Ginsburg-Landau Theory of Supersolid

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Outline of the talk: KITP, 2/10/2006

1. Introduction: History, PSU's experiments, NL-NS…….
2. Two-component Quantum GL theory of Supersolid
3. SF to **deconfined solid** transition and global phase diagram
4. The deconfined Solid to Supersolid transition
5. Discussions on PSU's experiments: He3 impurities
   Order from quenched disorder
6. The non-classical rotation inertia (NCRI) of the SF and SS
7. The energy of a supersolid vortex
8. Conclusions

Acknowledgement

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Also thank

All the discussions in the wonderful mini- workshop

1. Introduction

A **supersolid** is a state which has both off-diagonal long range order (ODLRO) and crystalline order.

C. N. Yang, 1962, ODLRO (= BEC)

Andreev and I. Lifshitz, 1969. Bose-Einstein Condensation (BEC) of vacancies leads to supersolid, classical hydrodynamics of vacancies.

G. V. Chester, 1970, Wavefunction with both BEC and crystalline order, a supersolid cannot exist without vacancies or interstitials

A. J. Leggett, 1970, Non-Classical Rotational Inertial (NCRI) ρ_s / ρ of supersolid He4, quantum exchange process of He atoms can also lead to a supersolid even in the absence of vacancies,

W. M. Saslow, 1976, improve the upper bound $\rho_s / \rho \leq 10^{-2}$

Over the last 35 years, a number of experiments have been designed to search for the supersolid state without success.

E. Kim and M. H. W. Chan, Nature, 15 Jan 2004; Science 24 Sep. 2004

Kim and Chan observed a marked 1 ~2 % NCRI of solid He4 at ~ 0.2 K, both when embedded in Vycor glass and in bulk He4.

He3 impurities $40\ ppB \sim 85\, ppM$ *decrease* the superfluid fractions but *increase* the SS to NS transition temperature considerably

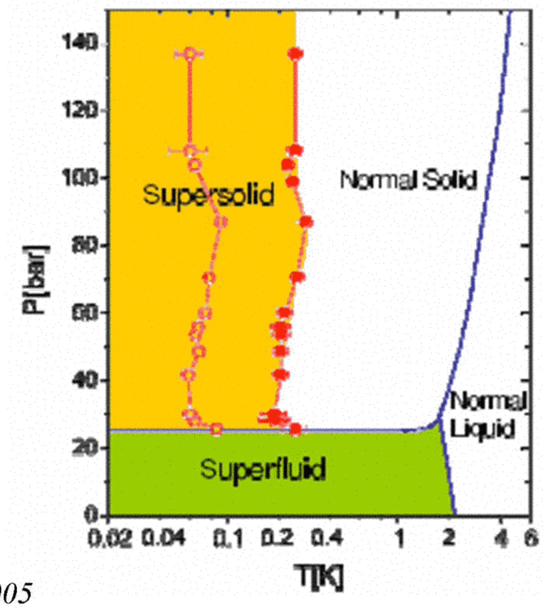
Goodkind, 2002, acoustic wave attenuation and heat wave measurements Solid He4 displays a phase transition at 200mK only if its strained or with He3 impurities ~ppM

A. Clark and M. H. W. Chan, March, 2005.

They also discovered $\sim 10^{-4}$ percent of superfluid density in solid hydrogen, even the 3d superfluid state of hydrogen is still elusive.

HD impurity concentration $\sim 10\ ppM$, it is extremely important to see if the above two unusual facts also hold for H2

Experimental global phase diagram of He4



Kim and Chan, 2005

PSU's experiments have rekindled great theoretical interests in the possible supersolid phase of He4

- D. M. Ceperley, B. Bernu, Phys. Rev. Lett. 93, 155303 (2004);
- W. M. Saslow, Phys. Rev. B 71, 092502 (2005);
- N. Prokof'ev, B. Svistunov, Phys. Rev. Lett. 94, 155302 (2005);
- D. E. Galli, M. Rossi, L. Reatto, Phys. Rev. B 71, 140506(R) (2005);
- N. Kumar, cond-mat/0507553;
- G. Baskaran, cond-mat/0505160;
- Xi Dai, Michael Ma, Fu-Chun Zhang, cond-mat/0501373 ;
- E. Burovski, E. Kozik, A. Kuklov, N. Prokof'ev, B. Svistunov, Phys. Rev. Lett., 94,165301(2005).
- D.T. Son, cond-mat/0501658.
- P. W. Anderson, W. F. Brinkman, David A. Huse, cond-mat/0507654
- A. T. Dorsey, P. M. Goldbart, J. Toner, cond-mat/0508271.
- ……

Very recently, there are two experimental groups one in US, one in Japan Confirmed PSU's experiments.

Density operator: $n(\vec{x}) = n_0 + \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\cdot\vec{x}}$

Complex order parameter: $\psi(x)$, $n(x) \neq |\psi(x)|^2$

In effective GL theory, ψ *and* n should be treated independently !

Liu and fisher, 1973

(1) Liquid to solid transition:

$$f_{L-NS} = \sum_{\vec{G}} \frac{1}{2} r_{\vec{G}} |n_{\vec{G}}|^2 - w \sum_{\vec{G}_1,\vec{G}_2,\vec{G}_3} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} \delta_{\vec{G}_1+\vec{G}_2+\vec{G}_3,0} + u \sum_{\vec{G}_1,\vec{G}_2,\vec{G}_3,\vec{G}_4} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} n_{\vec{G}_4} \delta_{\vec{G}_1+\vec{G}_2+\vec{G}_3+\vec{G}_4,0} + \cdots$$

(2) Liquid to superfluid transition:

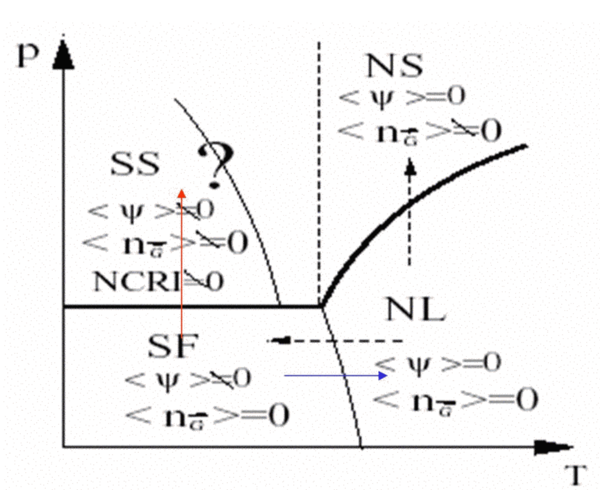
$$f_{L-SF} = K|\nabla\psi|^2 + t|\psi|^2 + u|\psi|^4 + \cdots$$

(3) Coupling: $f_{\text{int}} = v\,|\psi(x)|^2\, n(x) + \ldots\ldots$

A. T. Dorsey, P. M. Goldbart, J. Toner, cond-mat/0508271

In the NL, $t > 0, \psi$ has a gap, can be integrated out.

In the NL, $< n(x) >= n_0$, $n_{\vec{G}} = 0$ two sectors decouple



2. Two-component QGL theory

The theory based on two facts:

(1) There is a roton minimum in the superfluid state. It is well established

(2) The instability to solid formation is driven by the gap diminishing at the roton minimum. It has strong experimental supports

T. Schneider and C. P. Enz, 1971; *Yves Pomeau and Sergio Rica*, 1994.

P. Nozieres, April, 2004

How to choose the order parameter of the QGL theory ?

Superfluid: ψ Density wave operator: ρ

Primary complex order parameter: ψ

Descendent (or derivative) real positive order parameter: $\rho = |\psi|^2 \neq n(x)$

$\vec{B} = \nabla \times \vec{A}$ in quantum mechanics

AB phase is encoded in $\vec{A}$, but not in $\vec{B}$.

A new kind of quantum solid has:

the translational and rotational orders characterized by $\rho = |\psi|^2$

a hidden quantum phase order characterized by ψ

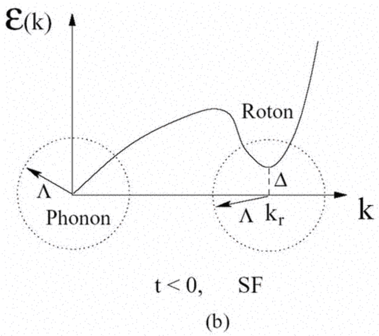
The most general QGL action at any d, T and P is:

$$\mathcal{S} = \frac{1}{2}\int d^d x d\tau [\kappa|\partial_\tau \psi|^2 + t|\psi|^2 + K|\nabla\psi|^2 + \boxed{L_1|\nabla^2\psi|^2} + \boxed{L_2|\nabla^3\psi|^2}] + u\int d^d x d\tau |\psi(\vec{x},\tau)|^4 + \cdots \qquad (1)$$

Due to the long-range Lennard-Jones potential between He4 atoms:

$$\boxed{L_1 < 0, L_2 > 0}$$

Look at the propagator: $\varepsilon(k) = t + K k^2 + L_1 k^4 + L_2 k^6$



Wide separation of two low energy modes in momentum scales

$$\psi(\vec{x},\tau) = \psi_1(\vec{x},\tau) + \psi_2(\vec{x},\tau)$$

$$\psi_1(\vec{x},\tau) = \int_0^\Lambda \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \psi(\vec{k},\tau) \qquad \psi_2(\vec{x},\tau) = \int_{|k-k_r|<\Lambda} \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \psi(\vec{k},\tau)$$

$$\psi_1, \quad k \sim 0 \qquad\qquad \psi_2, \quad k \sim k_r$$

One complex field at **two** length scales !

The two component QGL action in $(\vec{k},\omega)$ space:

$$\begin{aligned} \mathcal{S} &= \frac{1}{2}\int_0^\Lambda \frac{d^d k}{(2\pi)^d}\frac{1}{\beta}\sum_{i\omega_n}(\kappa\omega_n^2 + t + Kk^2)|\psi_1(\vec{k},i\omega_n)|^2 \\ &+ \frac{1}{2}\int_\Lambda \frac{d^d k}{(2\pi)^d}\frac{1}{\beta}\sum_{i\omega_n}(\kappa\omega_n^2 + \Delta + v_r(k-k_r)^2)|\psi_2(\vec{k},i\omega_n)|^2 \\ &+ u\int d^d x d\tau |\psi_1(\vec{x},\tau) + \psi_2(\vec{x},\tau)|^4 + \cdots \end{aligned} \qquad (2)$$

Where $\int_\Lambda = \int_{|k-k_r|}$

$$t \sim T - T_c, \quad T_c \sim 2.17K \qquad \Delta \sim p_c - p, \quad p_c \sim 25bar$$

One U(1) global symmetry: $\psi_1 \to e^{i\chi}\psi_1, \psi_2 \to e^{i\chi}\psi_2$

One P-H symmetry: $\psi_1 \to \psi_1^*, \quad \psi_2 \to \psi_2^*$

3. The SF to NS transition

In the SF state, $t < 0, \Delta > 0$

BEC: $< \psi_1 >= a \neq 0$ breaks U(1) symmetry, keeps the P-H symmetry

At mean field level, setting $\psi_1 = a$ into the QGL

and writing $\psi_2(\vec{x},\tau) = \phi_1(\vec{x},\tau) + i\phi_2(\vec{x},\tau)$

We find ϕ_1 is more massive than ϕ_2

Integrating out ϕ_1 leads to $n = 1, (d+1, d)$ quantum Lifshitz action:

$$\mathcal{S}_{sf-ss} = \frac{1}{2}\int_\Lambda \frac{d^dk}{(2\pi)^d}\frac{1}{\beta}\sum_{i\omega_n}(\kappa\omega_n^2 + \Delta_2 + v_r(k-k_r)^2)|\phi_2|^2 + u\int d^dx d\tau \phi_2^4 + \cdots \quad (5)$$

It keeps the P-H symmetry: $\phi_2 \rightarrow -\phi_2$

In contrast to classical liquid to solid transition,
there is no third power term ϕ_2^3

At mean field level, there is a 2nd order SF to NS transition:

$\Delta_2 > 0, < \psi_2 >= 0,$ SF phase

$\Delta_2 < 0, < \psi_2 >= \sum_{m=1}^{P} \Delta_m e^{i\vec{Q}_m \bullet \vec{x}}, Q_m \sim k_r$ Density Wave phase

Applying RG analysis developed by *R. Shankar*, 1994,
We found it is a fluctuation driven 1st order transition, consistent with the original picture in *S. A. Brazovskii*, 1975

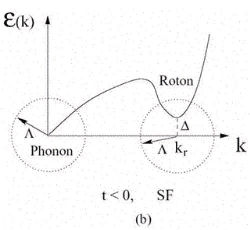
Normal solid (confined solid): $< \psi >= 0, < n_{\vec{G}} > \neq 0$

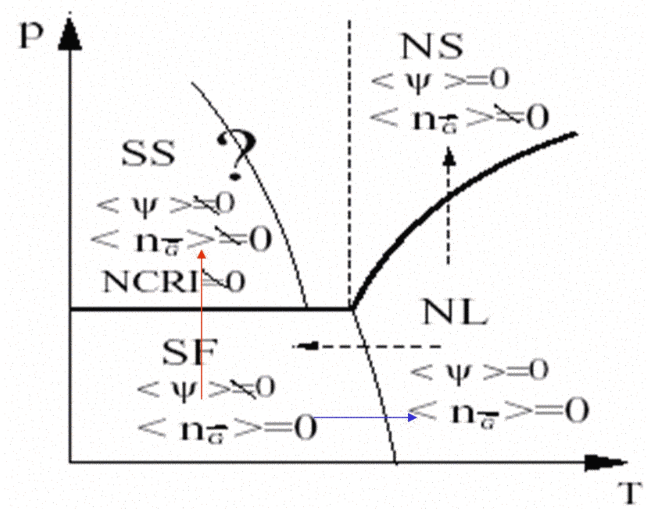
Density wave (deconfined solid): $< \psi > \neq 0, < n_{\vec{G}} > \neq 0$

In the NL, $t > 0, \psi$ has a gap, can be integrated out.

In the NL, $< n(x) >= n_0, \quad n_{\vec{G}} = 0$ two sectors decouple

$$\omega_q = \frac{q^2}{2mS_q}$$





4. NS to SS transition

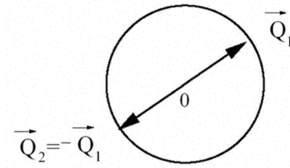
On both sides of the NS-SS transition, $\Delta < 0$, so ψ_2 is not critical

Setting $< \psi_2 >= \sum_{m=1}^{P} \Delta_m e^{i\vec{Q}_m \bullet \vec{x}}, Q_m \sim k_r$ into the QGL,

$$\begin{aligned} \mathcal{S} &= \frac{1}{2}\int_0^{\Lambda} \frac{d^d k}{(2\pi)^d}\frac{1}{\beta}\sum_{i\omega_n}(\kappa\omega_n^2 + t + Kk^2)|\psi_1(\vec{k}, i\omega_n)|^2 \\ &+ \frac{1}{2}\int_{\Lambda} \frac{d^d k}{(2\pi)^d}\frac{1}{\beta}\sum_{i\omega_n}(\kappa\omega_n^2 + \Delta + v_r(k-k_r)^2)|\psi_2(\vec{k}, i\omega_n)|^2 \\ &+ u\int d^d x d\tau |\psi_1(\vec{x},\tau) + \psi_2(\vec{x},\tau)|^4 + \cdots \end{aligned} \tag{2}$$

We study $P = 2,3,4,6,8$ respectively. $P = 1$ is trivial

(1) $P = 2, \quad \vec{Q}_2 = -\vec{Q}_1$



(1b)

ψ_2 is real, respects the P-H symmetry

The local density: $\rho^l_{DW-A} = 2\Delta^2(1+\cos 2\vec{Q}_1\cdot\vec{x})$

The GL action to describe DW-A to DW-B transition:

$$\mathcal{L}_{DW\,A-DW\,B} = \kappa|\partial_\tau\psi_1|^2 + K|\nabla\psi_1|^2 + t|\psi_1|^2 + u|\psi_1|^4 + u|\psi_1|^2|\psi_2|^2 + (\psi_2^\dagger\psi_1)^2 + h.c.\cdots$$

DW-A: $<\psi_1>=0, <\psi_2>\neq 0$ DW-B: $<\psi_1>\neq 0, <\psi_2>\neq 0, \ NCRI=0$

The diagonal term shift down the critical temperature to $t_c = -u|\psi_2|^2 < 0$

These off-diagonal **umklapp** terms $(\psi_2^\dagger\psi_1)^2 + h.c.$ break the global U(1) symmetry of ψ_1, exclude any possible superfluid component.

In fact, $(\psi_1)^{2m} + h.c., m \geq 1$ exist

The transition from DW-A to DW-B is in Ising universality class

There is no Goldstone modes at k=0, NCRI=0

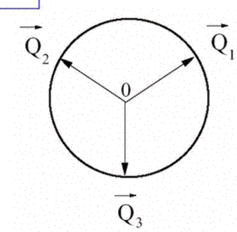
The only Goldstone modes are the lattice phonons in ψ_2 sector

Accompanying the transition is also a DW structure transition:

The local density: $\rho^l_{DW-B} = a^2 + 2a\Delta\cos\vec{Q}_1\cdot\vec{x} + 2\Delta^2(1+\cos 2\vec{Q}_1\cdot\vec{x})$

DW-B is in-commensurate to the underlying $n(x)$ lattice.

(2) $P = 3, \quad \vec{Q}_1 + \vec{Q}_2 + \vec{Q}_3 = 0$



(2b)

ψ_2 is complex, breaks the P-H symmetry

$\rho^l_{DW-A} = |\psi_2(\vec{x})|^2$ generates a 2d hexagonal lattice

$(\psi_2^\dagger\psi_1)^3 + h.c.$ exist and break the U(1) symmetry of ψ_1

No chance to get a SS either !

Lattice	P	$< \psi_2 >$	P-H symmetry	Linear Relation	Umklapp term	Scaling	Universality
$1d$ $chain$	2	real	unbroken	$\vec{Q}_2 = -\vec{Q}_1$	$(\psi_2^\dagger \psi_1)^2 + h.c.$	R	Ising
$square$	4	real	unbroken	$\vec{Q}_3 = -\vec{Q}_1, \vec{Q}_4 = -\vec{Q}_2$	$(\psi_2^\dagger \psi_1)^2 + h.c.$	R	Ising
sc	8	real	unbroken	$\vec{Q}_4 = \vec{Q}_1 - \vec{Q}_2 - \vec{Q}_3$	$(\psi_2^\dagger \psi_1)^2 + h.c.$	R	Ising
$hexagonal$	3	complex	broken	$\vec{Q}_1 + \vec{Q}_2 + \vec{Q}_3 = 0$	$(\psi_2^\dagger \psi_1)^3 + h.c.$	R	1st order
bcc	4	complex	broken	$\vec{Q}_4 = -\vec{Q}_1 - \vec{Q}_2 - \vec{Q}_3$	$(\psi_2^\dagger \psi_1)^4 + h.c.$	D-IR	3D XY
hcp	6	complex	broken	$\vec{Q}_4 = \frac{1}{3}\vec{Q}_1 - \frac{2}{3}\vec{Q}_2 - \frac{2}{3}\vec{Q}_3$	$(\psi_2^\dagger \psi_1)^6 + h.c.$	D-IR	3D XY

Bulk Solid He4: hcp He4 in Vycor: bcc Para-H2: hcp

$\vec{Q}$'s must have the same length k_r Independent of $c/a = x$

$\vec{k}$'s can have different lengths

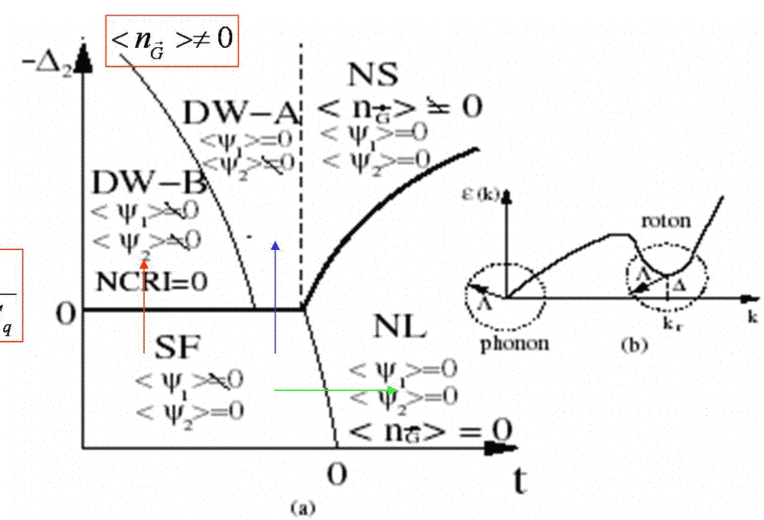
Even a spherical roton surface can lead to a **hcp** lattice.

ψ_1 stands for the ring exchange process

Any clean lattice has NO chance to get a SS ! But bcc and hcp lattices are quite close !

Theoretical global phase diagram of perfect He4

$$\omega_q = \frac{q^2}{2mS_q}$$



5. Discussions on PSU's experiments

He4 takes a hcp lattice with $c/a \sim 1.63$ which may depends on the pressure

The LR and the U(1) SBT term are independent of this ratio !

There is No supersolid in a perfect hcp lattice ! But hcp lattice is quite close !

However, the U(1) SBT need a perfect lattice, it is very sensitive to any lattice distortion !

There are two possible ways to eliminate the U(1) SBT

(1) Applying anisotropic pressures to distort the hcp lattice

(2) Adding He3 impurities distort the hcp lattice very effectively !

He3 impurities eliminate the U(1) symmetry breaking terms, also introduce a very weak random mass through $\rho_2 = |\psi_2|^2$ term:

$$\mathcal{L}_{NS-SS} = \kappa|\partial_\tau \psi_1|^2 + K|\nabla\psi_1|^2 + t|\psi_1|^2 + u|\psi_1|^4 + u_1|\psi_1|^2 \boxed{|\psi_2|^2} + \cdots \quad (7)$$

3d XY model in a weak random potential.

3d XY specific exponent $\alpha = -0.012 < 0$, from Harris criterion, a weak random mass is irrelevant !

However, the random mass will decrease the superfluid density !

In conclusion, He3 impurities play a dual role:

They kill the off-diagonal terms very effectively, introduce a random mass through the diagonal term, does not change 3d XY universality class, but decrease the superfluid density .

Order from quenched disorder: $U(1) \rightarrow U(1)_1 \times U(1)_2$

HD impurities in solid hydrogen play the same dual role !

In PSU's experiments, $40\,ppB < He3 < 85\,ppM$

$HD \sim 10\,ppM$ in H2 solid

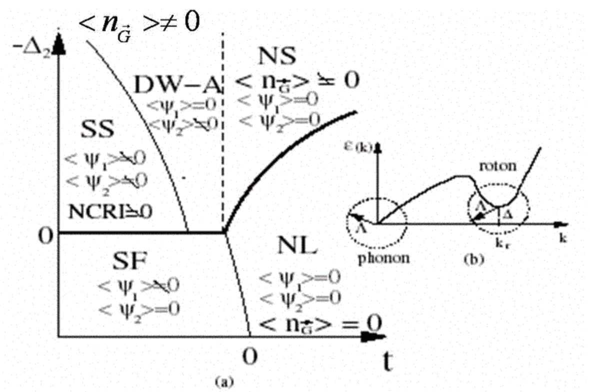
There is a magneto-roton in BLQH, for existence of excitonic supersolid, see

Jinwu, Ye, cond-mat/0310512, cond-mat/0407088

Theoretical global phase diagram of distorted solid He4

He3

$\omega_q = \frac{q^2}{2mS_q}$



Goodkind, 2002, acoustic wave attenuation and heat wave measurements
Solid He4 displays a phase transition at 200mK only if its strained or with He3 impurities ~ppM

7. NCRI is SF and SS

In the SF state, the superfluid density (NCRI) is :

$$\Delta \gg \Delta_c, \ \rho_s(T=0) = \rho = \int d^d x |\psi(\vec{x},\tau)|^2 = \int d^d x (|\psi_1(\vec{x},\tau)|^2 + |\psi_2(\vec{x},\tau)|^2)$$

ψ_2 contributes to NCRI ! even it does not contributes to BEC !

(1) At finite T, due to phase fluctuations:

$$\rho_s(T) = \rho_s(T=0) - cT^2 \text{ at } d=3.$$

(2) Let's look at the roton fluctuations: $< \phi_2^2(\vec{x},\tau) >_{T=0} \sim \log \frac{\Lambda}{\Delta}$

It is **IR divergent** signifying the instability to lattice formation

At $T << \Delta << \Lambda$, $< \phi_2^2(\vec{x},\tau) >_T - < \phi_2^2(\vec{x},\tau) >_{T=0} \sim (\log \frac{\Lambda}{\Delta}) e^{-\frac{\Delta}{T}}$

Exponentially suppressed at finite T

In the SS side: $\rho = |\psi_1|^2 + |\psi_2|^2 + 2(\psi_1^* \psi_2 + h.c.)$

Superfluid: $\psi_1, \quad k \sim 0$ Density wave: $\psi_2, \quad k \sim k_r \sim 1/a$

BEC: $< \psi_1 >$ Density: $\rho^l{}_{DW} = |\psi_2|^2$

NCRI: $\rho^l{}_s = |\psi_1|^2$ $a =$ Lattice constant

What does the crossing term $K = 2(\psi_1^* \psi_2 + h.c.)$ mean ?

It is the local transfer rate between the SF and the DW component !

As expected, $\int d^d x K = 0$, because the SS reaches equilibrium.

8. SF and SS vortex

In SF : $E_v^{SF} = \frac{\rho_s^{SF} h^2}{4\pi m^2} ln \frac{R}{\xi_{SF}}$ $v_c^{SF} \sim 30cm/s$

m : He4 atom mass :Я System size

$\zeta_{SF} \sim a$: core size of a SF vortex

In SS: $E_v^{SS} = \frac{\rho_s^{SS} h^2}{4\pi m^2} ln \frac{R}{\xi_{SS}}$

We expect: $\xi_{SS} \sim 1/\Lambda \gg 1/k_r \sim a \sim \zeta_{SF}$

Compared to E_v^{SF} , there are two reductions: $\rho_{SS} << \rho_{SF}, \quad \zeta_{SS} >> \zeta_{SF}$

Lead to very low critical velocity: $v_{SS}^c \sim 30\mu m/s$

Using $\rho_{SS} \sim 10^{-2} \rho_{SF}$, *we find* $\zeta_{SS} \sim 2 \times 10^4 \zeta_{SF}$

At $x = 0.3\, ppM, \quad d_{imp} \sim 450\, \mathring{A}$

Three widely separated length scales: $\xi_{SS} \gg d_{imp} \gg \xi_{SF}$

is sufficiently dense to effectively eliminate the off-diagonal umklapp term

is also sufficiently dilute to dilute to introduce an uncorrelated random mass through the diagonal term

In principle, the specific measure measurement should see λ peak

3D XY correlation length $\xi \sim t^{-\nu} \gg \xi_{SS}$

The critical regime $t = \frac{T - T_{SS}}{T_{SS}}$ is much much narrower than the NL to SF Transition !

This make the λ peak observation very difficult ! But a jump in specific heat should be detected.

9. Conclusions

1. I construct a simple and novel two components Quantum Ginsburg Landau (QGL) theory which put SF and DW on the same footing !
2. Map out the global phase diagrams
3. Discuss all the possible classical and quantum phase transitions.
4. A perfect 3d lattice excludes any superfluid component, but bcc and hcp lattice are quite close
5. He3 impurities can distort lattices effectively and leads to a Supersolid

   Order from quenched disorder !
6. Establish a clear physical picture of SS in both momentum and real space
7. NCRI in Supersolid, local transfer rate between SS and DW
8. There length scales $\zeta_{SS} \sim 2\times10^4 \zeta_{SF} >> d_{imp} >> \zeta_{SF} \sim 3\, \mathring{A}$