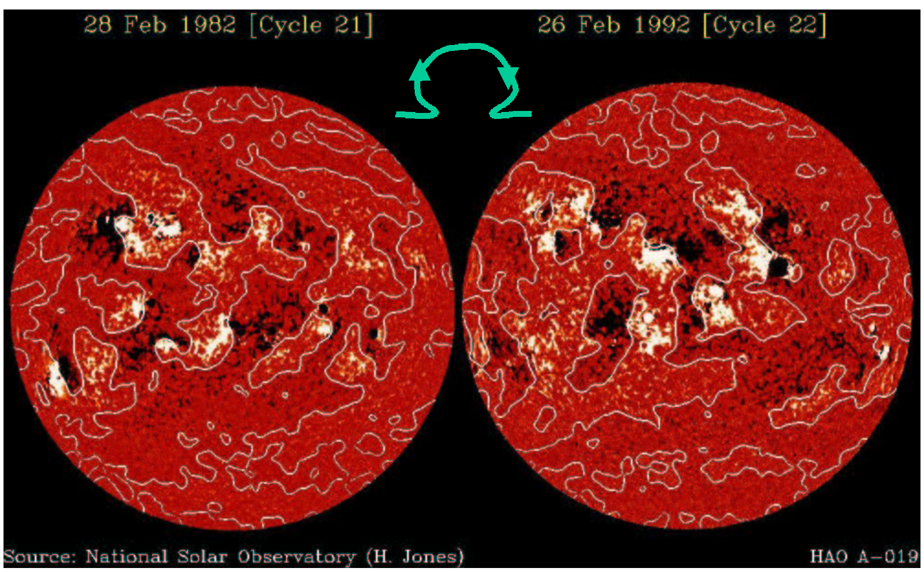
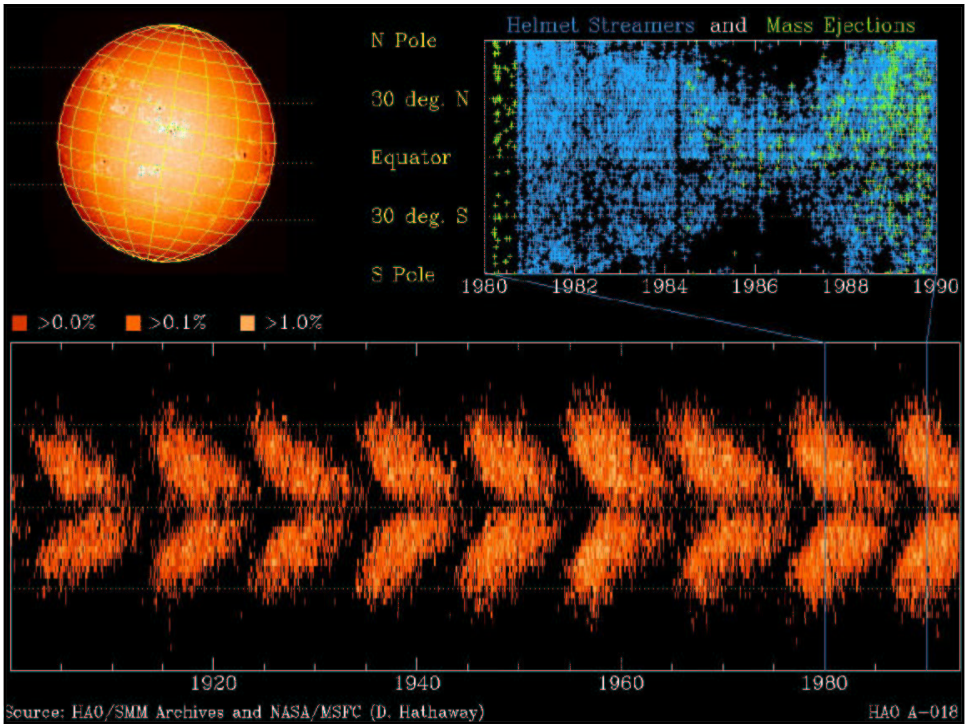
The nonlinearity of large scale dynamos

Axel Brandenburg
(*ITP* & *Nordita, Copenhagen*)
collaboration with Eric Blackman (Rochester)

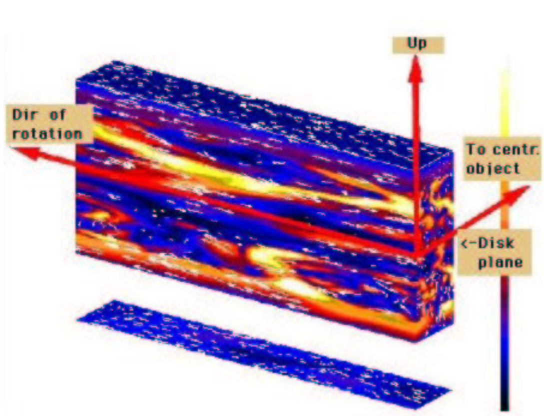
Order out of disorder: Hale's polarity law

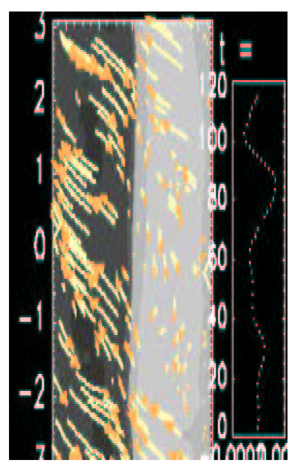




Local simulation of an accretion disc

An example of collective behavior in a simulation





Nonlinearity of Large Scale Dynamos 4

Motivation for mean-field theory

- Not an excuse not to do global simulation!
  - Still need to understand results
  - Example: differential rotation
    - still Reynolds stress & baroclinic term
- Karlsruhe dynamo experiment
  - Cartoon picture understanding: model for earth
  - Mean field theory: inclined dipole (S1-field)
- Local disc simulations: S0-osc *versus* A0-st
- Sun: we see collective behavior

Nonlinearity of Large Scale Dynamos 5

Observe collective behavior → mean-field theory

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla\times\left(\overline{\mathbf{U}}\times\overline{\mathbf{B}} + \overline{\mathbf{u}\times\mathbf{b}} - \eta\overline{\mathbf{J}}\right)$$

$$\mathrm{E} = \alpha\overline{\mathbf{B}} - \eta_t\overline{\mathbf{J}}$$

$$\alpha = -\tfrac{1}{3}\tau\langle\boldsymbol{\omega}\cdot\mathbf{u}\rangle$$

pseudo scalar

$$\frac{\partial \overline{B}_x}{\partial t} = -\alpha\frac{\partial \overline{B}_y}{\partial z} + (\eta+\eta_t)\frac{\partial^2 \overline{B}_x}{\partial z^2}$$

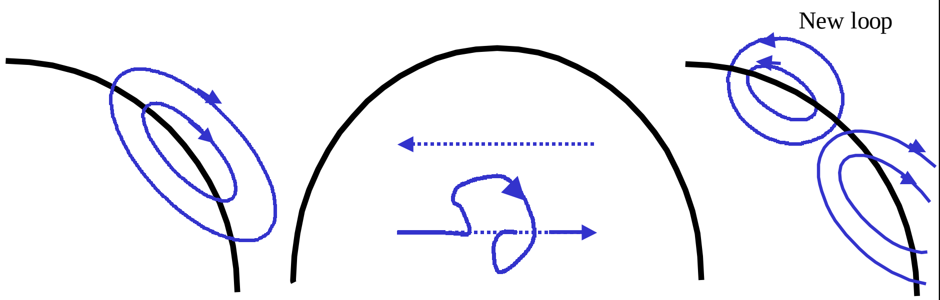
$$\frac{\partial \overline{B}_y}{\partial t} = -\Omega'\frac{\partial \overline{B}_x}{\partial z} + (\eta+\eta_t)\frac{\partial^2 \overline{B}_y}{\partial z^2}$$

$$\overline{B}_x = \overline{B}_{x0}\exp(ikz+\lambda t)$$

(e.g. growing dynamo wave)

6

The old cartoon picture



Differential rotation
(prehelioseism: faster inside)
↙
?need meridional circulation

Cyclonic convection;
Buoyant flux tubes
→ α-effect

Equatorward
migration

Quenching of alpha

- Isotropic box simulations:
  - η_t is "catastrophically" quenched
    (Cattaneo & Vainshtein 1991, because 2D: Gruzinov & Diamond 1994)
  - α is "catastrophically" quenched
    (Vainshtein & Cattaneo 1992, Gruzinov & Diamond 1994, +others)

$$\alpha = \frac{\alpha_0}{1+R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$

does not exclude:-

$$\alpha = \frac{\alpha_0 + \eta_t R_m \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} / B_{eq}^2}{1+R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$

- Astrophysical simulations:
  - α small even kinematically (R_m dependence?)
  - ***B*** is definitely strong

What's wrong?

- Imposed field approach suspect?
  - <***B***> forced to stay unchanced, **J**=0 always!
- Something wrong with forcing?
  - thermal/magnetic buoyancy different
- Boundary conditions? (Blackman & Field 2000)
  - but alpha should be determined locally?
- Need to determine α and η_t simultaneously

$$\alpha = \frac{\alpha_0 + \eta_t R_m \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} / B_{eq}^2}{1 + R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$

Nonlinearity of Large Scale Dynamos 9

Helically forced MHD

Induction Equation:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mathbf{J}$$

Magn. Vector potential

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -c_s^2 \nabla \ln \rho + \frac{1}{\rho}\left[\mathbf{J} \times \mathbf{B} + \mu\left(\nabla^2 \mathbf{u} + \tfrac{1}{3}\nabla\nabla \cdot \mathbf{u}\right)\right] + \mathbf{f}$$

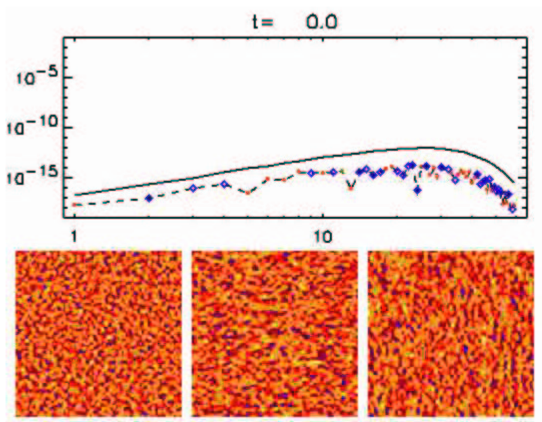
Momentum and Continuity eqns

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\nabla \cdot \mathbf{u}$$

+ forcing function: polarized waves

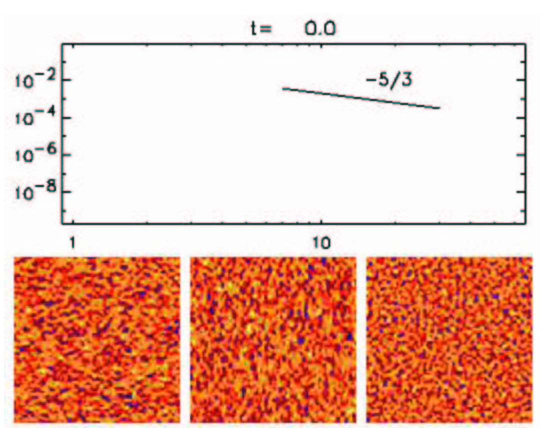
Nonlinearity of Large Scale Dynamos 10

Spectra and slices of B



Nonlinearity of Large Scale Dynamos 11

Magnetic Prandtl number = 100



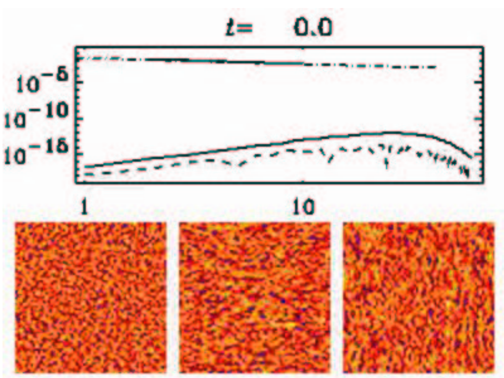
Nonlinearity of Large Scale Dynamos 12

What causes large scale field?

- Inverse cascade of magnetic helicity
  - Frisch et al. (1975), Pouquet et al. (1976)
  - Intrinsically nonlinear
- Alpha-effect (nonlocal in k-space)
  - Steenbeck, Krause & Radler (1966)
  - Exists already in linear approximation

Nonlinearity of Large Scale Dynamos 13

Large scale separation



Nonlocal inverse
Transfer in k-space

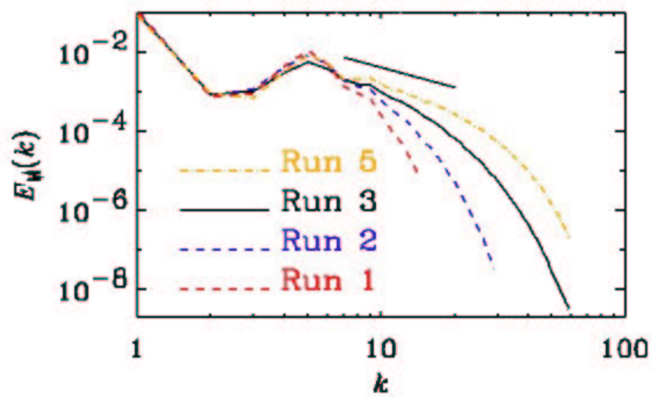
consistent with

$$k_{\max} = \alpha / (2\eta_T)$$

→Evidence for
Alpha-effect.

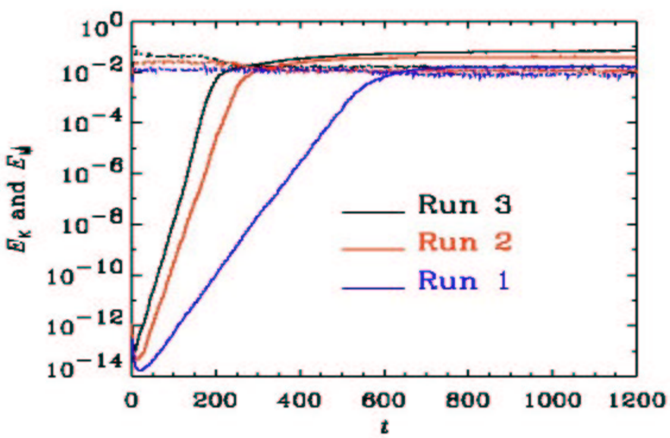
Nonlinearity of Large Scale Dynamos 14

Convergence at large scales



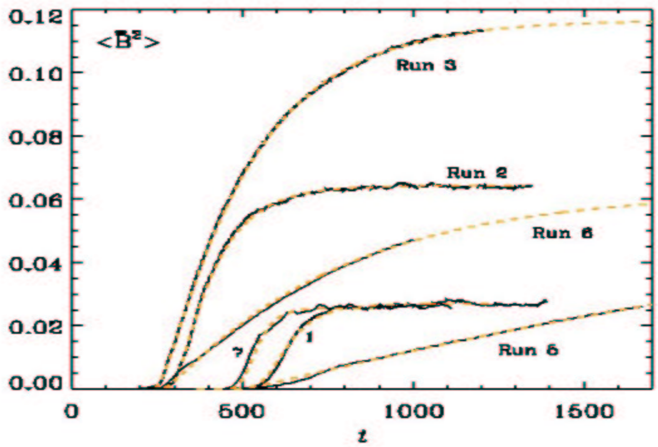
Nonlinearity of Large Scale Dynamos 15

Faster growth if R_m is large



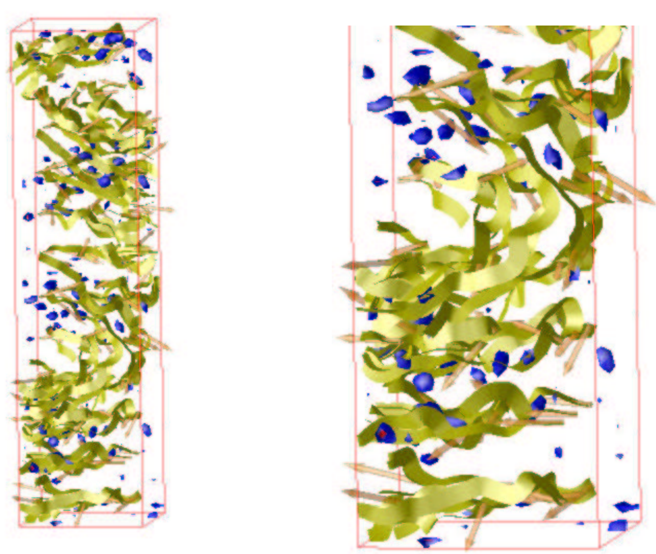
Nonlinearity of Large Scale Dynamos 16

Saturation slow-down



Nonlinearity of Large Scale Dynamos 17

Large and small scale helical fields



Saturation behavior explained by magnetic helicity conservation

$$\frac{d}{dt}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle + \text{surface terms}$$

Steady state, closed box $\langle \mathbf{J}\cdot\mathbf{B}\rangle = 0$

Small scale and large scale current helicity in balance

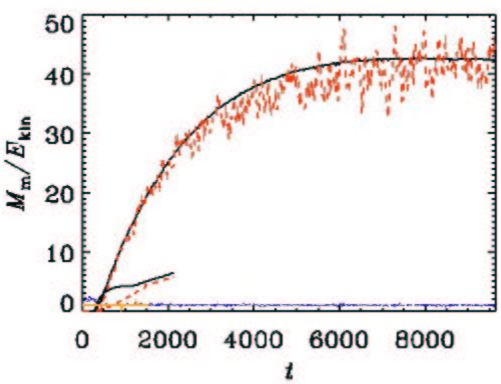
$$\langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle + \langle \mathbf{j}\cdot\mathbf{b}\rangle = 0$$

$$\longrightarrow \quad k_1\langle \overline{\mathbf{B}}^2\rangle = k_\mathrm{f}\langle \mathbf{b}^2\rangle$$

Nonlinearity of Large Scale Dynamos 19

With hyperdiffusivity

Brandenburg & Sarson (2002, PRL)



$$k_1^3\langle \overline{\mathbf{B}}^2\rangle = k_\mathrm{f}^3\langle \mathbf{b}^2\rangle$$

for ordinary hyperdiffusion $\propto \eta_2 k^4$

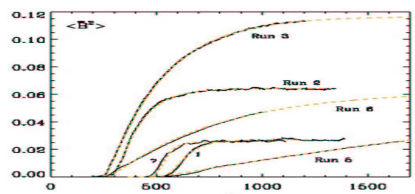
Nonlinearity of Large Scale Dynamos 20

Slow-down explained by magnetic helicity conservation

$$\frac{d}{dt}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

$$k_1^{-1}\frac{d}{dt}\left\langle \overline{\mathbf{B}}^2\right\rangle = -2\eta k_1\left\langle \overline{\mathbf{B}}^2\right\rangle + 2\eta k_\mathrm{f}\left\langle \mathbf{b}^2\right\rangle$$

$$\longrightarrow \quad \left\langle \overline{\mathbf{B}}^2\right\rangle = \left\langle \mathbf{b}^2\right\rangle \frac{k_f}{k_1}\left[1-e^{-2\eta k_1^2 (t-t_\mathrm{s})}\right]$$



molecular value!!

21

Resistivity important?

$$\left|\frac{dH}{dt}\right| \le 2\eta^{1/2}\left\langle \eta\mathbf{J}^2\right\rangle^{1/2}\left\langle \mathbf{B}^2\right\rangle^{1/2} = 2\eta^{1/2}Q^{1/2}M^{1/2}$$

$$\text{assume}\left|\frac{dH}{dt}\right| \approx \omega|H|, \quad Q \approx \omega M$$

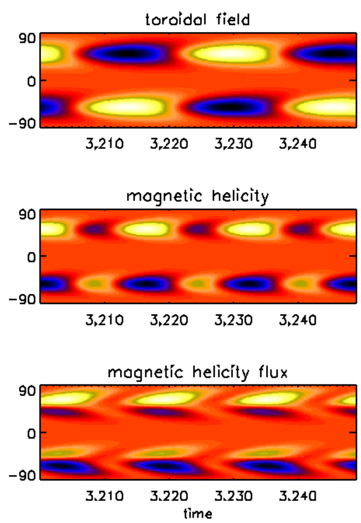
$$|H|/2M \le (2\eta/\omega)^{1/2}$$

$10^{-5}R_\mathrm{sun} \approx 7$ km

(from models)

10-300km

→ resistivity is important, and not infinitely small



model

22

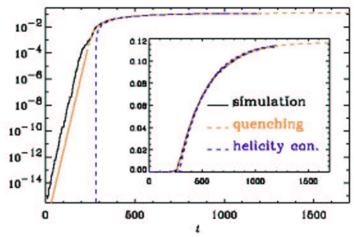
Comparison with quenched mean-field models

$$\left\langle \overline{\mathbf{B}}^2 \right\rangle = \left\langle \mathbf{b}^2 \right\rangle \frac{k_f}{k_1} \left[1 - e^{-2\eta k_1^2 (t - t_s)}\right]$$

Ought to be satisfied also for magnetically driven instabilities!

$$\alpha = \frac{\alpha_0}{1 + a\overline{\mathbf{B}}^2 / B_{eq}^2}$$

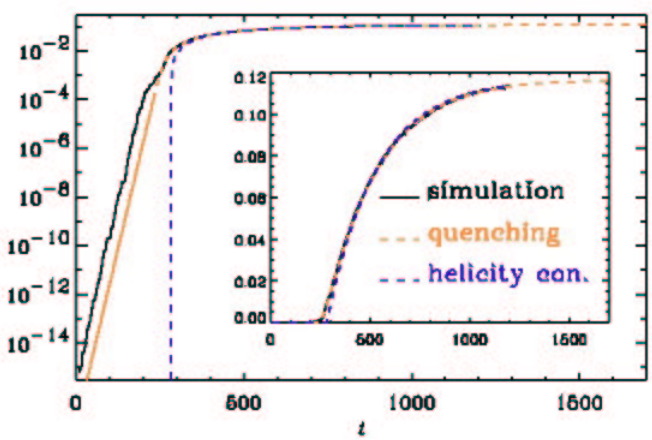
$$a = \lambda / (\eta k_f k_1)$$



Nonlinearity of Large Scale Dynamos 23

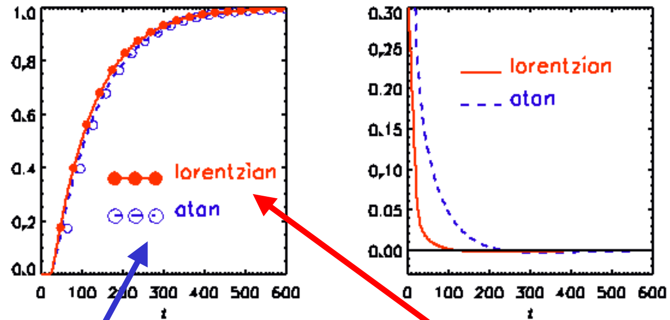
Excellent fit!

$$\left\langle \overline{\mathbf{B}}^2 \right\rangle = \left\langle \mathbf{b}^2 \right\rangle \frac{k_f}{k_1} \left[1 - e^{-2\eta k_1^2 (t - t_s)}\right]$$



Nonlinearity of Large Scale Dynamos 24

Other quenchings ruled out



$$q = \frac{1}{\overline{\mathbf{B}}^2}\left(1 - \frac{\tan^{-1}\sqrt{3\overline{\mathbf{B}}^2}}{\sqrt{3\overline{\mathbf{B}}^2}}\right) \qquad \alpha = \frac{\alpha_0}{1+\alpha_B \overline{\mathbf{B}}^2 / B_{eq}^2}$$

Nonlinearity of Large Scale Dynamos 25

Taking magnetic helicity seriously

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

Two-scale assumption

$$\mathrm{E} = \alpha\overline{\mathbf{B}} - \eta_t \overline{\mathbf{J}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle = +2\langle \mathrm{E}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle \qquad \alpha = \alpha_K + \alpha_M$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{a}\cdot\mathbf{b}\rangle = -2\langle \mathrm{E}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle \mathbf{j}\cdot\mathbf{b}\rangle \qquad \alpha_K = -\tfrac{1}{3}\tau\langle \boldsymbol{\omega}\cdot\mathbf{u}\rangle$$

→ Dynamical α-quenching (Kleeorin & Ruzmaikin 1982)

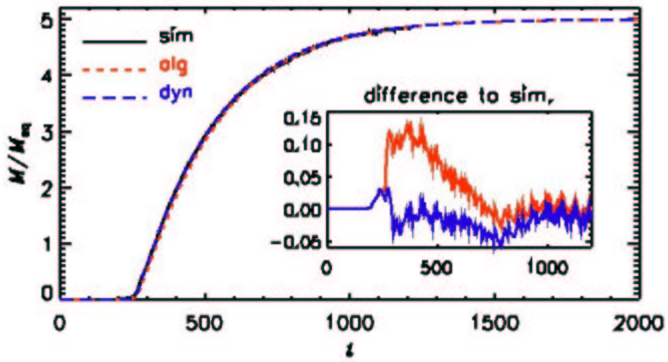
$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_M = -2\eta k_f^2\left(a\frac{\langle \mathrm{E}\cdot\overline{\mathbf{B}}\rangle}{B_{eq}^2} + \alpha_M\right)$$

$$a = \lambda/(\eta k_\mathrm{f} k_1) \qquad \alpha_M = \tfrac{1}{3}\tau\langle \mathbf{j}\cdot\mathbf{b}\rangle$$

Steady limit: consistent with VC92

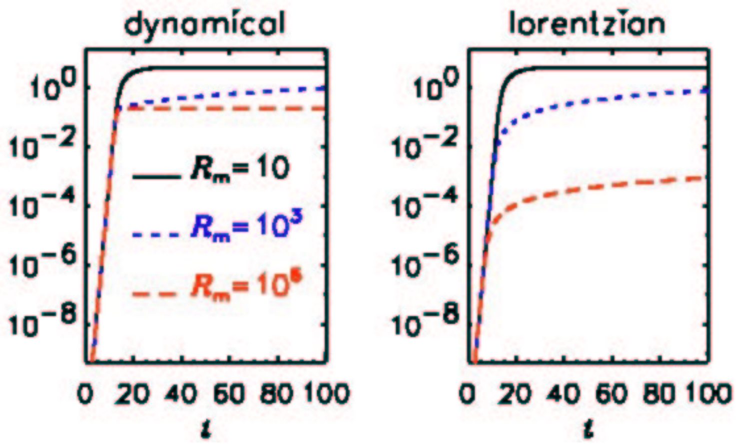
$$\alpha = \frac{\alpha_0 + \eta_t R_m \overline{\mathbf{J}}\cdot\overline{\mathbf{B}} / B_{eq}^2}{1 + R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$

Dynamical quenching in α^2-dynamo



→ Agrees better with simulations

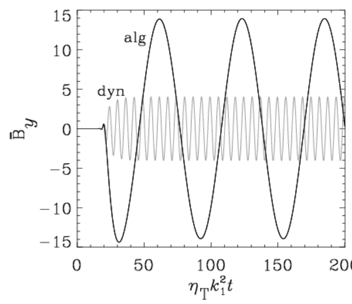
Dynamical quenching in α^2-dynamo

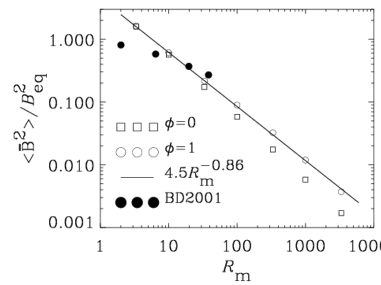


→ Dynamical quenching allows faster growth

Universality

- With lorentzian, *a* needs to be adjusted
- With dynamical quenching, $a=\lambda/(\eta k_1 k_f)$



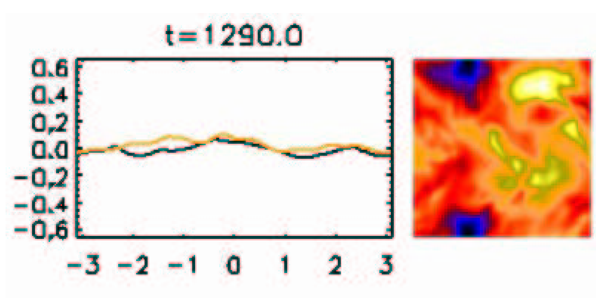


αΩ-dynamo: shorter periods

Open boundaries: right amplitude

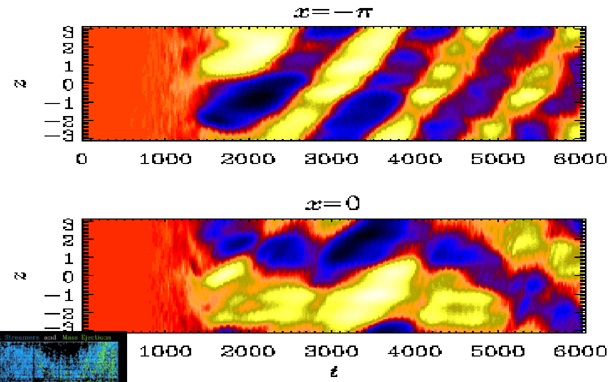
Nonlinearity of Large Scale Dynamos 29

Additional effect of shear



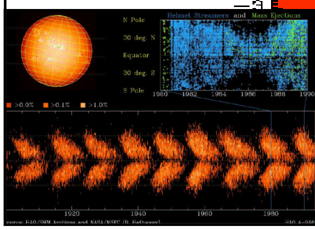
Nonlinearity of Large Scale Dynamos 30

Additional effect of shear



Negative shear

Positive shear



Nonlinearity of Large Scale Dynamos 31

Conclusions

- Solar large-scale field → collective effect
- Mean-field theory → α-effect ill understood
- α-effect, or nonlocal inverse cascade
  - – Resistive time scale
  - – Large scale field is still strong
  - – Many quenchings can be ruled out
  - – Dynamical quenching gives best description

Nonlinearity of Large Scale Dynamos 32