Reconnection of Twisted Flux Tubes

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Motivation

- Sonnerup (1974) 2.5D theory for reconnection of two sheets of equal field strengths:

  Field reconnects at any angle,

  speed of reconnection slows down as sin(theta/2).

  Is this true in 3D?
- Flare models for colliding/reconnecting flux tubes: Gold & Hoyle (1960), Heyvaerts et al. (1977), Sturrock et al. (1984), Priest et al. (1994), Farnik et al. (1996).

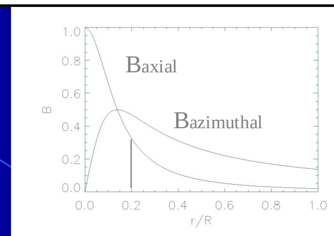
  Are they viable?
- Flux tubes are building blocks of magnetic field:

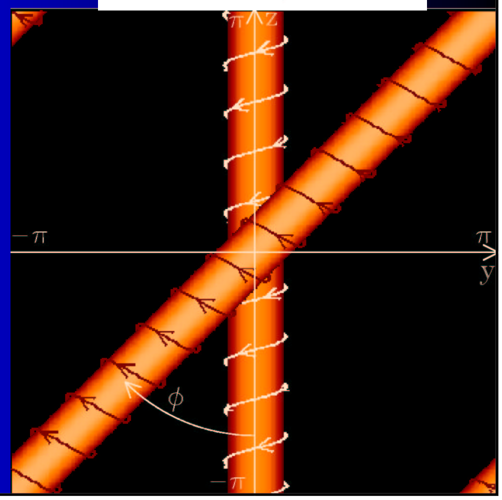
  in convection zone, solar atmosphere, solar wind or Earth's magnetosphere.

  Flux tube reconnection should be generally applicable.

Simulation setup

- Two identical Gold-Hoyle magnetic flux tubes
  Twist: qL/2pi = 10.
- Back tube R twisted
- Front tube R or L twisted,
- Collision angle phi = n pi/4:
  to give RLn or RRn
- Visco-resistive MHD, Lundquist numbers ~ 600
- Periodic box, spectral code
- Push tubes together with initial stagnation point flow at 1/40 of Alfven speed on axis



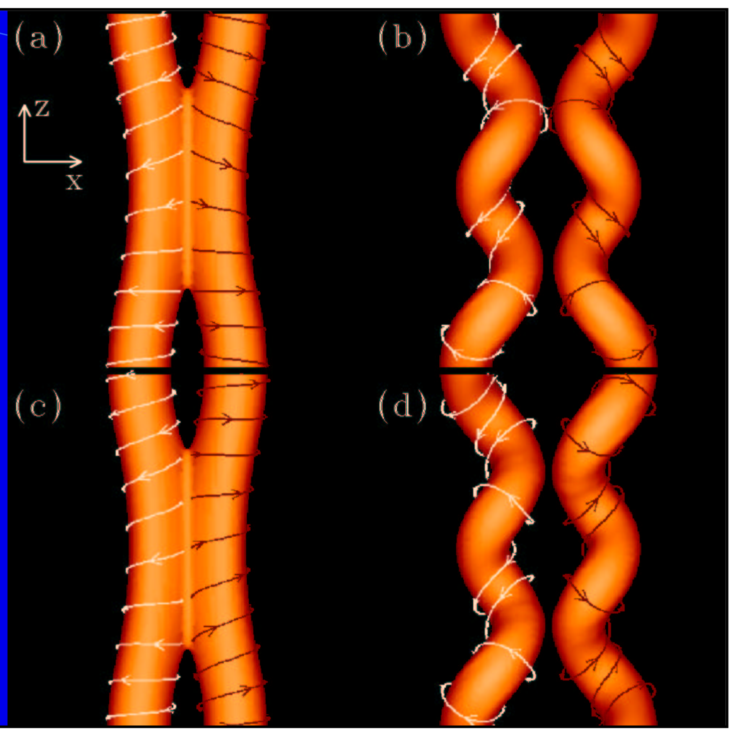


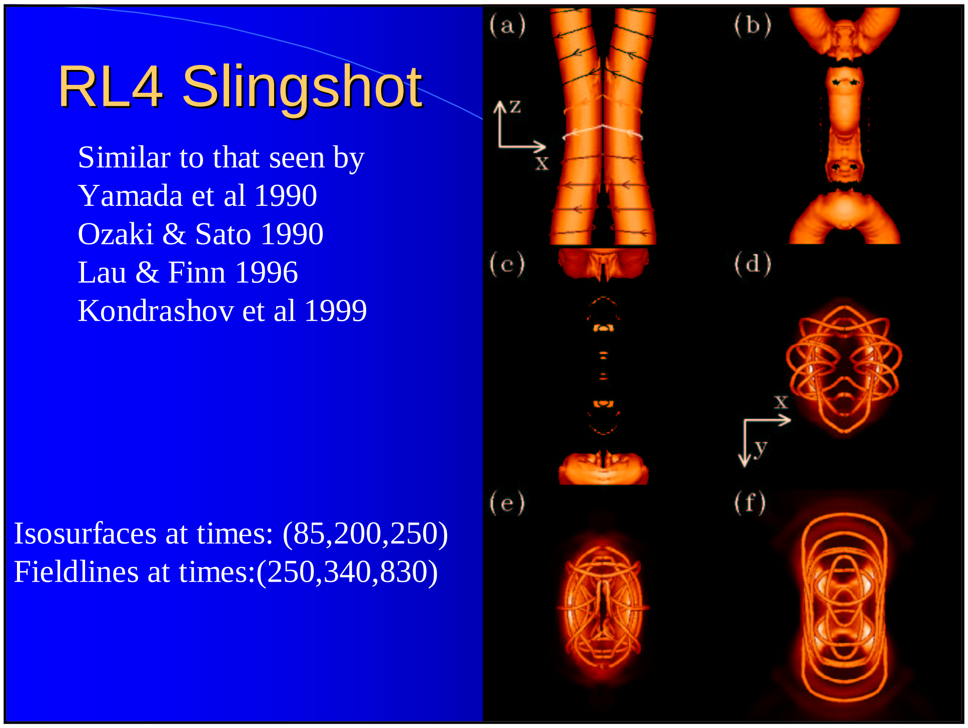
RL0, RR4 Bounce

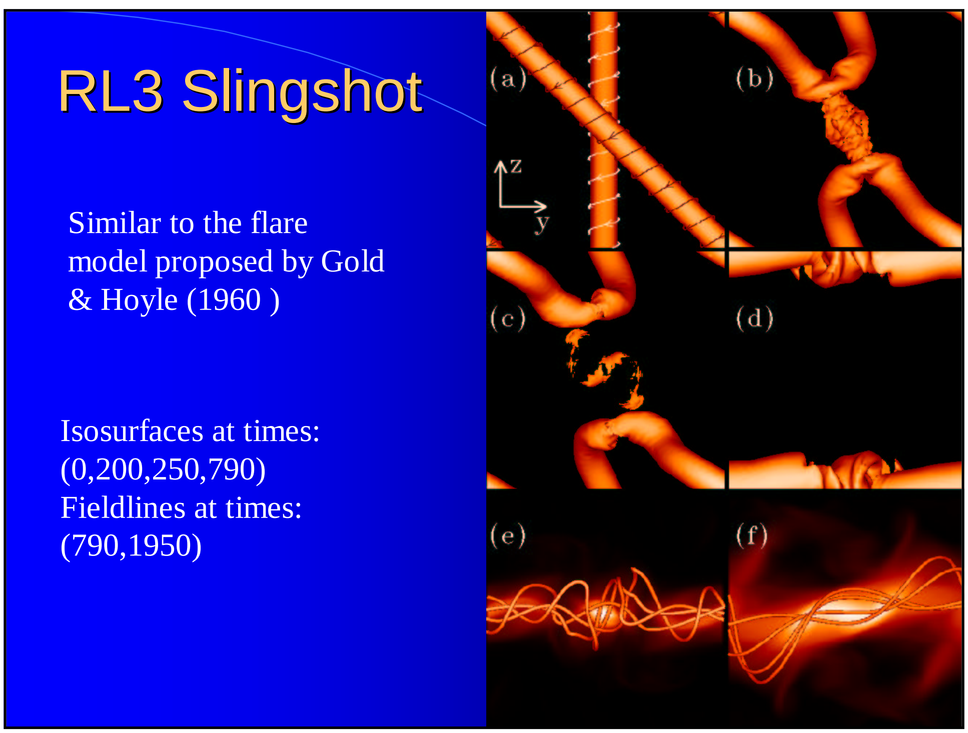
Top: RL0

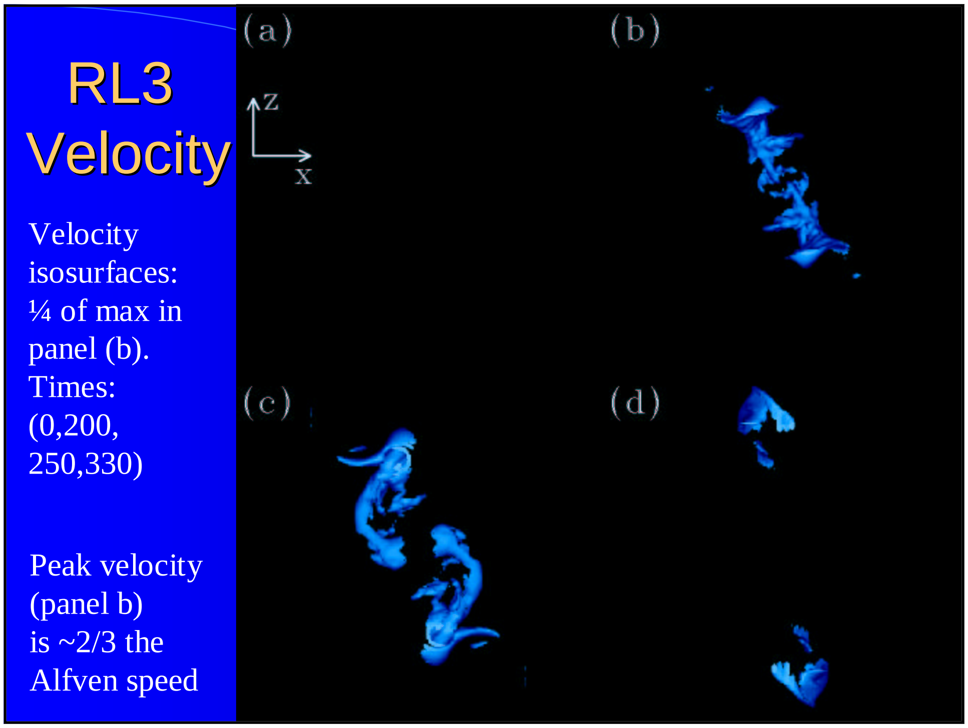
Bottom: RR4

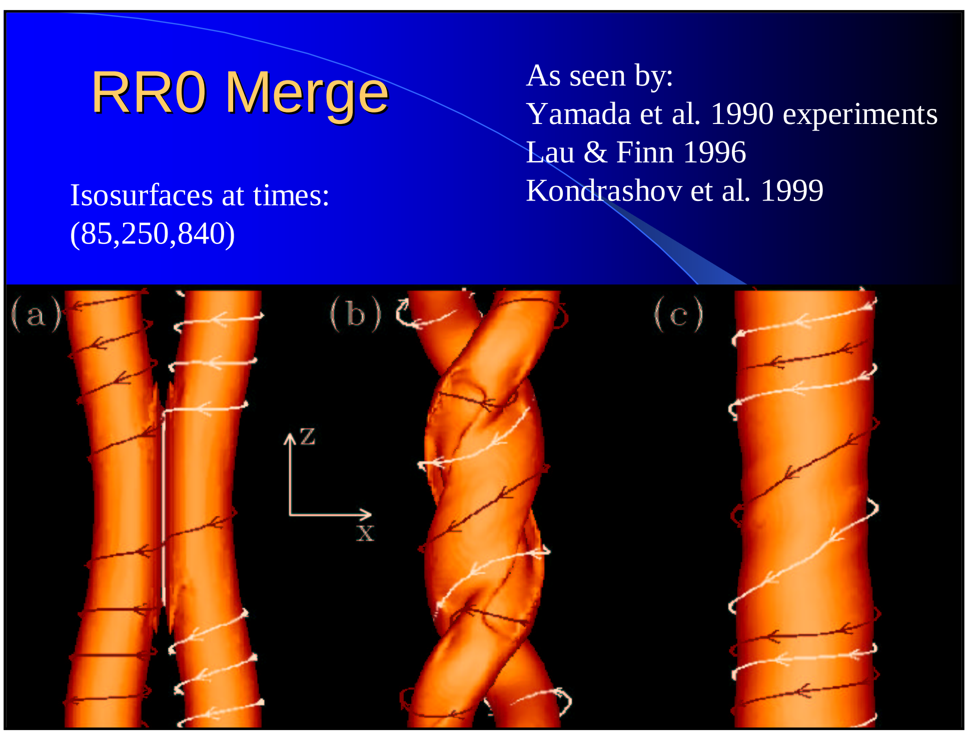
Magnetic isosurfaces: 1/3 of max at 85 and 280 Alfven crossing times

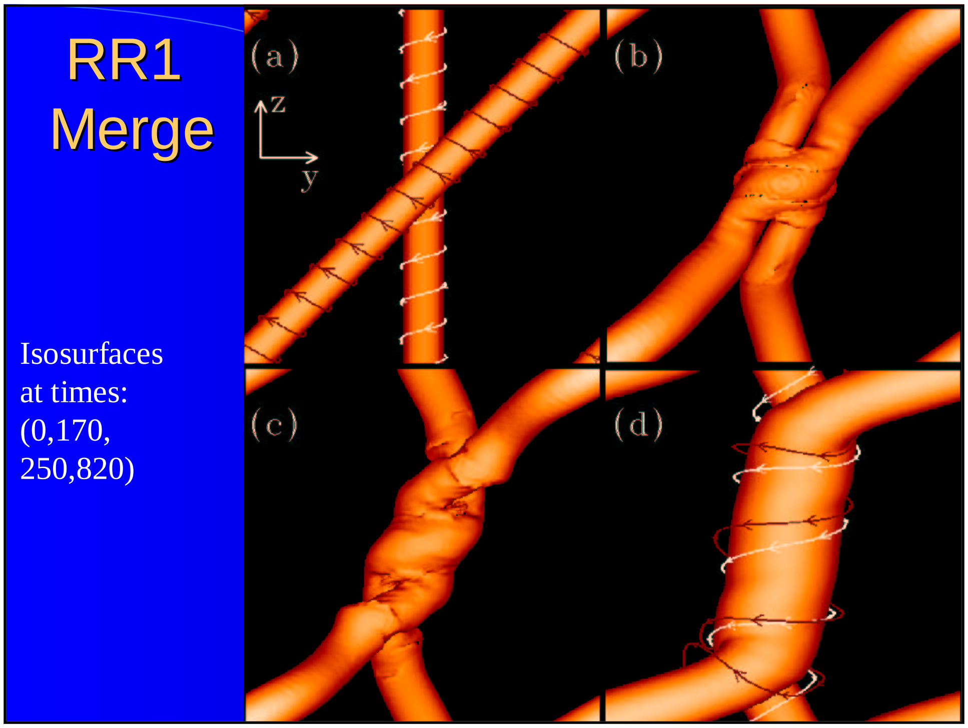


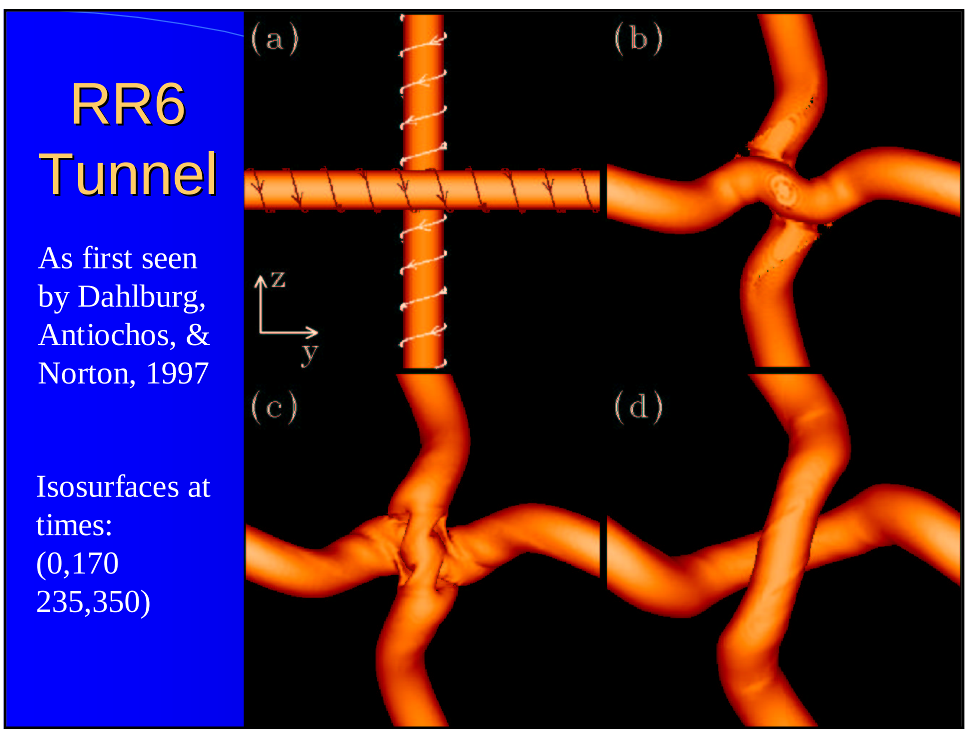

RL4 Slingshot
Similar to that seen by
Yamada et al 1990
Ozaki & Sato 1990
Lau & Finn 1996
Kondrashov et al 1999
Isosurfaces at times: (85,200,250)
Fieldlines at times:(250,340,830)
(a)
(b)
(c)
(d)
(e)
(f)
z
x
x
y


RL3 Slingshot
Similar to the flare
model proposed by Gold
& Hoyle (1960)
Isosurfaces at times:
(0,200,250,790)
Fieldlines at times:
(790,1950)
(a)
(b)
(c)
(d)
(e)
(f)
z
y


RL3
Velocity
Velocity isosurfaces: ¼ of max in panel (b). Times: (0,200, 250,330)
Peak velocity (panel b) is ~2/3 the Alfven speed
(a)
(b)
(c)
(d)
z
x


RR0 Merge
Isosurfaces at times: (85,250,840)
As seen by:
Yamada et al. 1990 experiments
Lau & Finn 1996
Kondrashov et al. 1999
(a)
(b)
(c)
z
x

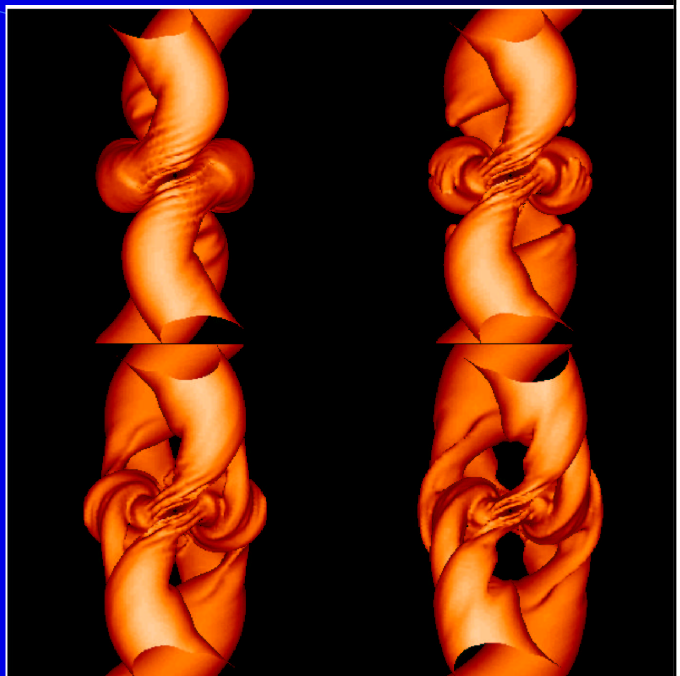

RR1
Merge
Isosurfaces
at times:
(0,170,
250,820)
(a)
z
y
(b)
(c)
(d)


RR6
Tunnel
As first seen
by Dahlburg,
Antiochos, &
Norton, 1997
Isosurfaces at
times:
(0,170
235,350)
(a)
(b)
z
y
(c)
(d)

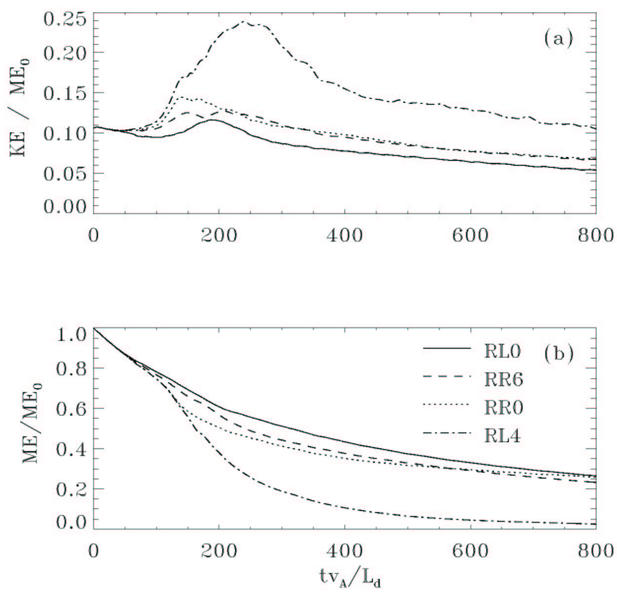
Closeup of Tunnel

Both flux tubes reconnect twice: front and back from this view

Isosurfaces at times: (210,235, 245,260)



Global Energetics



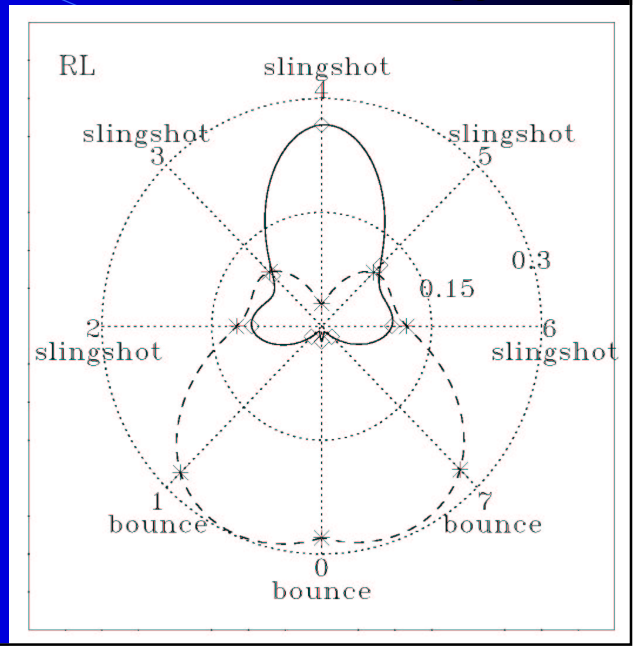
Next: look at peak kinetic energy and final magnetic energy for each simulation

RL0: bounce
RR6: tunnel
RR0: merge
RL4: slingshot

RL Reconnection Energy

Solid curve:
twice the peak
kinetic energy

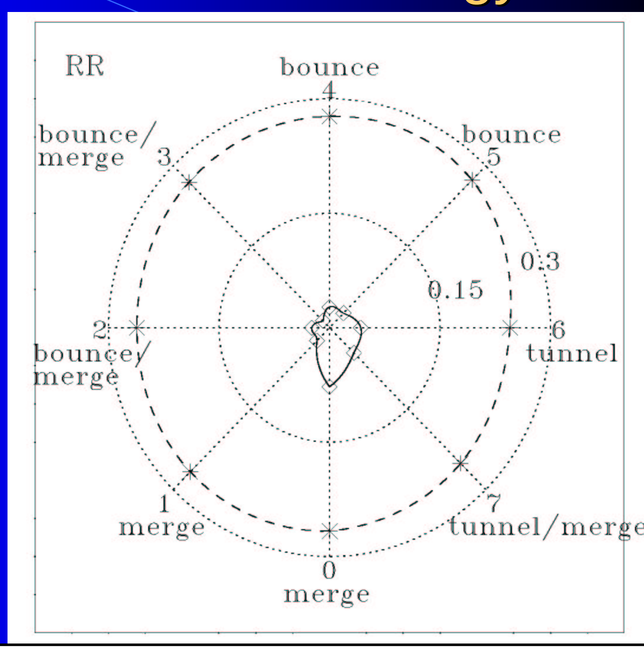
Dashed curve:
magnetic energy
remaining at end of
simulation
(750 Alfven times)



RR Reconnection Energy

Solid: kinetic

Dashed: magnetic



Relative Helicity for a pair of flux tubes

Following Berger 1984, Wright & Berger 1989

$$H_r = \int_V (\mathbf{A} + \mathbf{A}')\cdot(\mathbf{B} - \mathbf{B}')\, d^3x = 2H_m + 2H_t = const.$$

$$H_m = \frac{\delta}{2\pi}\Phi^2$$

Mutual helicity depends on the angle by which the tubes rotate about each other.

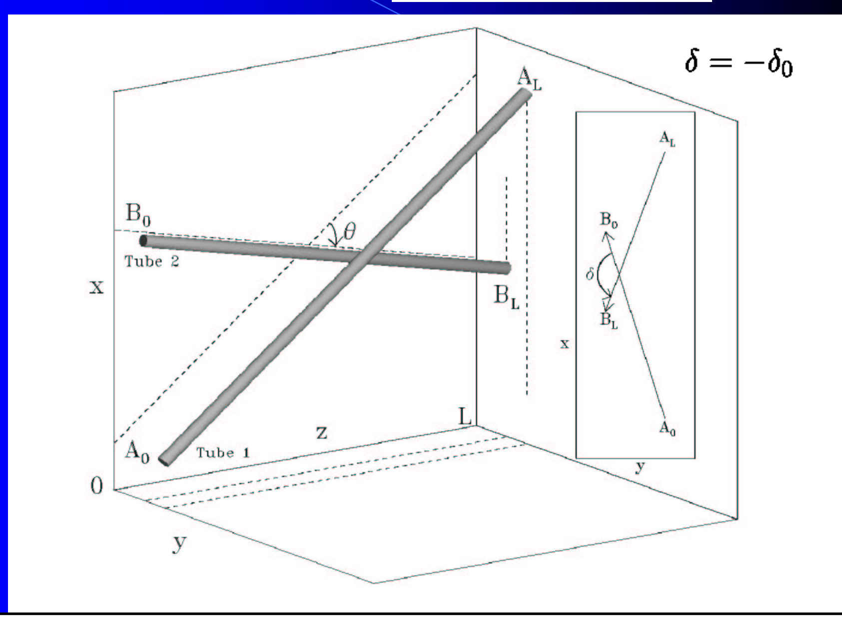
$$H_t = \frac{qL}{2\pi}\Phi^2 = N\Phi^2$$

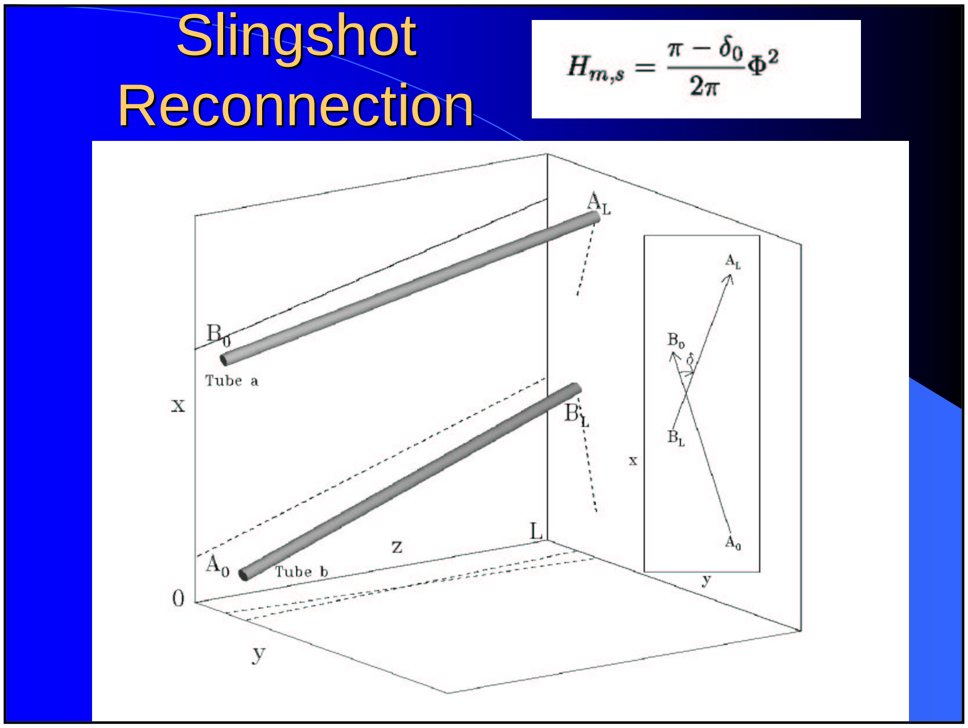
Twist helicity depends on the angle by which the fieldlines rotate about the tube axis.

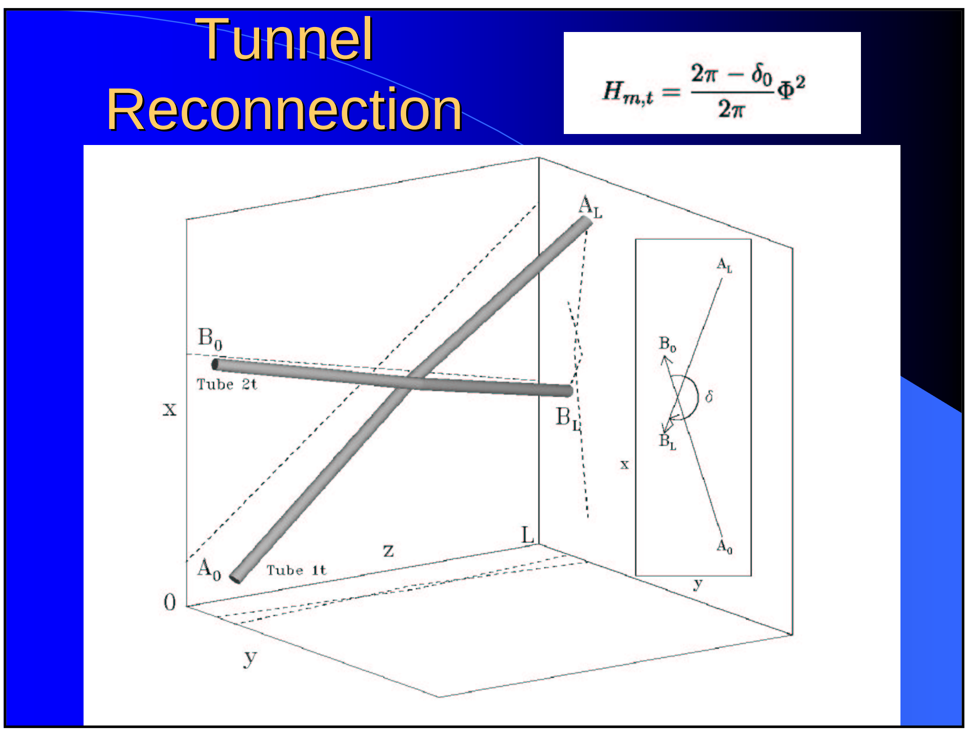
So that: $H_r = \Phi^2\left(2N + \frac{\delta}{\pi}\right)$

Assume helicity is conserved in reconnection: sum of mutual and twist helicity is constant.

Initial State: $H_{m,0} = \frac{-\delta_0}{2\pi}\Phi^2$




Slingshot
Reconnection
$H_{m,s} = \frac{\pi - \delta_0}{2\pi}\Phi^2$
A_L
B_0
Tube a
x
B_L
A_0
Tube b
z
L
0
y
x
y


Tunnel
Reconnection
$H_{m,t} = \frac{2\pi - \delta_0}{2\pi}\Phi^2$
A_L
B_0
Tube 2t
x
B_L
A_0
Tube 1t
z
L
0
y
δ
x
y

Helicity conservation determines the post-reconnection twist:

Initial helicity

$$H_0 = \left(2N_0 - \frac{\delta_0}{\pi}\right)\Phi^2$$

Slingshot helicity

$$H_s = \left(2N_s - \frac{\delta_0}{\pi} + 1\right)\Phi^2$$

Tunnel helicity

$$H_t = \left(2N_t - \frac{\delta_0}{\pi} + 2\right)\Phi^2$$

Helicity conservation:

$$H_0 = H_s = H_t$$

Tubes each lose half a turn of twist

$$N_s = N_0 - \frac{1}{2}$$

Tubes each lose a full turn of twist

$$N_t = N_0 - 1$$

Assumptions for equilibrium states

Flux tube field profile: Same form before and after reconnection, but constants can change.

$$B_{axial} = bf(\lambda r)$$
$$B_{azimuthal} = qrB_{axial}$$

Helicity conservation

$$N_0 = N_s - \frac{1}{2} = N_t - 1$$

Mass conservation

$$\rho L \pi R^2 = \rho L_0 \pi R_0^2$$

Flux conservation

$$\Phi = \Phi_0 = b_0 \int_0^{R_0} 2\pi f(\lambda_0 r) r dr = \frac{2\pi b_0}{\lambda_0^2}\Gamma_{11}(\lambda_0 R_0)$$
$$= \frac{2\pi b}{\lambda^2}\Gamma_{11}(\lambda R)$$

For identical initial tubes can show:

$$\lambda R = \lambda_0 R_0,$$

Predicted Energies for RR6

Initial state:

$$E'_0 = \Gamma_{12} + \Gamma_{32}$$

Slingshot:

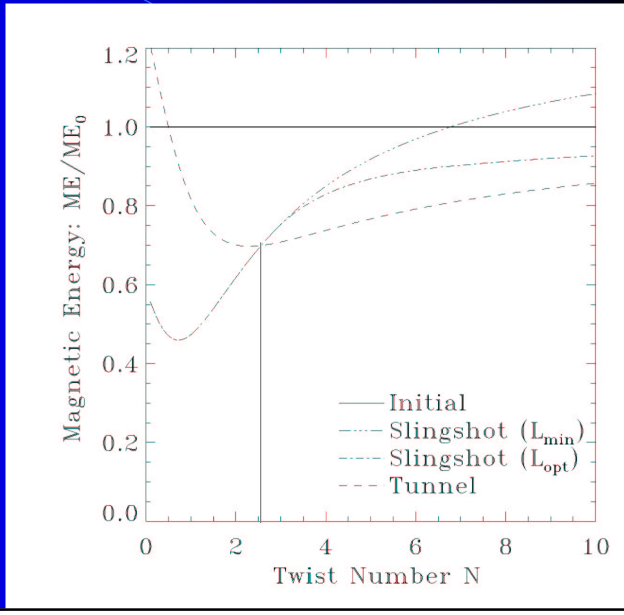
$$E'_s = \frac{L_s^2}{L_0^2}\Gamma_{12} + \frac{L_0}{L_s}\frac{(N_0 - 1/2)^2}{N_0^2}\Gamma_{32}$$

Tunnel:

$$E'_t = \Gamma_{12} + \frac{(N_0 - 1)^2}{N_0^2}\Gamma_{32}$$

Normalization:

$$E' = 2q_0^2 E/(L_0 b_0^2)$$



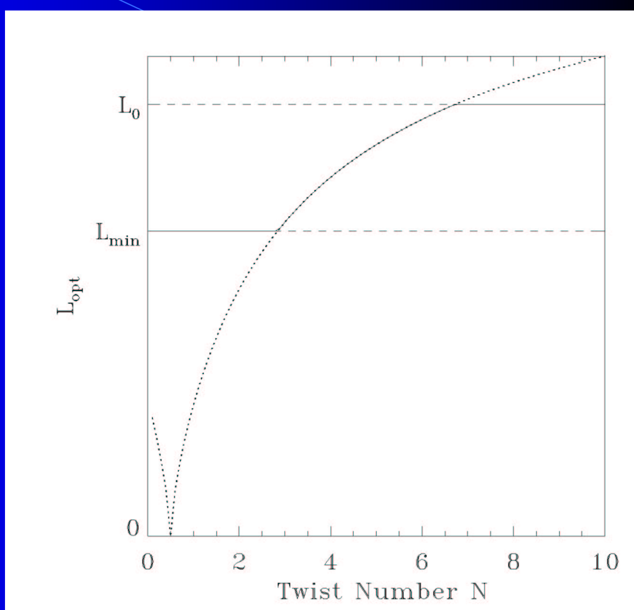
Optimal Length for Slingshot

1) Shortest length allowed by geometry:

$$L_{min} = L_0/\sqrt{2}$$

2) Optimal length, minimizing energy

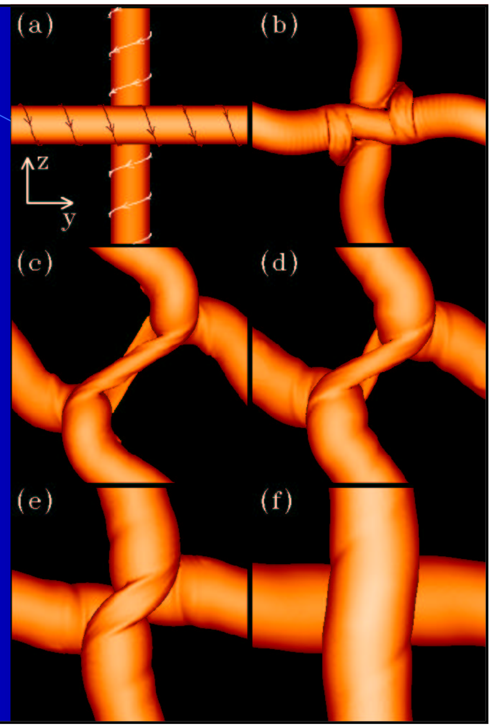
$$L_{opt} = L_0\left(\frac{N_s^2\Gamma_{32}}{2N_0^2\Gamma_{12}}\right)^{1/3}$$



RR6 N=6: Tunnel

- Still tunnels at N=6
- Comes close to slingshot, however

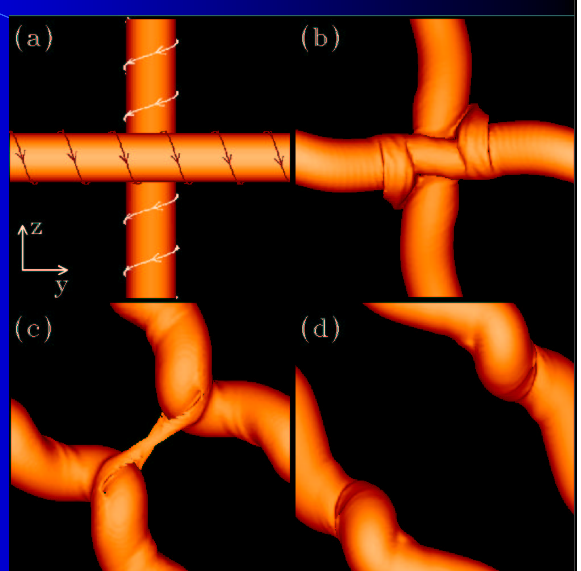
Isosurfaces at times:
(0,150,
480,789,
1115,2700)



RR6 N=5.5: Slingshot

- Transition at N=5.5 higher than prediction of N=2.5
- Final state kinked: minimum energy length is longer than the shortest possible length

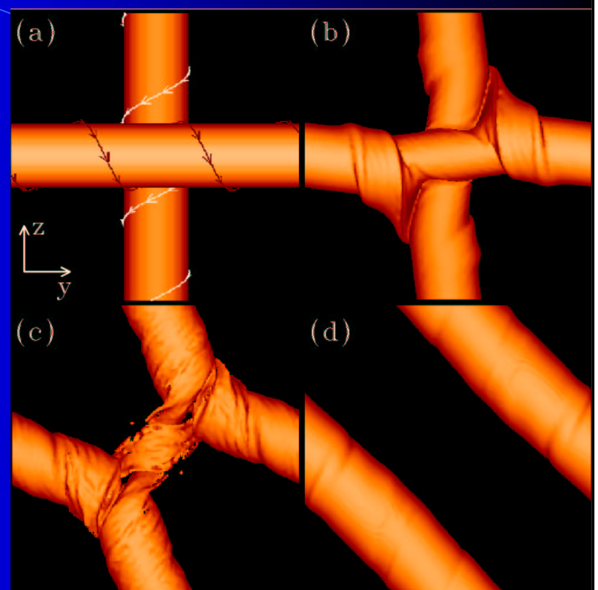
Isosurfaces at times:
(0,150,290,450)



RR6 N=3: Slingshot

- Tubes no longer kink at this twist: optimal length is shortest length

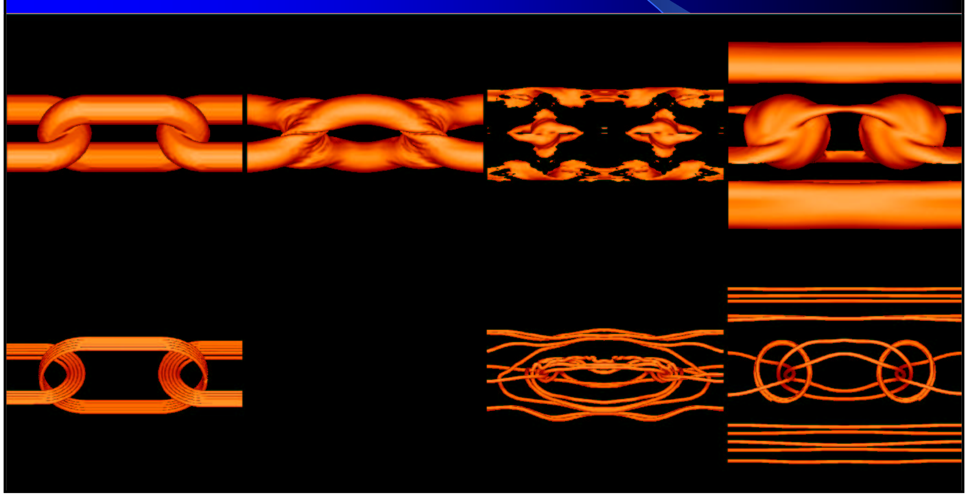
Isosurfaces at times: (0,140,220,500)

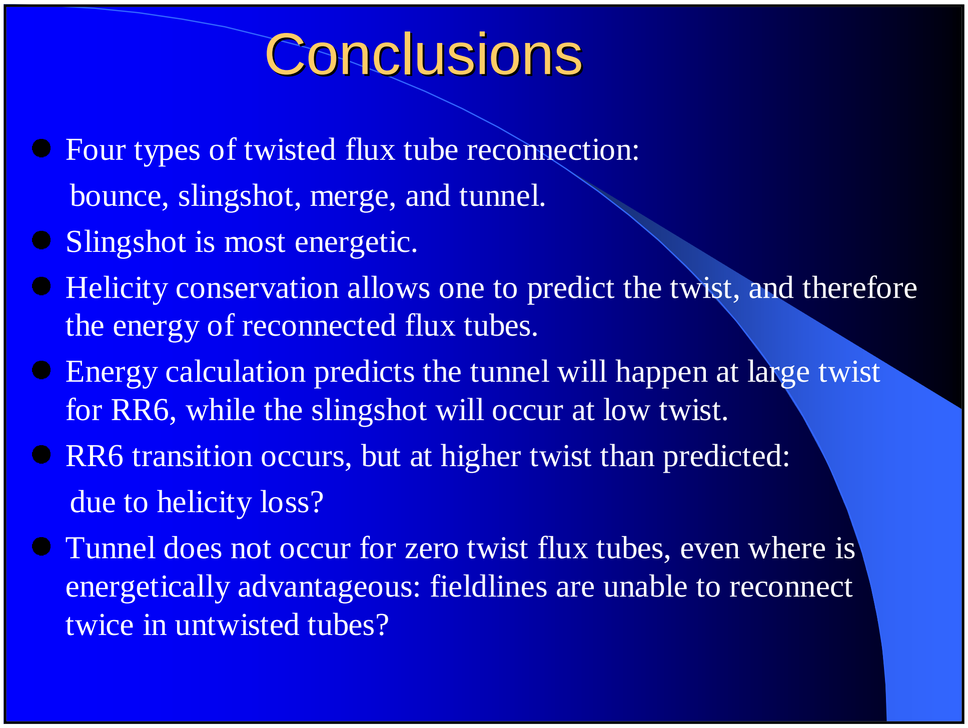


Zero twist reconnection

- Energetically, tubes should tunnel
- In fact ~ 2/3 of flux slingshots
- Tunnel is inaccessible to fieldlines?

Isosurfaces at times ~(0,2,12,60)




Conclusions
Four types of twisted flux tube reconnection:
bounce, slingshot, merge, and tunnel.
Slingshot is most energetic.
Helicity conservation allows one to predict the twist, and therefore the energy of reconnected flux tubes.
Energy calculation predicts the tunnel will happen at large twist for RR6, while the slingshot will occur at low twist.
RR6 transition occurs, but at higher twist than predicted:
due to helicity loss?
Tunnel does not occur for zero twist flux tubes, even where is energetically advantageous: fieldlines are unable to reconnect twice in untwisted tubes?