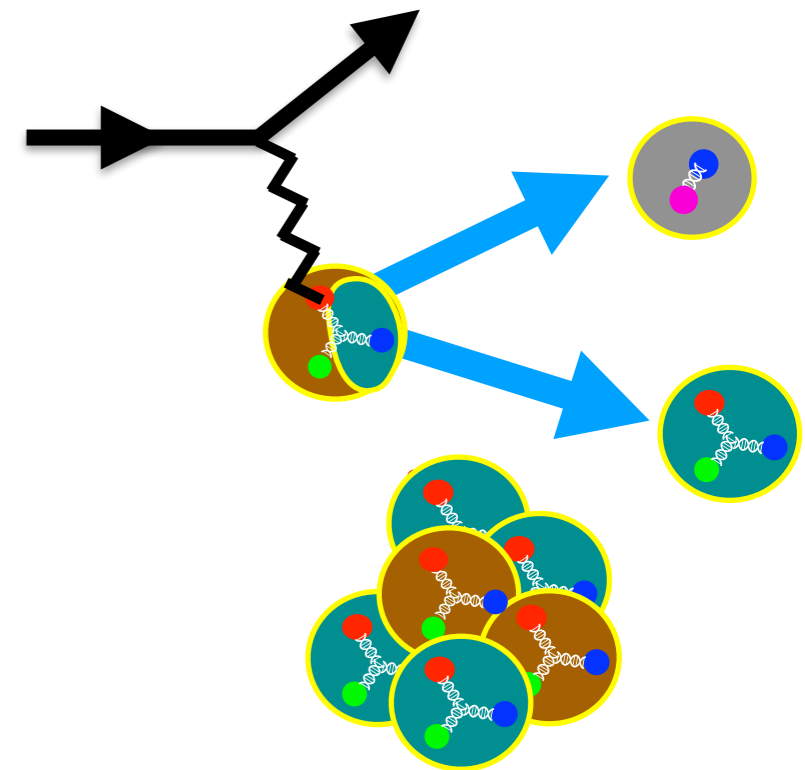
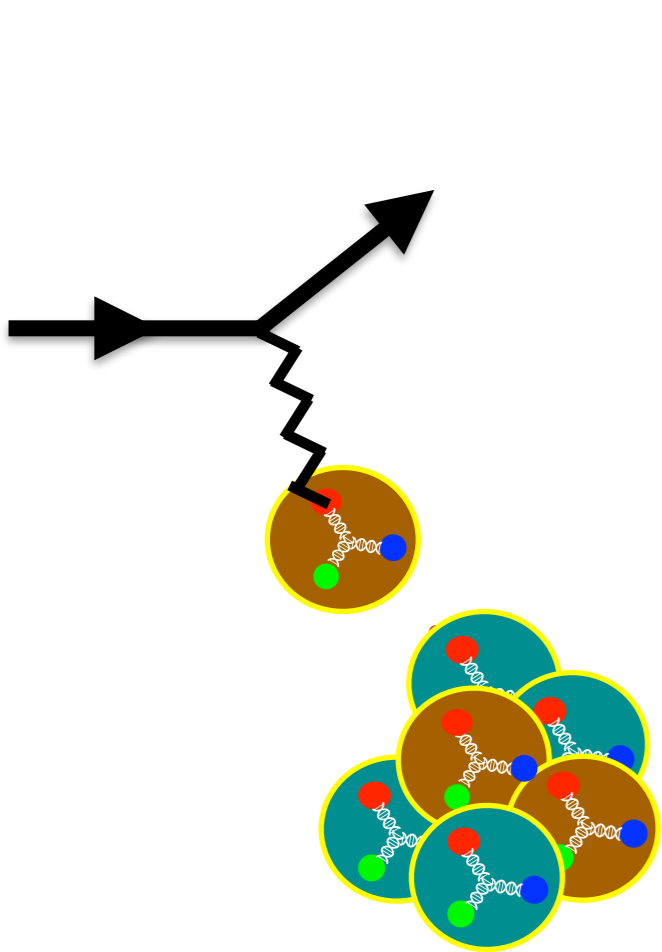


Lattice QCD and Neutrino Interactions

Michael Wagman



Snowmass Theory Frontier Conference

Kavli Institute for Theoretical Physics

February 24, 2022

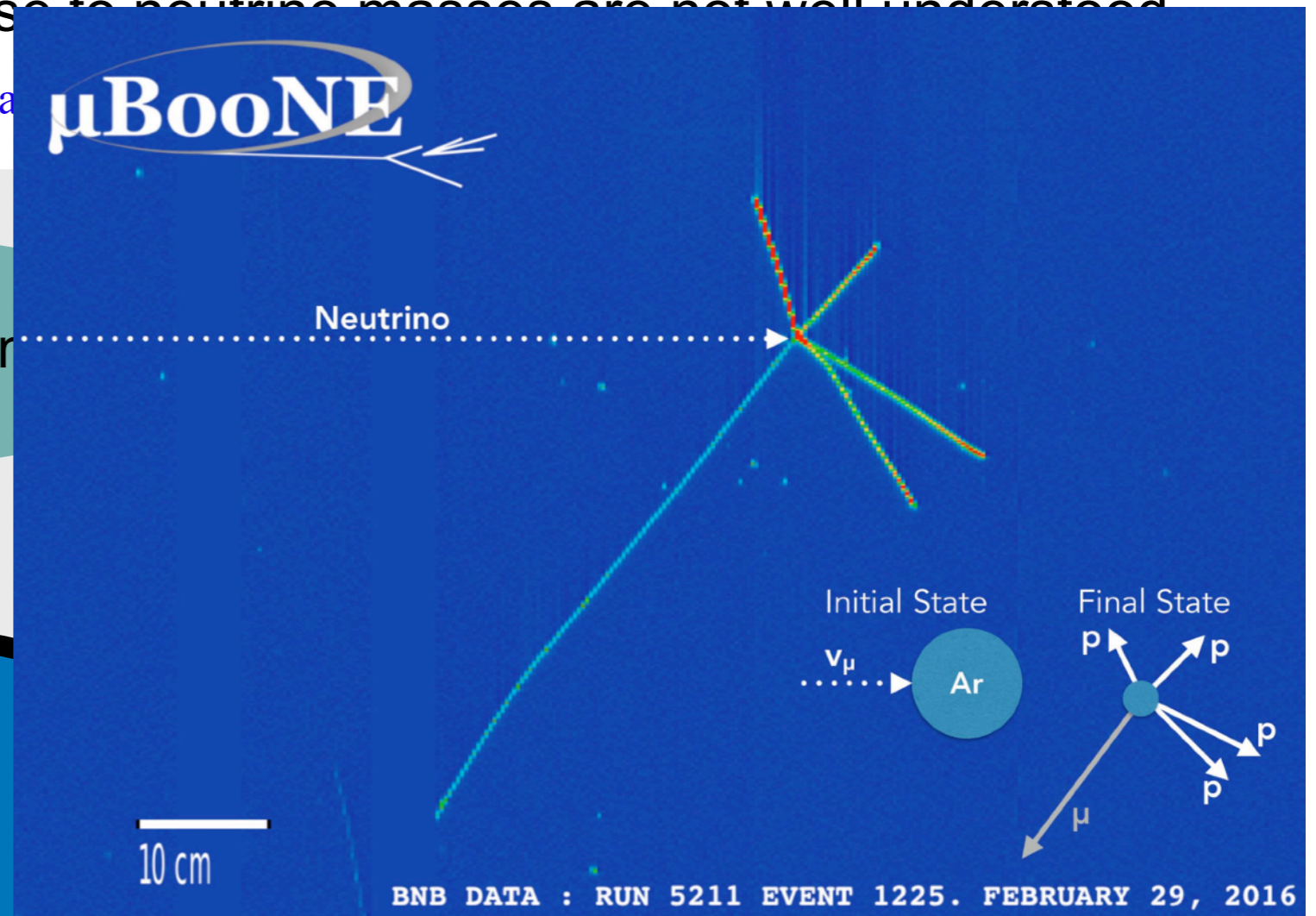


Fermilab

Neutrinos, quarks, and new physics

Observations of neutrino oscillations demonstrate that neutrinos have mass, but the fundamental interactions giving rise to neutrino masses are not well understood.

See talks by Pedro Machado later today; Alex...

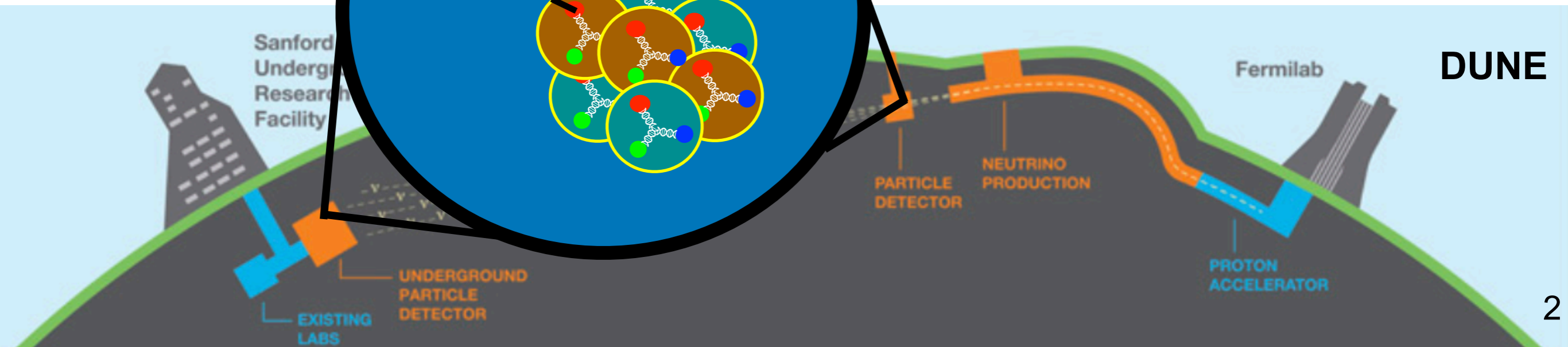


CP violation

Sterile neutrinos?

Next-generation a
parameters go

precedented accuracy



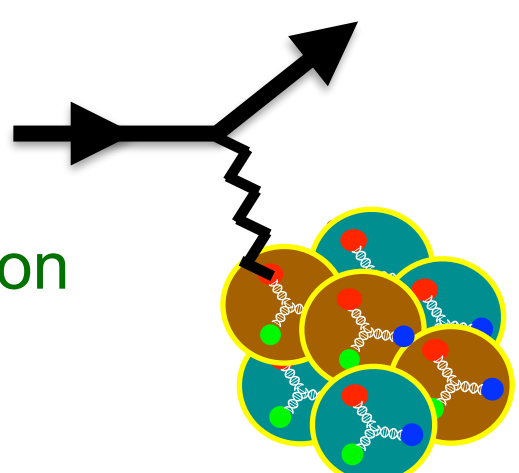
Precision neutrino physics

Comparisons of near + far detector neutrino fluxes can be used to precisely determine oscillation parameters, discover neutrino CP violation, ...

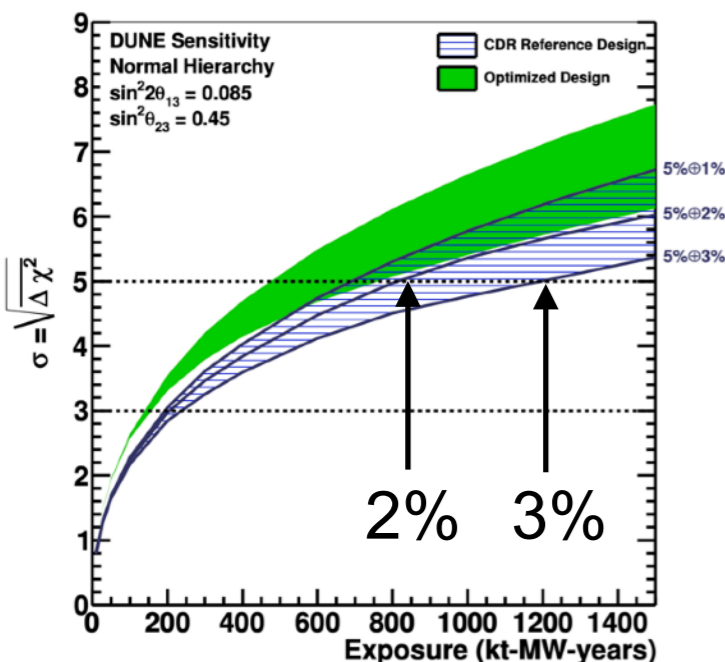
Relating measured final-state event rates to incoming neutrino flux requires precise knowledge of νA cross-section

$$\frac{N_{\text{near}}}{N_{\text{far}}} = \frac{\int dE_\nu \Phi_{\text{near}}(E_\nu) \sigma(E_\nu)}{\int dE_\nu \Phi_{\text{far}}(E_\nu) \sigma(E_\nu)}$$

Near-detector neutrino flux Cross-section
Experimentally measured event rates Far-detector flux (depends on oscillation parameters)



50% CP Violation Sensitivity

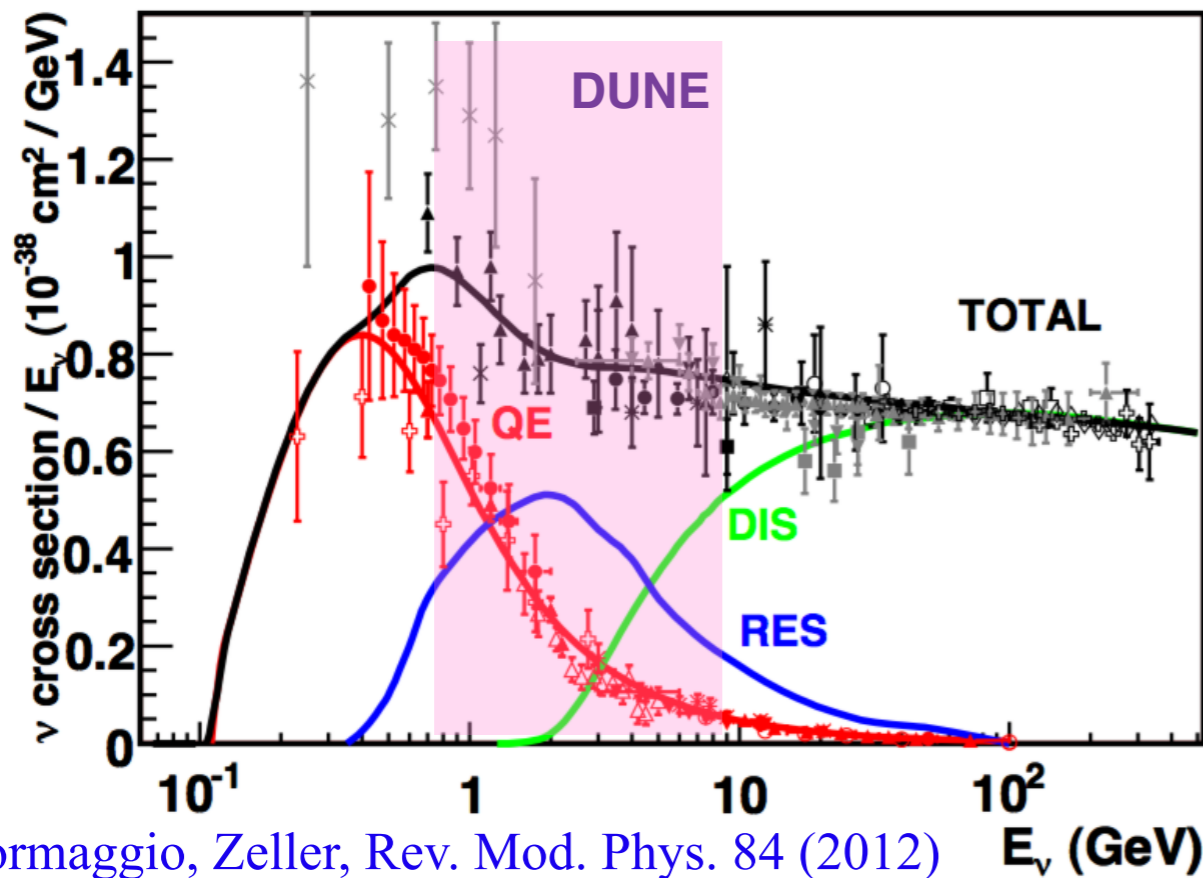


DUNE aims to have few-percent cross-section uncertainty

- 2% vs 3% cross-section uncertainty estimated to change exposure required to discover CP violation by 50%

[Acciarri et al \(DUNE\) arXiv 1512.06148](#)

Neutrino-nucleus scattering



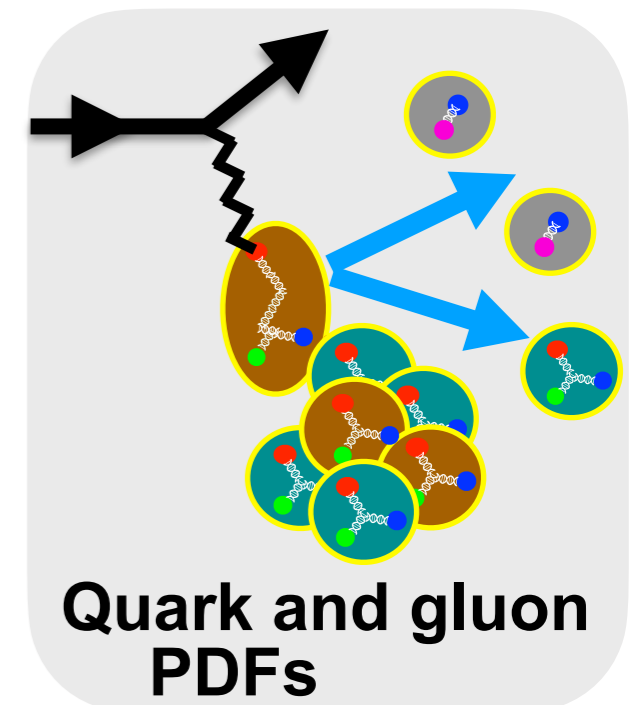
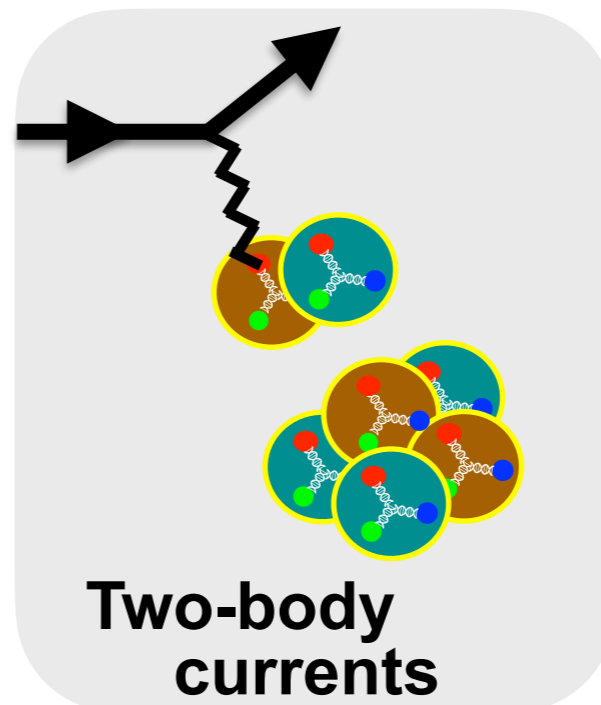
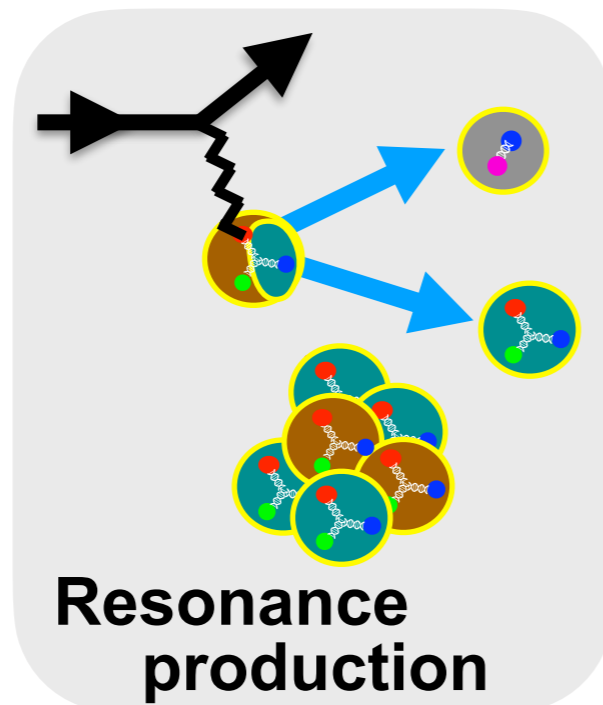
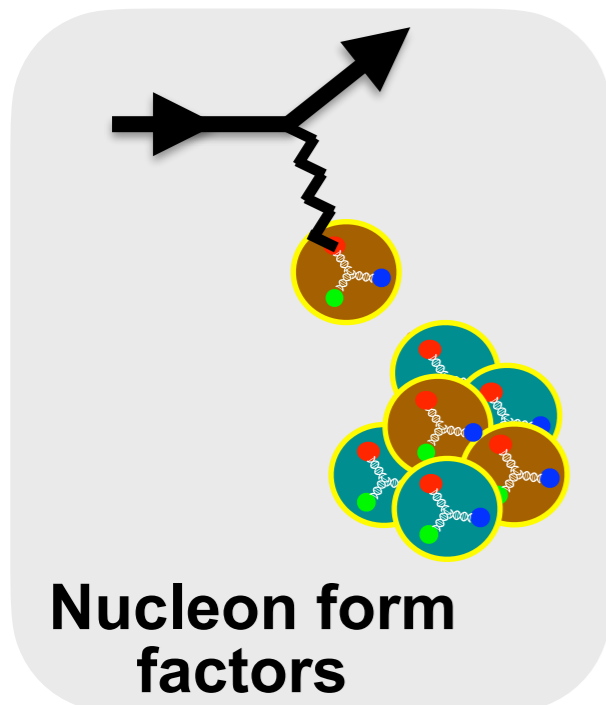
Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

Accelerator neutrino fluxes cover a wide range of energies where different processes dominate cross-section:

- Quasi-elastic nucleon scattering
- Resonance production
- Deep inelastic scattering

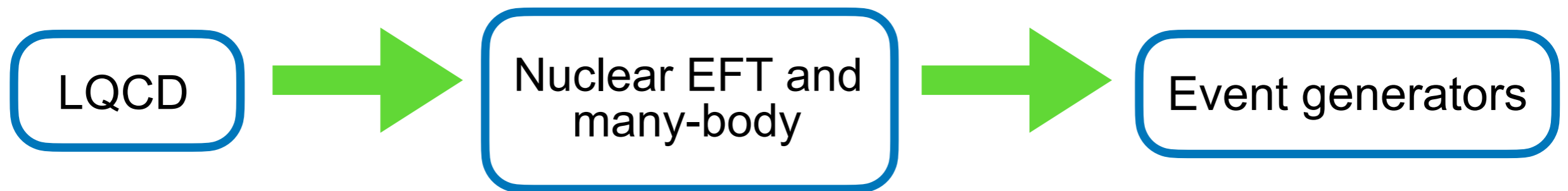
Theory input required to decompose cross section into such processes and therefore predict its energy dependence

Effective theories for different energies require different inputs



Lattice QCD, EFT, and νA

LQCD can provide results for few-nucleon observables that can be matched to nuclear EFTs and models that can make predictions for larger nuclei



Results provided by LQCD and experiment are complementary

Easy for LQCD:

- Axial vs vector currents
- Isovector vs isoscalar
- Pions

Hard for LQCD:

- Large baryon number
- Real-time dynamics
- Multi-hadron states
- (Light quark masses)

Lattice QCD and νA

νA scattering amplitudes factorize into leptonic and hadronic parts

$$\mathcal{M}_{\nu A \rightarrow \ell f} \propto (\bar{u}_\ell \gamma_\mu \gamma_5 u_\nu) \langle f | \bar{q} \gamma_\mu \gamma_5 q | A \rangle + \dots$$

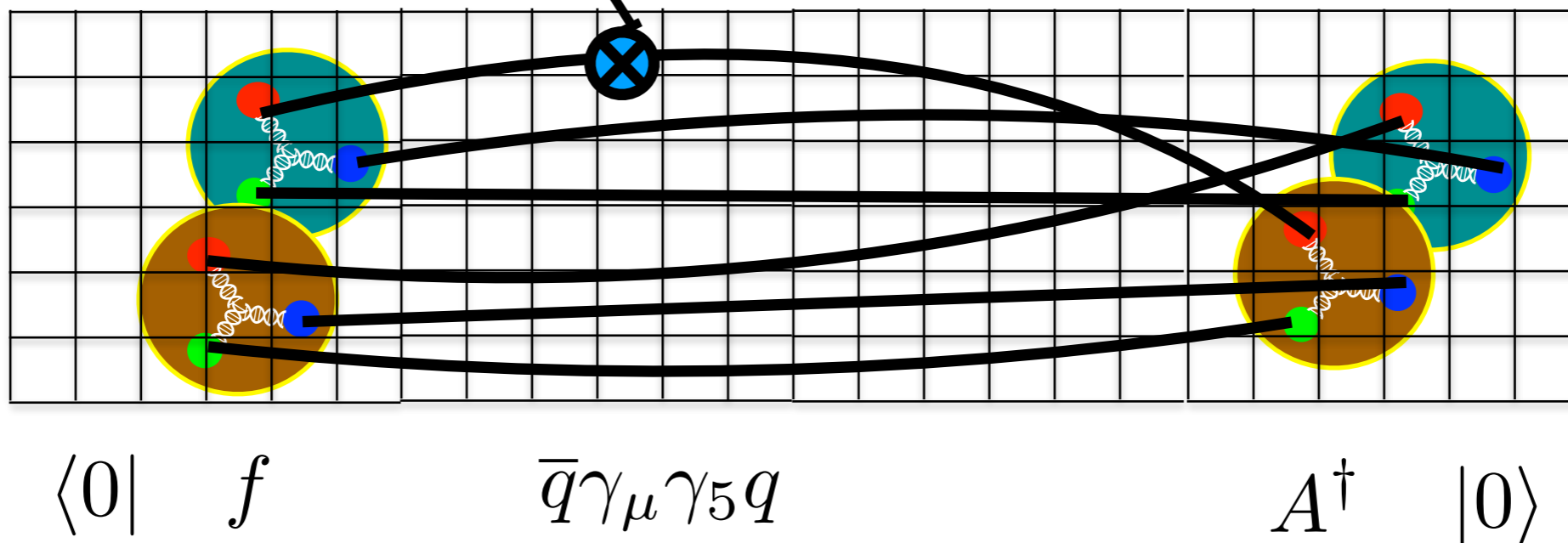
Generic Euclidean hadronic matrix elements calculable (in principle) using lattice QCD

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\bar{q} \mathcal{D}q e^{-S_{QCD}(U, q, \bar{q})} \mathcal{O}(U, q, \bar{q}) \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}(U_i)$$

Quark fields integrated out
analytically, propagators
obtained with matrix inversion

(Dirac matrix size $\sim 10^9 \times 10^9$)

Monte Carlo sample
gluon fields with
probability $\propto e^{-S}$

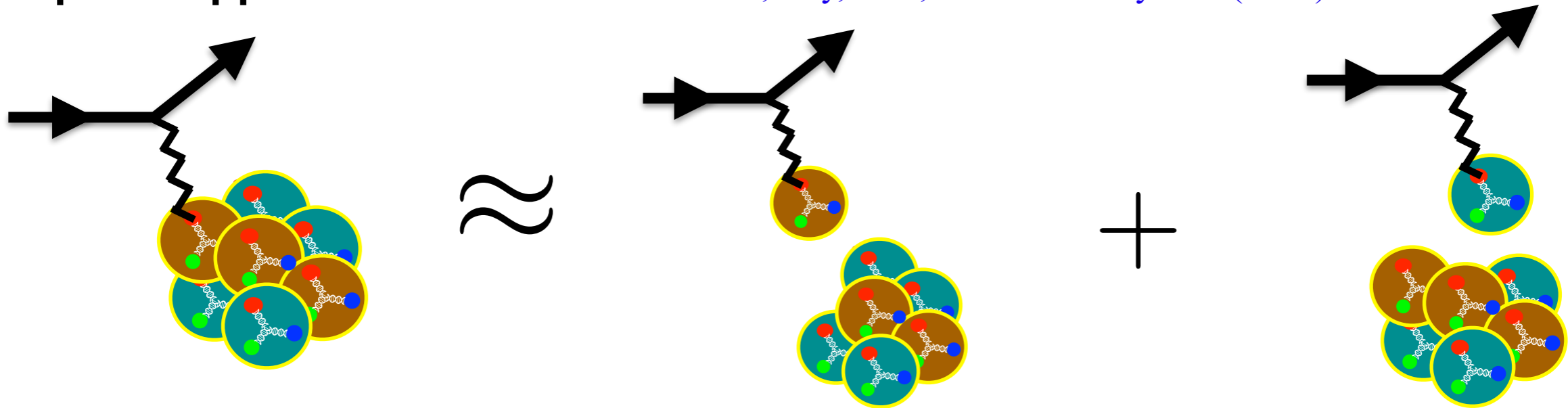


Nucleon form factors

Single-nucleon currents dominate at low energies where nucleons are effective degrees of freedom for nuclei, multi-nucleon currents provide corrections

Impulse approximation

Benhar, Day, Sick, Rev. Mod. Phys. 80 (2008)



Nucleon vector and axial form factors are key inputs to nuclear EFTs / models

$$\langle N(p') | A_\mu(q) | N(p) \rangle = \bar{u}(p') \left[G_A(Q^2) \gamma_\mu \gamma_5 + \frac{\tilde{G}_P(Q^2)}{2M_N} q_\mu \gamma_5 \right] u(p)$$

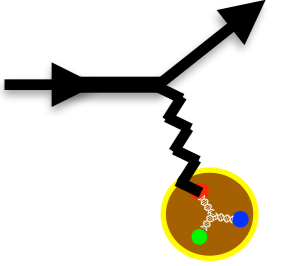
Electron scattering experiments can precisely measure vector form factors, which provides important validation

See e.g. Khachatryan et al. [CLAS and e4v] Nature 599 (2021)

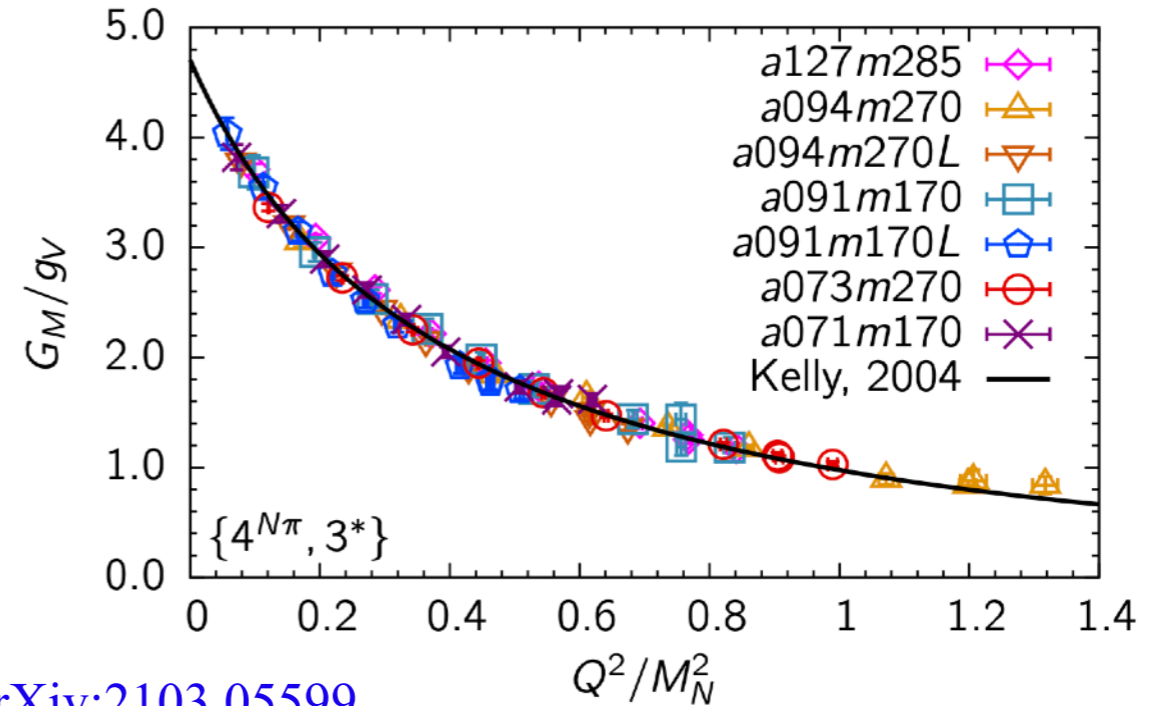
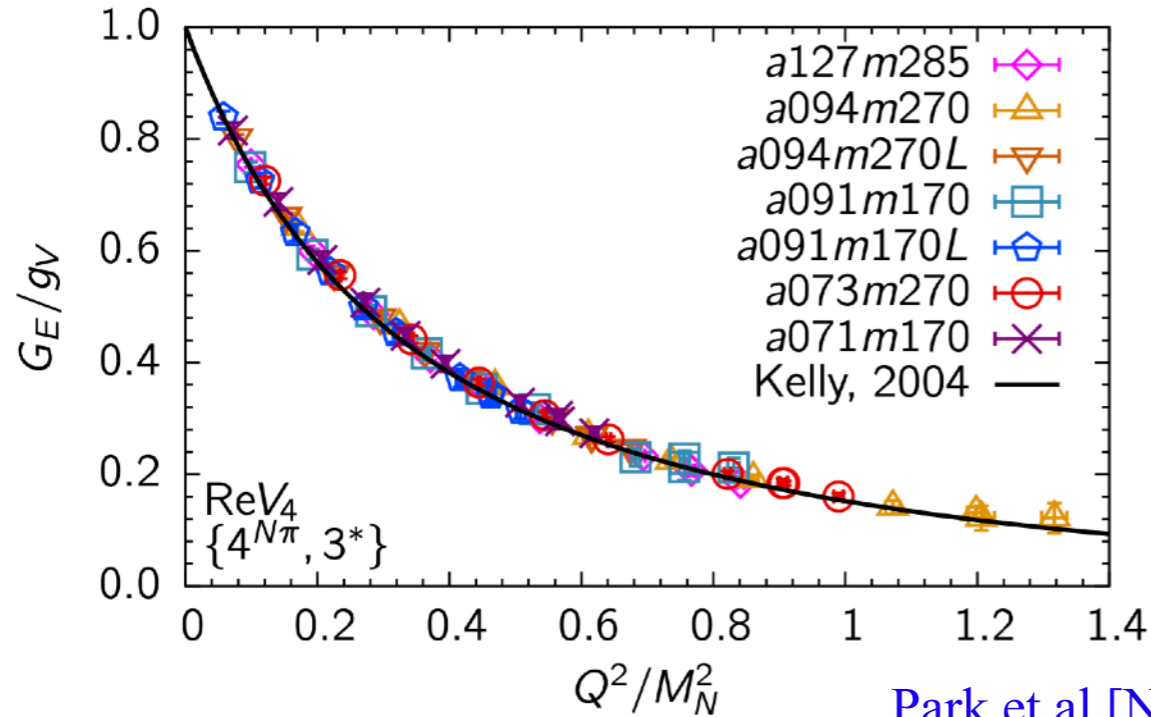
Corrections to impulse approximation can be included systematically

Review: Rocco, Front. Phys. 29 (2020)

Form factors and LQCD



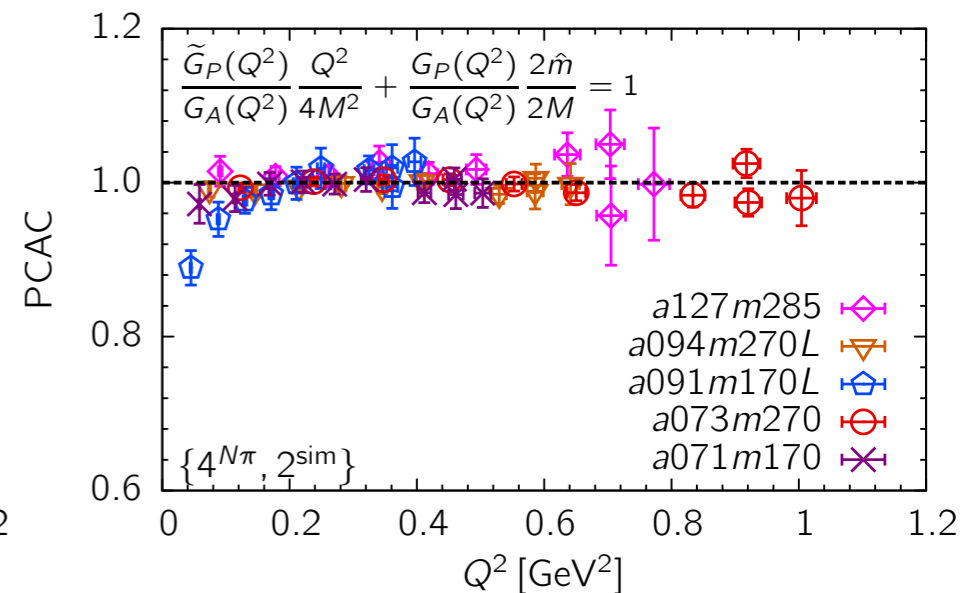
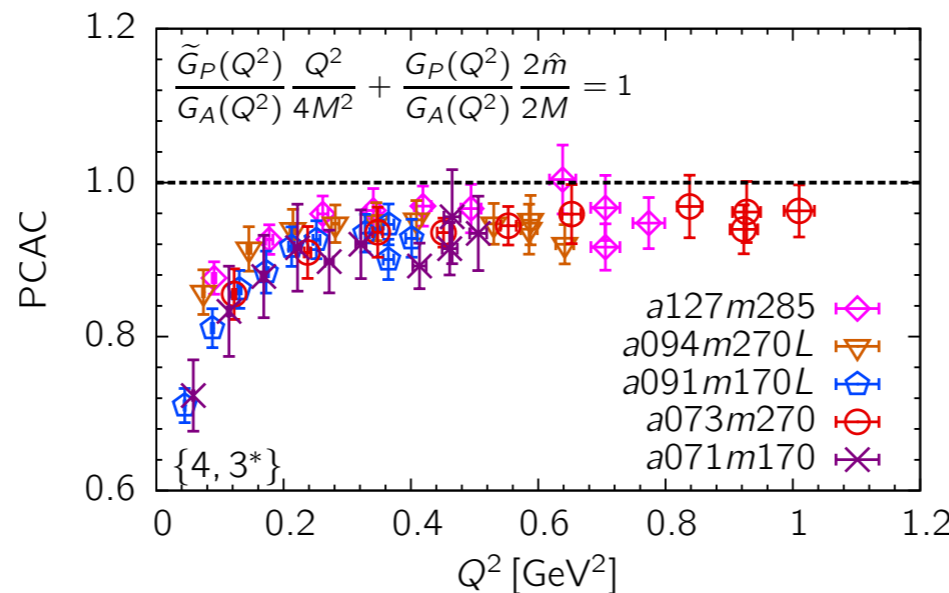
Vector and axial form factors recently calculated using nearly physical quark masses:



Park et al [NME], arXiv:2103.05599

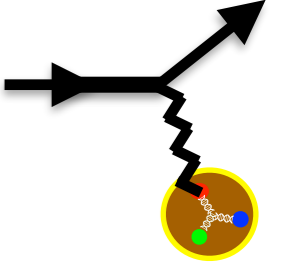
LQCD nucleon electric and magnetic form factor results agree with phenomenological parameterizations after accounting for excited-state and discretization effects

Careful treatment of $N\pi$ excited states required to reproduce consequences of axial ward identities that assume ground-state dominance

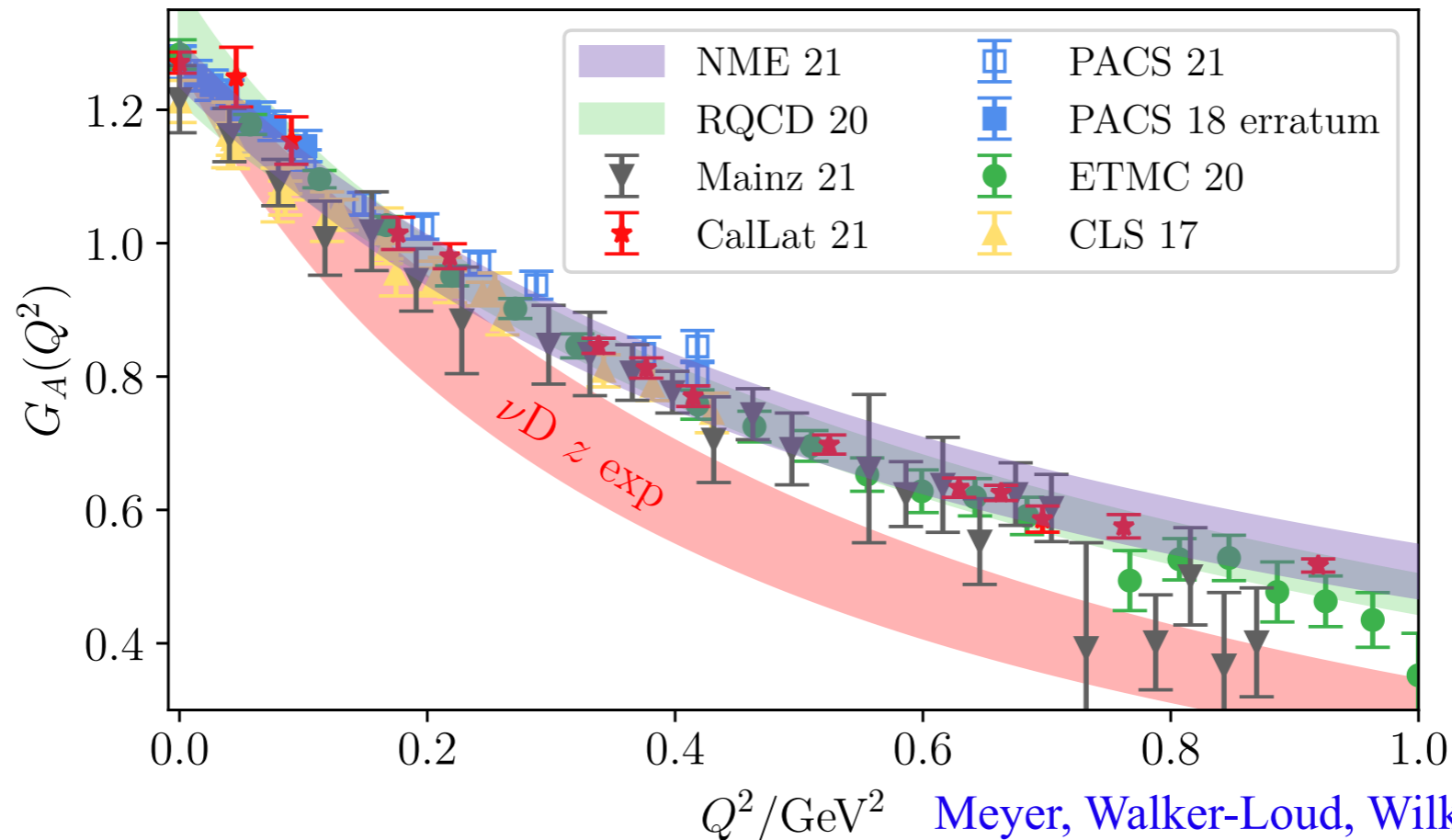


Park et al [NME], arXiv:2103.05599

Axial form factors



Recent axial form factor calculations include physical quark masses, continuum / infinite-volume extrapolations, and excited-state fits that account explicitly for $N\pi$ states



Differences between LQCD and experimental axial form factor determinations could arise from challenging LQCD systematic uncertainties (excited states, lattice spacing, ...)

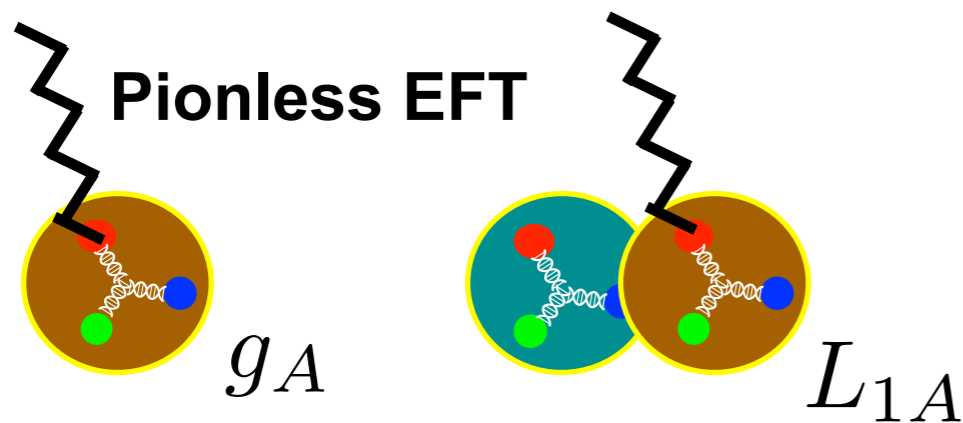
Differences could also arise from underestimated uncertainties in phenomenological form factor determinations using deuterium bubble chamber data

Axial currents in nuclei

QCD interactions make nuclei differ from collections of nucleons

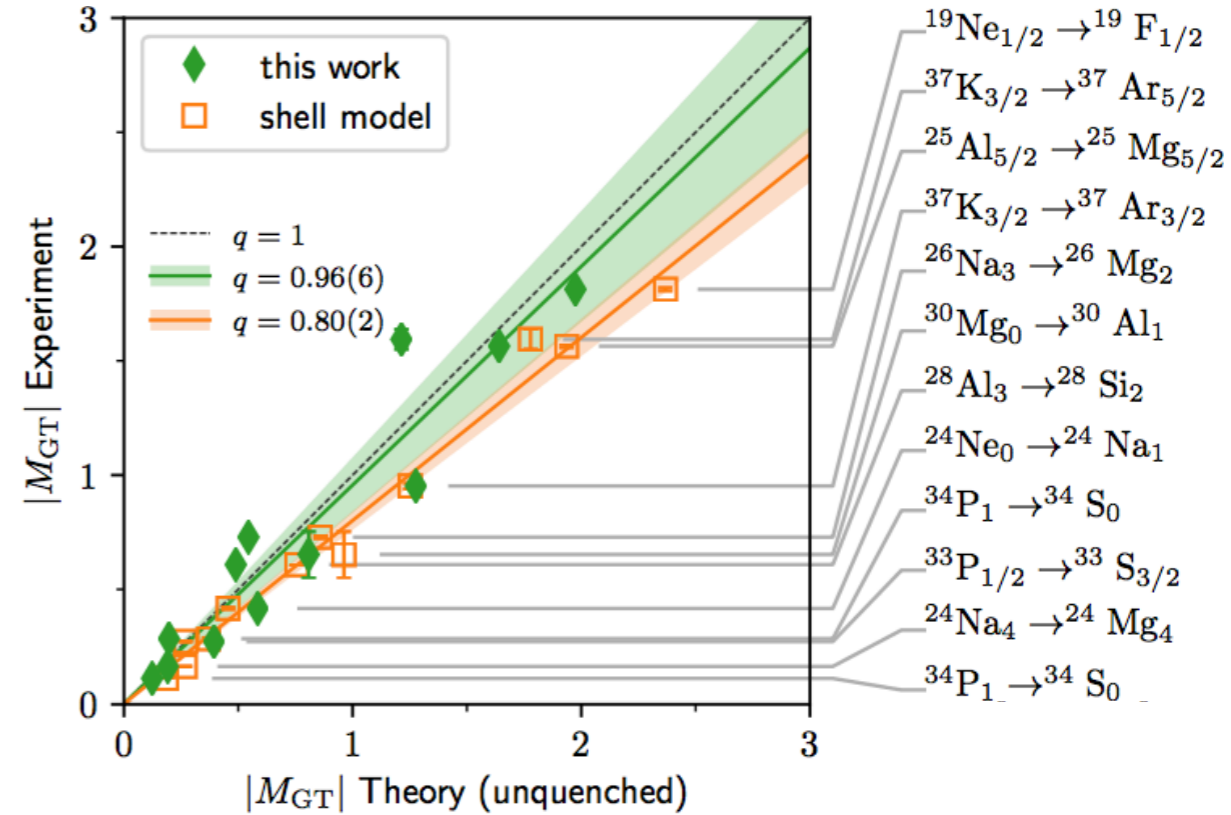
Modern nuclear theory calculations including multi-nucleon correlations and currents with e.g. chiral EFT can reproduce experiment without “quenching” g_A

Few-nucleon LQCD results can constrain two-body currents in nuclear models and EFTs



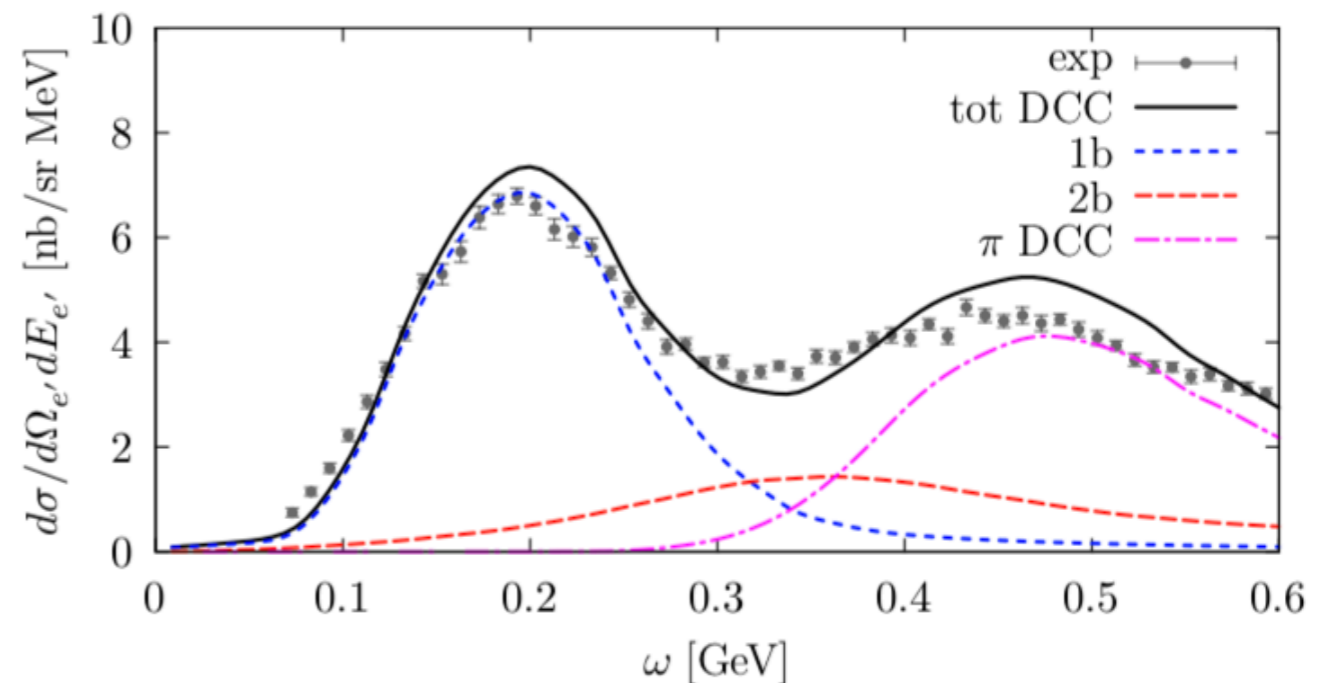
Percent-level accuracy for neutrino-nucleus cross sections will require more precise two-body current inputs

Gysbers et al, Nature Phys. 15 (2019)

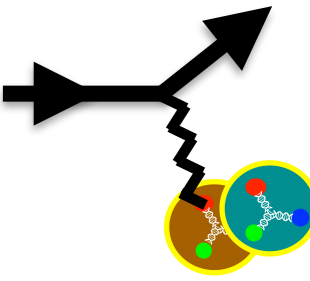


Rocco, Nakamura, Lee, Lovato, PRC 100 (2019)

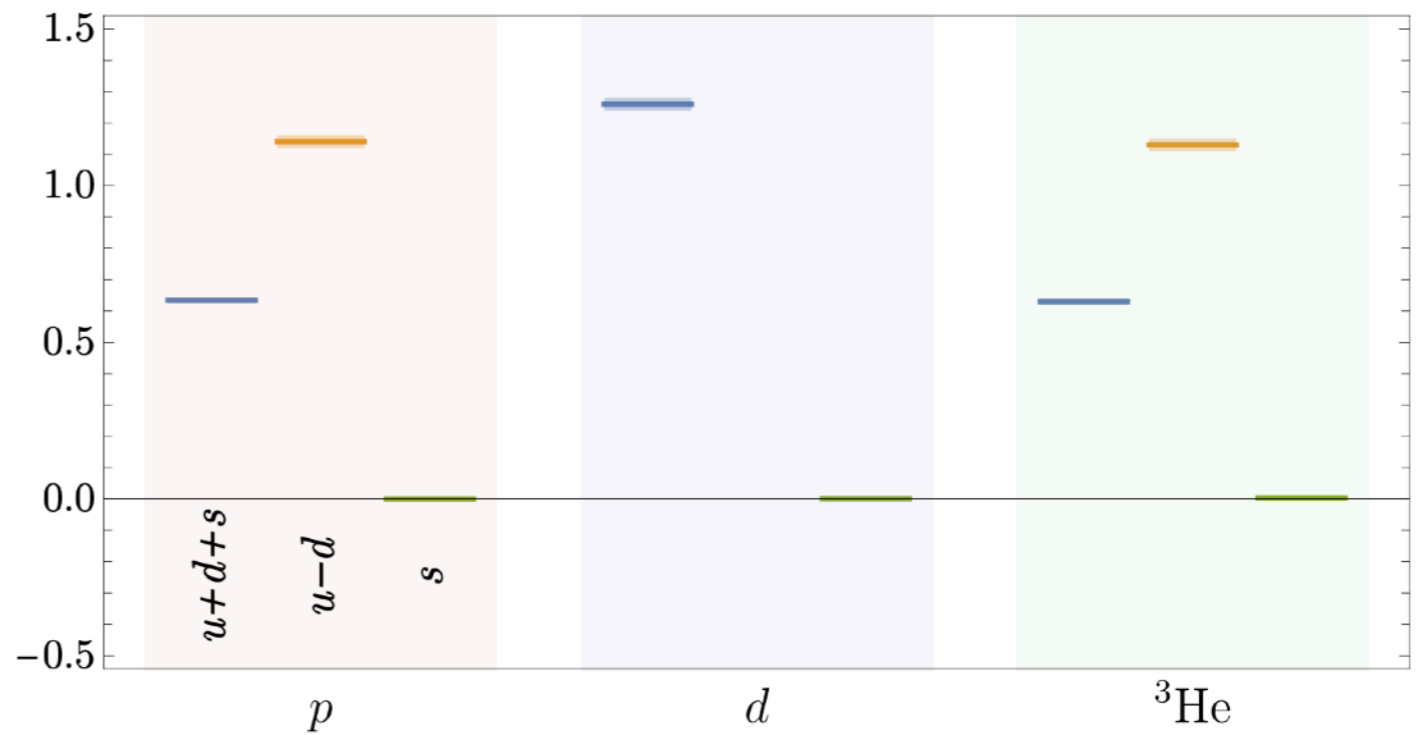
$E_e = 961 \text{ MeV}, \theta_e = 37.5^\circ$



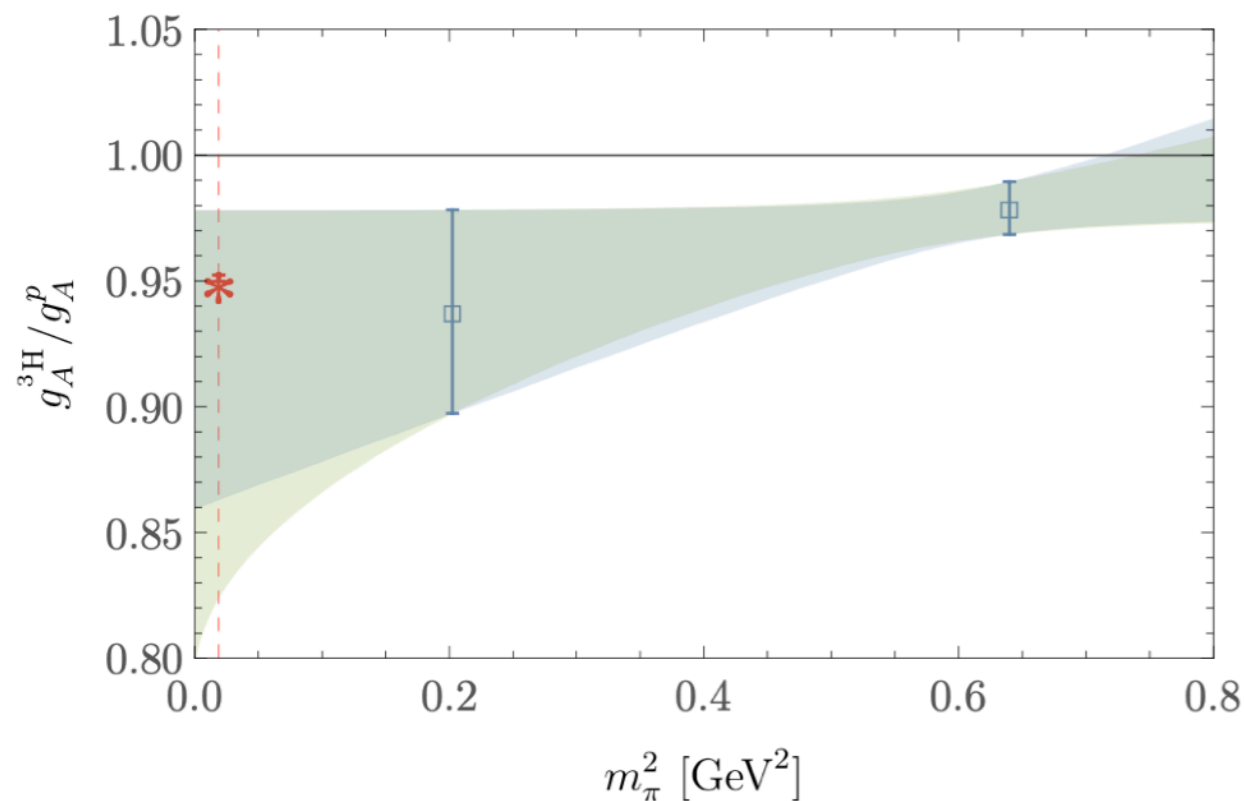
Two-body currents in LQCD



Flavor decomposition of axial matrix elements of two and three nucleon systems computed with $m_\pi = 806$ MeV



Chang, MW et al [NPLQCD], PRL 120 (2018)



Parreño, MW et al [NPLQCD] PRD 103 (2021)

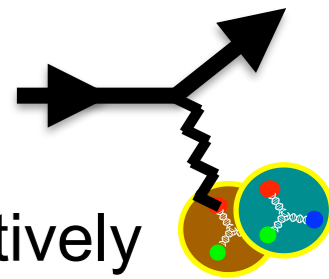
Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation of triton axial charge to physical point

Several systematic uncertainties remain, but encouraging agreement with experiment seen

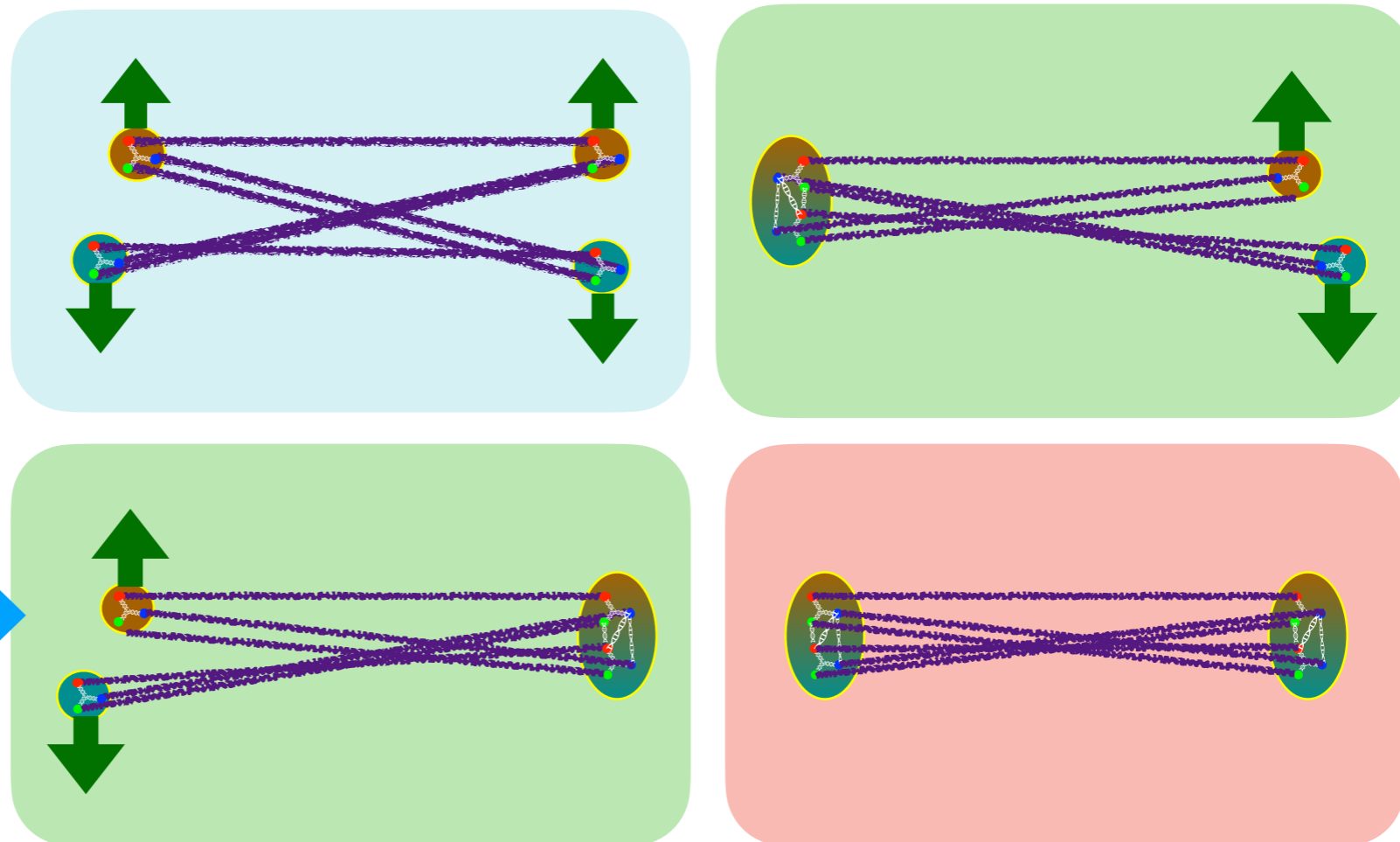
Matching to finite-volume pionless EFT used to constrain L_{1A}

Detmold and Shanahan, PRD 103 (2021)

Variational methods



Excited-state effects from unbound multi-nucleon scattering states are not effectively suppressed in computationally accessible Euclidean correlation functions



LQCD nuclear matrix element calculations so far

First calculations of plane-wave nucleon-nucleon (upper left) correlation functions and symmetric correlation-function matrices indicate that excited-state effects lead to large systematic uncertainties on calculated energy levels

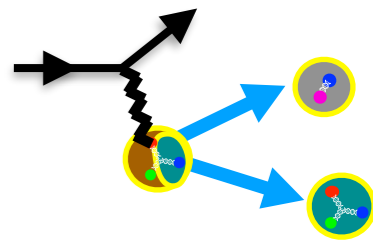
[Francis et al, PRD 99 \(2019\)](#)

[Green et al, PRL 127 \(2021\)](#)

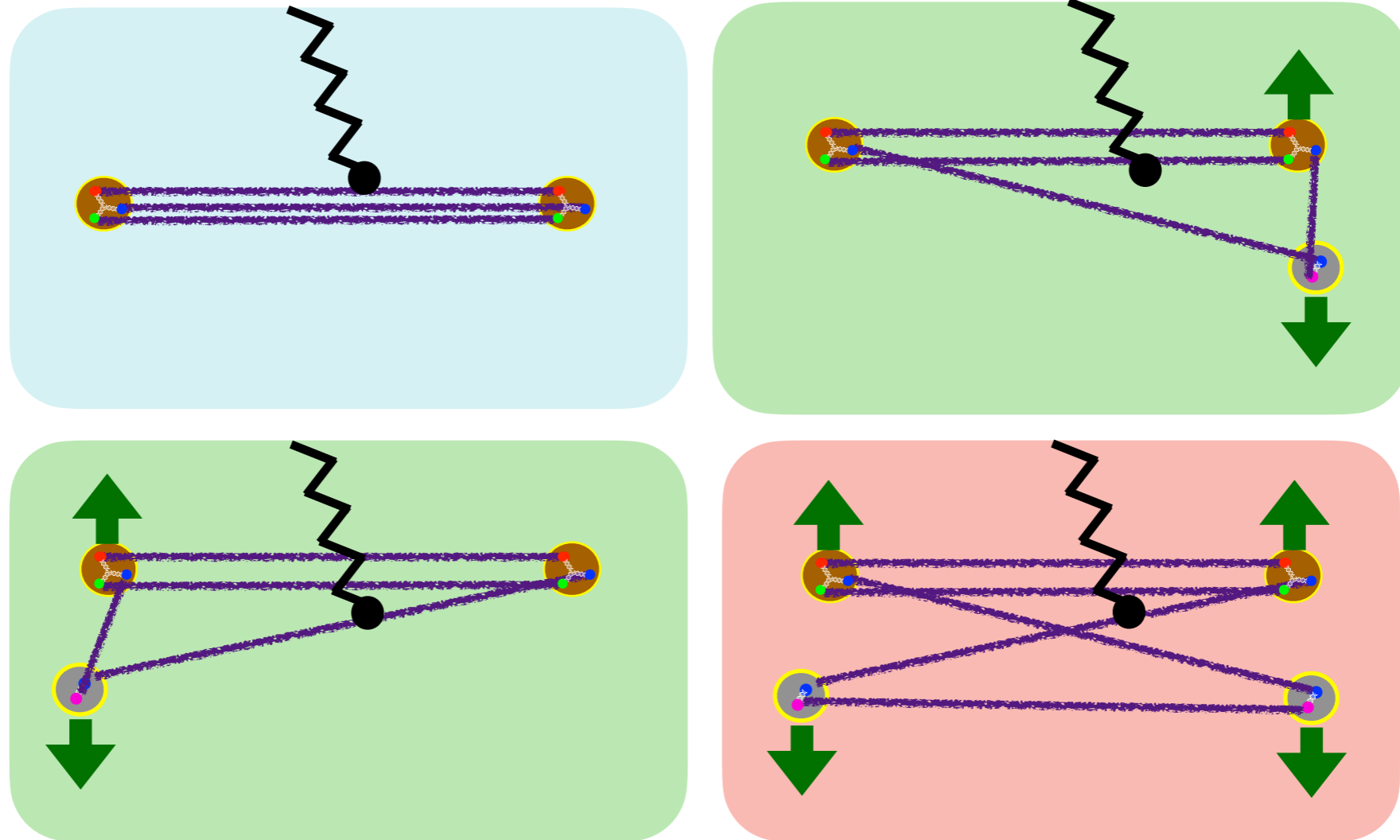
[Hörz et al, PRC 103 \(2021\)](#)

[Amarasinghe, MW et al, arXiv:2108.10835](#)

Variational methods for $N\pi$



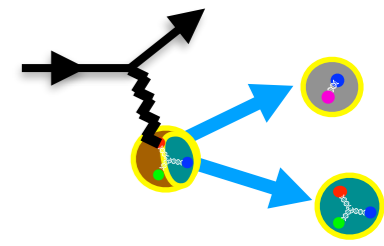
Analogous variational methods can be applied to study $N\pi$ systems



Same calculation can be used to explicitly remove excited-state contamination from elastic nucleon form factors and access pion-production amplitudes

$N \rightarrow \Delta$ transition form factors can be calculated if $I=1/2$ and $I=3/2$ operators included

$N\pi$ systems in LQCD

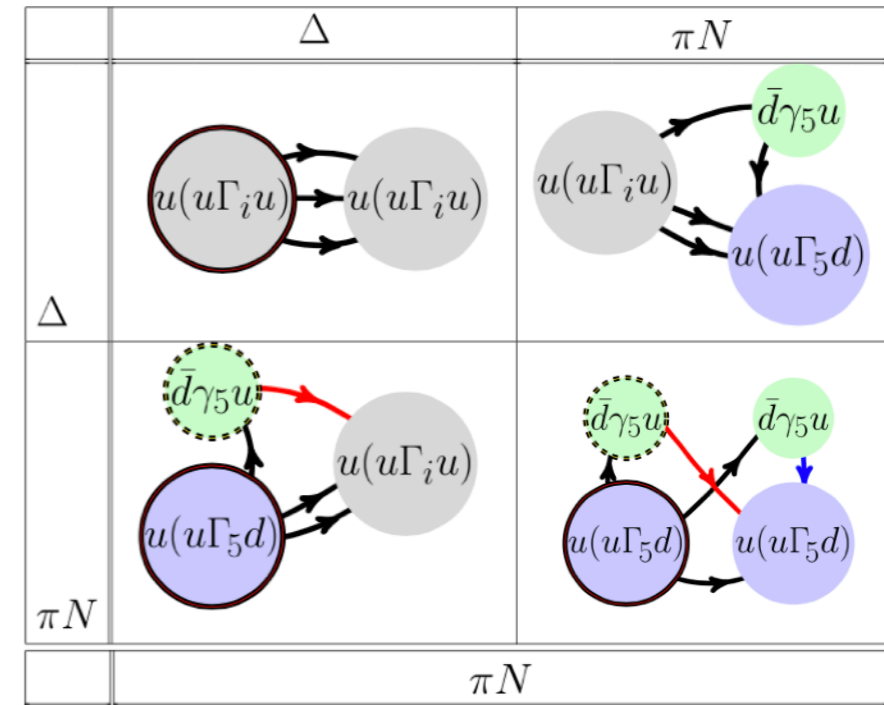


The $I=J=3/2$ sector has been explored using variational calculations including both localized $\Delta \sim qqq$ and $N\pi$ operators

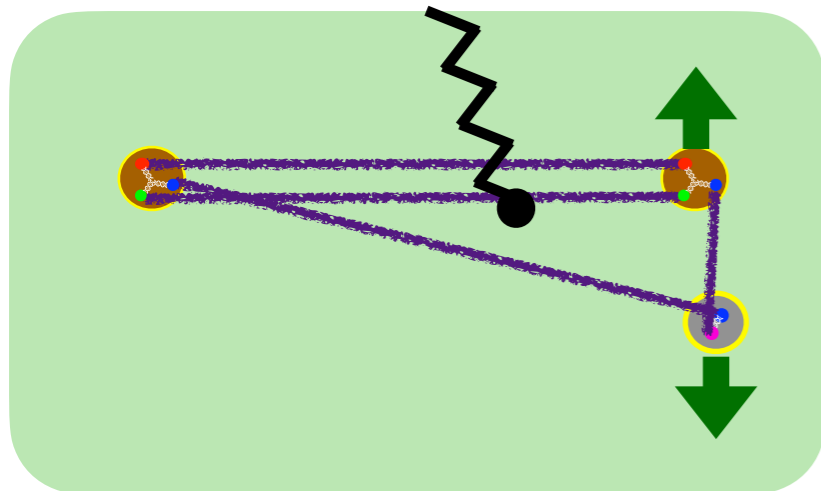
Andersen, Bulava, Hörz, Morningstar, PRD 97 (2018)

Silvi, Paul, Alexandrou, Krieg, Leskovec, Meinel, Negele, Petschlies, Pochinsky, Rendon, Syritsyn, and Todaro, PRD 23 (2021)

Finite-volume energy spectra are mapped through generalizations of Lüscher's quantization condition to constraints on P-wave $N\pi$ scattering phase shifts

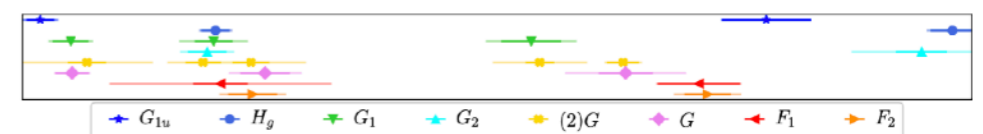
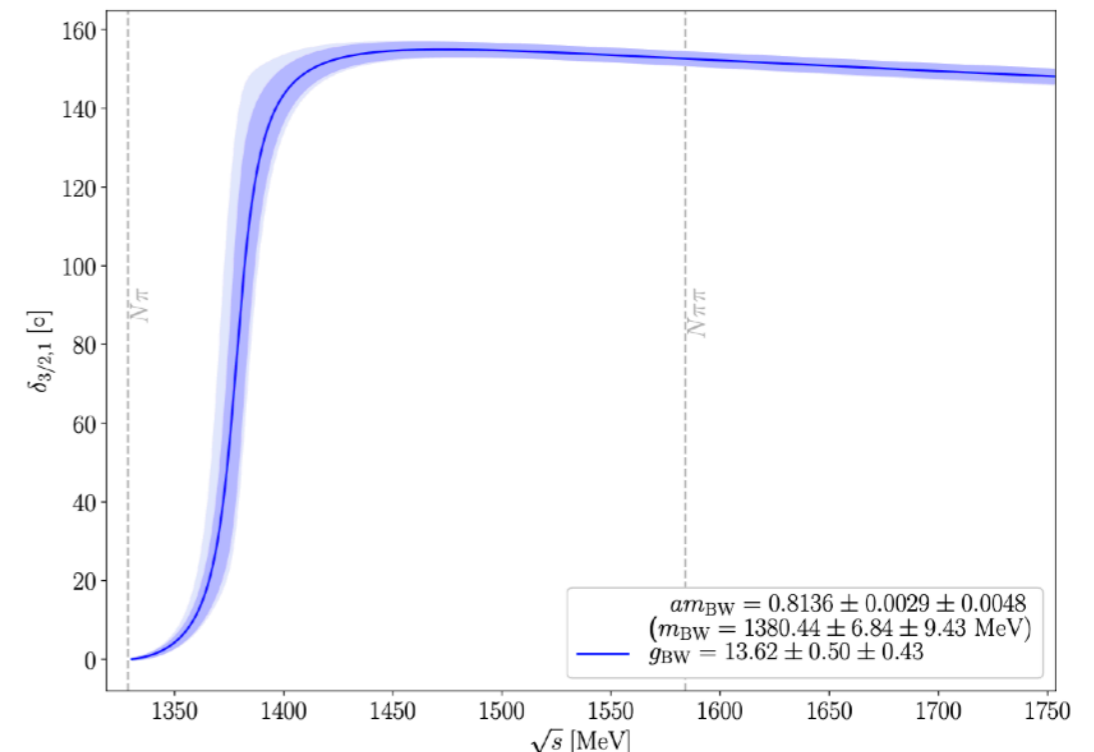


Silvi et al, PRD 23 (2021)

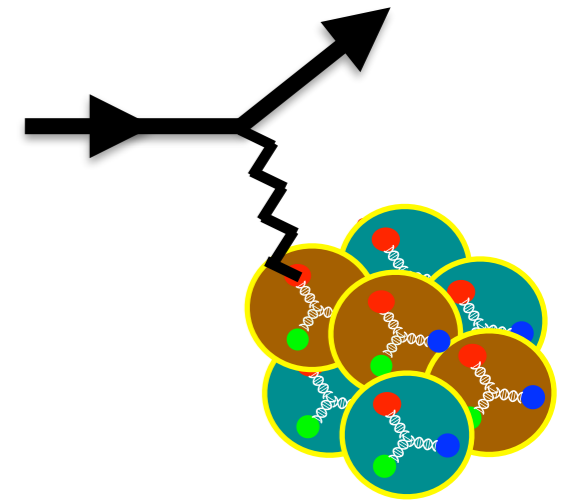


Calculations of $N \rightarrow \Delta$ transition form factors under active exploration

Barca, Bali, and Collins, arXiv:2110.11908 [hep-lat]



The hadron tensor



LQCD methods for calculating exclusive cross-sections break down above multi-particle thresholds (≤ 3 hadrons is state-of-the-art)

Reviews: Briceño, Dudek, and Young, *Rev. Mod. Phys.* 90 (2018)

Hansen and Sharpe, *Ann. Rev. Nucl. Part. Sci.* 69 (2019)

How can we use LQCD to constrain higher energies, e.g. shallow inelastic scattering region?

The hadron tensor:

$$\frac{d^2 \sigma}{dE' d \cos \theta} \propto L_{\mu\nu} W^{\mu\nu}$$

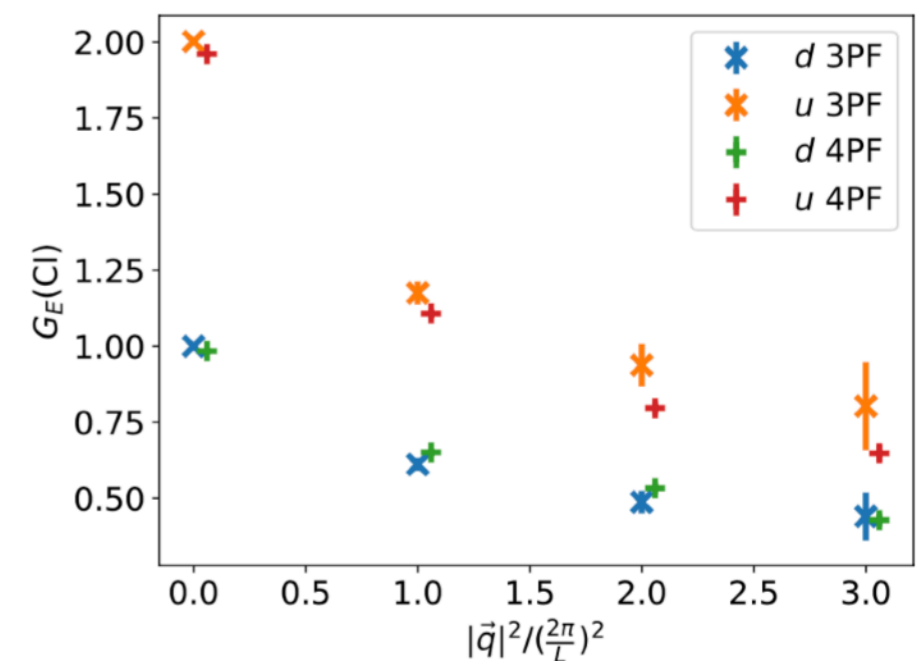
↙ Lepton tensor (perturbative)
 ← Hadron tensor (nonperturbative QCD)

Liu and Dong, *PRL* 72 (1994)

Aglietti et al, *Phys. Lett. B* 432 (1998)

Liu, *PRD* 62 (2000)

LQCD hadron tensor calculations require a challenging inverse Laplace transform, calculations exploring several different methods underway



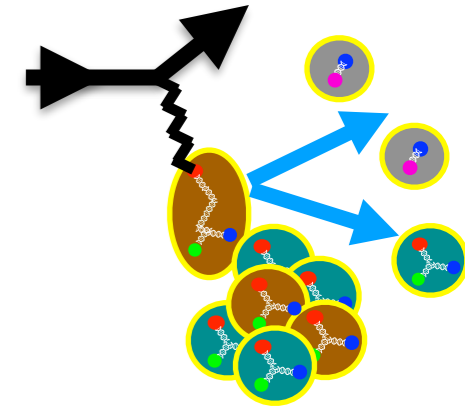
Liang, Draper, Liu, Rothkopf, and Yang [χ QCD] *PRD* 101 (2020)

Fukaya, Hashimoto, Kaneko, and Ohki, *PRD* 102 (2020)

Proof-of-principle results demonstrate consistency between nucleon hadron tensor and form factor calculations for elastic scattering kinematics

Liang, Liu, and Yang [χ QCD] *EPJ Web Conf.* 175 (2018)

DIS and LQCD



LQCD can also compute quantities relevant to neutrino DIS

Large momentum effective theory connects
Euclidean matrix elements to light-cone PDFs

See talk by Huey-Wen Lin, up next!

Current LQCD results can improve global
analyses of isovector polarized PDFs that are
relevant for weak interactions in neutrino DIS

Chen, Cohen, Ji, Lin, Zhang, Nucl. Phys. B 911 (2016)

Alexandrou, Cichy, Constantinou, Jansen, Scapellato, Steffens,
PRL 121 (2018)

Alexandrou et al, PRL 126 (2021)

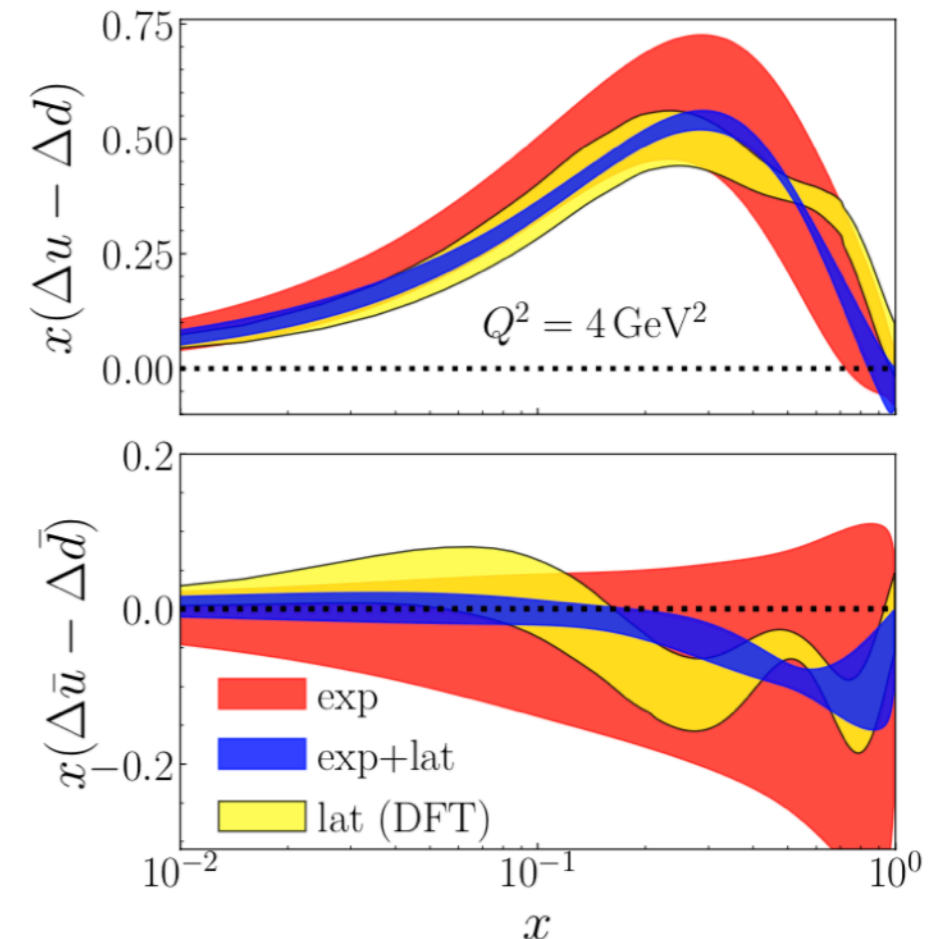
Alexandrou et al, PRD 103 (2021)

Alexandrou et al, PRD 104 (2021)

Exploratory calculations of PDF moments of two- and three-nucleon systems
performed, could inform models of EMC effect in neutrino DIS

Winter, MW et al [NPLQCD], PRD 96 (2017)

Detmold, MW et al [NPLQCD] PRL 126 (2021)

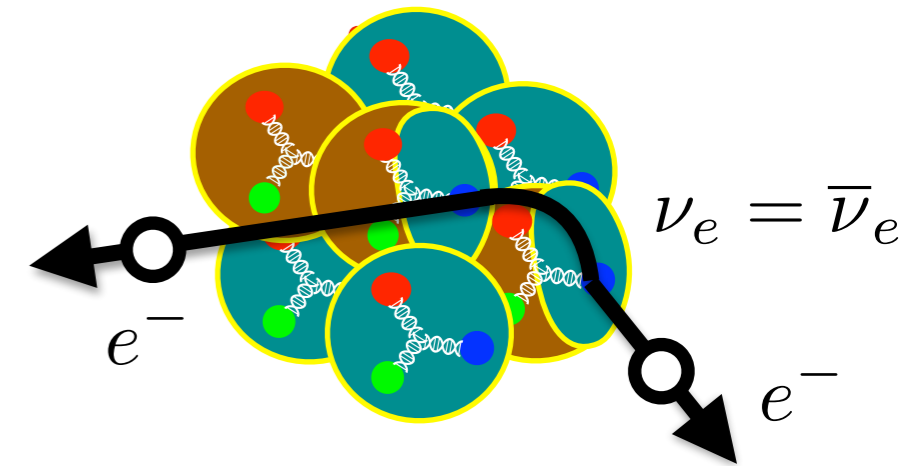


Bringewatt et al [JAM], PRD 103 (2021)

Lepton-number violation

Low-energy signature of lepton-number violation: $0\nu\beta\beta$

Experimental data on the half-lives of nuclei where double- β but not single- β decay is allowed can be used to constrain Majorana masses



$$(T_{1/2}^{0\nu})^{-1} = G^{0\nu} |\mathcal{M}^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$

Phase space

Nuclear matrix element

Effective Majorana mass

Recent chiral EFT calculations discovered that a short-distance contact operator is needed for renormalizability, efforts ongoing to constrain it and add it to many-body calculations

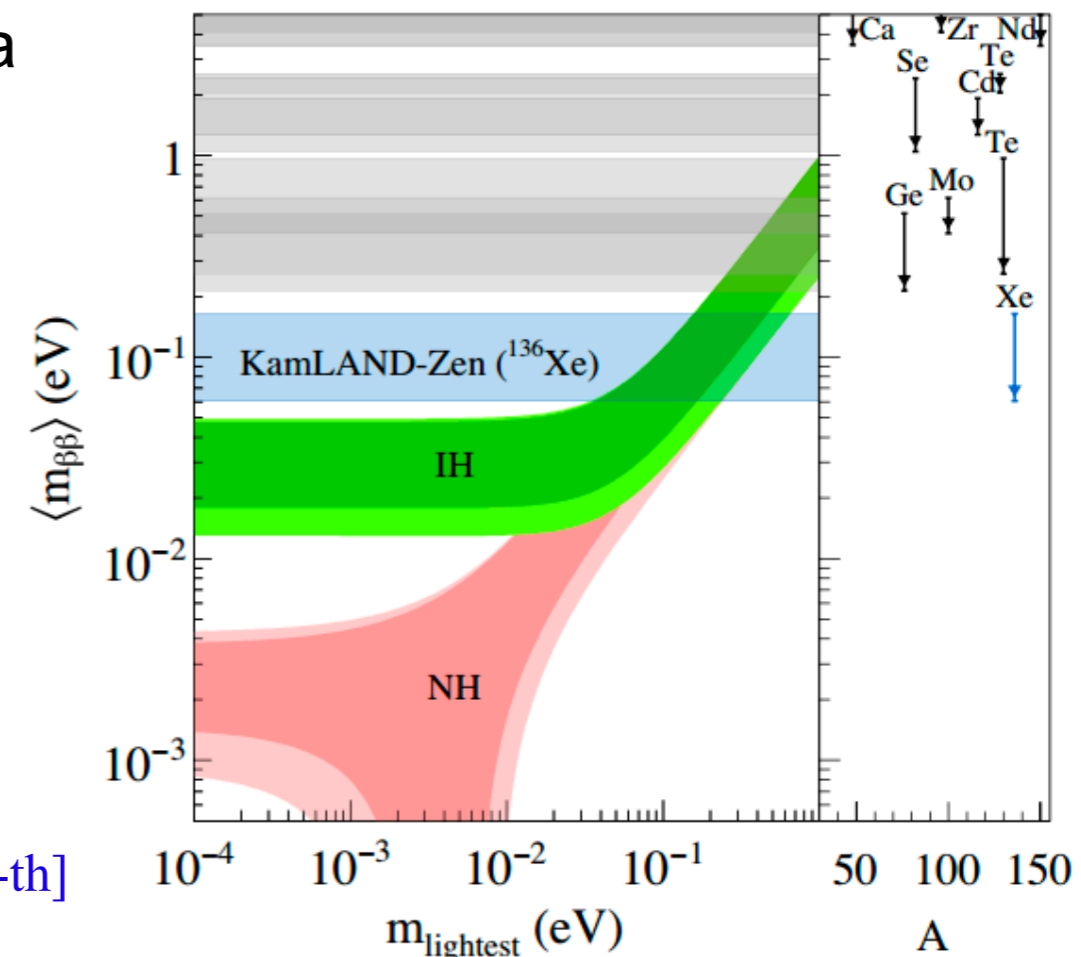
Cirigliano, Dekens, de Vries, Graesser, and Mereghetti PRL 120 (2018)

Cirigliano, Dekens, de Vries, and Hoferichter PRL 126 (2021)

Wirth, Yao, and Hergert, PRL 127 (2021)

Weiss, Soriano, Lovato, Menedez, and Wiringa arXiv:2112.08146 [nucl-th]

...



Gando et al (KamLAND-Zen) PRL 117 (2016)

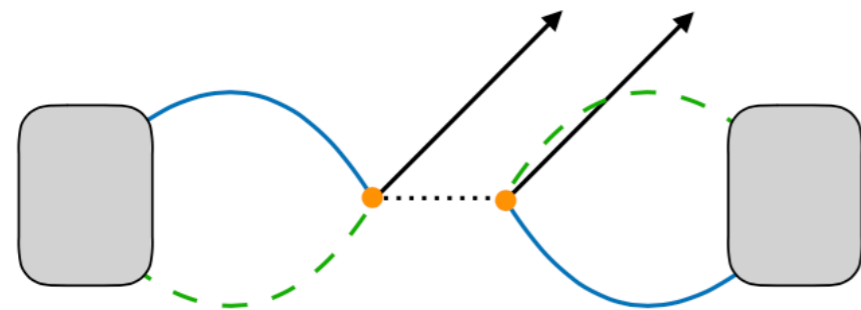
LQCD and $0\nu\beta\beta$

Pion analogs such as $\pi^- \rightarrow \pi^+ e^- e^-$ calculated with physical quark masses

Short distance [Nicholson et al, PRL 121 \(2018\)](#)

Long distance [Feng, Jin, Tuo, PRL 122 \(2019\)](#)

[Tuo, Feng, and Jin, PRD 100 \(2019\)](#)



[Detmold and Murphy, arXiv:2004.07404](#)

Two-nucleon matrix elements for $2\nu\beta\beta$ calculated by matching LQCD and EFT

$$\langle pp | A_3^+ A_3^+ | nn \rangle$$

[Shanahan, MW et al \[NPLQCD\], PRL 119 \(2017\)](#)

[Tiburzi, MW et al \[NPLQCD\], PRD 96 \(2017\)](#)

Future LQCD calculations can help precisely determine contact operators appearing at LO in chiral EFT descriptions of $0\nu\beta\beta$

Formalism for relating LQCD correlation functions to physical matrix elements under active development

[Briceño et al, PRD 21 \(2020\)](#)

[Feng, Jin, Wang, Zhang, PRD 103 \(2021\)](#)

[Davoudi and Kadam, PRL 126 \(2021\)](#)

Questions

