Bosonic action:

$$F_b[\theta] = \frac{\pi\bar{\nu}D}{2}\sum_{\mathbf{p}}\int\frac{d\mathbf{r}d\mathbf{r}'}{L^d}[\nabla_{\mathbf{r}}\theta(\mathbf{r};\mathbf{r}';\mathbf{p})]^2$$

$$+\,2\tilde{s}\sum_{\mathbf{p}}C_p\int\frac{d\mathbf{r}}{L^d}\left[\cosh\theta(\mathbf{r};\mathbf{r};\mathbf{p})-1\right].$$

Self-consistency equation

$$\frac{\delta\nu}{\bar{\nu}} = \frac{\tau\Delta}{\pi}\sum_{\mathbf{p}}C_p\int\frac{d\mathbf{r}}{L^d}\left[\cosh\theta(\mathbf{r};\mathbf{r};\mathbf{p})-1\right]$$

Saddle-point equation for boson action

$$\nabla^2_{(\mathbf{r})}\theta(\mathbf{r};\mathbf{r}';\mathbf{p}) - \kappa_p \sinh\theta(\mathbf{r};\mathbf{r};\mathbf{p})\delta(\mathbf{r}-\mathbf{r}') = 0$$

$$\kappa_p = \tilde{s}\, C_p /(\pi \bar{\nu} D)$$

MK equation, for comparison:

$$\nabla^2\theta\,(r) + \kappa^2 \sinh\theta(r) = 0\,; \quad \kappa^2 = i\omega/D$$

Formal solution

$$\theta(\mathbf{r};\mathbf{r}';\mathbf{p}) = -\kappa_p \mathcal{D}(\mathbf{r};\mathbf{r}')\sinh\theta(\mathbf{r}';\mathbf{r}';\mathbf{p})$$

Green’s function of Laplace operator

Self-consistency at **r = r`** → $\theta(\mathbf{r};\mathbf{r};\mathbf{p})$

Trivial solution: $\theta(\mathbf{r};\mathbf{r};\mathbf{p}) = 0$

Non-trivial solution for small κ_p :

$$\theta\text{sign}[\text{Re}(\theta)] \approx \ln[-1/(\kappa_p \mathcal{D}(\mathbf{r};\mathbf{r}))]$$

Saddle-point action leads to:

$$\mathcal{P}(\nu) \propto \exp\left(-\frac{\pi \bar{g}}{2\ln(L/l)} \ln^2\left[\frac{\delta\nu}{\bar{\nu}\tau\Delta}\right]\right)$$

Conclusions (DOS)

A working field-theoretical representation has been developed for DOS distribution functions.
The basic element – integration over bi-local primary superfields.

Slow functional (in the spirit of a non-extended nonlinear σ-model) has been derived.
Non-perturbative secondary saddle-point approach has led to a log-normal asymptotics of distribution functions.

The formalism opens a way to study the complete statistics of fluctuations in disordered conductors