Recent Developments in Supersymmetric QFT

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Major Themes

- Classification of superconformal field theories

- Exact calculations: Defects, BPS States, Localization, …

- Connections between SUSY and Non-SUSY dynamics

Disclaimer: Supersymmetry is a huge subject! This talk is an incomplete survey of a few exciting directions, with even less complete referencing!

The Classification Problem

Goal: Classify superconformal field theories

Why is it interesting? :

- SCFTs are highly symmetric universality classes. Understanding them is a starting point to classifying the possible dynamics of QFTs

- Supersymmetric QFTs exhibit extremely rich mathematical structure. New examples might lead to new mathematics

- Holographically, such a classification teaches us about AdS vacua, and hence about non-perturbative constraints of quantum gravity

The Status of Maximal SUSY

d=6: N_Q= 16 theories admit a conjectural ADE classification [Witten]. The A series is constructed by M5-branes.

Evidence for classification via superconformal bootstrap. Minimal c theory realized by two M5s [Beem-Lemos-Rastelli-van Rees]

d=4: N_Q= 16 conjectural classification in terms of gauge theories, (i.e. N=4 SYM) Can it be made into a theorem?

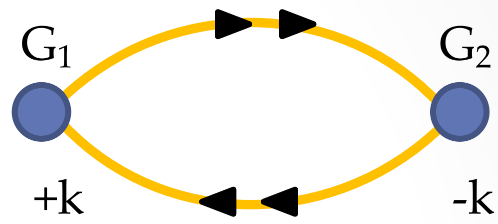
Every d=4, N_Q= 16 theory has an exactly marginal complex coupling τ with fixed Zamolodchikov metric [Papadodimas]: $ds^2 \sim \frac{|d\tau|^2}{Im(\tau)^2}$

Infinite distance limit should have higher-spin conserved currents. More generally conjectured any infinite distance limit in conformal moduli space [Perlmutter-Rastelli-Vafa-Valenzuela]

Maximal SUSY Zoo in 3d

Bagger-Lambert-Gustavsson:

$$BLG_k = \frac{SU(2)_k \times SU(2)_{-k}}{Z_2}$$



Maximal susy for any k. Small k describes the worldvolume theory of two M2 branes. But for general k there is no known string theory embedding!

Aharony-Bergman-Jafferis-Maldecena:

$ABJM_{N,1} = U(N)_1 \times U(N)_{-1}$ $\quad ABJM_{N,2} = U(N)_2 \times U(N)_{-2}$ $\quad ABJ_{N+1,N,2} = U(N+1)_2 \times U(N)_{-2}$

Low energy limit of SYM with gauge group U(N), SO(2N), and SO(2N+1)

List both incomplete (exceptional groups!) and redundant (dualities!)

[Bashkirov-Kapustin, Agmon-Chester-Pufu]: $BLG_3 + \text{Free Theory} = ABJM_{3,1}$

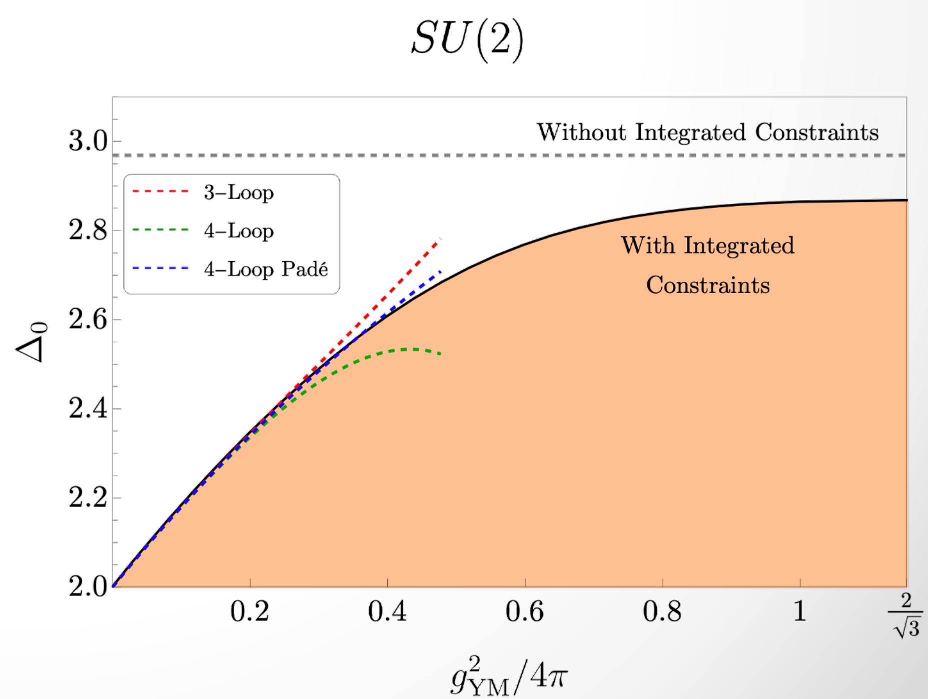
Recent Results in 4d N=4

New results from integrated correlators. Extracted by localization from sphere partition function Binder-Chester-Pufu-Wang, Dorigoni-Green-Wen

$$\int d^4x_i \,\langle L(x_1)\, L(x_2)\, M(x_3)\, M(x_4)\rangle \sim (\tau_2)^2 \;\partial_{\bar{\tau}}\partial_\tau\partial_m\partial_m Z_{S^4}\Big|_{m=0} = \sum_{p,q\in\mathbb{Z}} \int_0^\infty dt\; e^{-\frac{\pi t|p+q\tau|}{\tau_2}}\; B(t)$$

L, M are the Lagrangian and mass deformations, $B(t)$ rational function

These four-point functions can be used for direct bootstrap analysis to give upper bounds on the dimension of the lightest non-BPS scalar (Konishi at weak coupling) Chester-Dempsey-Pufu



Recent Results in 4d N=4

S-duality continues to be a source of new results. Collier-Perlmutter

Assuming $SL(2,\mathbb{Z})$ invariance, correlators constrained by harmonic analysis.

The coupling dependence is fixed by its average value and inner product with Eisenstein series $E_s(\tau)$ and Maass forms $\phi_n(\tau)$

$$\Theta(\tau) = \overline{\Theta} + \int_{Re(s)=1/2} ds \ (\Theta, E_s)\, E_s(\tau) + \sum_{n=1}^{\infty} (\Theta, \phi_n)\phi_n(\tau)$$

Deep idea quantitatively linking perturbative and non-perturbative physics

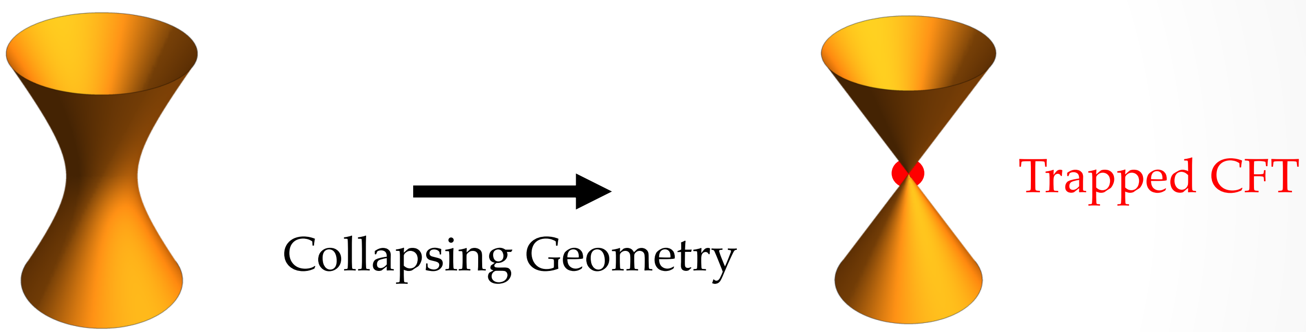
Suggests that ensemble averages over couplings at large N are equivalent to infinite 't Hooft coupling limit. Both holographically dual to IIB supergravity on $AdS_5 \times S^5$.

Example of supersymmetric QFT providing fundamental conceptual insight

The d>4 Universe

Theories in dimension larger than four are mysterious. Various approaches

Geometric: string theory solutions can support an SCFT.



6d: elliptic Calabi-Yau singularities Heckman-Morisson-Rudelius-Vafa

5d: Calabi-Yau singularities via collapsed surface Intriligator-Morrison-Seiberg

Open problems: classify such singularities. Relate 6d and 5d construction
Heckman, Katz, Kim, Vafa, Bhardwaj, Jefferson, Shafer-Nameki, Closset, Wang

Holographic: All AdS_7/AdS_6 solutions of type II string theory are known
Appruzi-Fazzi-Passias-Rosa-Tomasiello

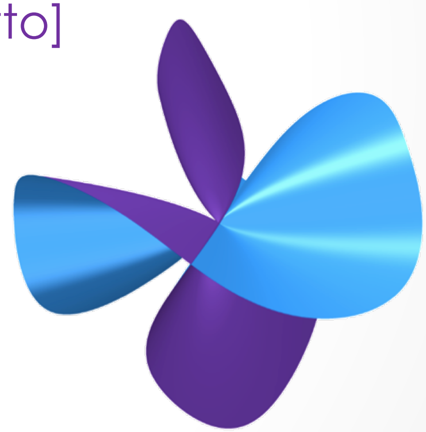
Geometry ↔ QFT

Throughout the zoo of supersymmetric field theory, there is a rich and enduring interplay with geometry and topology

Constructing field theories via string theory or compactification leads to geometric interpretation of supersymmetric correlators, particles, etc.

For 4d theories modernized by the class S construction involving the reduction of M5 branes on Reimann surfaces [Gaiotto]

Recent highlight is the proposed construction of Ricci flat metrics on 4-manifold K3 using BPS states and wall crossing formulas [Kachru-Tripathy-Zimmet]



For 3d theories an active area relating topology of knots and 3-manifolds with ideas of higher symmetry and vertex operator algebras [Gukov-Hsin-Pei, Gukov, Cheng-Chun-Feigin-Ferrari-Gukov-Harrison-Passaro]

Moduli Space Methods

Moduli spaces of vacua provide a powerful complementary technique

In 6d, examining the tensor branch, characterized by dynamical strings leads to universal results on the conformal anomalies and RG flows

$$T^{\mu}_{\mu} = a\, Euler(R) + \sum_{i=1,2,3} c_i\, W^2$$

The a and c_i give exact results about 2, 3, 4 pt correlation functions. Characterized by a universal formula, anomaly multiplet, relating to more computable standard ‘t Hooft anomalies Cordova-Dumitrescu-Intriligator

Can prove that expectations of the renormalization group hold for a large class of supersymmetric flows

$$a \geq 0\,, \qquad\qquad a_{UV} > a_{IR}$$

Interplay between developments in higher symmetry and supersymmetry

Moduli Space in 4d N=2

Rich moduli space structure occurs in 4d with half maximal supersymmetry

Coulomb Branch: Directly constrain the [Seiberg-Witten] geometry. Works best for low-rank theories. Systematized [Argyres, Lottito, Long, Lu, Martone, Whittig]

Higgs Branch: There is a tight interplay between the geometry of the Higgs branch and the protected set of local operators encoded in the chiral algebra of [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

The chiral algebra is a VOA constructed from (twisted) correlators of local operators restricted to a 2d plane. The Higgs branch is an associated variety constructed by truncating to commutative algebra [Beem-Rastelli]

Interplay between two approaches: match between VOAs and rank 1 and 2 moduli spaces [Kaidi-Martone-Rastelli-Weaver]

Both ideas connect moduli space geometry with CFT correlation functions. Desirable to develop for other supersymmetric theories.

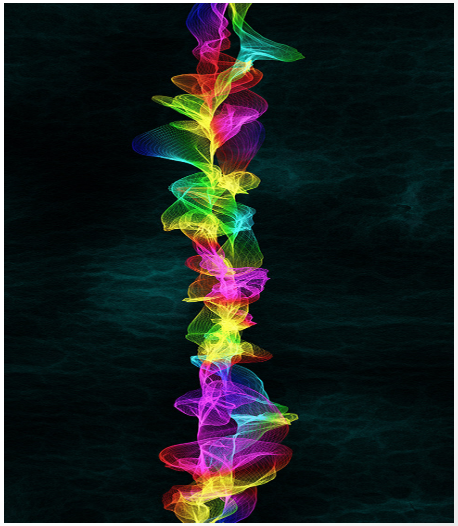
Supersymmetric Defects

While many aspects of QFT depend only on the local operators and their correlation functions there is an increasing appreciation of the importance of extended operators i.e. defects (line, surface, domain wall/boundaries)

For example extended operators are crucial for understanding generalized concepts of symmetry in quantum field theory

SUSY QFTs provide a controllable arena to learn about these objects

Partial classification of SUSY defects partition functions via localization. Analogs of a-theorems and anomalies for defect RG flows [Wang, Agmon-Wang]



Supersymmetric examples of defects with exotic fusion algebras from S-duality and higher form symmetry [Ohmori-Kaidi-Zheng]

Connections Between SUSY and Non-SUSY Dynamics

Desirable to directly link the deep understanding of supersymmetric dynamics and dualities to non-supersymmetric QFT

Done successfully in 3d QFT:

- **3d N=4 mirror symmetry:** U(1) gauge theory with 2 hypers of charge 1 is self-dual and flows to a theory with enhanced SU(2)xSU(2) global symmetry [Intriligator-Seiberg]

- **Non Susy Analog:** U(1) gauge theory with 2 electrons of charge 1 is self-dual and flows to a theory with SU(2)xSU(2) global symmetry [Xu-You, Hsin-Seiberg]

Direct link between large N Chern-Simons matter dualities in [Jain-Minwalla-Yokoyama, Gur-Ari-Yacoby] and explored at small N in [Kachru-Mulligan-Torroba-Wang] using soft supersymmetry breaking

Soft SUSY Breaking

Add a SUSY breaking mass term to a SUSY Lagrangian to remove fields

$$L = L_{susy} + M^2\ |\phi|^2$$

Ideally, the mass operator should be positive definite and part of a short multiplet (so we can track it) and preserve all symmetries

This problem of 4d soft SUSY breaking was explored in the heyday of duality

Challenge:

- The regime of parametric control is small M (using supersymmetry)
- The regime of most interest is large M (the non-supersymmetric theory)

Use to gain qualitative insight about IR dynamics of non-susy theories

4d Adjoint QCD

Example 4d adjoint QCD with two Weyl flavors of adjoint quarks λ^i.
The index i is a doublet under a chiral SU(2) global symmetry

Lore: confinement and chiral symmetry breaking via a chiral condensate

$$< Tr(\lambda^i \lambda^j) > \neq 0$$

This leads to a non-linear sigma model of Nambu-Goldstone bosons on S^2

Candidate phases constructed by embedding into pure N=2 SYM.
Deforming the IR [Seiberg-Witten] solution at monopole points leads to condensation of BPS monopoles.

Direct link to a confining phase of adjoint QCD by dual Higgs mechanism

[Cordova-Dumitrescu, D'Hoker-Dumitrescu-Gerchkovitz-Nardoni]

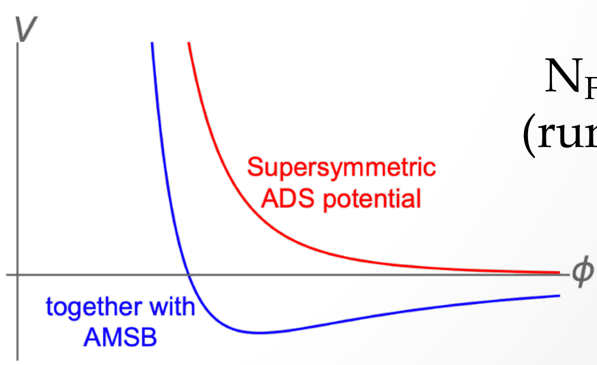
The Chiral Lagrangian from Seiberg Duality

A target for these ideas is to understand the dynamics of QCD using our detailed understanding of N=1 SUSY QCD from Seiberg duality.

Ask for range of N_F and N_C where one has chiral symmetry breaking or a CFT
New ideas from Murayama, Csaki, Telem, Gomes, Noether, Varier

Deform theory by an operator giving bare mass to gauginos and squarks. This can be tracked to IR using the fact that the operator in question appears in the supercurrent multiplet ("anomaly mediation")

In IR this deformation, together with moduli space potential, generates a minimum with chiral symmetry breaking if $N_F \leq \frac{3}{2} N_C$
Murayama



$N_F < N_c$
(runaway)

Exciting to understand the reach of these developments

Conclusions

- The universe of supersymmetric field theory is vast and growing. Each theory is a potential playground for mathematical physics
- New exact results in supersymmetry are teaching us about the guts of quantum field theory and quantum gravity
- There is more to explore in the relationship between supersymmetric and non-supersymmetric theories

Thanks for Listening!