CATACLYSMIC VARIABLES

AND THE TYPE Ia PROGENITOR PROBLEM

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Cataclysmic Variables

CVs are characterized by a low-mass star/BD (donor) losing mass to an accreting WD. There is a rich variety of “subclasses” (associated with disk or WD)

- Classical Novae
- Dwarf Novae
- SU Ursae Majoris
- Z Camelopardalis
- Recurrent Novae
- Nova-Like
- SW Sextantis
- Polars & Int. Polars
- **SuperSoft Sources**
Whelan & Iben (1973) first proposed that CVs could be Type Ia progenitors.

\[ M_{wd} > M_{ch} \Rightarrow \text{good standard candles} \]

**Single Degenerate Channel:**

- Candidate progenitors observed (SSXSSs, Symbiotics, CVs)
- Fine tuning of accretion rate is needed to avoid nova and/or CE (small volume in the phase space)
- Absence of H in the spectra
Cataclysmic Variables

• CVs are semi-detached that transfer mass by RLOF binaries (can be non-conservative)
• CVs have $\sim 70$ min $< P_{\text{orb}} < \sim 12$ hours
• White dwarf masses of $\sim 0.3$ to $1.4 \, M_\odot$
  • $\sim 10$-20% are magnetic (10-250 MG).
• Donor (secondary) have masses from $\sim 1.2$ to $\sim 0.02 \, M_\odot$

Any model must be able to explain the salient features:
1) Orbital Period Distribution
2) Mass-Transfer Rates
3) Morphology
CV $P_{\text{orb}}$ Distribution

- 80 min
- Period Gap

Histogram showing the distribution of orbital periods with a peak at 80 minutes and a period gap.

$P_{\text{orb}}$ [h] on the x-axis and N on the y-axis.
Inferred Mass Transfer Rates

Approximate Uncertainty

Patterson Relation (1984)

\[ M_2 = 6 \times 10^{-12} \left( \frac{P_{\text{orb}}}{\text{1hr}} \right)^{3.3\pm0.3} M_\odot \text{yr}^{-1} \]
The donor overflows its Critical Equipotential

\[ \Phi = -G \left( \frac{M_1 m}{s_1} + \frac{M_2 m}{s_2} \right) - \frac{1}{2} m \omega^2 r^2 \]

We now calculate \( \Phi \) along the red line with \( y = 0 \):

Matter flows through the inner Lagrange point (\( L_1 \)) from the donor star (\( M_2 \)) to the compact accretor (\( M_1 \)). The critical Roche equipotentials intersect the \( L_1 \) point.
Drivers of Mass Transfer

• The donor must expand wrt the Roche Lobe (or the RL must shrink)
• CASE 1: Nuclear evolution

\[ \tau_{\text{nuc}} \approx 10^{10} \frac{M}{M_\odot} \frac{L_\odot}{L} \text{yr} \approx 10^{10} \left( \frac{M}{M_\odot} \right)^{-2.5} \text{yr} \]

• The donor expands on its nuclear timescale
• Mass transfer can be initiated on SGB or RGB
Drivers of Mass Transfer

- CASE 2: Thermal Timescale Mass Transfer (TTMT)

\[ E_{th} = -0.5E_{grav} \sim GM^2/R, \text{ so } \tau_{th} \sim GM^2/RL \]

\[ \tau_{KH} \approx 3.1 \times 10^7 \left( \frac{M}{M_\odot} \right)^2 \frac{R_\odot}{R_L} \text{ yr} \approx 3.1 \times 10^7 \left( \frac{M}{M_\odot} \right)^{-2} \text{ yr} \]

- Donors with radiative envelopes can temporarily shrink due to mass loss, but expand on a KH timescale to re-establish thermal equilibrium

\[ \dot{M} \approx M_{\text{donor}}/\tau_{KH} \approx 10^{-6} - 10^{-8} M_\odot \text{ yr}^{-1} \]
CASE 3: Angular Momentum Loss (AML)

- **Gravitational Radiation**
  \[
  \left( \frac{\dot{J}_{GR}}{J} \right) = -1.3 \times 10^{-8} \left( \frac{M_T}{M_\odot} \right)^{1/3} \left( \frac{M_1}{M_\odot} \right) \left( \frac{M_2}{M_\odot} \right) \left( \frac{P_{orb}}{1 \text{ hr}} \right)^{-8/3} \text{ yr}^{-1}
  \]

- **Magnetic Braking by a MSW (Verbunt-Zwaan Law)**
  \[
  \left( \frac{\dot{J}_{MB}}{J} \right) = -7.1 \times 10^{-6} \left( \frac{M_T}{M_\odot} \right)^{1/3} \left( \frac{M_1}{M_\odot} \right)^{-1} \left( \frac{R}{R_\odot} \right)^\gamma \left( \frac{P_{orb}}{1 \text{ hr}} \right)^{-10/3} \text{ yr}^{-1}
  \]

- **Systemic Mass Loss**
  \[
  \dot{M}_1 = -\beta \dot{M}_2 \quad \text{or} \quad \dot{M}_1 + \dot{M}_2 = (1 - \beta) \dot{M}_2
  \]

  \[
  \delta J_{\delta M_T} = \delta M_T \alpha \left( A^2 \omega \right) = (1 - \beta) \delta M_2 \alpha \left( A^2 \omega \right), \quad \alpha > 0
  \]

  \[
  \frac{\dot{J}_{\delta M_T}}{J} = \alpha (1 - \beta) \left( \frac{M_T}{M_1} \right) \left( \frac{\dot{M}_2}{M_2} \right)
  \]

- **Total AML:**
  \[
  \frac{\dot{J}_{\text{orb}}}{J} = \frac{\dot{J}_{GR}}{J} + \frac{\dot{J}_{MB}}{J} + \frac{\dot{J}_{\delta M_T}}{J} = \frac{\dot{J}_{\text{dis}}}{J} + \frac{\dot{J}_{\delta M_T}}{J}
  \]
From Ed’s talk on Friday:

\[
\frac{R_L}{A} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \approx 0.46 \left( \frac{q}{1 + q} \right)^{1/3}, \quad q \equiv \frac{M_2}{M_1} \leq 0.8
\]

\[
\frac{\dot{R}_L}{R_L} \approx \frac{\dot{A}}{A} + \frac{1}{3} \frac{\dot{M}_2}{M_2} - \frac{1}{3} \frac{\dot{M}_T}{M_T}
\]

\[
J_{\text{orb}} = M_1 M_2 \sqrt{\frac{G A}{M_1 + M_2}} \quad \Rightarrow \quad 2 \frac{\dot{J}_{\text{orb}}}{J} = 2 \frac{\dot{M}_2}{M_2} + 2 \frac{\dot{M}_1}{M_1} - \frac{\dot{M}_T}{M_T} + \frac{\dot{A}}{A}
\]

\[
\frac{\dot{R}_L}{R_L} = 2 \frac{\dot{J}_{\text{orb}}}{J} - \frac{5}{3} \frac{\dot{M}_2}{M_2} - 2 \frac{\dot{M}_1}{M_1} + \frac{2}{3} \frac{\dot{M}_T}{M_T}
\]
Rate of Mass Transfer

The equations describing the mass-transfer can be approximated as follows:

\[
\frac{\dot{R}_L}{R_L} = -\frac{\dot{M}_2}{M_2} \left( \frac{5}{3} - 2\beta q - \frac{2}{3}(1 - \beta) \frac{q}{1 + q} \right) + 2 \frac{\dot{J}_{\text{dis}}}{J} - 2\alpha(1 - \beta)(1 + q) \left( -\frac{\dot{M}_2}{M_2} \right)
\]

internal J redistribution < 0 OR > 0  
dissipation < 0  
systemic J loss < 0

If the system remains in contact ⇒ \( R_L (t) = R_2 (t) \)

\[
\frac{\dot{R}_L}{R_L} = \frac{\dot{R}_2}{R_2} \approx \xi_{ad} \frac{\dot{M}_2}{M_2} + \frac{\dot{R}_{2,\text{nuc}}}{R_2} + \frac{\dot{R}_{2,\text{th}}}{R_2}
\]

where \( \xi_{ad} \equiv \left[ \frac{d \ln(R)}{d \ln(M)} \right]_{\text{ad}} \)

The equations describing the mass-transfer can be approximated as follows:

\[
-\frac{\dot{M}_2}{M_2} \approx \frac{\dot{R}_{2,\text{nuc}}}{R_2} + \frac{\dot{R}_{2,\text{th}}}{R_2} - 2\frac{\dot{J}_{\text{dis}}}{J} D(\alpha, \beta, q, \xi_{ad})
\]
Dynamical Instability

Mass transfer is ONLY stable if numerator and denominator > 0

N.B.: If $D < 0$ then the binary system is dynamically unstable

$\Rightarrow$ CE phase/merger

$$D(q, \alpha, \beta, \xi_{ad}) = \left[ \frac{5}{3} + \xi_{ad} - 2\beta q - \frac{2q(1 - \beta)}{3(1 + q)} - 2\alpha(1 - \beta)(1 + q) \right]$$

Sign of $D$ very much depends on the value of $\alpha$

Importance wrt Type Ia SNe was noted by Di Stefano, Nelson, Rappaport, Lee and Wood (1995); Han and Podsiadlowski (2004)
Binary Evolution

- Assumptions: 1) donor unevolved; 2) Mass lost from the system due to CNx; 3) Interrupted mag. braking

Howell, Nelson & Rappaport (2001)
Orbital Period Gap

- Assumptions: 1) MB severely attenuated when donor becomes fully convective (IMB); 2) Mass lost from the system due to CNe

Let $f$ represent a thermal ‘bloating factor’: $R_2 = f a M_2^b$

Roche Geometry Constraint:

$$f = \left( \frac{P_{\text{upper}}}{P_{\text{lower}}} \right)^{2/3} \sim 1.2 - 1.3 \Rightarrow \text{thermally evolving}$$
Initial Conditions: $M_{wd} = 1.0 \, M_\odot$, $M_{donor} = 1.5 \, M_\odot$

A sharp bifurcation in $P_{\text{orb}}$ is possible.
Formation of CVs

Start with primordial binary

Iben & Tutukov (1991)

Yungelson (2005)

Most Probable End State
CE Evolution

Webbink (1984)
de Kool (1990)
Taam & Sandquist (1998)

Based on a ‘first principles’ energy argument:

\[ \alpha_{\text{CE}} \frac{GM_2}{2} \left( \frac{M_{\text{core}}}{a_f} - \frac{M_1}{a_i} \right) = \frac{GM_{\text{env}} \left( M_{\text{env}} + 3M_{\text{core}} \right)}{R_1} \]

\( \alpha_{\text{CE}} \) is the efficiency of the deposition of E in removing the CE.
Derive the properties of CVs (and other IBs) in the present epoch given their formation throughout the history of the Galaxy.

**Initial Distribution**

\[ n_0(M_{10}, M_{20}, P_0) \, dM_{10} \, dM_{20} \, dP_0 \]

**Final Distribution**

\[ n_f(M_{1f}, M_{2f}, P_f) \, dM_{1f} \, dM_{2f} \, dP_f \]
1) Efficiency of CE process
   - Separation of orbit

2) Choice of initial mass of primary

3) Correlation of masses

4) Birth rate function (BRF)

Large number of uncertainties!
Evolution of 10 million model CVs. This model represents the present-day population of CVs in the Milky Way assuming an age of 10 GYr.

Major Predictions:
1) > 95% of all CVs have short orbital periods (<2 hr).
2) Donors immediately above the period gap are ~25% less massive than would be inferred if the donor was on the MS.
Mass versus $P_{\text{orb}}$

Relative Logarithmic Probability
Synthesized distribution matches observed one reasonably well (once selection effects are accounted for).

Caveats: 1) $P_{\text{min}}$ theoretical is < 80 min 2) Expect more CVs near $P_{\text{min}}$ (“spike”) 3) ~10 times more novae above “gap” (factor of ~100 discrepancy?)
Population Synthesis of CNe

Nelson 2002
Nelson et al. 2004

Pop. Synthesis yields a rate of ~10 – 100 CNe/yr in our Galaxy

Nova Cygni 1992

see also Townsley & Bildsten 2004
Theoretically predicted nova frequencies (densities expressed per hour of orbital period). Case (a): solid lines correspond to $q^{1/4}$ and dashed lines to $q^0$ ($\alpha = 0.3$ for both sets of curves). Case (b): solid lines correspond to $\alpha = 0.3$ and dashed lines to $\alpha = 1$ ($q^{1/4}$ for both sets of curves). As in Figure 1, the blue curves correspond to hot WD’s and the red curves to cool ones.
The Transition to Steady Burning

Mass transfer rate of $1 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}$: (i) **Black curve**: $M_{WD} = 0.95 \, M_\odot$; (ii) **Red curve**: $M_{WD} = 1.0 \, M_\odot$; (iii) **Blue curve**: $M_{WD} = 1.1 \, M_\odot$. Setting $M_{WD} = 1.0 \, M_\odot$, and increasing $\dot{M}$ yields the following: (iv) **Green curve**: $6 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}$; (v) **Pink curve**: $5 \times 10^{-7} \, M_\odot \, \text{yr}^{-1}$. The inset shows the evolution of case (iv) on an appropriately short time scale.
Profile of a Thermonuclear Runaway

Thermal profile of a 0.7 $M_{\odot}$ CO WD undergoing accretion at $1 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$. Each curve corresponds to an evolutionary time ($\Delta t$) measured relative to the first model in the sequence. Log $T(K)$ is plotted against the log of the mass fraction (as measured from the surface).

Nelson 2005
Quasi-Steady Burning

Temporal evolution of the luminosity of an accreting 1.0 $M_\odot$ CO WD undergoing accretion at $5 \times 10^{-7} M_\odot$ yr$^{-1}$. The WD quickly attains a state of quasi-steady H-burning.
Temporal Evolution of Supersofts

SSXSs can be regarded as “Super CVs”

Van den Heuvel et al. (1991) developed the model of steady H burning on the surface of WDs

Di Stefano & Nelson 1996
Di Stefano et al. 1995

Synthesis produced a “Type Ia” frequency that was too small by a factor of ~20

The observationally inferred SN Ia rate is ~0.3 century$^{-1}$
Recent Progenitor Results

Dynamical Instability

Synthesis produced a Type Ia frequency that was too small by a factor of \( \sim 3 \)

Han and Podsiadlowski 2004