Geometry and Entanglement

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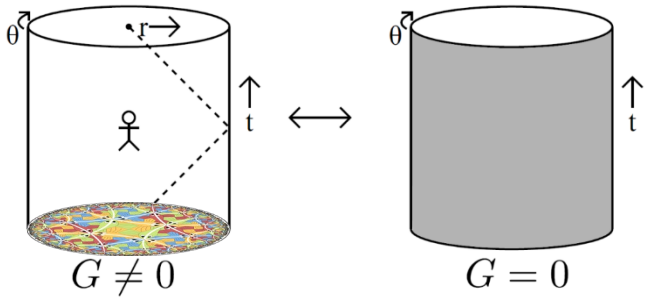
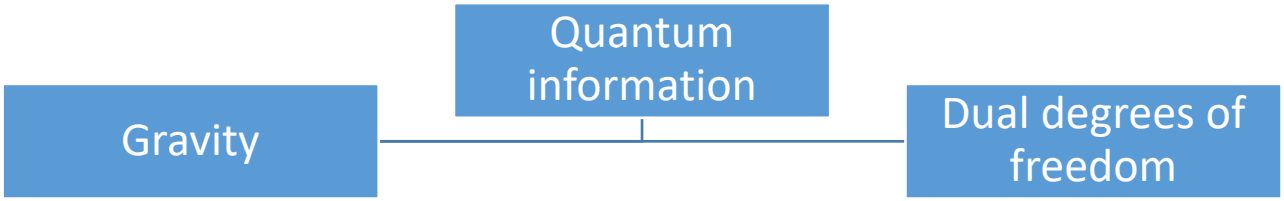
- The last 25 years have seen the discovery of non-perturbatively precise theories of quantum gravity: AdS/CFT and matrix models .
- In these theories, the gravitational spacetime emerges holographically from the collective behavior of dual, non-gravitational degrees of freedom.
- This emergence is sharpest in AdS/CFT:



- But even there, the basic mechanism has only been clarified recently.
- Insights from a quantum information perspective have been central to these recent developments.



- These connections grew out of a better understanding of the fine-grained gravitational entropy:

$$S = -\mathrm{Tr}\,\rho \log \rho$$

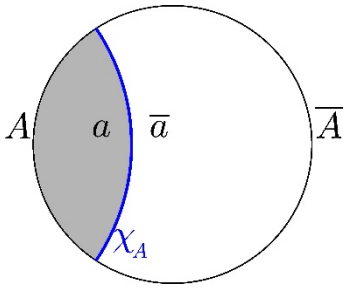
- Also known as the von Neumann entropy or entanglement entropy.
- ρ: density matrix of a subsystem.
- I will focus on AdS/CFT, where the subsystems are chosen to be subregions.

Gravitational entropy formula

At leading order, the entropy of a boundary subregion A is given by the area of a bulk extremal surface χ_A:

[Ryu & Takayanagi '06]

$$S = \text{ext}\frac{\text{Area}}{4G}$$



- Similar to the entropy of a black hole.
- Works in static as well as dynamical spacetimes.

[Hubeny, Rangamani & Takayanagi '07]

- Generalized to higher-derivative gravity and Renyi entropy.

[XD '13; Camps '13; Miao & Guo '15; XD '16; XD & Marolf '20; …]

- Quantum corrections now understood.

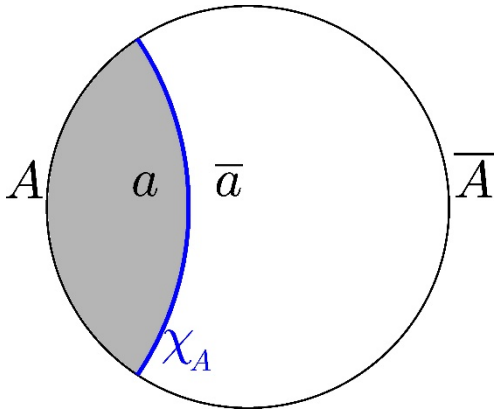
[Faulkner, Lewkowycz & Maldacena '13; Engelhardt & Wall '14; XD & Lewkowycz '18; …]

- Has been derived by semiclassical gravitational path integrals.

[Lewkowycz & Maldacena '13; XD, Lewkowycz & Rangamani '16; …]

Entanglement wedge

- A bulk region bounded by the extremal surface.
- On a time slice, it is between the boundary subregion A and the extremal surface χ_A:

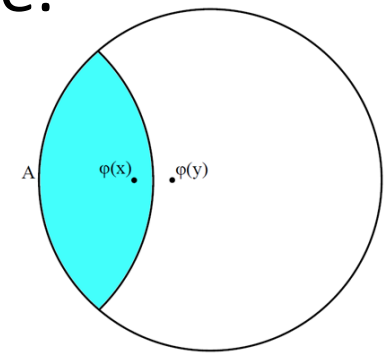


- A key notion in describing the central concept of subregion-subregion duality.

[Bousso, Leichenauer & Rosenhaus; Czech, Karczmarek, Nogueira & van Raamsdonk; Bousso, Freivogel, Leichenauer, Rosenhaus & Zukowski; Wall; Headrick, Hubeny, Lawrence & Rangamani; XD, Harlow & Wall; …]

Subregion-subregion duality

"The quantum information in a boundary subregion is exactly the information needed to describe its entanglement wedge."



- In particular, bulk operators in the entanglement wedge can be reconstructed as boundary operators on that subregion.
- This "entanglement wedge reconstruction" refines how spacetime emerges from the boundary.

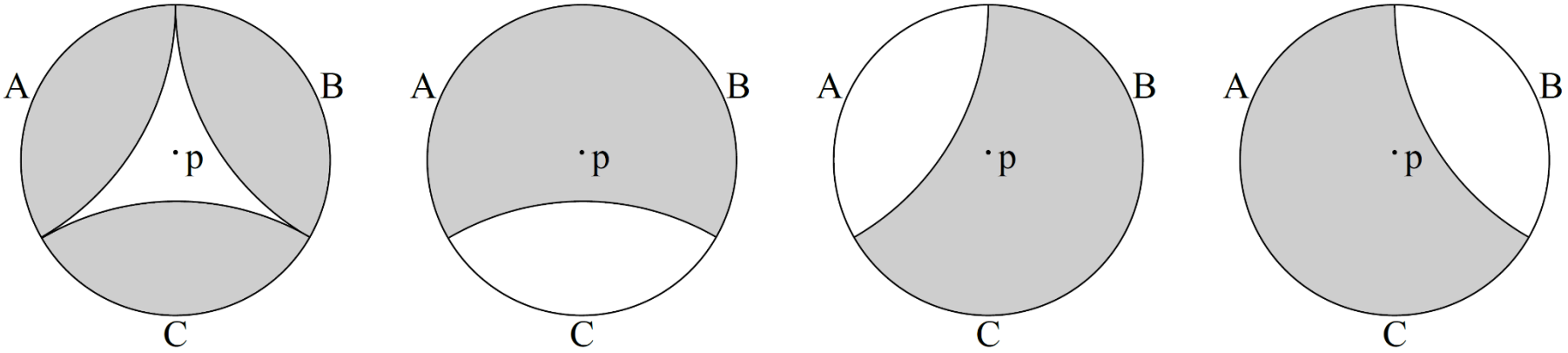
[Bousso, Leichenauer & Rosenhaus; Czech, Karczmarek, Nogueira & van Raamsdonk; Bousso, Freivogel, Leichenauer, Rosenhaus & Zukowski; Wall; Headrick, Hubeny, Lawrence & Rangamani; XD, Harlow & Wall; …]

Holographic code

- A striking aspect of subregion-subregion duality:

  It functions as a quantum error-correcting code!

- Information about the bulk is stored redundantly on the boundary.



- Another example of the importance of the quantum information perspective.

[Almheiri, XD, Harlow '14]

- So why does subregion-subregion duality work?

- The answer lies in the quantum corrections to the gravitational entropy formula:

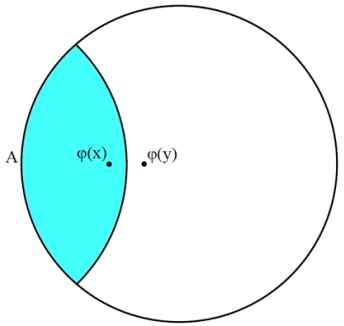
$$S = \text{ext}\,\frac{\text{Area}}{4G_N}$$

Quantum corrections

These corrections come from bulk matter fields and gravitons.

- The answer is surprisingly simple:

$$S = \text{ext}\frac{\text{Area}}{4G_N} \quad \Rightarrow \quad S = \text{ext}\left(\frac{\text{Area}}{4G_N} + S_{\text{bulk}}\right)$$



[Engelhardt & Wall '14]

- Replaces the extremal surface with a quantum extremal surface (QES).
- Called the QES formula.
- Matches one-loop FLM result. [Faulkner, Lewkowycz & Maldacena '13]
- Has been derived by semiclassical gravitational path integrals.

[XD & Lewkowycz '18; Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; …]

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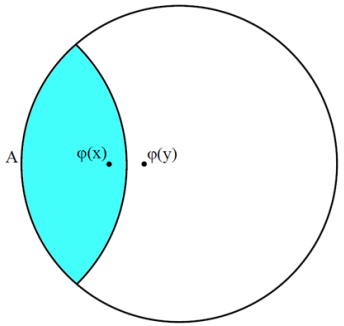


[Engelhardt & Wall '14]

- S_{bulk} defined in the entanglement wedge.
- A change to the state in the entanglement wedge would show up on the boundary subregion, at least entropy-wise.
- This strongly suggests subregion-subregion duality holds. Can be promoted to a proof.

[Jafferis, Lewkowycz, Maldacena & Suh; XD, Harlow & Wall]

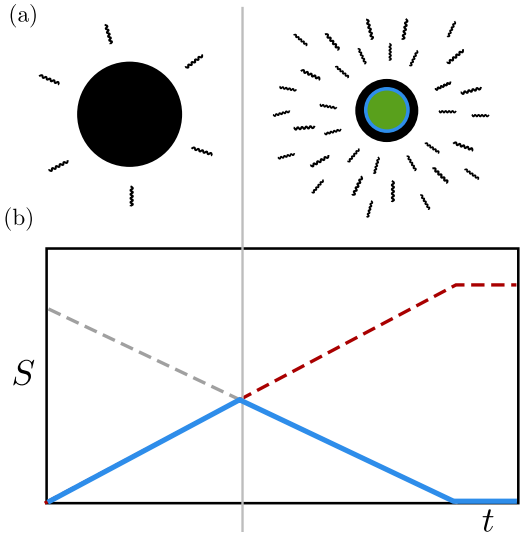
$$S = \text{ext}\left(\frac{\text{Area}}{4G_N} + S_{\text{bulk}}\right)$$

- Often, quantum correction S_{bulk} has a small effect compared to the area term.

- But sometimes, it has a dramatic effect.
- An important example: old black holes.

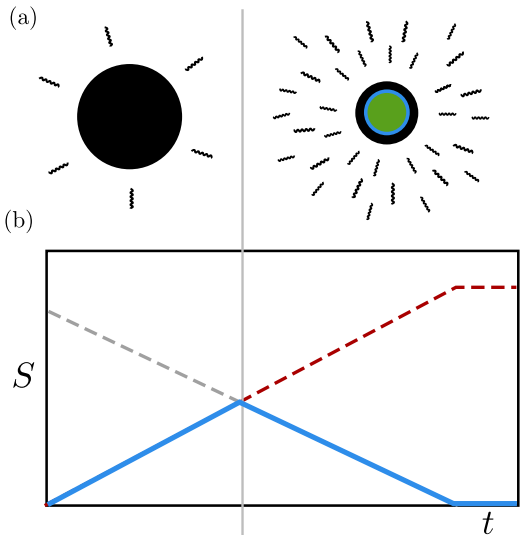
Entropy of Hawking radiation

- Hawking's calculation ➡ S grows monotonically.
- But unitary evaporation ➡ the Page curve:



- Remarkably, the QES formula agrees with unitarity!
- A new QES in the black hole interior becomes dominant at late stages of evaporation.

[Penington; Almheiri, Engelhardt, Marolf & Maxfield; Almheiri, Mahajan, Maldacena & Zhao; Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; …]

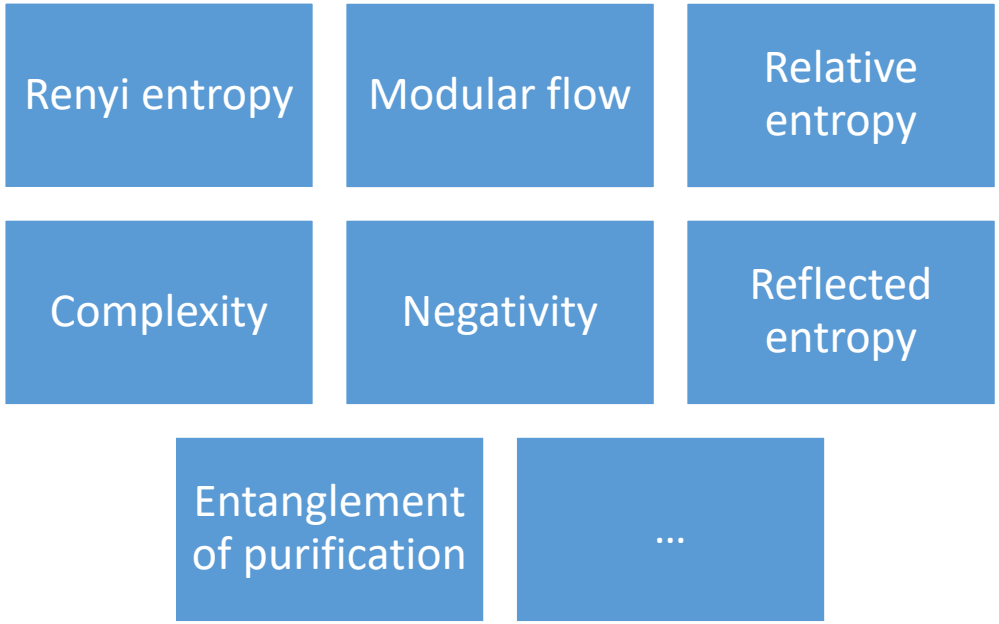


- The entanglement wedge now contains an ‘island’ in the black hole interior.

- According to subregion-subregion duality, the island is actually encoded in the Hawking radiation!

[Penington; Almheiri, Engelhardt, Marolf & Maxfield; Almheiri, Mahajan, Maldacena & Zhao; Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; …]

- I have mostly focused on the fine-grained gravitational entropy, but a lot of progress has also been made with other quantum information-theoretic concepts:



- All of this progress is gratifying but many mysteries remain.

- I will end with a few open questions.

Open questions

- It seems almost a miracle for the semiclassical gravitational path integral to be able to determine the fine-grained entropy of Hawking radiation. What else does it know?

- Relatedly, we might have expected string theory – as a well-established UV completion of gravity – to play a larger role in the interplay between quantum information and spacetime. So can we study gravitational entropy and bulk reconstruction in string theory? To what extend can we understand the gravitational entropy formula as arising from stringy edge modes?

Open questions

- How are basic features of the bulk gravitational theory represented by properties of the holographic code? Does this help clarify long-standing puzzles such as the origin of local physics on sub-AdS scales?

- How do we use all these ideas to further decode physics behind a horizon, or in cosmologies resembling our world?

- Perhaps quantum gravity in the lab will actually allow us to gain experimental insight into some of these issues?

Thank you!