HOW TURBULENT IS THE TACHOCLINE?

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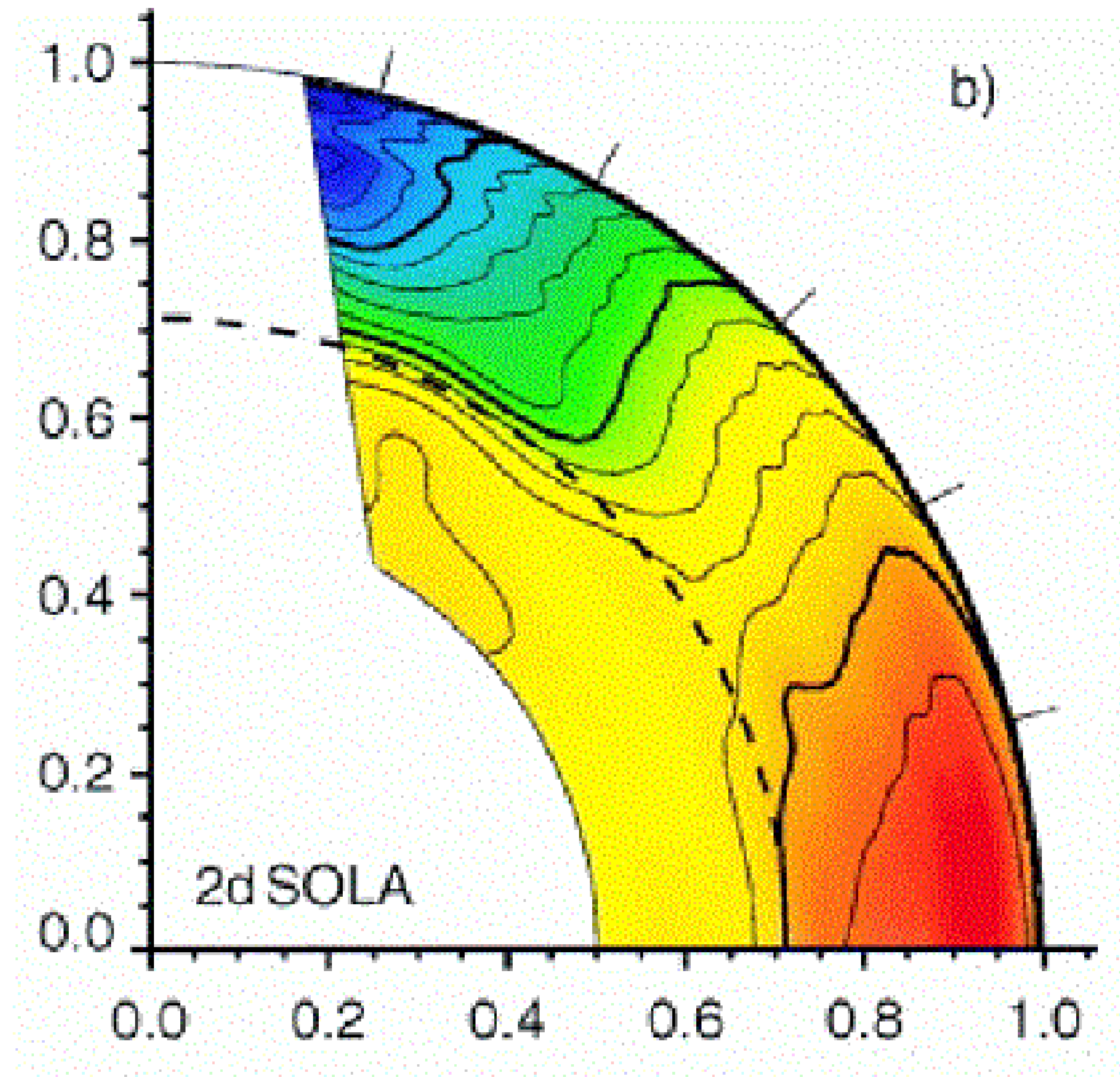
The solar tachocline: observed properties

Internal rotation from helioseismology (Schou et al. 1998):



Notations

$\Omega(r,\theta)$	angular velocity
$\Omega_0 = 2\pi \cdot 437\,\mathrm{nHz}$	angular veloc. of solar interior
$\Omega_{bcz} = 2\pi(456 - 72\cos^2\theta - 42\cos^4\theta\ \mathrm{nHz})$	angular veloc. in conv. zone
$\omega(r,\theta) = \Omega - \Omega_0$	residual rotation rate

$$\Delta\omega = \omega(r,\theta) - \int_0^{\pi/2} \omega(r,\theta)\sin\theta\,d\theta \qquad \bar{f} = \frac{1}{P_{cyc}}\int_t^{t+P_{cyc}} f\,dt$$

Observed features of the tachocline (Schou, Basu & Antia,...)

◊ Full thickness (ω reduced by 99 %): $w = 0.04 \pm 0.015 R_\odot$

$\Rightarrow$ Scale height $H = w/\ln 100 \lesssim 0.01\,R_\odot$

◊ Centered around $R/R_\odot = 0.691 \pm 0.003$. (Base of SCZ: 0.71 ± 0.003) at low latitudes, just below convective zone

◊ Evidence for prolate form: $R/R_\odot = 0.71 \pm 0.003$ at $\Phi = 60°$ at high latitude partly overlaps with conv. zone

◊ Marginal evidence for thicker tachocline at high latitudes: $w = 0.05 \pm 0.005$ at $\Phi = 60°$

The thin tachocline problem

Stationary solution of Navier-Stokes with isotropic viscosity

$\nu \nabla^2 \mathbf{v} = 0 \qquad \Rightarrow w \sim R_\odot.$

But: this state reached on viscous timescale $\gg t_\odot$ only: irrelevant?

Spiegel & Zahn 1992: Eddington-Sweet circulation more effective:

leads to $w \sim R_\odot$ in $< t_\odot$.

$\Rightarrow$ effective horizontal angular momentum transport needed

Candidates:

(a) HD

– Anisotropic viscosity (Spiegel & Zahn 1992)

– Other HD effects (Canuto 1998, Forgács-Dajka & Petrovay 2000)

Problems:

– tachocline seems HD stable (Charbonneau et al. 1999)

– such effects of this amplitude never observed

(b) MHD

— Permanent internal remnant magnetic field: 10^{-4} G sufficient

(Rüdiger & Kitchatinov 1997; MacGregor & Charbonneau 1999)

Problem: only works if field lines do not cross the boundary

(but cf. Garaud 2001)

— Oscillatory (dynamo) magnetic field (Forgács-Dajka & Petrovay 2001, 2002)

Penetration of an oscillatory field into the radiative interior:

$$H_{\rm skin} = (2\eta/\omega_{\rm cyc})^{1/2}$$

$H_{\rm skin} \gtrsim H$ if $\eta \gtrsim 10^9\,{\rm cm^2/s}$. (NB $\eta \sim 10^{12}$–10^{13} in SCZ.)

$\Rightarrow$ *(a)* strongly turbulent ("fast") tachocline = dynamo field
(b) weakly turbulent ("slow") tachocline = internal field

Arguments for case *(a)*:

- ◇ tachocline partly coincides with quasiadiabatic layer (= SCZ + overshoot)
- ◇ Dynamo field of $\sim 10^5$ G *must* be stored in a layer of at least a few Mms.
- ◇ 3D MHD instabilities (Gilman & Dikpati 2000)

Estimates

Horizontal shearing flow imposed on top a region with oscillatory horizontal field:

$$v_{y0} = v_0 \cos(kx) \qquad B_x = B_p \cos(\omega t)$$

Notations: $V_p = B_p(4\pi\rho)^{-1/2} \qquad b = B_y(4\pi\rho)^{-1/2}$

E.o.m. and induction:

$$\partial_t v = V_p \cos(\omega t)\partial_x b + \nu\nabla^2 v$$

$$\partial_t b = V_p \cos(\omega t)\partial_x v + \eta\nabla^2 b$$

Solutions:

$$v = \overline{v}(x,z) + v'(x,z)f(\omega t) \qquad b = b'(x,z)f(\omega t + \phi)$$

Splitting into mean and fluctuating parts:

$$0 = V_p\overline{\cos(\omega t)f(\omega t+\phi)}\partial_x b' + \nu\nabla^2\overline{v}$$

$$\partial_t v' = V_p[\cos(\omega t)f(\omega t+\phi)]'\partial_x b + \nu\nabla^2 v'$$

$$\partial_t b' = V_p[\cos(\omega t)f(\omega t)\partial_x v'], +\eta\nabla^2 b'$$

Estimates:

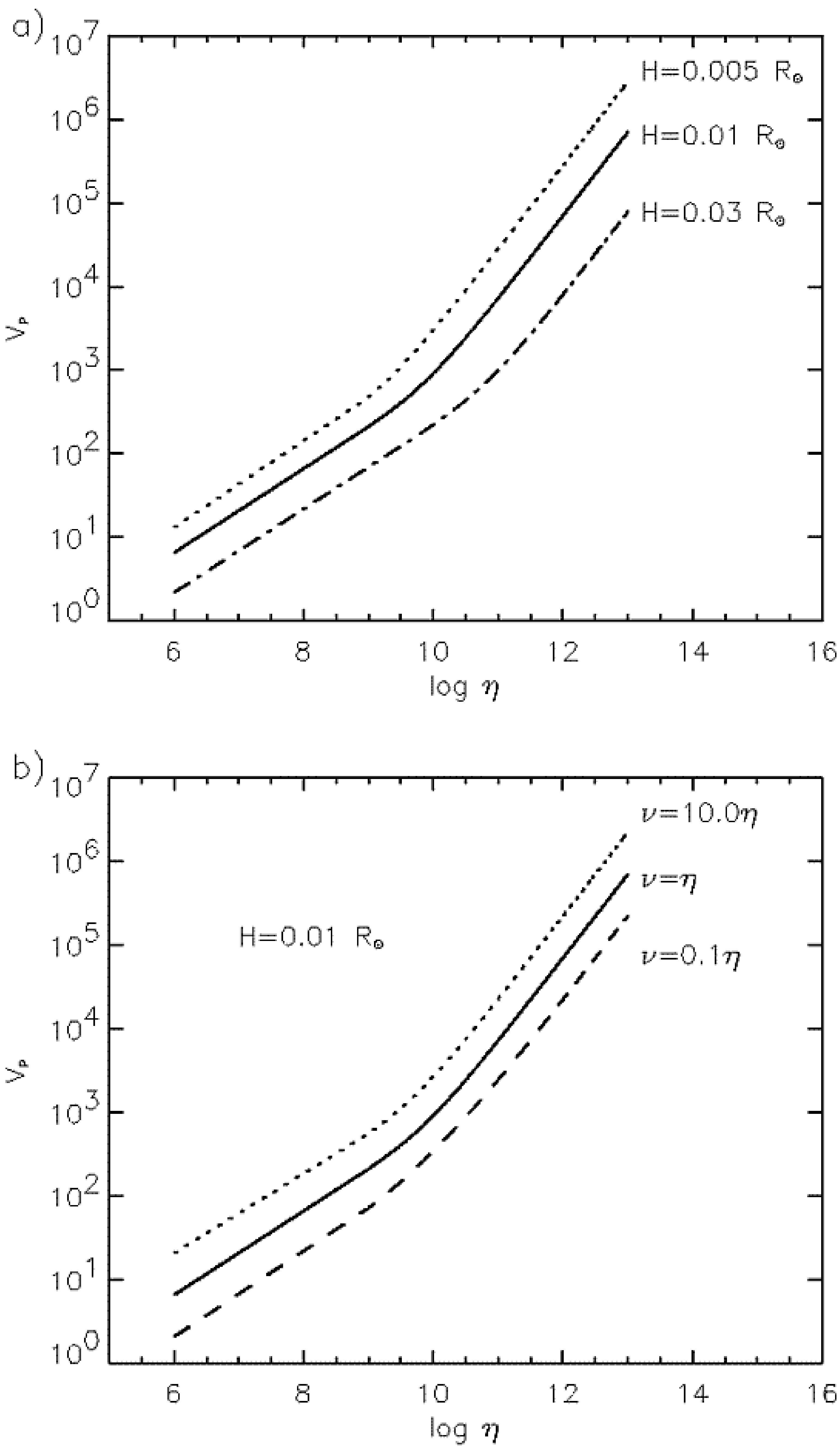
$$V_p b'/R \sim \nu\overline{v}/H^2$$

$$(\omega + \nu/H^2)v' \sim V_p b'/R$$

$$\omega b' \sim (V_p + v')V_p/R + \eta b'/H^2$$

From these

$$V_p^2 = \frac{\nu R^2 \omega}{H^2} \frac{(1+\eta/\omega H^2)(1+\nu/\omega H^2)}{1+2\nu/\omega H^2}$$



Numerical solution for the solar case

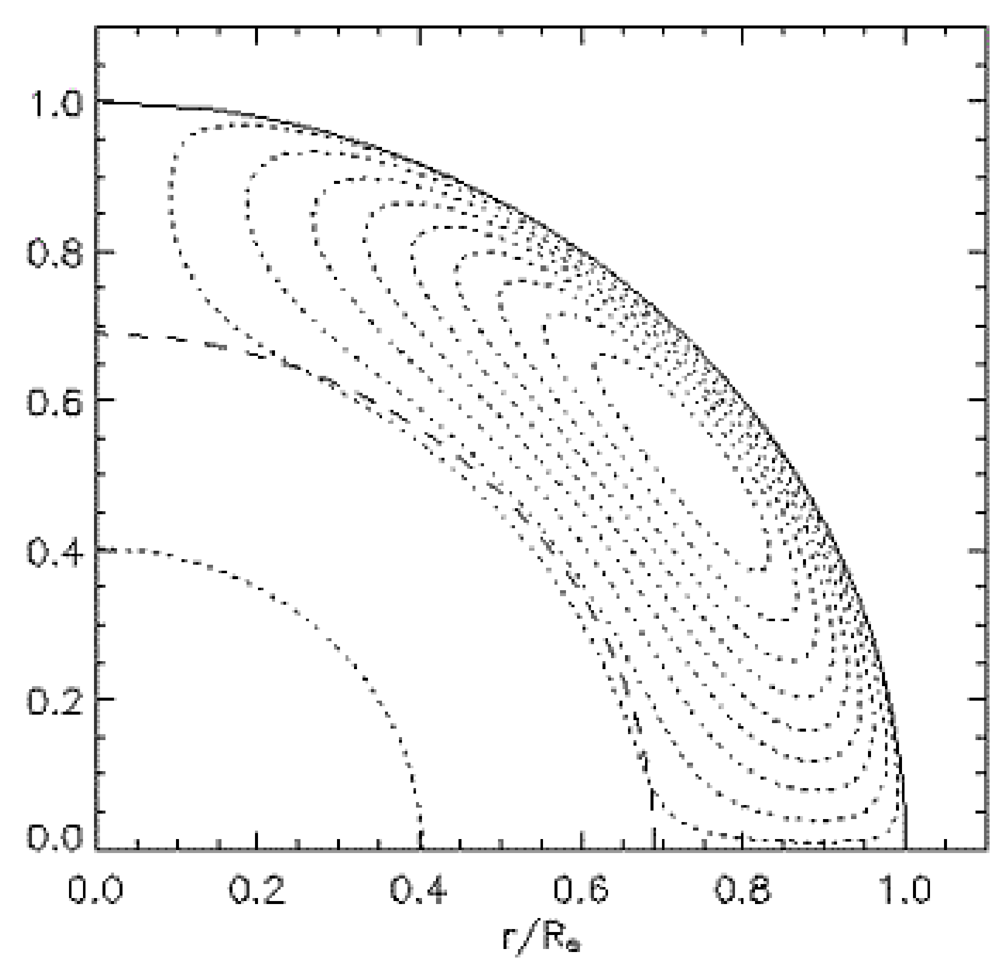
$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = \mathbf{g} - 2(\vec{\mathbf{\Omega}}_0 \times \mathbf{v}) - \frac{1}{\rho}\nabla\left(\mathbf{p} + \frac{\mathbf{B^2}}{8\pi}\right) + \frac{1}{4\pi\rho}(\mathbf{B}\cdot\nabla)\mathbf{B} + \frac{1}{\rho}\nabla\cdot\hat{\tau}$$

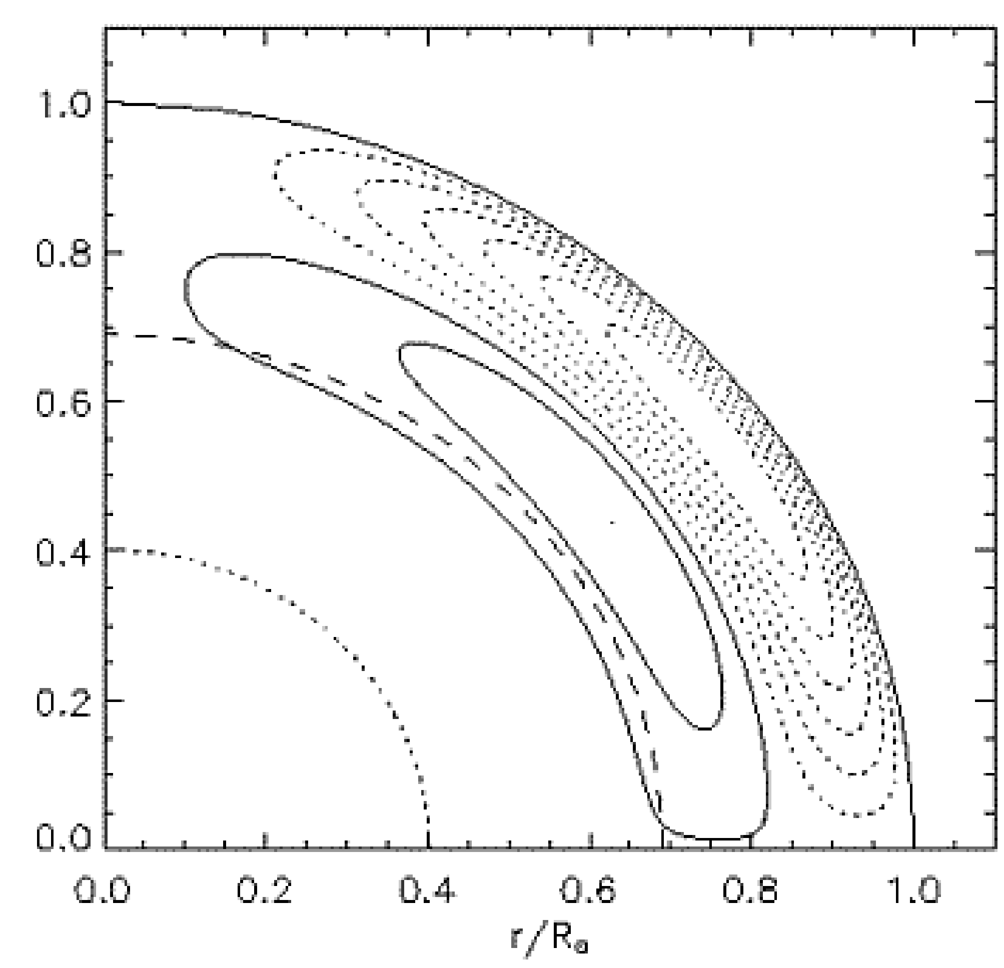
$$\partial_t \mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times(\eta\nabla\times\mathbf{B})$$

$$\nabla\cdot(\rho\mathbf{v}) = \mathbf{0} \qquad \nabla\cdot\mathbf{B} = \mathbf{0}$$

$$\mathbf{v} = \omega(\mathbf{r},\theta,\mathbf{t})\,\mathbf{r}\sin\theta\,\mathbf{e}_\phi + \mathbf{v_m} \qquad \mathbf{B} = \mathbf{B}(\mathbf{r},\theta,\mathbf{t})\,\mathbf{e}_\phi + \nabla\times[\mathbf{A}(\mathbf{r},\theta,\mathbf{t})\,\mathbf{e}_\phi]$$

Meridional circulation: zero, or prescribed





Boundary conditions:

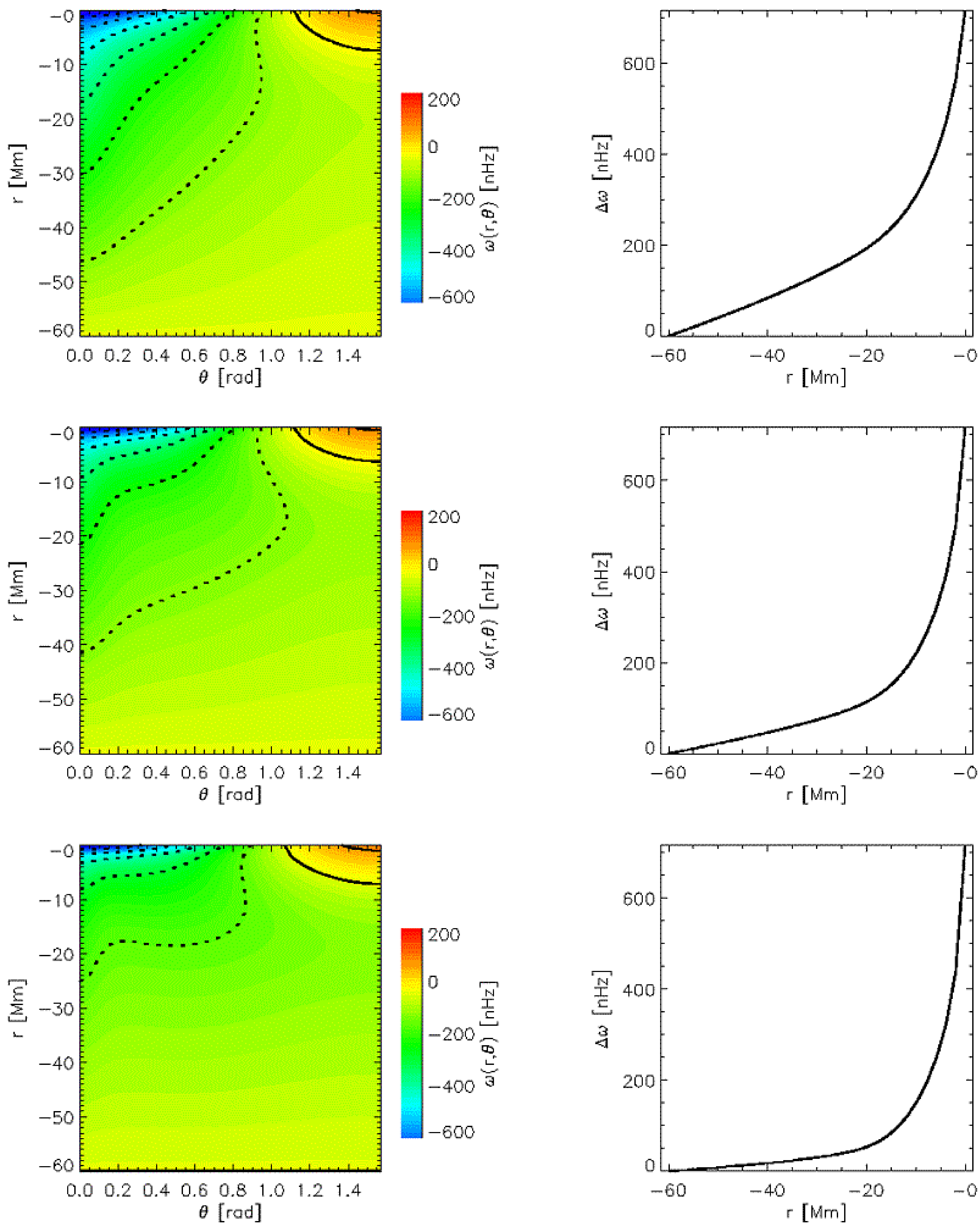
Axial and equatorial (dipole) symmetry.

Bottom: $\omega = B = A = 0$ (rigidly rotating perfect conductor)

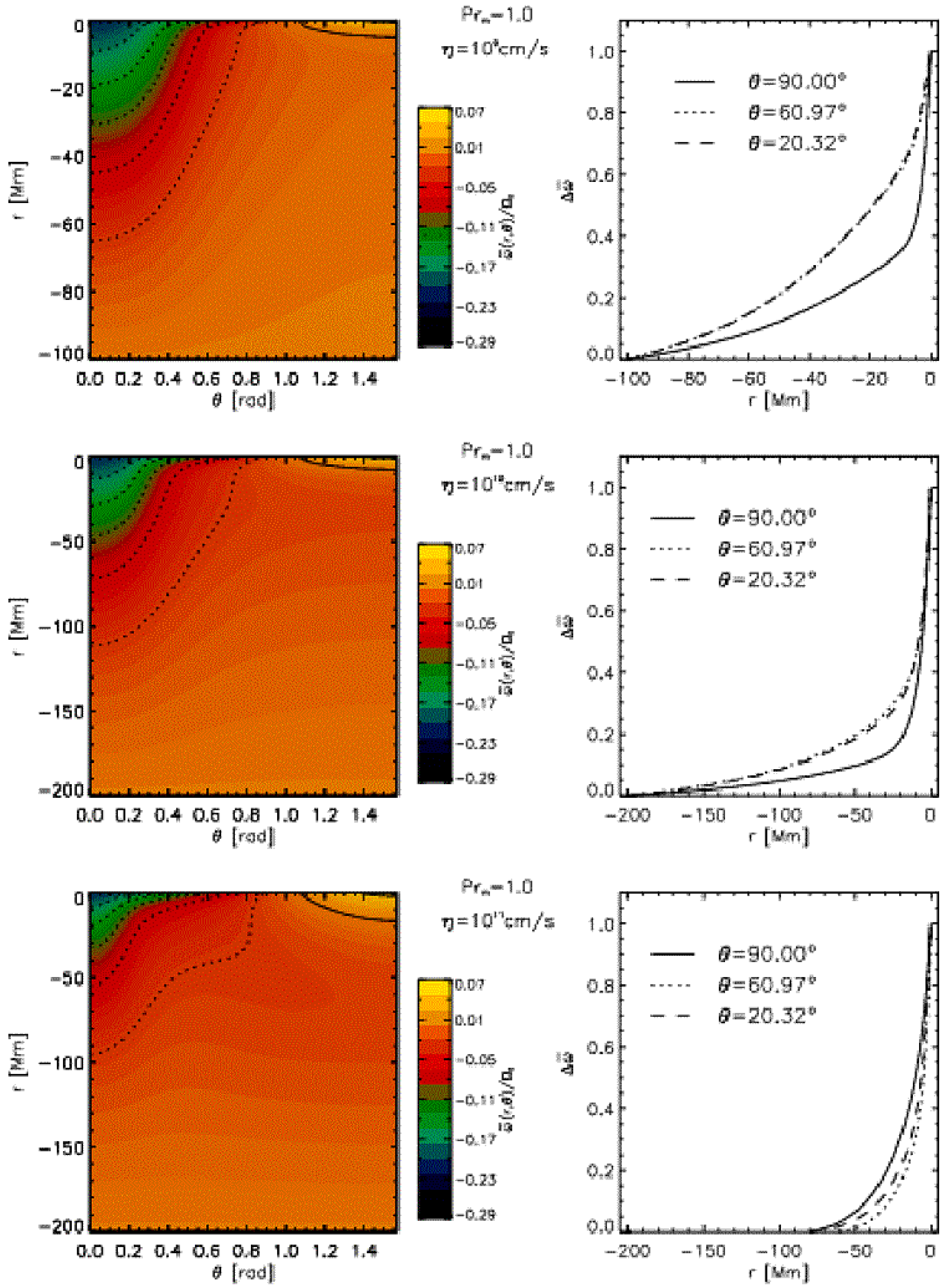
Top: $\omega = \Omega_{bcz} - \Omega_0 \qquad A = A_0 \sin\theta\cos(\omega_{cyc} t) \qquad B = 0$

Parameters: η, $\mathrm{Pr}_m = \nu/\eta$, diff. rot amplitude ($\pm 3\,$cm/s, or zero).

Mean solutions: varying B_p
($B_p = 1600, 2000, 2400$ G)

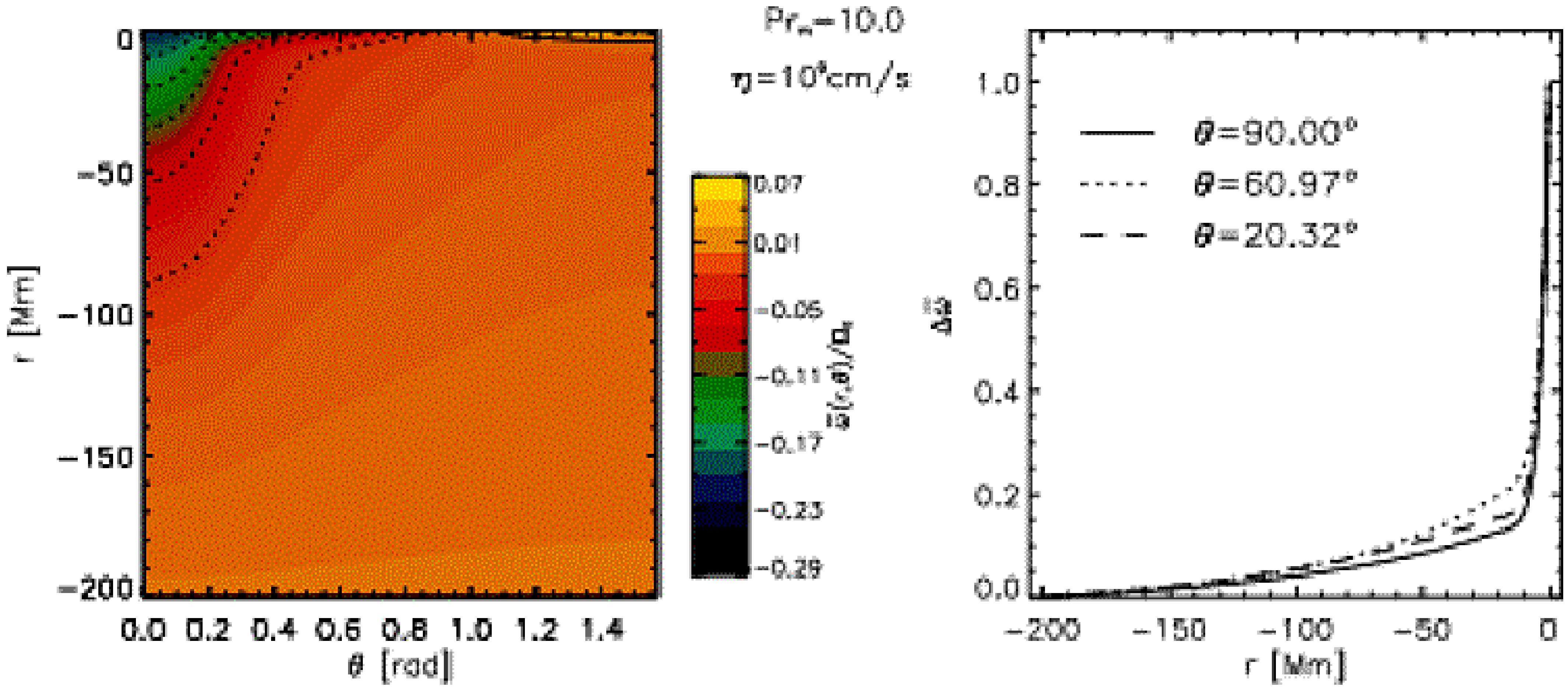


Mean solutions: varying η, $\mathrm{Pr_m} = 1$
$(\log \eta = 9, 10, 11)$

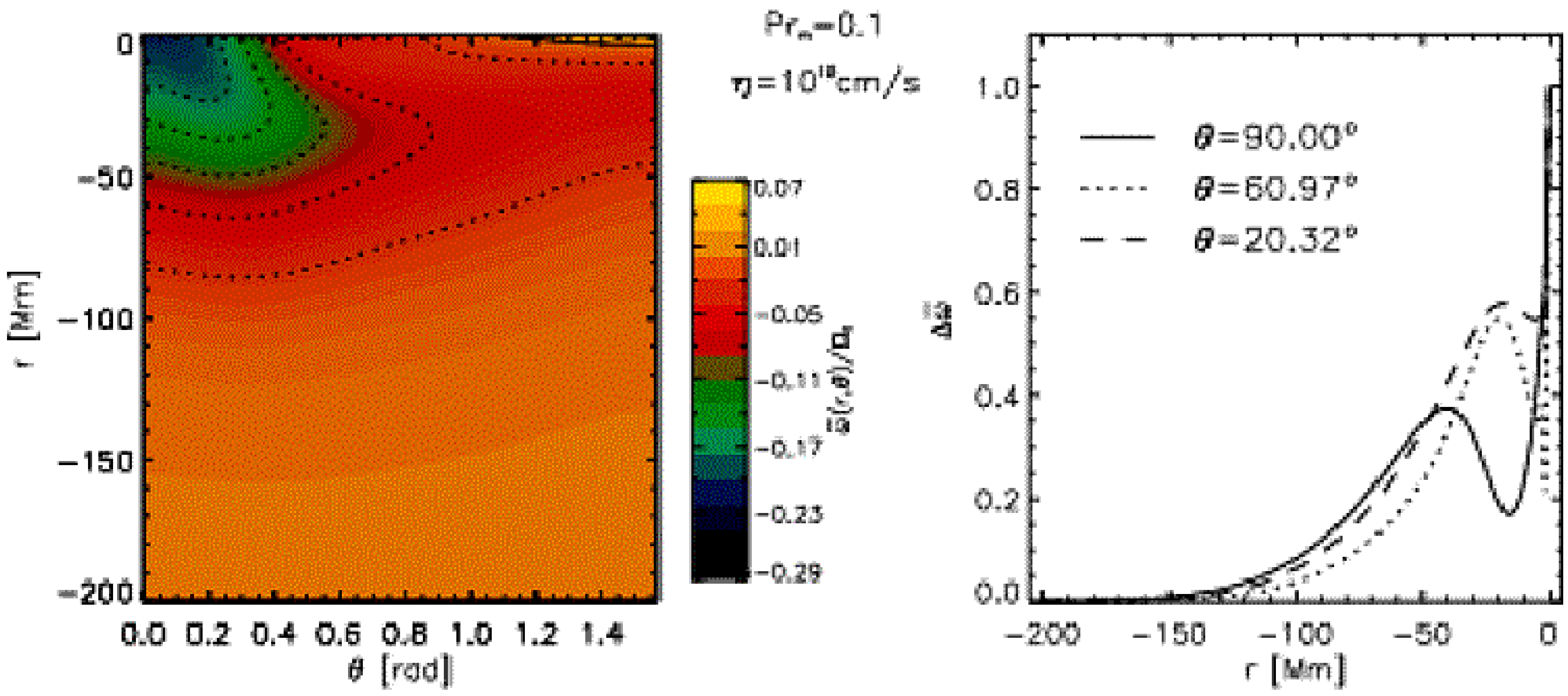


Mean solutions: varying $\mathrm{Pr_m}$

$$(\log\eta, \log\nu) = (9, 10);\ B_p \simeq 16000G:$$

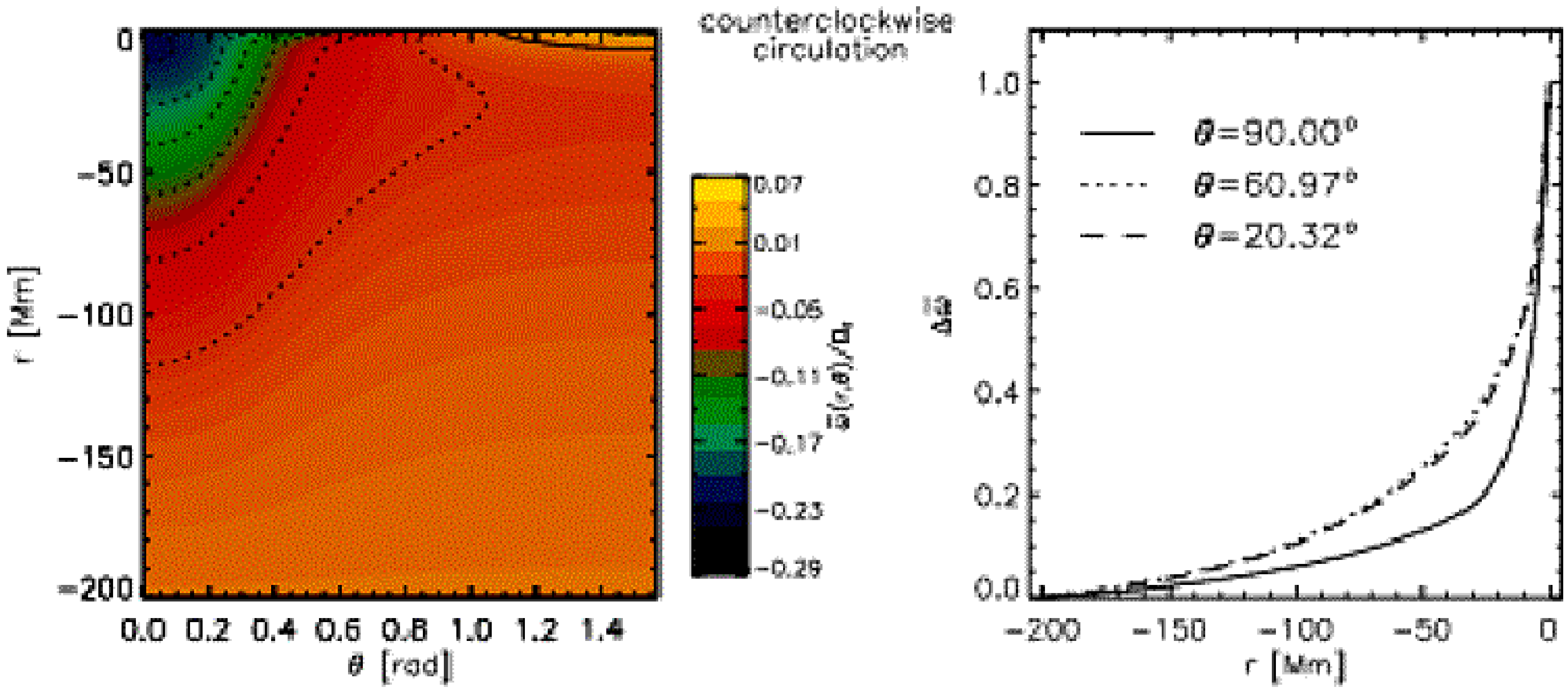


$$(\log\eta, \log\nu) = (10, 9);\ B_p \simeq 1900G:$$

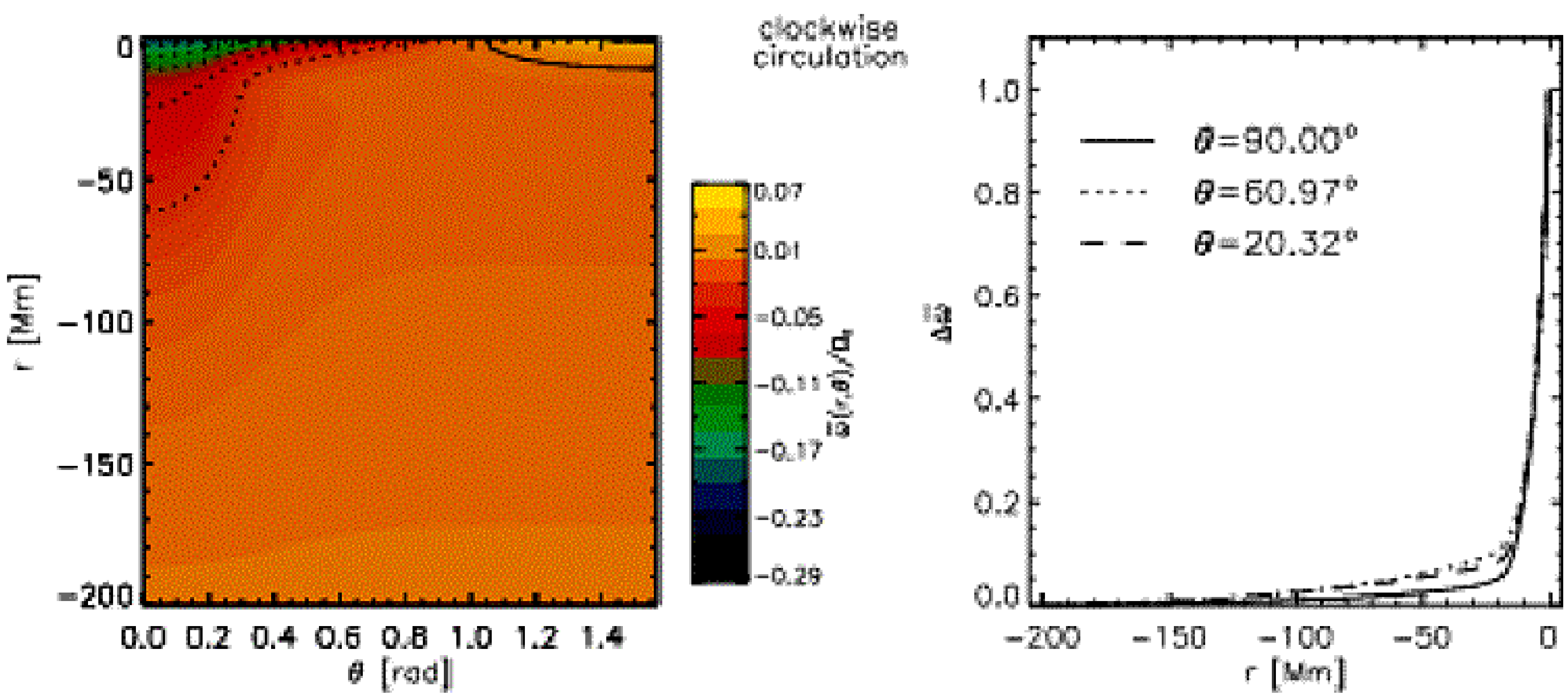


Mean solutions: meridional flow, $B_p \simeq 3200G$

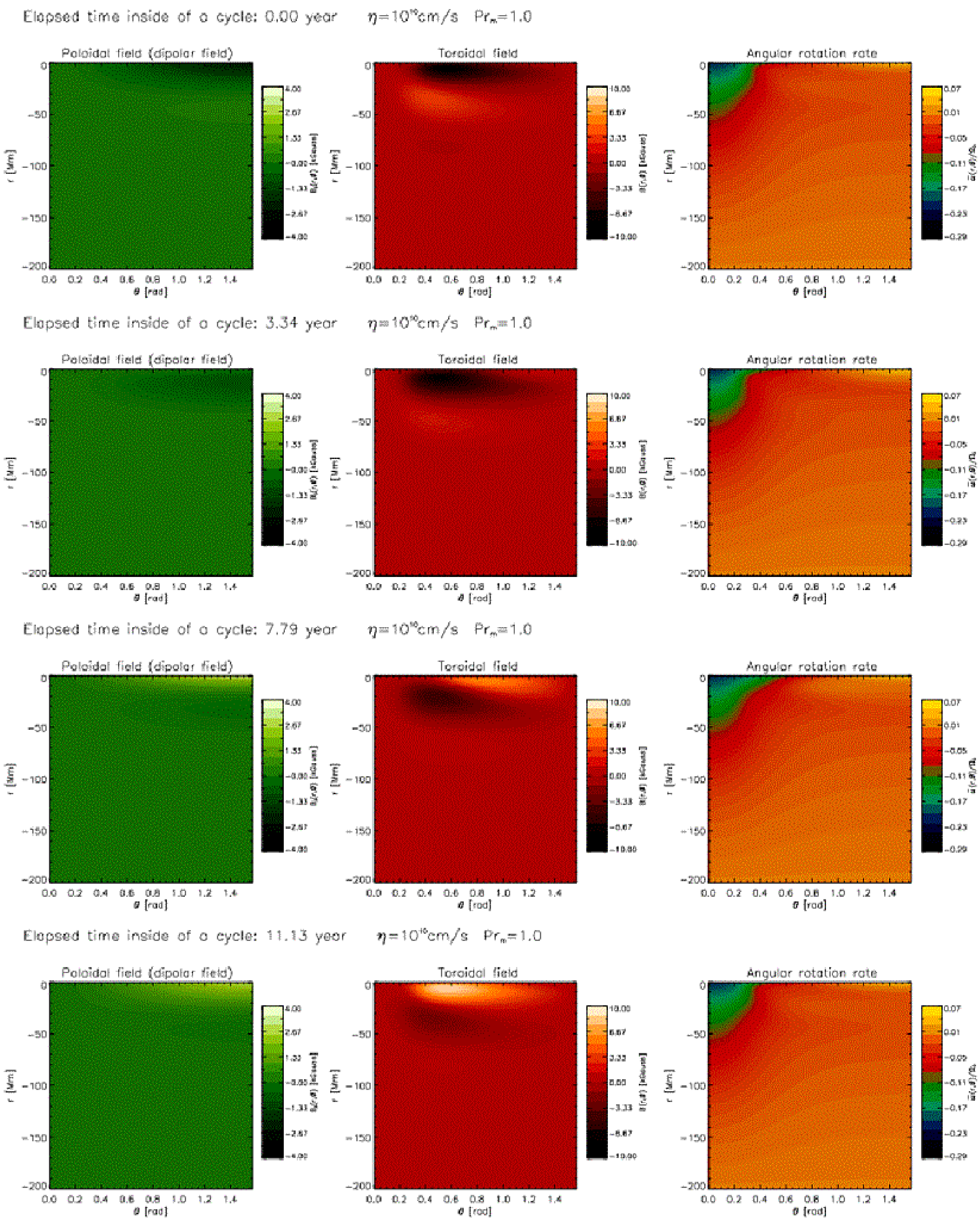
$v \simeq +3\,\mathrm{cm/s}$:

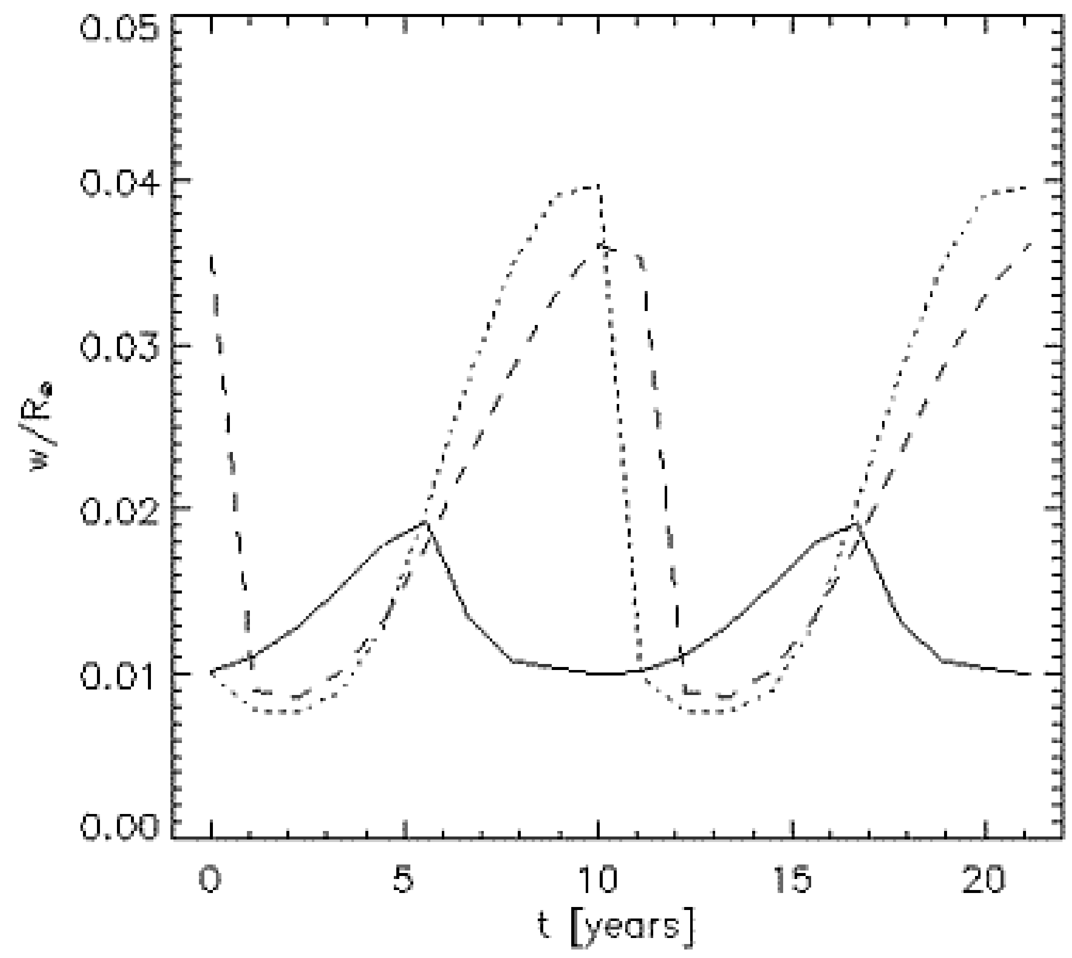


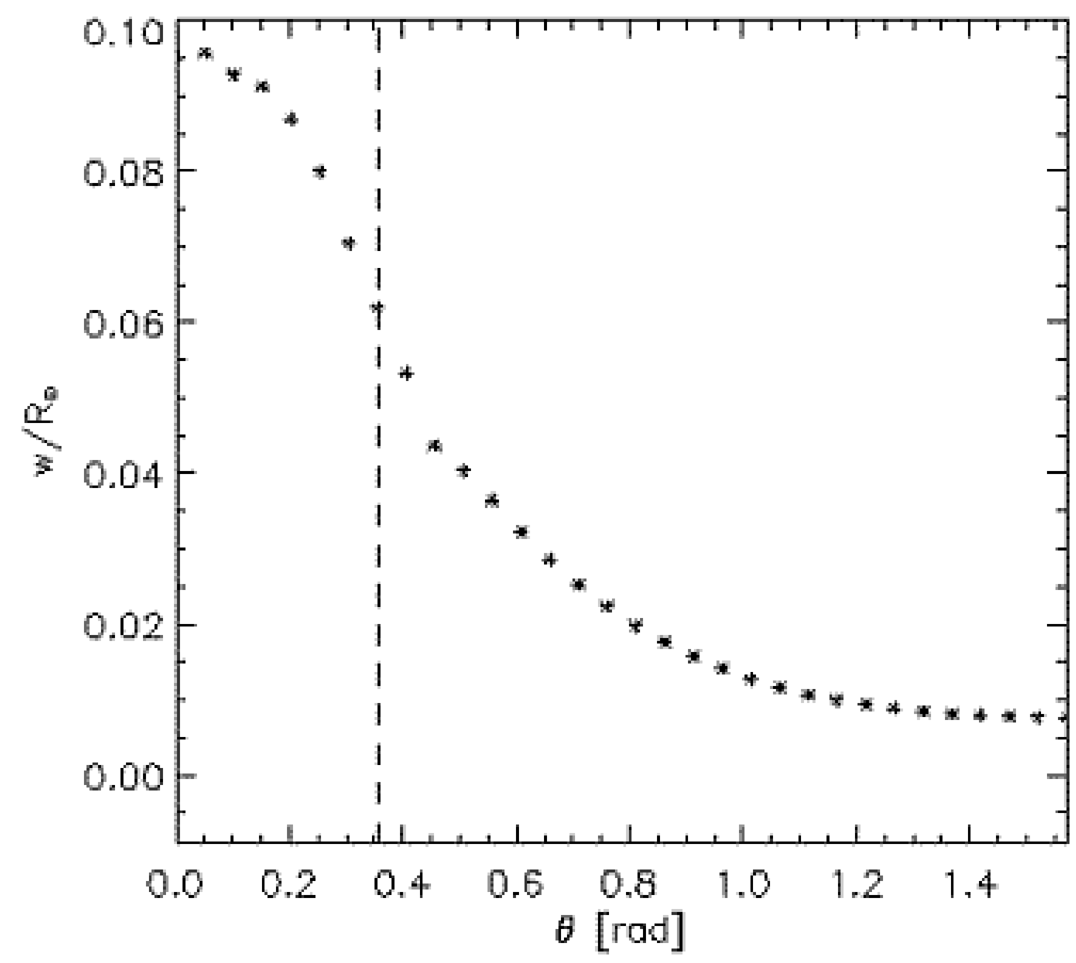
$v \simeq -3\,\mathrm{cm/s}$:



Time dependence ($B_p = 2400$ G)







Conclusion

- Oscillatory poloidal field is able to confine the tachocline
- Confining field $B_{\rm conf}$ increases with ν, ν/η, and with equatorward flow speed
- $B_{\rm conf}$ quite reasonable, 10^3–10^4 G, for all parameter combinations
- w increases with latitude, and strongly depends on cycle phase.
  Less strong dependence for higher ν and/or ν/η.

Some speculation:

⇒ possibility of non-kinematic "Ω-effect".

Non-kinematic (i.e. flux tube) α-effect also likely

⇒ Is the solar dynamo "homeostatic"?
Would it be supercritical at all without the non-kinematic effects?

Work to be done:

- Other field geometries, migrating field
- Turbulence treated more realistically.
  Cf. work on nonlinear phase of instabilities (Cally 2001, Miesch 2001)

Animations: `http://astro.elte.hu/kutat/sol/fast1/fast1e.html`