EFTs and Scattering Amplitudes

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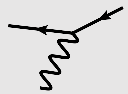
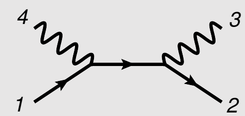
Turning the traditional QFT approach upside-down

Lagrangian → Feynman Rules  → Amplitude 

Unbroken symmetries

Spont. Broken symmetries

Ward identities
(e.g. conservation of some global charge)

Soft theorems
(e.g. Adler zero in pion EFT)

Compute the amplitudes of given class of models efficiently w/o reference to the Lagrangian

Explore space of possible QFT with given spectrum and symmetries

Derive general results about QFTs / EFTs

Reverse!

Modern On-shell Approach
Start with the on-shell amplitudes & impose consistency conditions in order to

Effective Field Theory (EFT)

EFT-ology I:
To given order in the derivative-expansion, include all higher-derivative gauge-invariant local operators permitted by the symmetries.

Lagrangian formulation: How many gauge-invariant local operators are there subject to 1) integration-by-parts *and* 2) the EOM *and* 3) field redefinitions?

On-shell amplitudes methods are VERY efficient for this.

On-shell local operators in 1-1 correspondence on-shell matrix elements

A**mplitudes formulation:** How many independent on-shell matrix elements are there modulo momentum conservation and Bose/Fermi symmetry of identical states?

Examples

1 $\partial^{2k}\phi^4$ Abelian => Bose symmetry => symmetric degree k polynomials in s,t,u indep. under to $s+t+u=0$

Such polynomials are of the form $(stu)^{n_1}(s^2+t^2+u^2)^{n_2}$

So, count of indep. operators is number of ways to write $k = 3n_1 + 2n_2$

Example $\partial^{22}\phi^4$ k=11 n_1 odd -> n_1=1 or 3 => there are **2** such indep. operators.

So: Counting easy. Direct construction of local matrix elements easy. Basis changes easy.

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2 $D^k\bar{\psi}F^2\phi^2\psi$

Spinor-helicity formalism makes this very efficient: $\langle\psi^+\psi^-\gamma^+\gamma^-\phi\phi\rangle$ massless

k =	1,	3,	5,...
dim =	10,	12,	14,...
# operators =	2,	21,	114,...
Comp time =	0.1s,	1.6s,	5min,...

Same principles (and more machinery) for MASSIVE particles. Useful application: SMEFT

Application in SMEFT

Many other applications of these ideas in formal theory, such as for local counterterms for UV divergences in perturbative supergravity, higher-derivative corrections to chiral perturbation theory, Galileons, finite local counterterms in Born-Infeld, monopoles, dark matter…

Elvang, Freedman, Kiermaier; Beisert, Morales; Mitchell; Hadjiantonis, Jones, Paranjape; Bern, Parra-Martinez, Roiban; Csaki, Hong, Shirman, Telem, Terning; Falkowski, Isabella, Machado;…

Example
3- and 4-pt SMEFT operators systematically characterized by

Aoude, Durieux, Kitahara, Machado, Shadmi, Weiss (2018-21)

using the massive spinor helicity formalism of

Arkani-Hamed, Huang, and Huang (2017)

Some comparisons in certain sectors so far with Lagrangian approaches, for example w/ Henning Lu Melia Murayama. Plus in follow-up papers.

Further expanded technique and analysis by Accettulli Huber + De Angelis (2022) and De Angelis (2022).

Anomalous dimension mixing matrix

Under RG, operators mix.

Important for interpretation of experimental results to understand how. Mereghetti 's talk

Under RG, operators can mix. $\Delta\mathcal{L} = \sum_i c_i \mathcal{O}_i$

$$16\pi^2 \frac{\partial c_i}{\partial \log \mu} = \gamma_{ij}^{\rm UV} c_j$$

$\gamma_{ij}^{\rm UV}$ is the anomalous-dimension matrix

Surprising **1-loop non-renormalization results for SMEFT** dim 6 operators. Alonso, Jenkins, Manohar (2014)
(Grojean, Jenkins, Manohar, Trott; Elias-Miro, Espinosa, Masso, Pomarol (2013))

Explained by Cheung and C-H Shen (2015) using on-shell amplitudes methods to characterize the possible local operators at dim 5 and 6.

Using on-shell unitarity methods to get anomalous dimensions and beta functions from Caron-Huot and Wilhelm (2016) , new non-renormalization theorems derived for **dim 5 through 7 SMEFT operators** by Bern, Parra-Martinez, and Sawyer (2019). **2-loop** SMEFT anomalous dim's Bern, Parra-Martinez, and Sawyer (2020). Mixing matrix at Dim 8 in Accettulli Huber + De Angelis (2022).

EFT geometry

Higgs EFT formulated geometrically in terms of curvature on the scalar manifold

Alonso, Jenkins, Manohar (2015)

Picked up recently from an on-shell amplitudes perspective:

Alonso, Jenkins, Manohar (2016)
Cheung, Helset, Parra-Martinez (2021+22)
T. Cohen, N. Craig, X. Lu, Sutherland (2022)

Geometry-kinematics duality

EFT-ology II:
The higher-derivative operators appear with generic coefficients naturally expected to be of order ~1 in units of the scale of the UV physics.

… so if these coefficients are not ~1 (say <<1 or >>1 or even 0) we have some explanation to do.

UV-completable (i.e. non-swampland) models have constraints on the Wilson coefficients.

Exploring those bounds are the subject of the S-matrix bootstrap / EFT-hedron / weak gravity conjecture via amplitudes

Adams, Arkani-Hamed, Dubovski, Nicolis, Rattazzi; Arkani-Hamed, T-C Huang, Y-t Huang; Vafa, Ooguri; Arkani-Hamed, Y-t Huang , J-Y Liu, Cheung, Remmen, Jones, McPeak, Caron-Huot, …

Bottom-up bootstrap of string theory via amplitudes:

Arkani-Hamed, Y-t Huang, Vieira, Penedones, Guerrieri, Komargodski, Sever, Zhiboedov, Alonso, Rodina, Eberhardt, Mizera, Liu, Wang, Van Duong, Mazáč, Rastelli, Simmons-Duffin, Bellazzini, Miro, Rattazzi, Riembau, Riva, Tolley, Wang, S-Y Zhou, Parra-Martinez,…

Related: Snowmass white paper on bootstrapping string theory by Gopakumar, Perlmutter, Pufu, Yin

EFTs … and so much more

Soft theorems in EFTs and bootstrapping exceptional EFTs

Cheung, Trnka, Elvang, Jones, Naculich, Hadjiantonis, Paranjape, Helset, Parra-Martinez, Z Yin,C-H Shen. I. Low, Kampf, Novotný, …

Celestial amplitudes & EFTs

Arkani-Hamed, Pate, Raclariu, Strominger

Double-copy in EFTs

BCJ-based	Carrasco, Rodina, Zekioglu
KLT bootstrap	HH Chi, Elvang, Herderschee, Jones, Paranjape
Connecting (4pt)	Durieux, Grojean, Bonnefoy, Machado, Roosmale Nepveu

Gravitional physics, LIGO

Solon’s talk

On-shell amplitudes methods in EFTs

Very active and growing field of research, attracting a lot of young researchers

Impact both on the front of

Advancing our understanding of Quantum Field Theory on the formal side

And direct applications to particle physics in SMEFT, Higgs EFT, …

Those are the pillars of our field: the interplay between

the pursuit of the mathematical truth and beauty

&

experimental + pheno particle physics and description of Nature