The Structure and Dissipation of Hierarchical Current Sheets

with a discussion of

When and How to Apply Adaptive Meshes, and When Not

Åke Nordlund
Niels Bohr Institute for Astronomy, Physics, and Geophysics, Univ. of Copenhagen

Power Laws

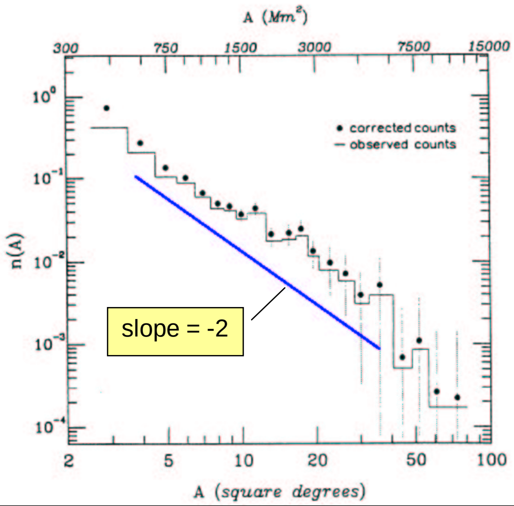
Power laws are common in astrophysics. The occurrence of power laws in general indicates that the same type of phenomenon can be observed over a range of scales, with no obvious preferred scale.

- Emerging flux (Zwaan, Harvey, Martin, …)
  - About the same flux per logarithmic interval
- Flare event size (Dennis, Crosby, …)
  - Somewhat decreasing fluency with log E
- Velocity power spectra
  - Surprisingly similar for sub- and super-sonic turbulence

Emerging Flux

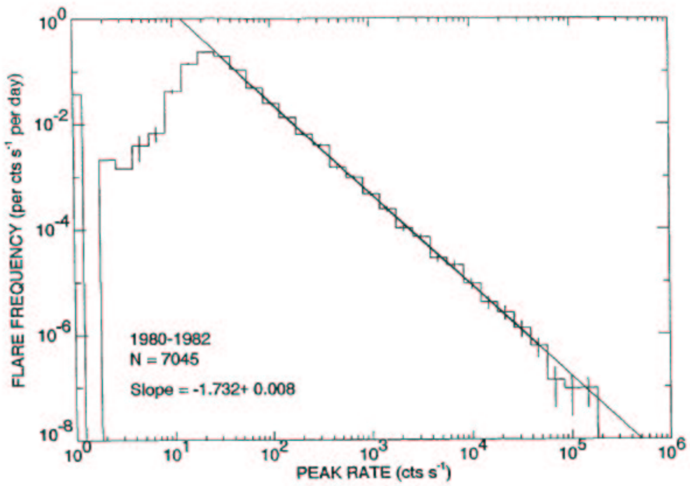
KAREN L. HARVEY AND CORNELIS ZWAAN

Solar Physics **148:** 85–118, 1993.



Flare Peak Rate Distribution

(Crosby, Aschwanden & Dennis, SPh 1993)



Dissipative Structures

- Sub-sonic turbulence
  - Vortex tubes, sheaths a
    - Fractal dimension ~ 1
- Super-sonic turbulence
  - Shocks (oblique, corrugated)
    - Fractal dimension ~ 2
- Low beta, driven magnetic dissipation
  - Current sheets (curved, fragmented)
    - Fractal dimension ~ 2

> So, what do dissipative structures have to do with power laws?

Turbulence break-throughs

- Turbulence scaling (She & Leveque 1994)
  - Analytic predictions for structure functions and power laws
    - Follow from co-dimension of dissipative structures
- Wavelet analysis (Farge et al 2001)
  - Turbulence ≈ hierarchy of vortex tubes
    - A few % of the wavelet coefficients contain almost all of the energy and enstrophy spectra, the rest is ~ white noise

She & Leveque

(Phys Rev Letters, 72, 336, 1994)

VOLUME 72, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JANUARY 1994

Universal Scaling Laws in Fully Developed Turbulence

Zhen-Su She and Emmanuel Leveque*
Department of Mathematics, University of Arizona, Tucson, Arizona 85721
(Received 12 July 1993)

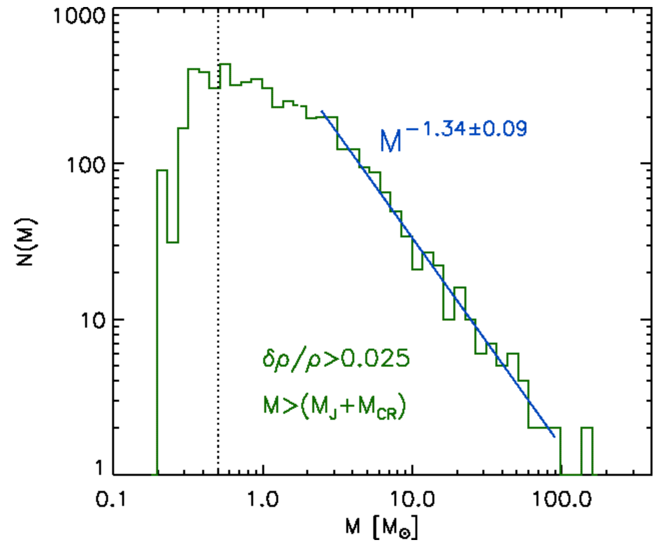
The inertial-range scaling laws of fully developed turbulence are described in terms of scalings of a sequence of moment ratios of the energy dissipation field ϵ_ℓ coarse grained at inertial-range scale ℓ. These moment ratios $\epsilon_\ell^{(p)} = \langle \epsilon_\ell^{p+1} \rangle / \langle \epsilon_\ell^p \rangle$ $(p = 0, 1, 2, ...,)$ form a hierarchy of structures. The most singular structures $\epsilon_\ell^{(\infty)}$ are assumed to be filaments, and it is argued that $\epsilon_\ell^\infty \sim \ell^{-2/3}$. Furthermore, a universal relation between scalings of successive structures is postulated, which leads to a prediction of the entire set of the scaling exponents: $\langle \epsilon_\ell{}^p \rangle \sim \ell^{\tau_p}$, $\tau_p = -\frac{2}{3}p + 2[1 - (\frac{2}{3})^p]$ and $\langle \delta v_\ell^p \rangle \sim \ell^{\zeta_p}$, $\zeta_p = \frac{p}{9} + 2[1 - (\frac{2}{3})^{p/3}]$.

Supersonic Turbulence

- Initial Mass Function (Padoan & Nordlund, astro-ph/0011465)
  - Analytic theory ⇒ Stellar IMF
    - Clump size determined by MHD shock jump conditions
    - Velocity power spectrum ~ $k^{-1.8}$
    - Salpeter IMF slope 1.36 = 3/(4-1.8)
- Supersonic Turbulence (Boldyrev, Nordlund & Padoan, astro-ph/0111345)
  - Analytic theory for structure functions & power spectrum of super-sonic turbulence
    - From application of She-Leveque formalism
    - Assuming dissipative structures have dimension ~ 2
    - Numerical structure functions up to 10th order

Initial Mass Function

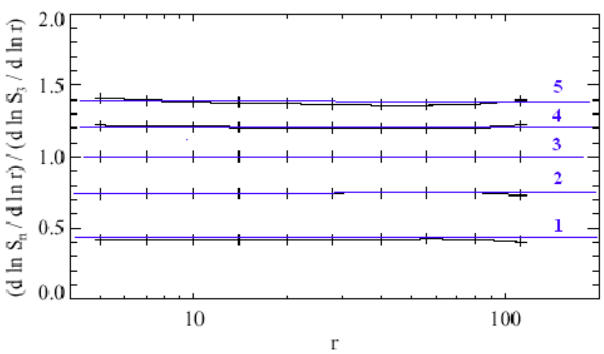
(from numerical experiment, Padoan & Nordlund, 2001)



Supersonic Turbulence Structure Functions

(Boldyrev, Nordlund & Padoan, astro-ph/0111345)

$$\zeta(p) = \frac{p}{9} + 1 - \left(\frac{1}{3}\right)^{p/3}$$



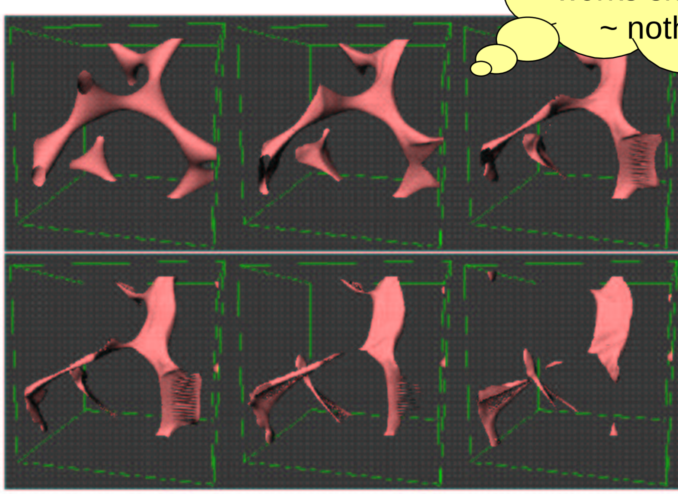
Driven, low-β magnetic dissipation

- Field Line Braiding (Galsgaard & Nordlund JGR 1996a)
  - Dissipative structures: hierarchy of curved current sheets
    - Scaling law for dissipation
    - Winding number ~ unit width Gaussian
- Driven Kink (Galsgaard & Nordlund JGR 1996b)
  - Also develops hierarchy of current sheets
    - Will not sustain winding number > 1+
    - Consistent with scaling law
- Driven **B** with Nulls (Galsgaard & Nordlund JGR 1996c)
  - Fate of null points: hierarchy of current sheets
    - Nulls are quickly lost and forgotten
    - Large scale arcade (due to driving shear pattern)
    - Consistent with scaling law

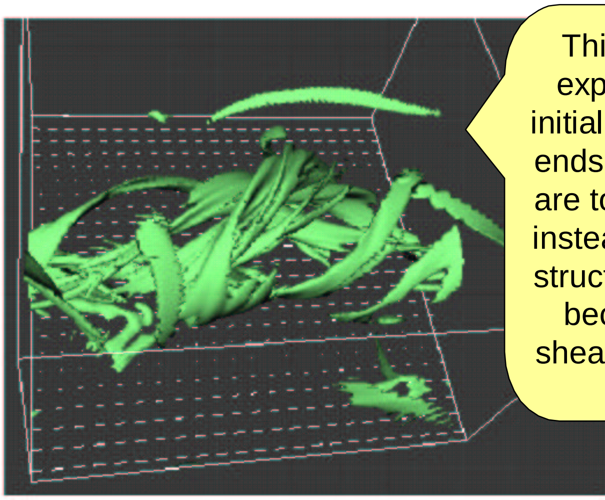
The null point issu

(Galsgaard & Nordlund JGR

The region where a linear expansion of B works shrinks to ~ nothing!



Arcade along boundaries

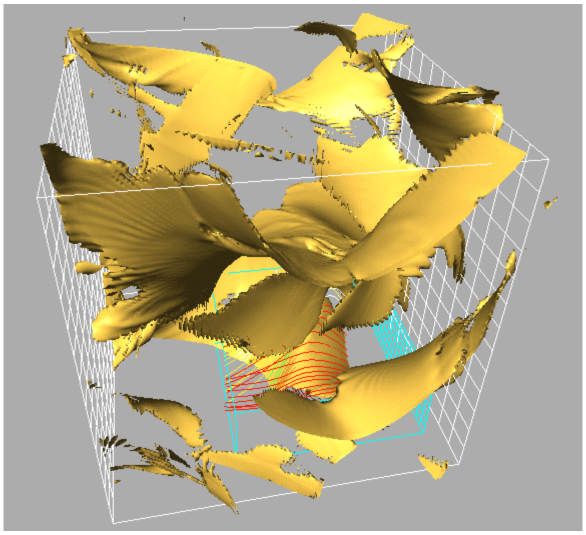


This is how the experiment with initially 8 null points ends up – the nulls are totally lost, and instead an archade structure develops, because of the shearing boundary motions

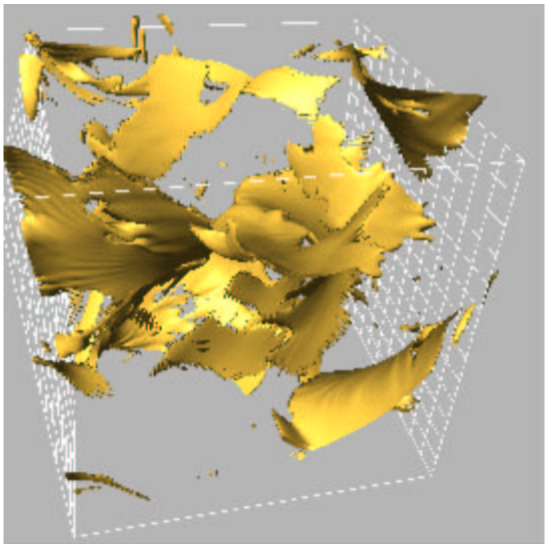
Skeletons?

- The skeleton approach is not useful
  - Force-free fields tend to "squish" nulls; shrinking the linear region to ~ nothing
  - The driver pattern + field line connectivity determines where dissipation occurs
    - There is more to "connectivity" than static topology..

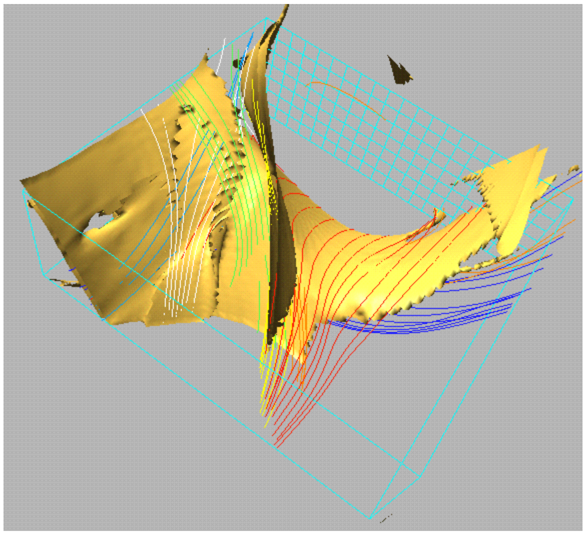
Current sheet hierarchy: 3-D



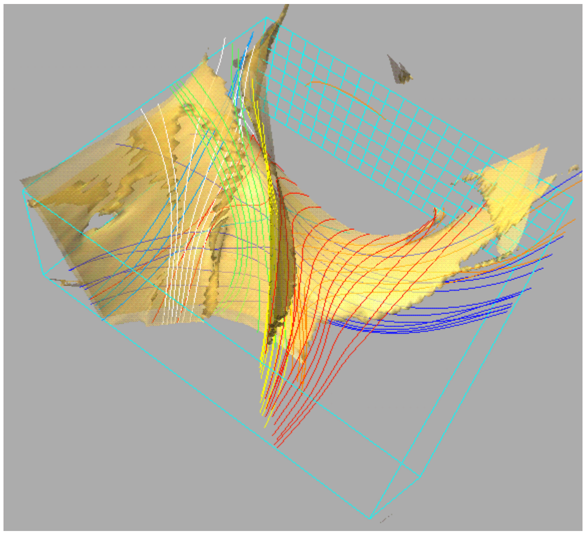
Current sheet hierarchy: evolution



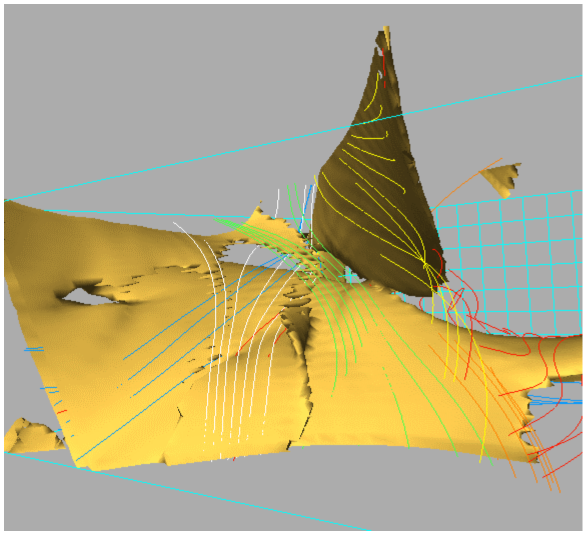
Current sheet hierarchy: detail



Current sheet hierarchy: translucent



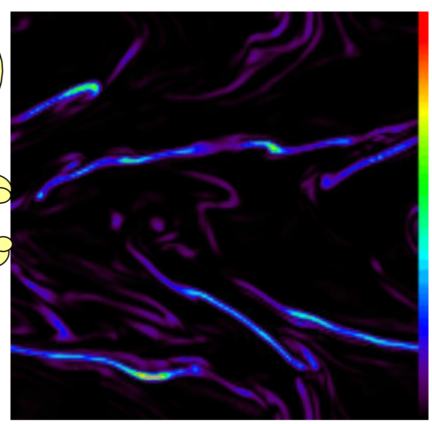
Current sheet hierarchy: close-up

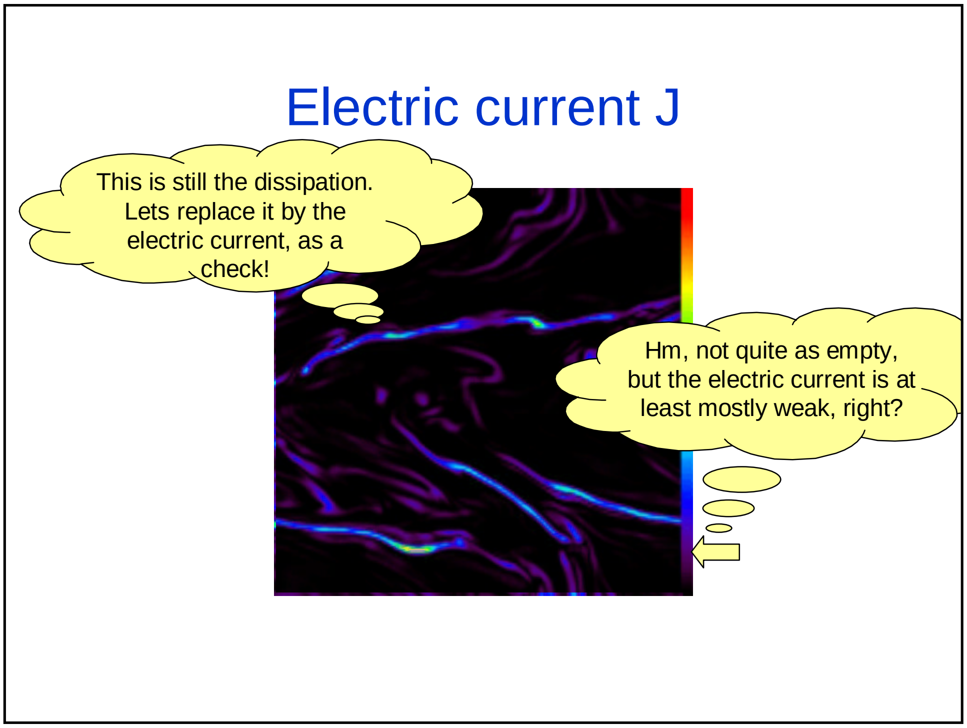


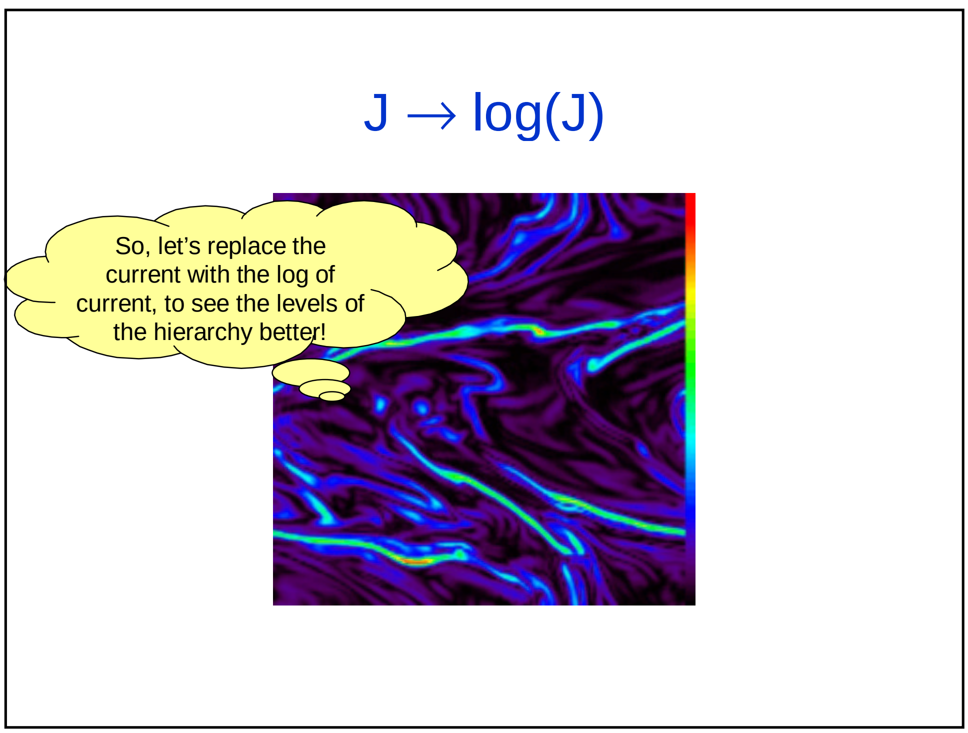
Scan through hierarchy: dissipation

Note that all features rotate as we scan through – this means that these currents sheets are all curved in the 3rd dimension.

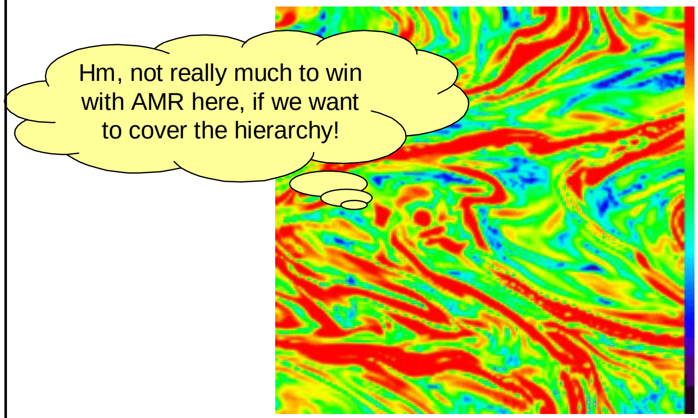
Hm, the dissipation looks pretty intermittent– large nice empty areas to ignore with an AMR code, right?



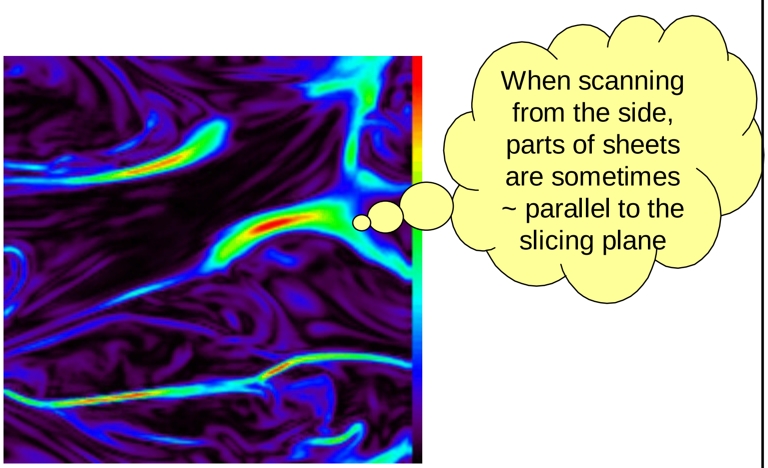

Electric current J
This is still the dissipation. Lets replace it by the electric current, as a check!
Hm, not quite as empty, but the electric current is at least mostly weak, right?


J → log(J)
So, let's replace the current with the log of current, to see the levels of the hierarchy better!

Log of the current



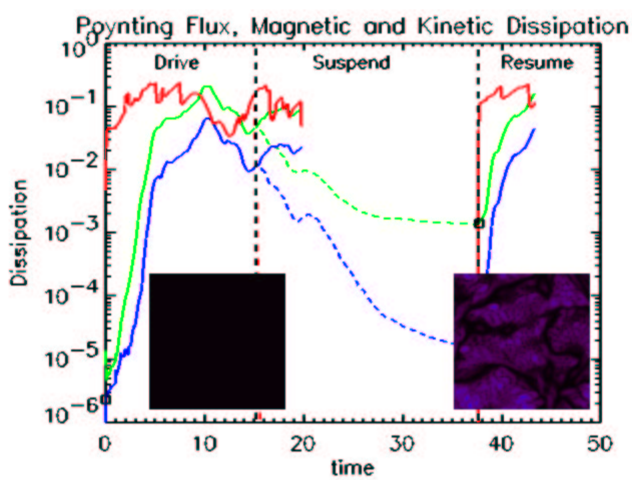
Scan through dissipation: across



Drive / suspend / resume

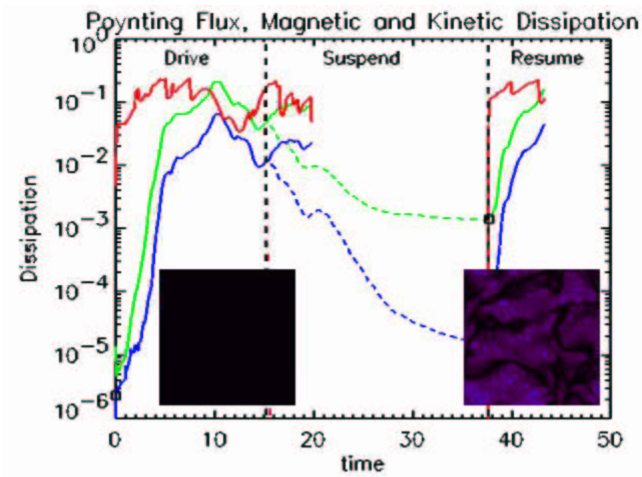
- Nordlund & Galsgaard 2002 (in prep)
  - – Suspended driving: current sheets die quickly
  - – Suspended state: force-free, near marginal state
    - At high Lundquist number: very long lived
  - – Resumed driving: current sheets turn on quickly
    - Explicit demonstration: repeat the initial motions

Drive / suspend / resume



Drive / suspend / resume

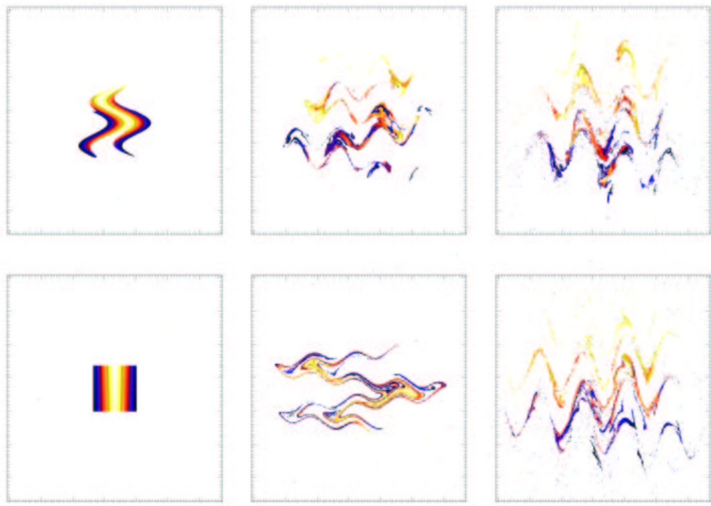
(movie)



What do we conclude?

- A driven magnetic plasma dissipates in a marginal state
  - Winding number distribution ~ Gaussian of unit width
  - Dissipation levels ranging from very small to very large can be sustained at nearly the same winding numbers (angles)
    - Explains the form of the scaling laws
- When driving is turned off, dissipation in current sheets quickly eats the (small) energy reserve, and leaves the plasma in a ~ force free state
  - Very long lived (at high Lundquist number)
  - Very “near” marginal dissipative state!
- How does the “cascading” to small scales work?

Field line connectivity



VRML files

(See this link for information about VRML plugins)

- Quiescent field (VRML file)
  - Look along the field lines
    - Shear, but not enough twist to make current sheets
- Active field (VRML file)
  - Look either along the field lines, or spin it
    - Boundary stress has turned the shear into twist, and current sheets have depeloped

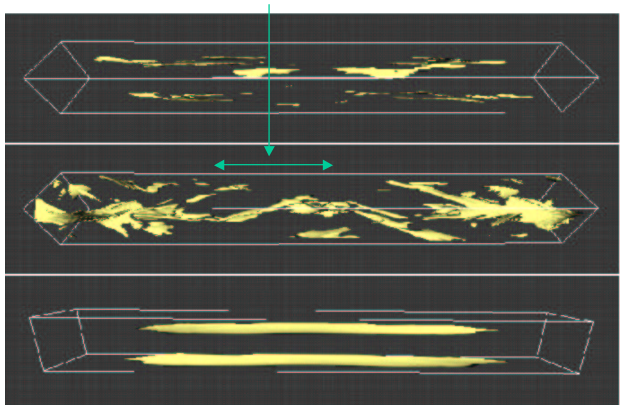
Field Line Connectivity

- The messed-up (braided) connectivity implies that even large scale motions on the boundaries immediately stresses small scale interlocking flux bundles
  - ⇒ Small scale current sheets

Stress, rather than velocity, cascades to small scales

Intermittency

- Depends on correlation time, length, …
  - – Correlation time (τ) x A-speed (c)< Length (L)
    - ⇒ Effective length, $L_{eff} \sim \tau\, c < L$



Distribution of dissipation

(Galsgaard & Nordlund 1996a)

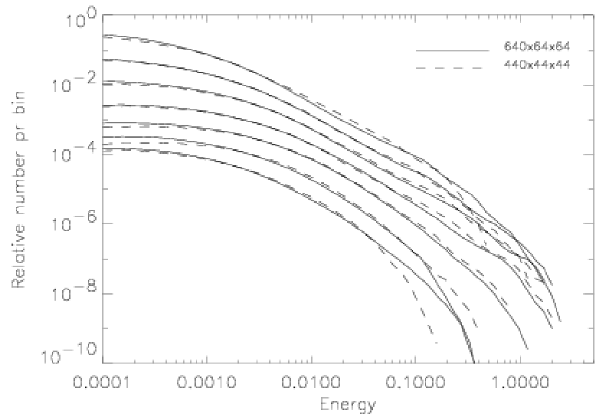


Figure 10. Histograms of the Joule dissipation

So what?

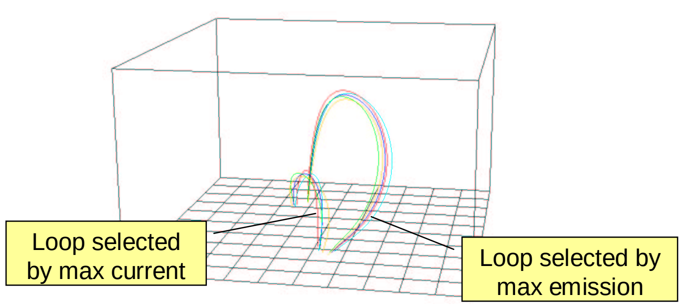
- The PDF of dissipation is not the same thing as the distribution of flare event size!
  - But nevertheless, both being power laws presumably is not just a coincidence
    - Somebody: find the connection!
- The index of the flare event size being ≈ the index of super-sonic turbulence is surely(?) a coincidence!
  - Nevertheless, intriguing …
    - Both have ≈ 2-D dissipating structures!

Location of dissipative structures

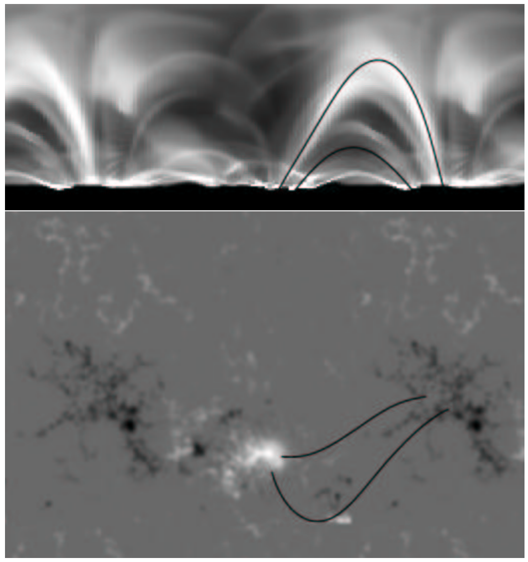
- Q: what determines the locations?
  - Topology skeleton?
  - Driving?
- A: driving, together with connectivity
  - Obvious in active region model. Need:
    - Good field lines; connecting strong fields
    - Good driver; non-parallel shears / twist

Corona conditions

- Let's take a look at the loops Gudiksen showed last Thursday!
  - Where do they connect to?
  - What would a potential field look like?

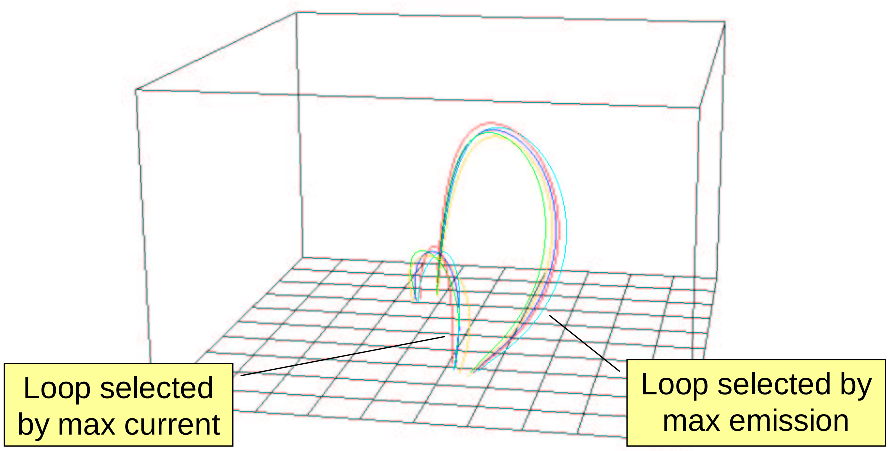


Connectivity



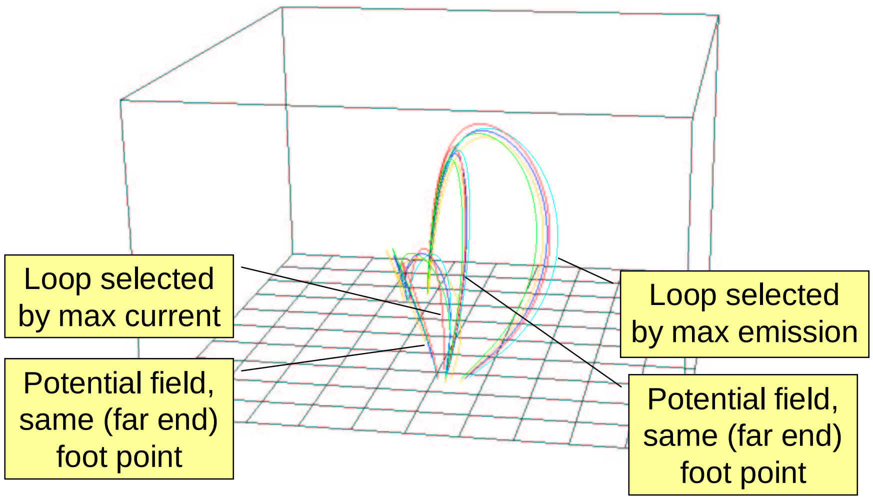
Tracing the magnetic field

(click on the image for VRML rendering)



Potential extrapolation

(click on the image for VRML rendering)



Winding up

Issues:

- Dissipative structures
  - Curved (twisted) current sheets in low-beta plasmas
  - Marginal state (reminiscent of self-organized criticality)
- What about adaptive meshes?

Adaptive mesh issues: where?

- Where to apply them
  - Top-down
    - Apply "wherever needed", "until resolved"
      - Doomed strategy: never converges
  - Bottom-up
    - Define smallest scale
    - Are there significant volumes with larger scales?
      - If not: pointless
      - If yes: typically peripheral regions
      - Question (big): what criterion to apply in central region

Adaptive mesh issues: when / not?

- When not to apply AMR
  - In initial, unrealistically smooth state
    - Develops large scale, Sweet-Parker type sheet
      - Unconstrained adaptive mesh will try to "resolve"
      - Evolution will grind to a halt: $\Delta x, \Delta t \to 0$
  - Instead, let hierarchy appear at low to intermediate resolution
      - Control via parameter: smallest scale / no. of levels

Conclusions

- Dissipating structures in low-β plasmas
  - Curved, hierarchical current sheets
  - Driven by stress from 3rd dimension; twist
  - Dissipate in a “marginal” state
    - TODO: Understand the relation to scaling properties such as PDFs and power laws
- AMR is a mixed blessing
  - Needs to be applied with care
  - Not to be let loose on smooth initial states