STUDYING THE ASYMMETRY OF BIPOLAR ACTIVE REGIONS BY MEANS OF THE THIN FLUX-TUBE APPROXIMATION

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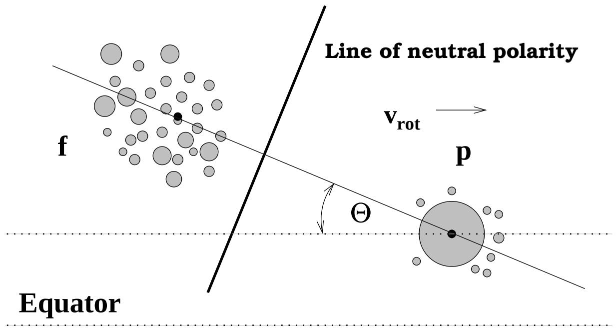
1. ASYMMETRIES OF BIPOLAR ACTIVE REGIONS



(a) p-spots move away from the line of neutral polarity faster than f-spots.

(b) f-spots lay closer to the lines of neutral polarity than p-spots.

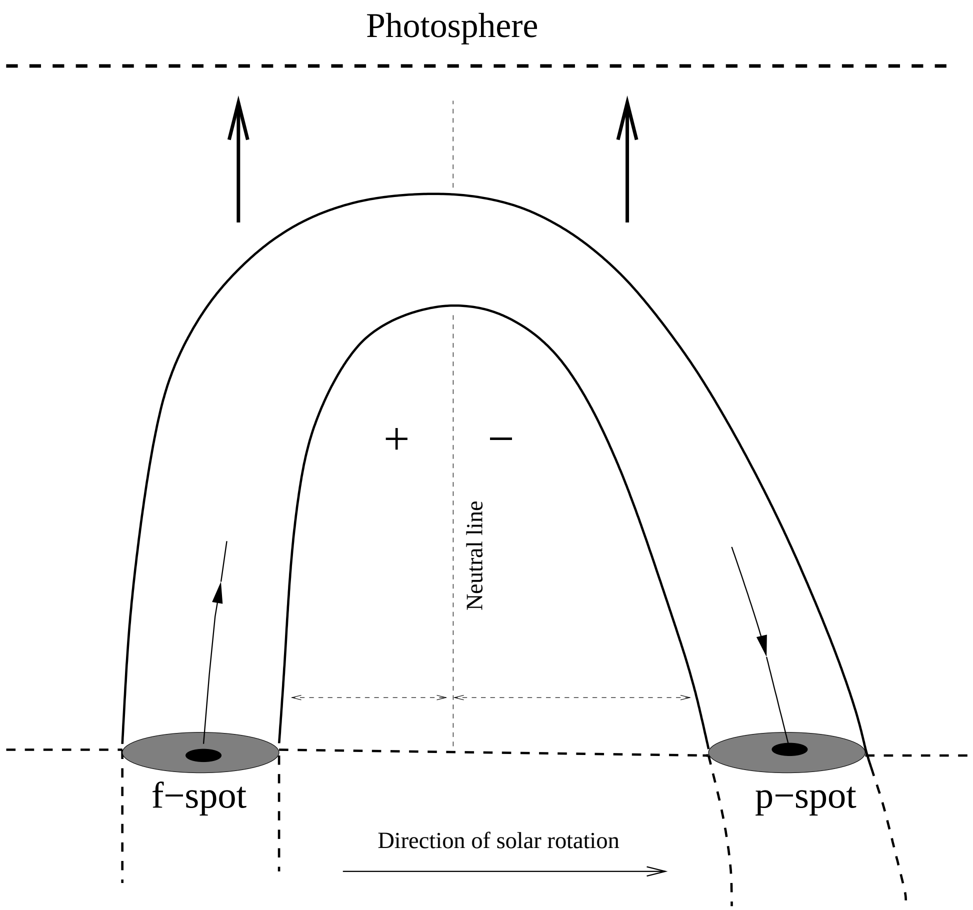
(c) p-spots are more stable than f-spots.

[Ref.: van Driel-Gesztelyi & Petrovay, *Solar Phys.* **126**, 285 (1990)]

These properties can be explained if one assumes that the magnetic field in f-spots is more vertical than in p-spots.
[Ref.: Wang & Sheely, *Astrophys. J.* **375**, 761 (1991)]

(d) Tilt angle of active region's main axis (by typically 10°) with respect to the equator.

Properties (a)–(c) can be explained if one assumes that the magnetic field in f-spots is more vertical than in p-spots. [Ref.: Wang & Sheely, *Astrophys. J.* **375**, 761 (1991)]



Geometrical asymmetry: Difference in inclination between p- and f-wing.

2. WHAT CAUSES THE GEOMETRICAL ASYMMETRY ?

The asymmetry has surely to do with the subphotospheric structure and evolution of the active region magnetic fields.

(a) Difference in inclination between p- and f-wing is a phenomenon with a local cause near the surface.

- van Driel-Gesztelyi & Petrovay, *Solar Phys.* **126**, 285 (1990).
- Petrovay *et al.*, *Solar Phys.* **127**, 51 (1990).
- Wang & Sheely, *Astrophys. J.* **375**, 761 (1991).
- Cauzzi, G., Canfield, R. C., & Fisher, G. H.: "A search for asymmetric flows in young active regions," *Astrophys. J.* **456**, 850 (1996).

(b) The difference in inclination results from the (nonlinear) magnetic flux evolution from the bottom of the convection zone.

- Moreno-Insertis, F., Caligari. P. & Schüssler. M.: "Active region asymmetry as a result of the rise of magnetic flux tubes," *Solar Phys.* **153**, 449 (1994).

⇒ Integrate the MHD equations (thin flux tube) starting from a toroidal flux tube at the bottom of the solar convection zone that becomes unstable.

3. MODEL AND BASIC EQUATIONS.

The thin flux tube approximation:

- Defouw, R.J. *Astrophys. J.* **209**, 266 (1976)
- Roberts, B., & Webb, A.R. *Solar Phys.* **56**, 5 (1978)
- Spruit, H.C.: *Astron. Astrophys.* **98**, 155 (1981)
- Ferriz-Mas, A., & Schüssler, M. *Geophys. Astrophys. Fluid Dynam.* **72**, 209 (1993)

Motion of a 1-D continuum (flux tube) in 3-D Euclidean space (rotating star).

"i" $\Rightarrow$ quantities inside the tube.

(a) Equation of motion:

$$\rho_i \frac{D\,\mathbf{v}_i}{D\,t} = -\,\mathbf{grad}\left(p_i + \frac{B^2}{8\pi}\right) + \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{4\pi} +$$

$$\rho_i[\mathbf{g} - \boldsymbol{\Omega}\wedge(\boldsymbol{\Omega}\wedge\mathbf{r})] + 2\rho_i\mathbf{v}_i\wedge\boldsymbol{\Omega}\,.$$

Define the Frenet basis of unit vectors $\{\mathbf{e}_t, \mathbf{e}_n, \mathbf{e}_b\}$ along the tube's axis. Equation of motion projected on this basis.

(b) Equation of continuity plus induction:

$$\frac{D}{Dt}\left(\frac{\rho_i}{B}\right) \quad + \quad \frac{\rho_i}{B}\left[\frac{\partial(\mathbf{v}_i\cdot\mathbf{e}_t)}{\partial s} - \kappa\,(\mathbf{v}_i\cdot\mathbf{e}_n)\right] \quad = \quad 0\,.$$

(c) Equation of energy: Isentropic evolution.

[Moreno-Insertis, F. Astron. Astrophys. **122**, 241 (1983): *For tubes with magnetic fluxes $\Phi \gtrsim 10^{19}$ Mx thermal insulation has to be assumed and not permanent thermal equilibrium.*]

(d) Constitutive relation: Ideal gas $p = \dfrac{\mathcal{R}}{\mu}\rho T\,.$

(e) Instantaneous lateral pressure balance:

$$p + \frac{B^2}{8\pi} = p_e\,.$$

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s = Arc-length at time t.

s_0 = Arc-length along the equilibrium path.

We take s_0 as Lagrangian coordinate $\Rightarrow$

$$\frac{D\,\mathbf{v}_i}{D\,t} = \left(\frac{\partial\mathbf{v}_i}{\partial t}\right)_{s_0}$$

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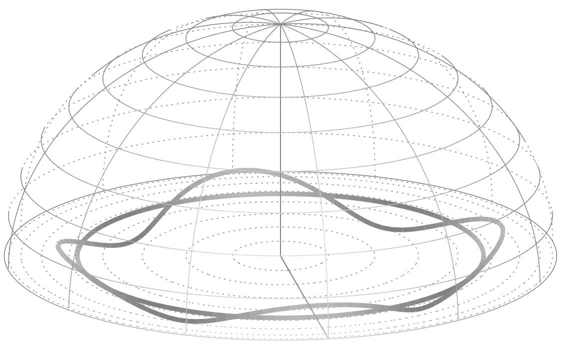
TOROIDAL FLUX TUBE IN A ROTATING STAR



Use cylindrical coordinates $(R, \phi, z) \Rightarrow \{\mathbf{e}_R, \mathbf{e}_\phi, \mathbf{e}_z\}$.

Choose non-inertial frame rotating with the constant angular velocity $\mathbf{\Omega} = \Omega \mathbf{e}_z$ of the matter in the equilibrium flux tube.

External matter rotates differentially (in steady state) with angular velocity

$$\mathbf{\Omega}_e(R, z) = \Omega_e(R, z)\mathbf{e}_z \,.$$

External velocity field (due solely to stellar rotation):

$$\mathbf{v}_e(R, z) = (\Omega_e - \Omega)\,\mathbf{e}_\phi \,.$$

Momentum eq. applied to the external medium:

$$\rho_e(\mathbf{v}_e \cdot \nabla)\mathbf{v}_e = -\mathbf{grad}\, p_e + \rho_e\,[\mathbf{g} - \mathbf{\Omega} \wedge (\mathbf{\Omega} \wedge \mathbf{r})] + 2\rho_e \mathbf{v}_e \wedge \mathbf{\Omega}$$

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4. ASYMMETRY AS A RESULT OF THE RISE OF MAGNETIC FLUX TUBES.

Numerical integration of the MHD equations (in the thin flux-tube approximation).

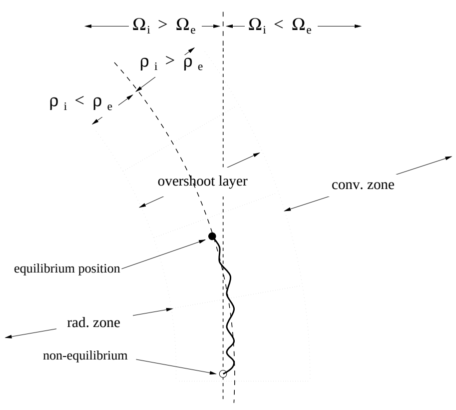
4.1. Initial conditions.

- Normal component of momentum equation yields the relationship between Ω y Ω_{e0}.
- Binormal component yields $\rho_{i0} = \rho_{e0}$.

$\Rightarrow$ **Non-buoyant flux tube** with

$$\Omega^2 = \Omega_{e0}^2 + \frac{v_A^2}{R_0^2} \quad \Rightarrow T_{i0} < T_{e0} \,.$$

[Ref.: Moreno-Insertis, F., Schüssler, M. & Ferriz-Mas, A. *Astron. Astrophys.* **264**, 686–700 (1992).]



An equilibrium toroidal flux tube just below the convection zone must have an azimuthal flow in the direction of solar rotation that is greater than the local rotation velocity of the unmagnetized surrounding plasma.

4.2. Results of numerical simulations.

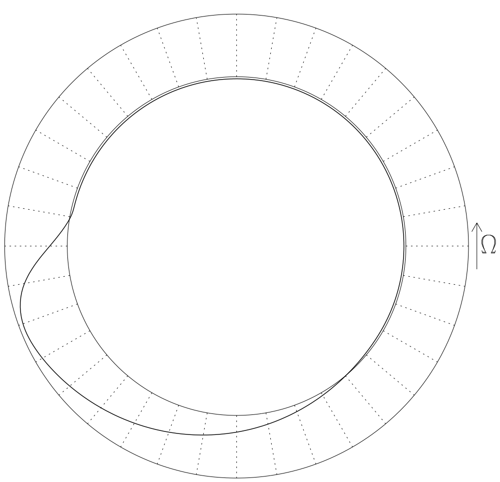
[Ref.: Moreno-Insertis, F., Caligari. P. & Schüssler. M. *Solar Phys.* **153**, 449 (1994).

Initial condition: toroidal flux tube at the bottom of the solar convection zone that becomes unstable.

• Flux tube initially in mechanical equilibrium is subject to small perturbations and becomes unstable.

• The tubes rises across the convection zone in $\simeq 30$ days and eventually reaches the photosphere.

• The follower wing of the highest loop is more inclined with respect to the horizontal than the preceding wing.



Time evolution of an unstable magnetic flux tube from the bottom of the convection zone to the surface. The direction of

5. LINEAR APPROACH

Is the asymmetry itself a result of the (non-linear) rise across the convection zone?

OR is there an asymmetry in the early stages (linear phenomenon)?

• Consider Lagrangian perturbations about the equilibrium path: $\mathbf{r}(s_0, t) = \mathbf{r}_0(s_0) + \boldsymbol{\xi}(s_0, t)$.

"0" $\Rightarrow$ equilibrium values. e.g.: $\Omega_{e0} \overset{\text{def}}{=} \Omega_e(r_0)$

• Linearize equations: $p_i = p_{i0} + p_{i1}$, $\rho_i = \rho_{i0} + \rho_{i1}$, ...

• Normal mode analysis $\boldsymbol{\xi} \sim \exp(i\omega t + im\phi_0) \quad \Rightarrow$

$$\left(\frac{\partial}{\partial t}\right)_{s_0} \equiv i\omega \qquad \text{and} \qquad \left(\frac{\partial}{\partial \phi_0}\right)_t \equiv im\,.$$

Dispersion relation:

$$P(\omega) = \omega^6 + c_4\,\omega^4 + c_3\,\omega^3 + c_2\,\omega^2 + c_1\,\omega + c_0 = 0\,.$$

Coefficients (real) determined by (r_0, λ_0), by B_0, Ω_{e0}, gradient of Ω_e, and the physical parameters at the equilibrium radius r_0 (as given by a solar model).

[Ref.: Ferriz-Mas & Schüssler, *Geophys. Astrophys. Fluid Dynam.* **72**, 209–247 (1993) and **81**, 233–265 (1995)].

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Flux tube lying in the equatorial plane.

Perturbations contained within the equatorial plane decoupled from those $\perp$ equatorial plane:

(a) Perturbations $\perp$ equatorial plane:

$$\omega^2 = (m^2 - 1)\,\frac{v_A^2}{r_0^2} - (\Omega_{e0}^2 - \Omega^2) = \left(m\frac{v_A}{r_0}\right)^2 \geq 0\,.$$

Instability only possible for $m = 0$ (poleward-slip instability).

(b) Perturbations contained within the equatorial plane:

$$\omega^4 + d_2\,\omega^2 + d_1\,\omega + d_0 = 0\,.$$

Analytical criterion for stability:

$$-\frac{4}{27}d_0\left(d_2^2 - 4d_0\right)^2 + d_1^2\left(\frac{d_2^3}{27} - \frac{4}{3}d_0 d_2 + \frac{d_1^2}{4}\right) < 0\,.$$

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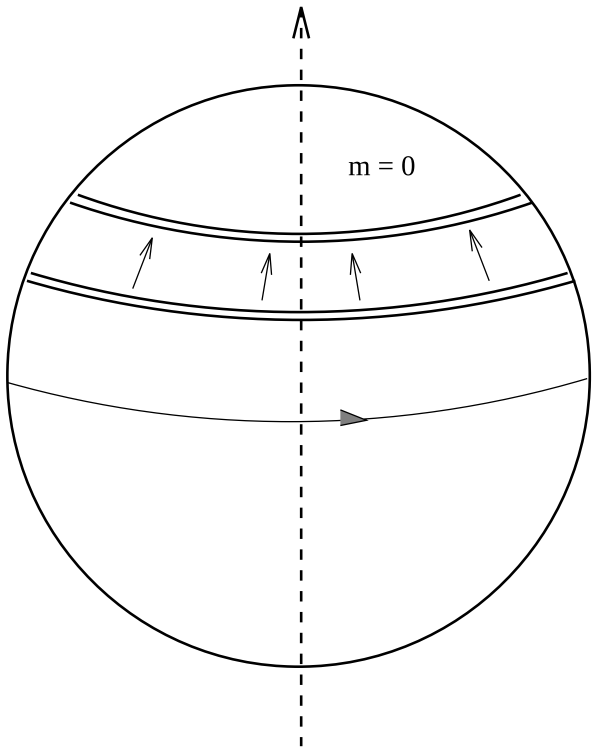
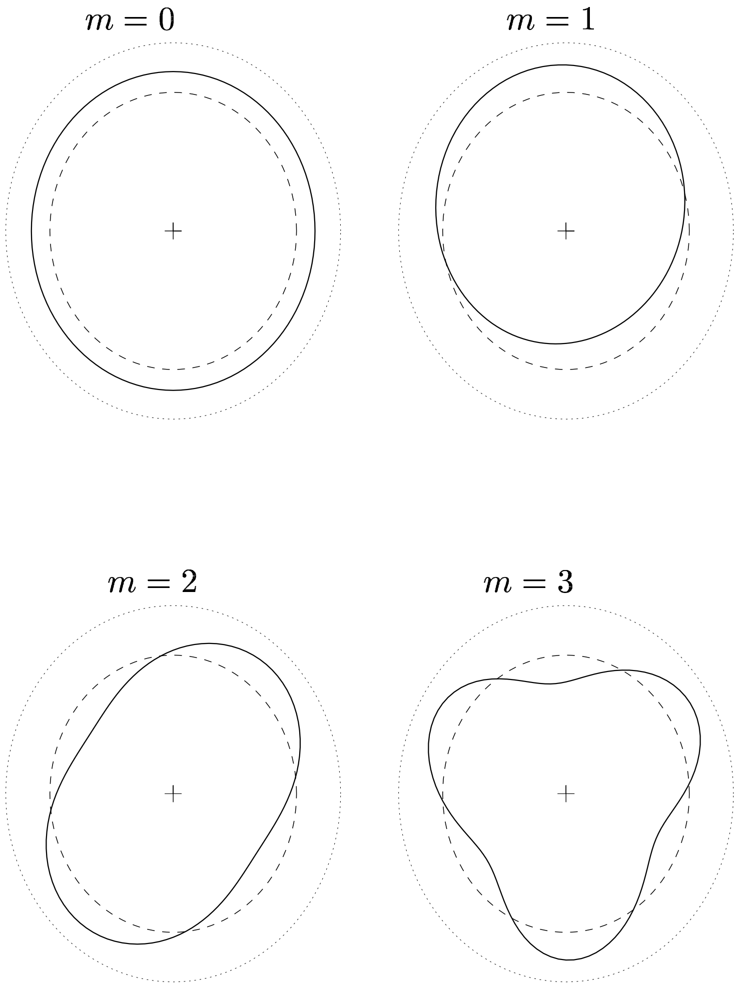
Perturbations $\perp$ equatorial plane



• Instability can set in for $m = 0$ (so-called "poleward-slip instability").

"Poleward-slip instability" (case without rotation) first discussed in Spruit & van Ballegooijen, *Astron. Astrophys.* **106**, 58 (1982)

Modes within the equatorial plane for m = 0, 1, 2, 3



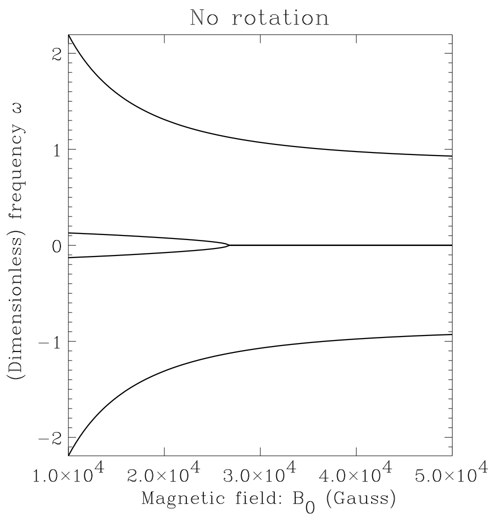
6. ROOTS OF THE DISPERSION RELATION

Algebraic fourth-order equation in the variable ω. Closed solutions in simple form for

- axisymmetric modes ($m = 0$)
- non-axisymmetric modes in the limit of rapid rotation.

(a) No rotation:

$$\omega_1 = +\sqrt{\omega_+^2} \quad \omega_2 = +\sqrt{\omega_-^2} \qquad \omega_\pm^2 = -\frac{d_2}{2} \pm \frac{1}{2}\sqrt{d_2^2 - 4d_0}\,.$$
$$\omega_3 = -\sqrt{\omega_-^2} \quad \omega_4 = -\sqrt{\omega_+^2}$$



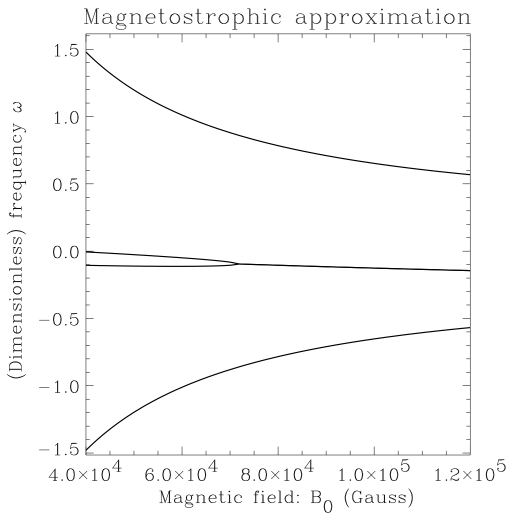
(b) Magnetostrophic approximation:

Limit of rapid rotation (or limit of weak fields) valid when $\omega_I^2 \gg \omega_A^2 \quad (4H_P^2\Omega^2 \gg v_A^2)$.

$$\omega_1 = +2\tilde{\Omega} \qquad \omega_4 = -2\tilde{\Omega}$$
$$\omega_2 = \omega_+ \qquad \omega_3 = \omega_- \,.$$

where the roots $\omega_\pm$ are given by

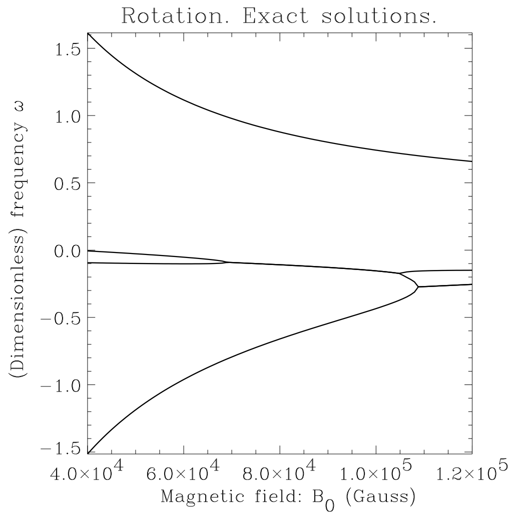
$$\tilde{\Omega}\,\omega_\pm = 2mf\left(f - \frac{1}{2\gamma}\right) \pm \frac{mf}{\sqrt{2}}\sqrt{2m^2f^2 - \frac{4}{\gamma}f + \frac{2}{\gamma}\left(\frac{1}{\gamma} - \frac{1}{2}\right) - \beta\delta + 4q_R\,\tilde{\Omega}_{e0}^2}\,.$$



(c) General case:

All coefficients of the dispersion relation are real $\Rightarrow$ the roots of $\omega^4 + d_2\omega^2 + d_1\omega + d_0 = 0$ are either real or pairs of complex conjugates.

For the bottom of the solar convection zone $H_P/r_0 \simeq 0.114$ so that $H_P/r_0 < 1/(2\gamma) \Rightarrow d_1 < 0$, and the number of real roots is determined by the signs of d_2 and d_0 alone.



7. FORMATION OF THE ASYMMETRY.

Fourier components: $\boldsymbol{\xi} = \hat{\boldsymbol{\xi}} \exp(i\omega t + im\phi_0)$.

Amplitude relation:

$$\hat{\xi}_\phi = i\frac{(\omega_I \omega - \omega_A \omega_H)}{(\omega^2 - \omega_A^2)}\,\hat{\xi}_r\,,$$

where

$$\omega_I \stackrel{\text{def}}{=} 2\Omega\,, \qquad \omega_A \stackrel{\text{def}}{=} v_A\left(\frac{m}{r_0}\right)\,,$$

$$\omega_H \stackrel{\text{def}}{=} 2v_A\left(\frac{1}{r_0} - \frac{1}{2\gamma H_P}\right) \quad < 0 \text{ for bottom solar C.Z.}$$

[H_P is the local pressure scale-height].

- In general, ω is complex: $\omega = \eta - i\,\sigma$.
- A mode is unstable if $\Im m(\omega) < 0$ (i.e., if $\sigma > 0$).

$$\hat{\xi}_\phi = i\frac{(\omega_I \omega - \omega_A \omega_H)}{(\omega^2 - \omega_A^2)}\,\hat{\xi}_r = (P - i\,Q)\hat{\xi}_r\,,$$

where

$$P = \frac{\omega_I \sigma(\eta^2 - \sigma^2 - \omega_A^2) - 2\eta\sigma\,(\omega_I \eta - \omega_A \omega_H)}{\left(\eta^2 - \sigma^2 - \omega_A^2\right)^2 + 4\eta^2\sigma^2}\,,$$

$$Q = \frac{2\eta\sigma^2\omega_I + (\omega_I\eta - \omega_A\omega_H)(\eta^2 - \sigma^2 - \omega_A^2)}{(\eta^2 - \sigma^2 - \omega_A^2) + 4\eta^2\sigma^2}\,.$$

7.1. Lagrangian description.

Taking $\Re e(\xi_r) = C\cos(\omega\,t + m\phi_0)$

(C real amplitude) and using the amplitude relation we have

$$\Re e(\xi_\phi) = C\exp(\sigma\,t)[P\cos(\eta t + m\phi_0) + +Q\sin(\eta t + m\phi_0)]\,.$$

7.2. Eulerian description.

$\boldsymbol{\xi} = \boldsymbol{\xi}(s_0, t)$ is the displacement vector as a function of the Lagrangian coordinate s_0 (or ϕ_0).

Call $\mathbf{X}$ the displacement as a function of the Eulerian coordinate ϕ:

$$\mathbf{X} = \mathbf{X}(\phi, t)\,.$$

Then

$$\Re e(\xi_\phi) = C\exp(\sigma\,t)[P\cos(\eta t + m\phi_0) + +Q\sin(\eta t + m\phi_0)] \quad \Rightarrow$$

$$\Rightarrow \Re e(X_\phi) = C\exp(\sigma\,t)\left[P\cos\left(\eta t + m\phi_0 - m\frac{X_\phi}{r_0}\right) + +Q\,\sin\left(\eta t + m\phi_0 - m\frac{X_\phi}{r_0}\right)\right].$$

Implicit assumption of small perturbation (linear stability analysis) $\Rightarrow |\boldsymbol{\xi}| = |\mathbf{X}| \ll r_0 \Rightarrow$

$\epsilon \stackrel{\text{def}}{=} \dfrac{X_\phi}{r_0}$ can be taken as a small parameter

Up to first order in the perturbations:

$$\Re e[X_\phi(\phi,t)] \simeq C\exp(\sigma\, t)\,(P\,\cos\theta + Q\,\sin\theta) + \\ + \frac{m}{r_0}C^2\exp(2\sigma t)\,\left[\frac{P^2-Q^2}{2}\sin 2\theta - PQ\cos 2\theta\right].$$

$$\Re e[X_r(\phi,t)] \simeq C\exp(\sigma t)\times \\ \times\left[P\cos\theta + \frac{m}{r_0}C\exp(\sigma t)(P\cos\theta + Q\sin\theta)\sin\theta\right] = \\ = A_1\,\cos\theta + A_2\sin^2\theta + A_3\,\sin\theta\,\cos\theta\,.$$

- The asymmetry is due to the term

$$(m/r_0)\,C^2\exp(2\sigma\, t)P\cos\theta\,\sin\theta$$

and is proportional to the factor P:

$$P = \frac{\omega_I\sigma(\eta^2-\sigma^2-\omega_A^2) - 2\eta\sigma\,(\omega_I\eta - \omega_A\omega_H)}{(\eta^2-\sigma^2-\omega_A^2)^2 + 4\eta^2\sigma^2}\,.$$

$$P = \frac{\omega_I\sigma(\eta^2-\sigma^2-\omega_A^2) - 2\eta\sigma\,(\omega_I\eta - \omega_A\omega_H)}{(\eta^2-\sigma^2-\omega_A^2)^2 + 4\eta^2\sigma^2}\,.$$

(a) No rotation $\Rightarrow$ P vanishes.

(b) Rotation in the stable regime:

$$\sigma \equiv 0 \Rightarrow \quad P \equiv 0.$$

(c) Instability in the magnetostrophic approximation:

Necessary condition is $\omega_I^2 \gg \omega_A^2$.

$$\eta = \mathrm{Re}\,(\omega_\pm) = \frac{\omega_A\,\omega_H}{\omega_I} \Rightarrow$$

$$P = \frac{\omega_I\sigma(\eta^2-\sigma^2-\omega_A^2)}{(\eta^2-\sigma^2-\omega_A^2)^2 + 4\eta^2\sigma^2}\,.$$

Only the case $\sigma > 0$ is of interest. Sign of P determined by sign of $(\eta^2-\sigma^2-\omega_A^2)$, but

$$\omega_I^2 \gg \omega_A^2 \Rightarrow \eta^2 - \sigma^2 - \omega_A^2 < 0 \Rightarrow P < 0\,.$$

$P < 0$ yields the correct asymmetry.

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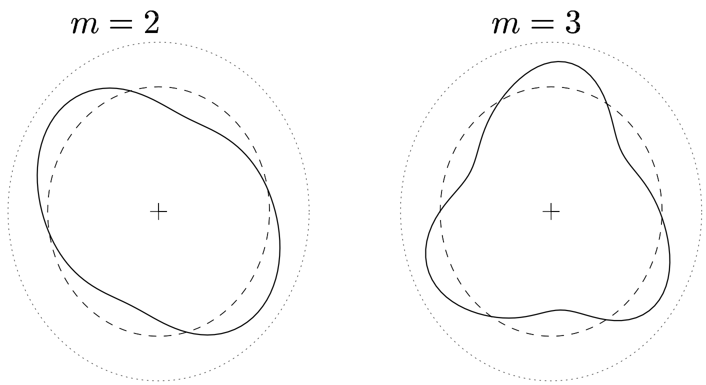
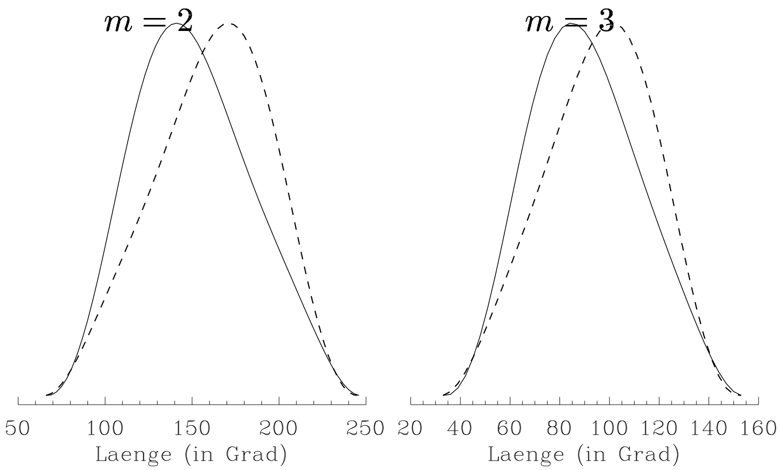
Asymmetry for unstable modes with m = 2 and m = 3





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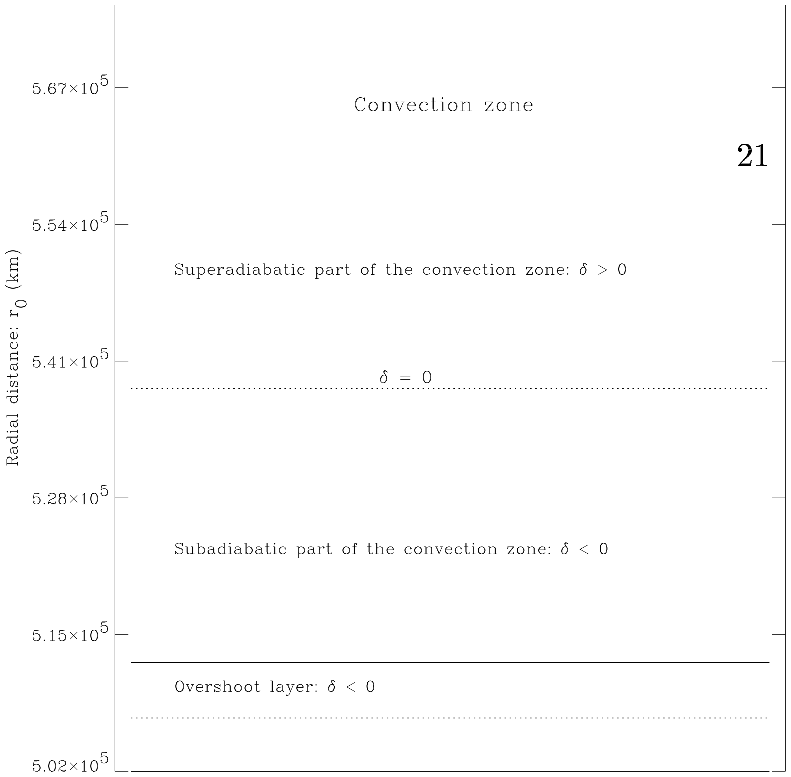

Beginning of the radiative zone

r_0 (km)	δ	p (dyn cm^{-2})	g (cm s^{-2})
5.591e+5	2.001e-7	1.502e+13	42073.27
5.385e+5	-1.781e-9	2.351e+13	45189.98
5.126e+5	-4.232e-7	3.891e+13	49582.96
5.073e+5	-9.071e-7	4.283e+13	50547.93
5.023e+5	-1.170e-4	4.689e+13	51500.07