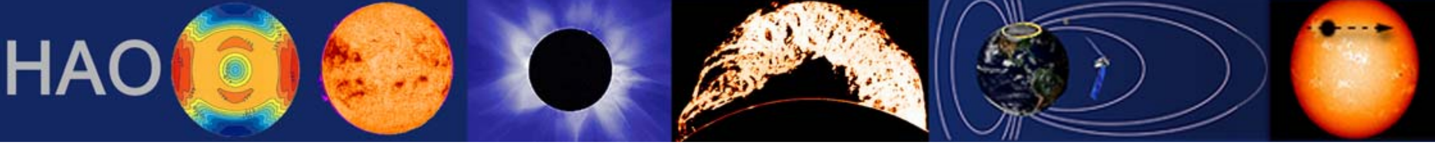


Solar Rotation: Observations, Theory, Modeling, and Implications for Solar Magnetism

Peter A. Gilman

High Altitude Observatory

Observational Challenges for the Next Decade of Solar Magnetohydrodynamics

16-18 January 2002

High Altitude Observatory (HAO) – National Center for Atmospheric Research (NCAR)

Major Classes of Solar Rotation Observations

Tracers:

- Spots — Tracking Individuals
- Magnetic patterns
  CaK images
- Supergranulation
- Coronal brightness

Pattern Autocorrelations

Advantage/Disadvantage:

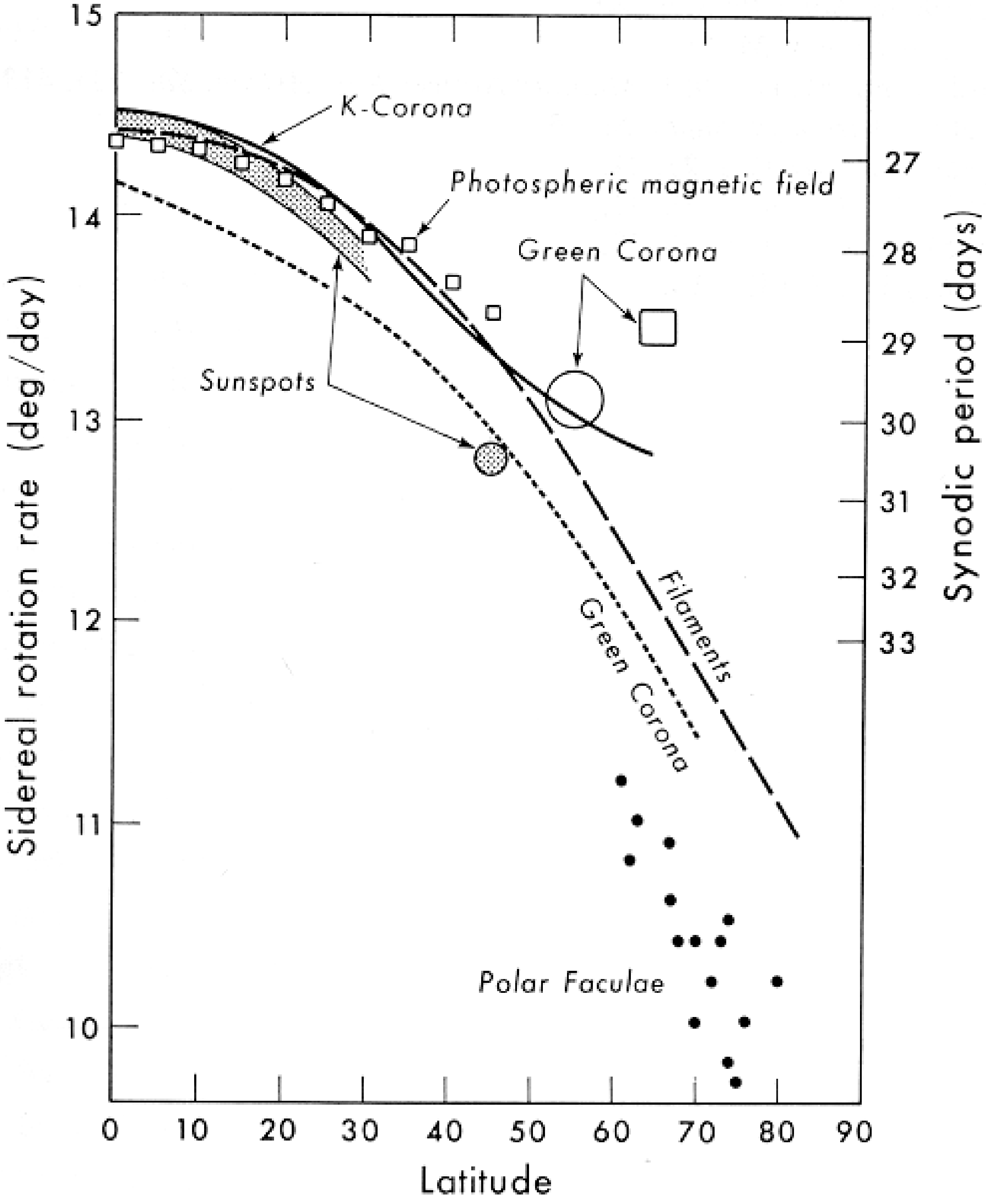
- Spots: long records/thin biased coverage
- Patterns: global coverage/ developments and decay; low resolution for coronal

Surface Doppler Shifts:

Advantage/Disadvantage: Global coverage, averaging, filtering/absolute calibration problems

Helioseismic:

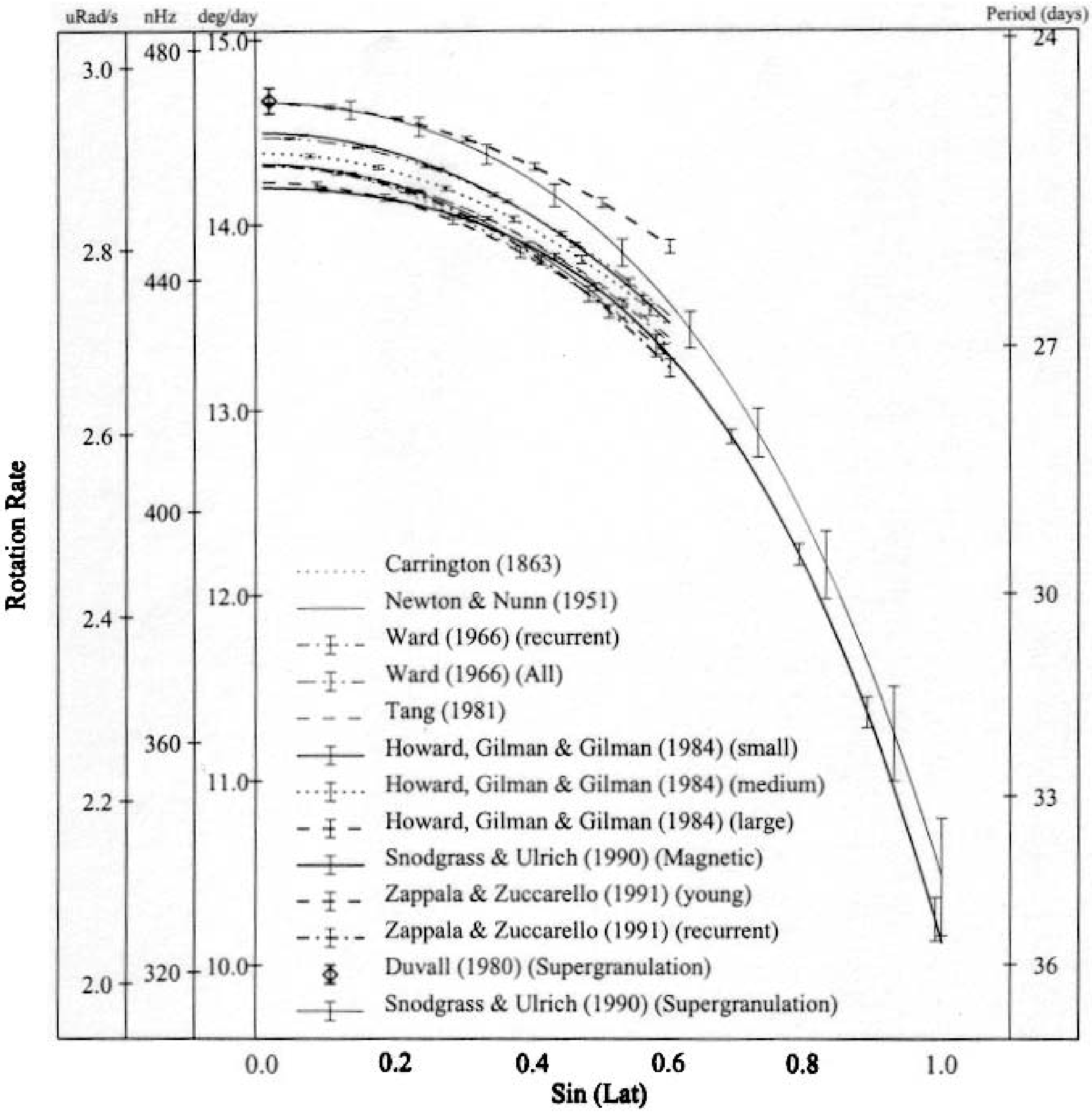
Advantage/Disadvantage: Depth and latitude/inversion ambiguities; kernel widths; pole problems



Differential rotation measured by various tracers at various levels in the solar atmosphere.

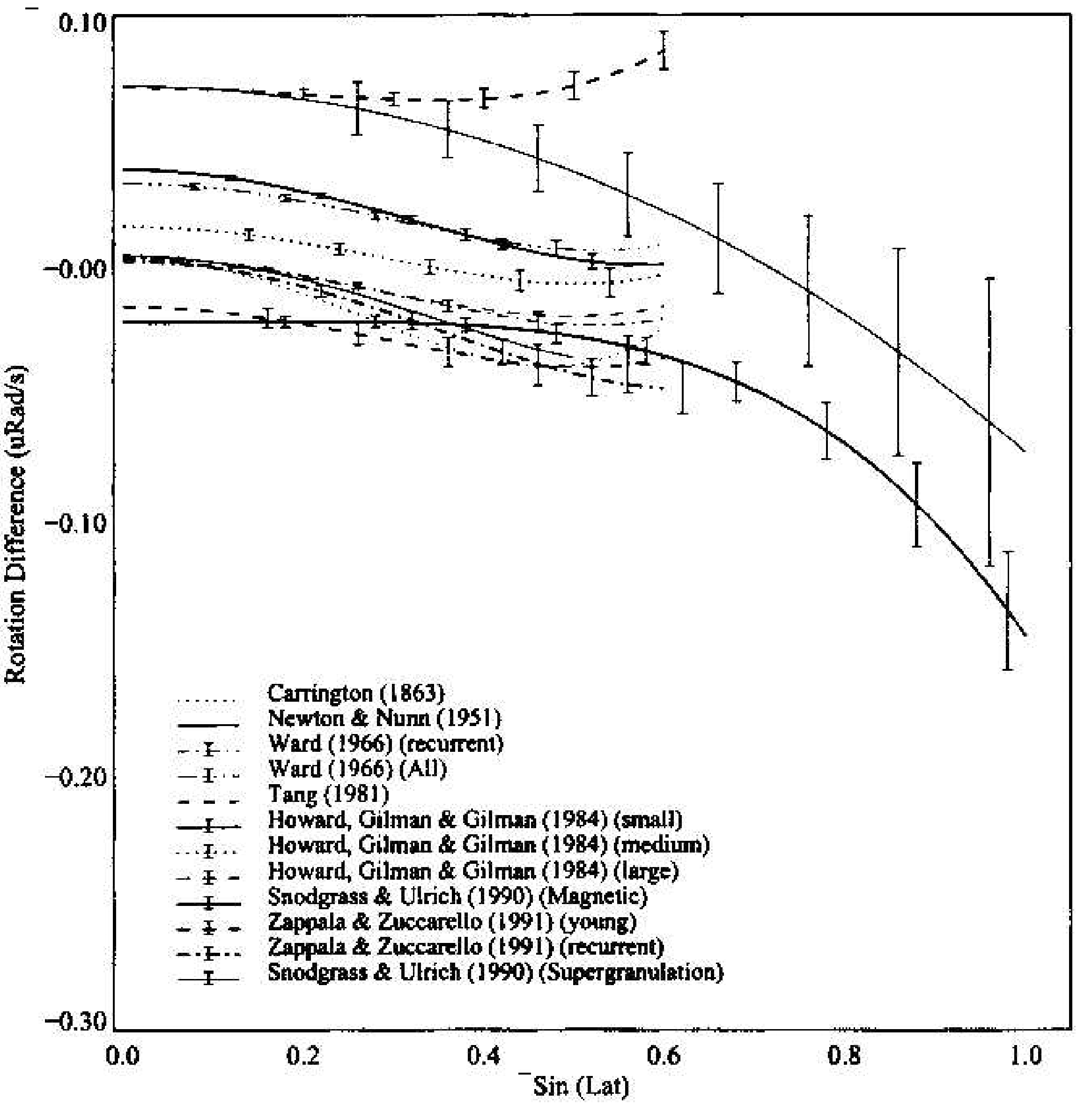
(Reprinted from Wilcox, J., Howard, R., Solar Phys. 13:251-60, 1970. With permission from the authors and D. Reidel Publishing Co.)

Tracer Rotation



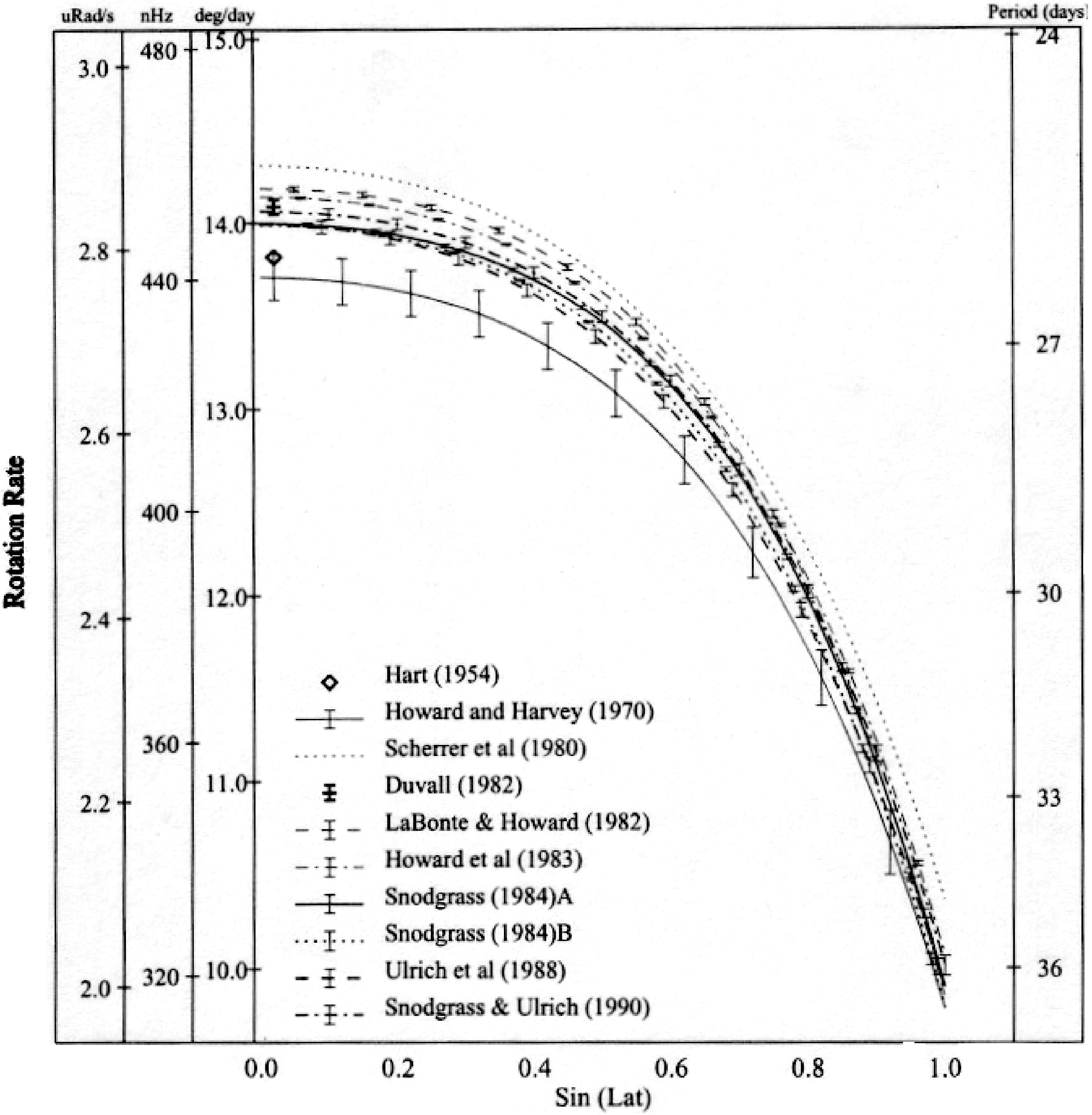
(Reprinted from Beck, John G., A Comparison of Different Rotation Measurements, Solar Physics, 191:47-70, 1999.)

Tracer Rotation Differences



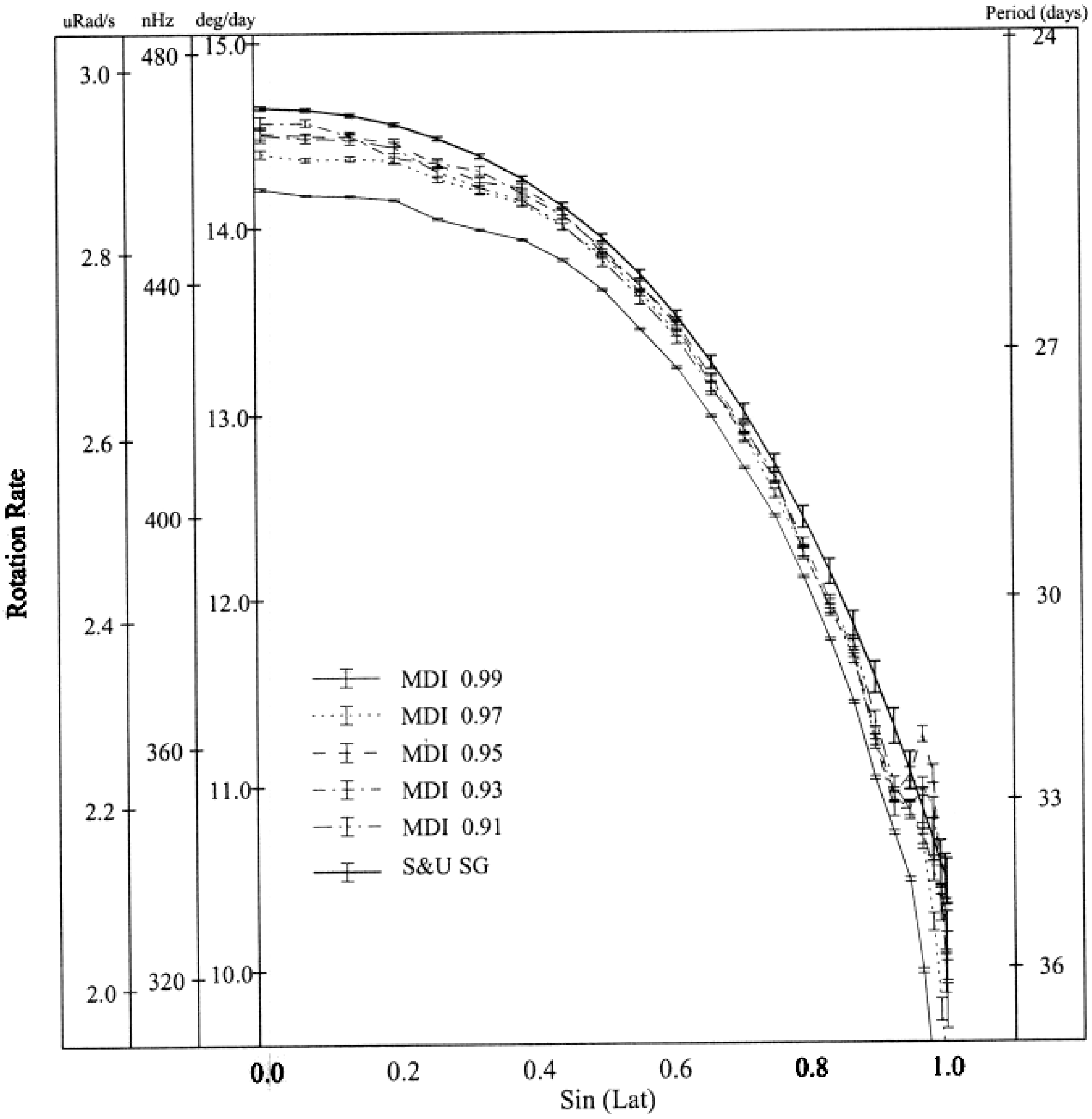
(Reprinted from Beck, John G., A Comparison of Different Rotation Measurements, Solar Physics, 191:47-70, 1999.)

Spectroscopic Rotation



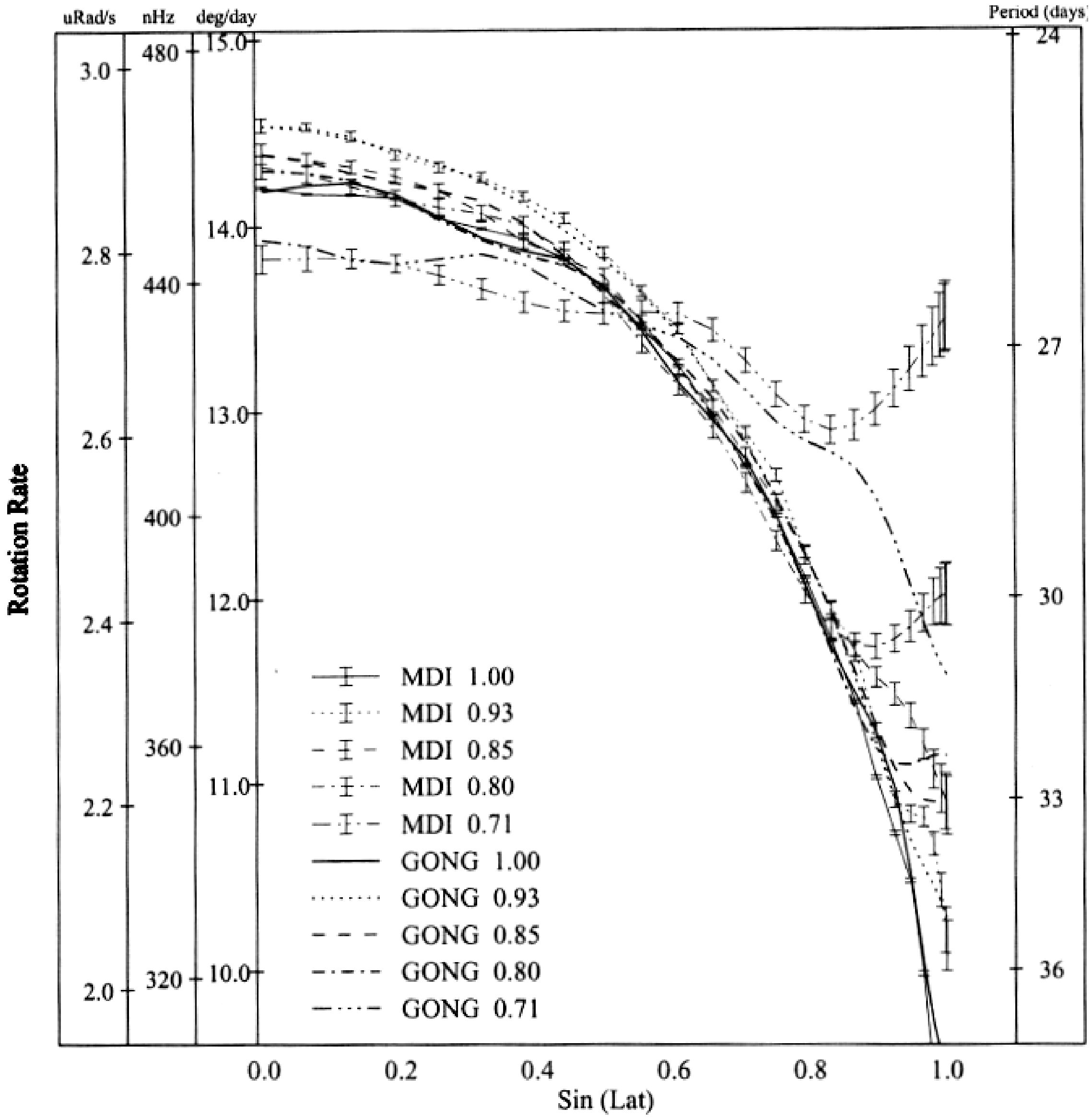
(Reprinted from Beck, John G., A Comparison of Different Rotation Measurements, Solar Physics, 191:47-70, 1999.)

Rotation Inversions and Supergranule Rotations



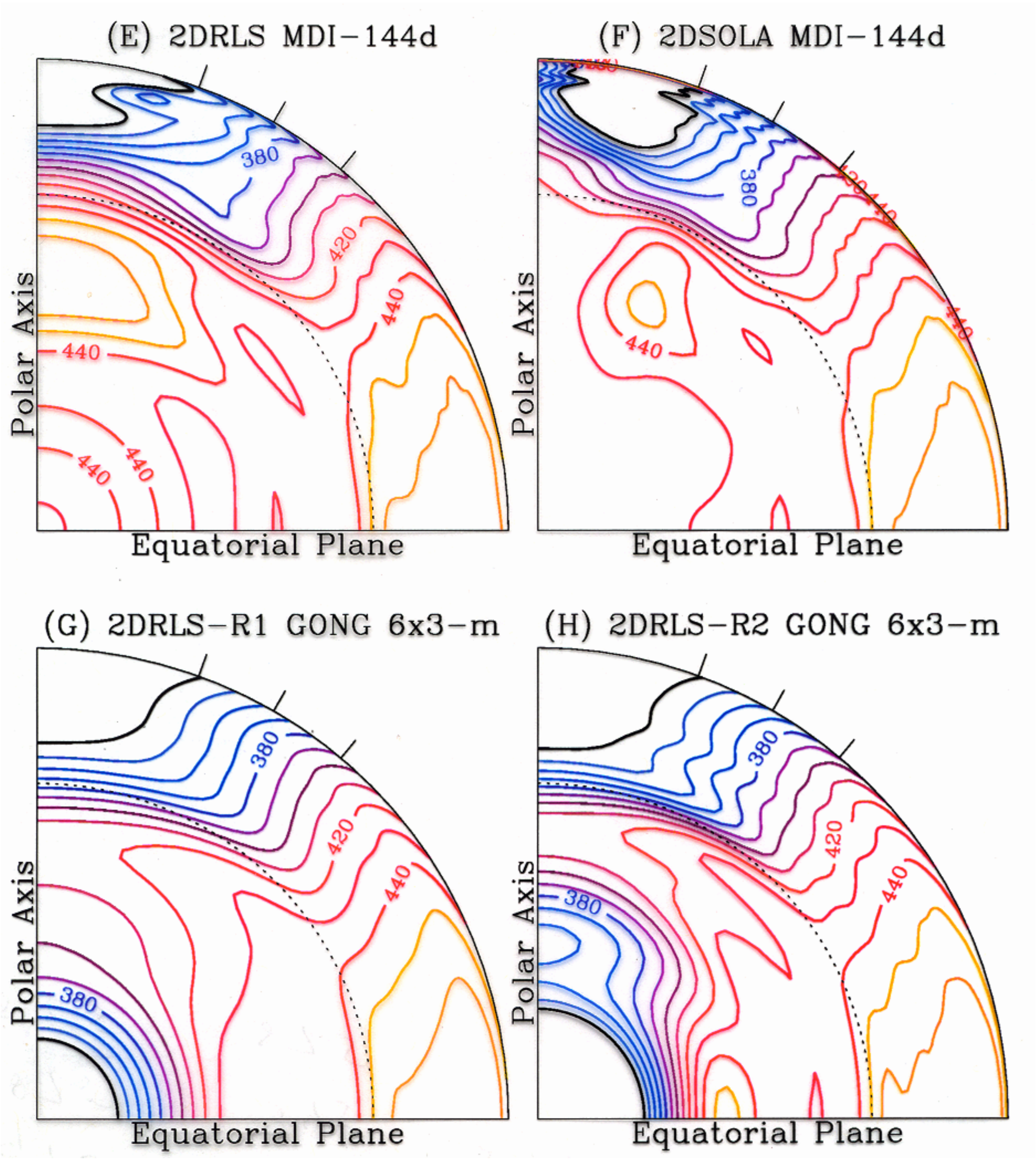
(Reprinted from Beck, John G., A Comparison of Different Rotation Measurements, Solar Physics, 191:47-70, 1999.)

MDI & GONG Differential Rotation Curves at Various Depths

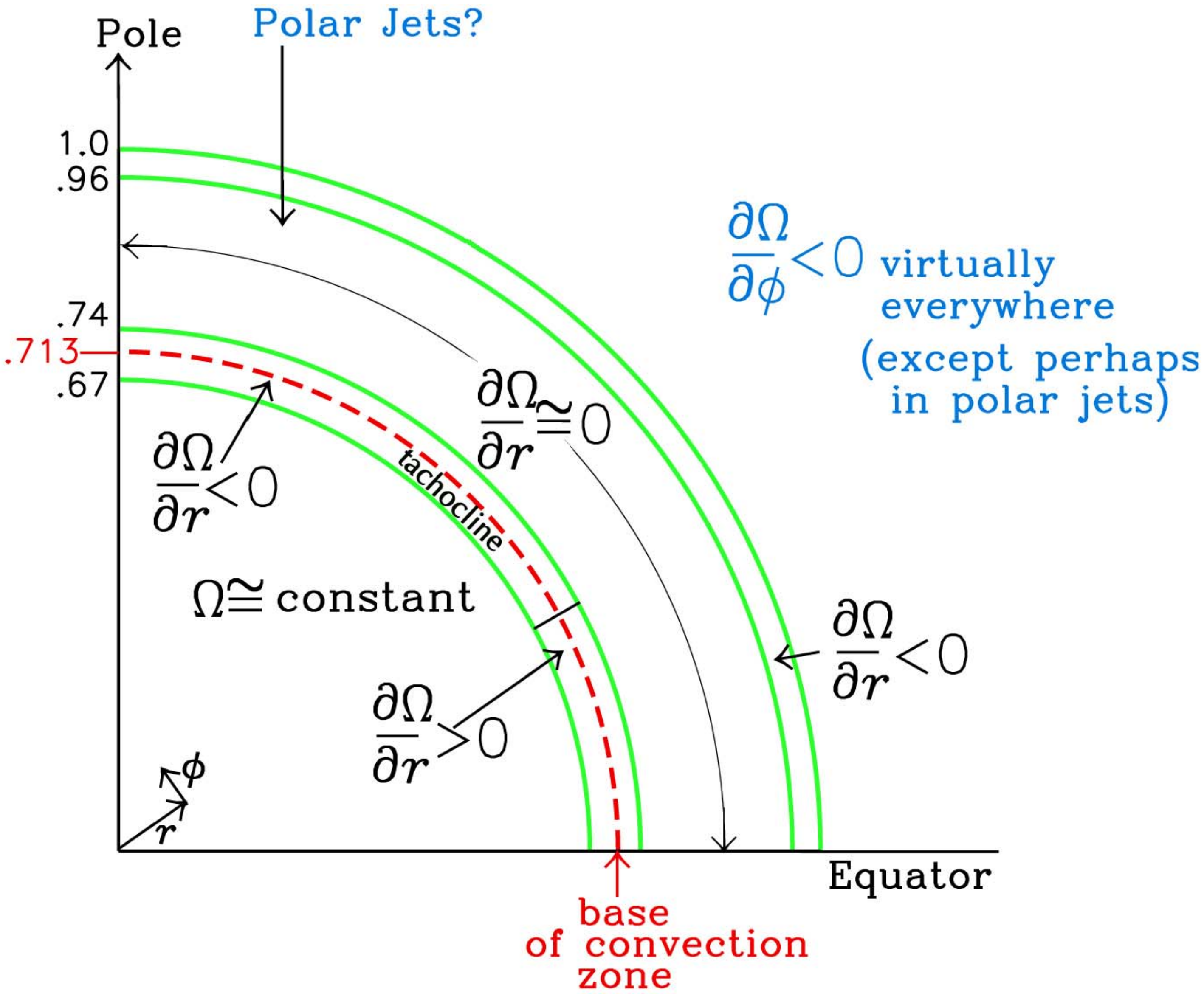


(Reprinted from Beck, John G., A Comparison of Different Rotation Measurements, Solar Physics, 191:47-70, 1999.)

Differential Rotation from Helioseismic Analysis



Angular Velocity Domains in Solar Convection Zone & Interior, from Helioseismology



Time Variations in Rotation

Evolutionary: 10^8 yrs?	spin down by solar wind torques
Sunspot cycle Envelope:	Maunder minima rotation
Sunspot Cycle: 10 yrs	• Torsional oscillations • Spot rotation changes
<< Sunspot Cycle: 1 yr	1.3 yr periodicity near tachocline from helioseismology

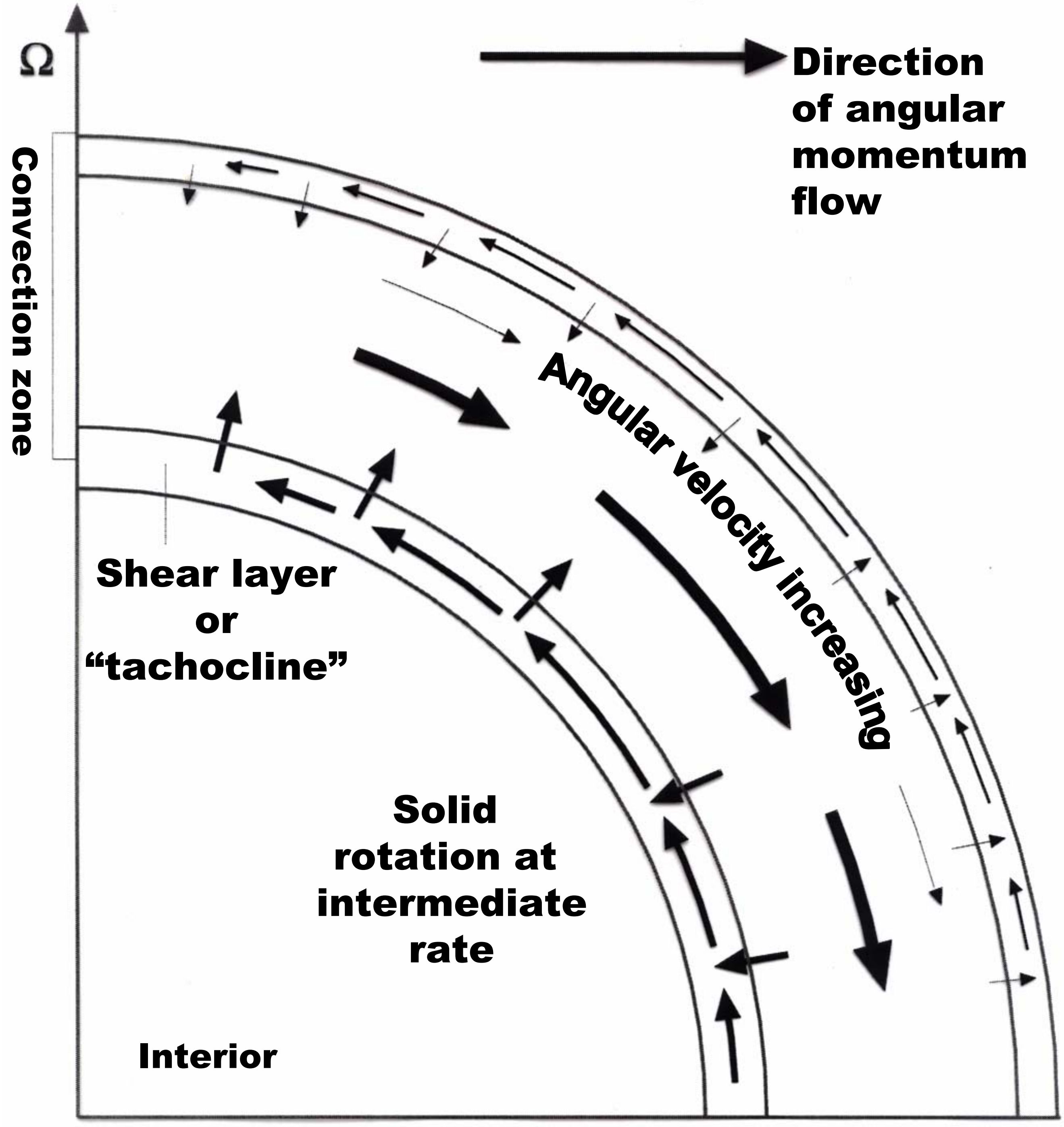
Irregular but real fluctuations on any time scale?

Amplitudes of all above variations < 1% average rotation (compare to differential rotation of 30%)

Rotation Domains and their Drivers

Interior (below convection zone):	Primordial remainder
Convection Zone:	In momentum balance with interior, internal Reynolds stresses dominate in setting detailed profile
Photosphere, Chromosphere, Corona:	Tied to convection zone by magnetic stresses, wave transfer
Solar Wind:	Angular momentum conservation leads to trailing spiral
Special Domains (in or near convection zone):	• Tachocline • Near photosphere • Polar regions

Angular Momentum Cycles in the Sun



Ω

Convection zone

Direction of angular momentum flow

Angular velocity increasing

Shear layer
or
“tachocline”

Solid rotation at intermediate rate

Interior

Equator

Rotation Regimes in the Sun

Could be common explanation in terms of angular momentum cycles, if we assume that

1. Primary driver is Reynolds stresses in the bulk of the convection zone transporting angular momentum from high latitudes to low
2. There is some leakage of momentum accumulated near the equator to layers above and below (by a variety of possible mechanisms, that may be different above and below)
3. Rotation is in a statistically steady state

It follows from 1-3 above that there must be return angular momentum to high latitudes both above and below (as well as in situ return to high latitude in bulk of convection zone, due to other scale of motion or other torques)

Time scales for all branches ~1 year, so long term, weak torques like the solar wind torque do not affect this picture (will return to tachocline later)

Mechanisms for Transporting Angular Momentum

$$\vec{v} = u\hat{\lambda} + v\hat{\phi} + w\hat{r}; \quad \vec{\mathrm{B}} = a\hat{\lambda} + b\hat{\phi} + c\hat{r}$$

- Reynolds stresses of convection (all scales); $\overline{u'v'}$ and $\overline{u'w'}$
- Meridional circulation (poleward ~ 20 m/sec flow upper part of convection zone)
- Maxwell stresses, e.g. $\overline{a'b'}, \overline{a'c'}$
- Gravity or other waves (that acquire Reynolds stresses)

All may be acting somewhere, and some, everywhere

Probable Dominant Torque Regimes

Surface Boundary Layer: Meridional circulation, small scale radial Reynolds stresses ($\overline{u'w'}$) - some tendency for radially moving fluid elements to conserve angular momentum. Maxwell stresses too small to link low latitudes with high though they nominally are through closed field lines that thread the corona.

Bulk of Convection Zone: convective scale and smaller scale Reynolds stresses $\overline{u'v'}$ and $\overline{u'w'}$, meridional circulation. Maxwell stresses play only a sporadic, localized role.

Tachocline: Small scale Reynolds & Maxwell stresses $\overline{u'w'}$, $\overline{a'c'}$; global scale Maxwell stresses $\overline{a'b'}$, $\overline{a'c'}$, $\bar{a}\bar{b}$, $\bar{a}\bar{c}$. Global scale Reynolds stresses likely smaller than Maxwell stresses. Meridional circulation? *(I will say more about tachocline later)*

What Matters in Determining Rotation Locally?

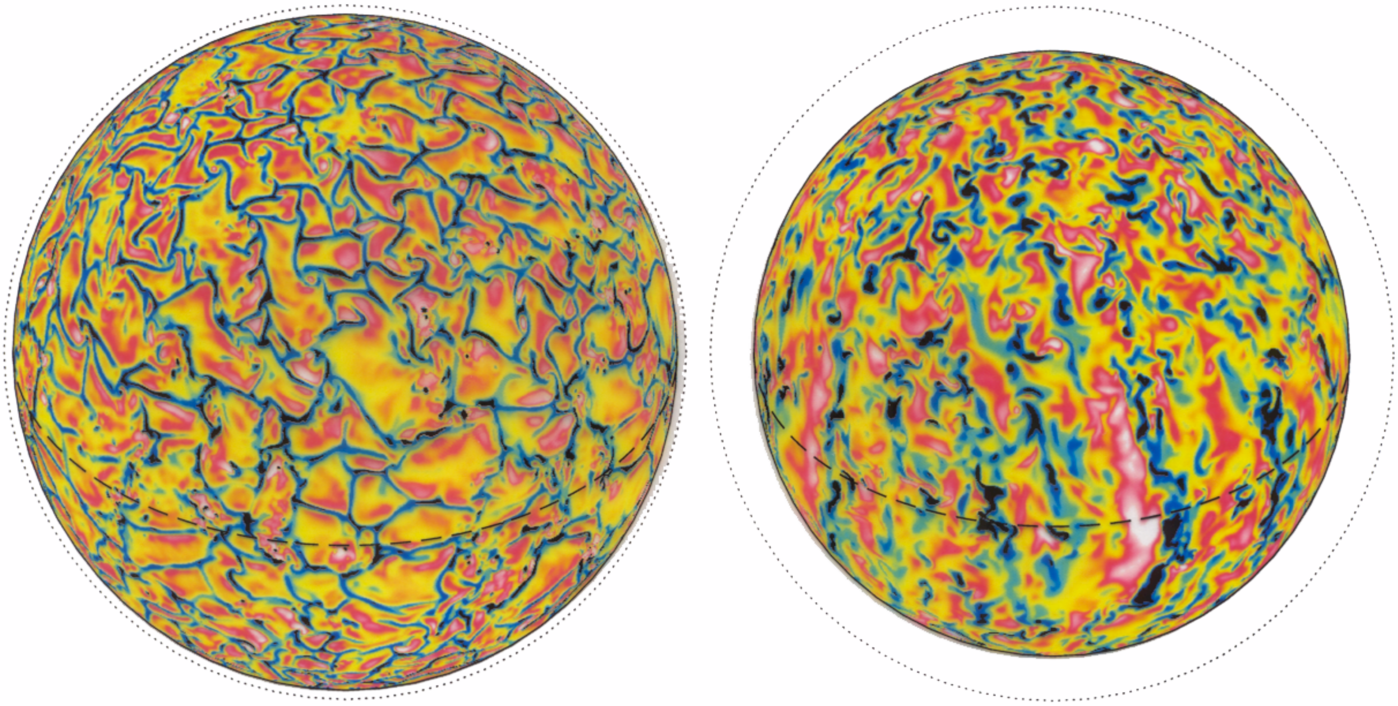
- Stably stratified or unstably stratified
- $\vec{j} \times \vec{B}$ force small or large

Interior Below Convection Zone:	Stable stratification magnetic field **small** (?)
Convection Zone:	Unstable stratification magnetic field **small** (except in big tubes)

Photosphere	Unstable	Field **Small** (except in tubes)
Chromosphere	Stable	Field **Small** (except in tubes)
Corona	Stable	Field **Large**
Solar Wind	Stable	Field **Small**

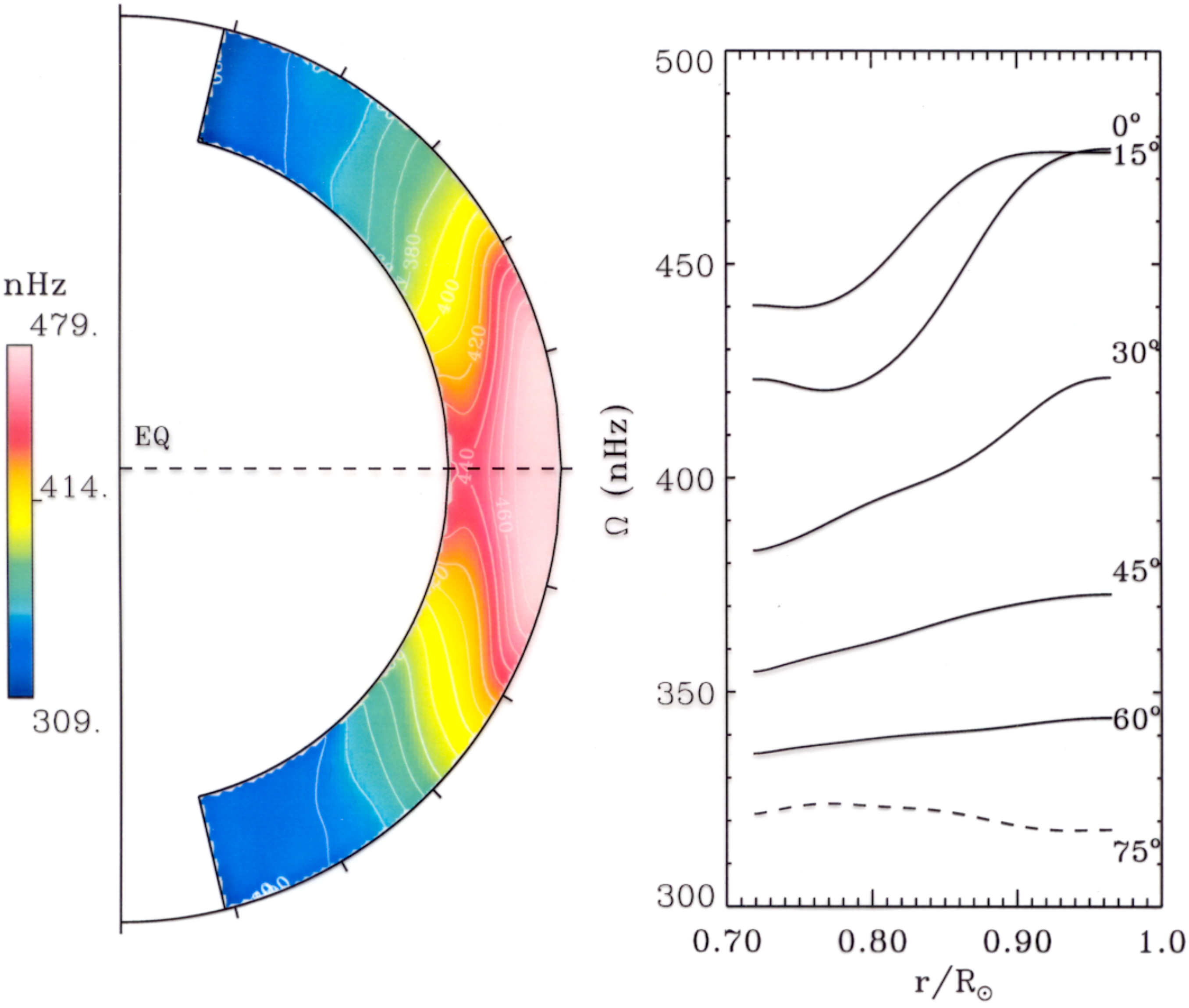
Tachocline	Stable (2 domains)	Field **Large**
Near Photosphere	Unstable	Field **Small** (except in tubes)
Polar Region	Unstable	Field **Small**

Recent Global Convection Simulation



(Brun & Toomre 2002.)

Differential Rotation from Recent Global Convection Simulation



(Brun & Toomre 2002.)

Links Between Solar Rotation & Magnetic Structure

Passive Effects of Average Differential Rotation:

- Change synoptic magnetic patterns *(shearing by differential rotation)*
- Thereby evolve coronal structures, cause disequilibrium – sudden changes
- Different tracers to rotate at different rates. Wakes around spots, etc.

Direct Effects of Rotation & Differential Rotation on Observed Patterns:

- Coriolis effect on emerging spots and active regions – latitude of emergence, tilts, leader-follower differences, etc.
- Instability of tachocline DR and TF setting templates for observed surface magnetic features. Active longitudes? Spot latitudes?
- Handedness of solar magnetic field
  Left Handed - Northern Hemisphere
  Right Handed - Southern Hemisphere

Properties of Solar Rotation We Need to Know for Solar Dynamo Models

Differential rotation in latitude and radius

Role of rotation in production of kinetic helicity (α-effect) for producing poloidal field from toroidal field

Three Candidate Locations:

- Near Surface (Babcock-Leighton type effect)
- Bulk of Convection Zone (influence by Coriolis forces on convection)
- Tachocline (global ~2D MHD instabilities and/or Coriolis influence on injected toroidal flux tubes)

Rotation and First Solar Dynamo Paradox

(1970s – 1980s)

- Mean field dynamo theory applied to sun required rotation increase inward
- Global convection models predicted rotation approximately constant on cylinders, but with equatorial acceleration ~30%

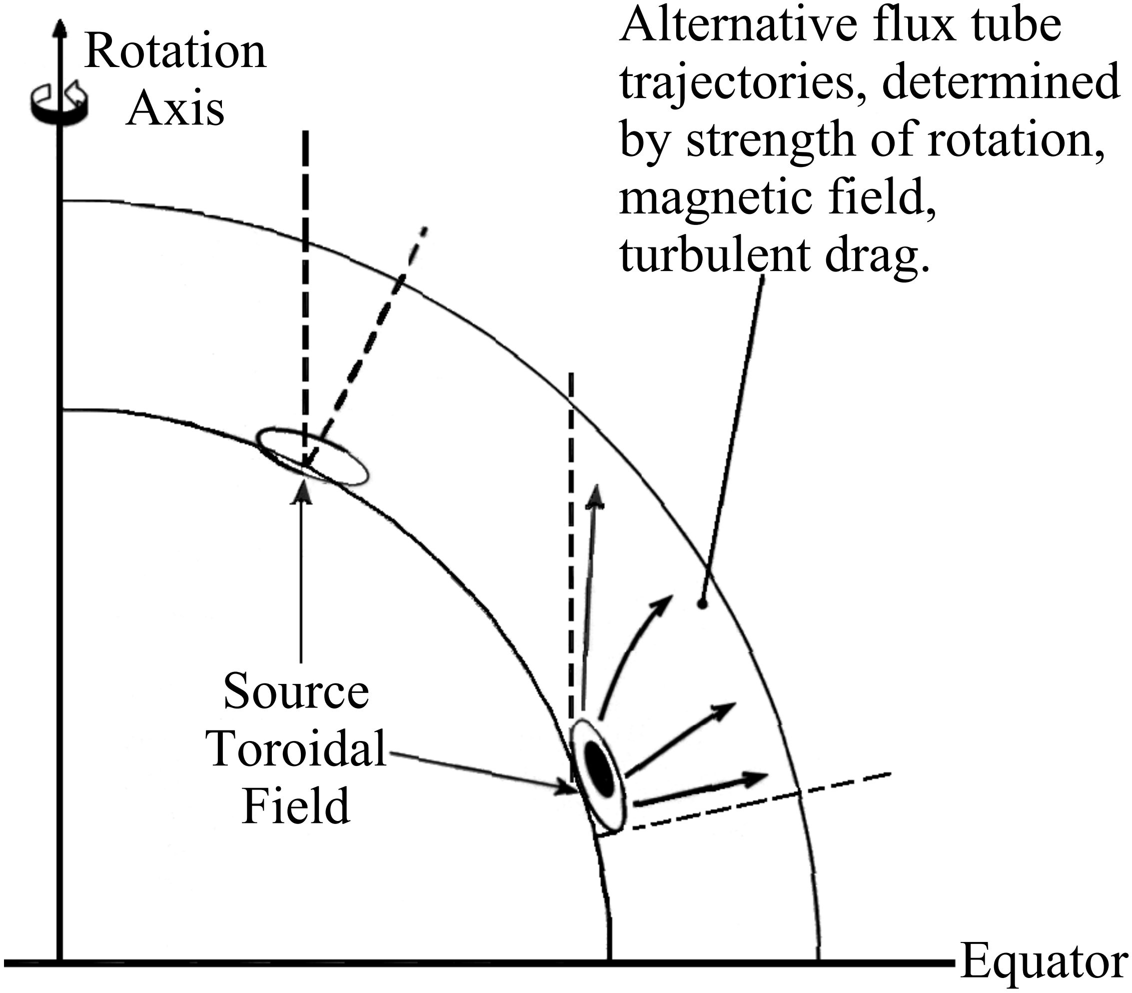
In 1970s prevailing view was that global convection theory must be wrong (I never shared that view).

In 1980s helioseismic inferences proved both were wrong, but dynamo theory more wrong than convection theory.

Conclusion

Move dynamo to base of convection zone

Schematic of Range of Trajectories of Rising Tubes



Alternative flux tube trajectories, determined by strength of rotation, magnetic field, turbulent drag.

Limiting Cases:

Strong Rotation	Weak Rotation
Weak Magnetic Fields	**Strong** Magnetic Fields
Weak Turbulence	**Strong** Turbulence

⬇ Trajectory Parallel to the Rotation Axis

⬇ Trajectory Radial

Choudhuri and Gilman, 1987, ApJ., **316**, 788.

Rotation and Second Solar Dynamo Paradox

(1980s – 1990s)

- To produce sunspots in low latitudes requires toroidal fields ~10^5 gauss at the base of the convection zone (influence of Coriolis forces on rising tubes)

- 10^5 gauss fields very hard to store – must be below convectively unstable layer (overshoot layer subadiabatic?)

- 10^5 gauss fields are 10^2 x equipartition – won't that suppress dynamo action? (but apparently does not in geo case!)

Resolution

Interface Dynamos

Flux Transport Dynamos

How Much Solar Rotation Theory is Needed to do Solar Dynamos?

Kinematic Dynamos:	NONE - Just need enough observational detail of rotation, meridional circulation
Ultimate MHD Dynamo:	Full theory for solar differential rotation and magnetic fields-still impractical; early attempts were dynamos, but missed radial rotation gradient and butterfly diagram
A Promising Intermediate Step:	Combine a global MHD theory of the tachocline with kinematics of the convection zone Requires focus on theory of coexisting toroidal field and differential rotation in the tachocline

The Solar Tachocline

Definitions:

Tachocline – where have strong radial gradient of rotation at convection zone base

Overshoot (undershoot) layer – where convection penetrates below level at which specific entropy gradient changes sign in mixing length stellar structure models

(Some people distinguish between "overshoot" & "penetration")

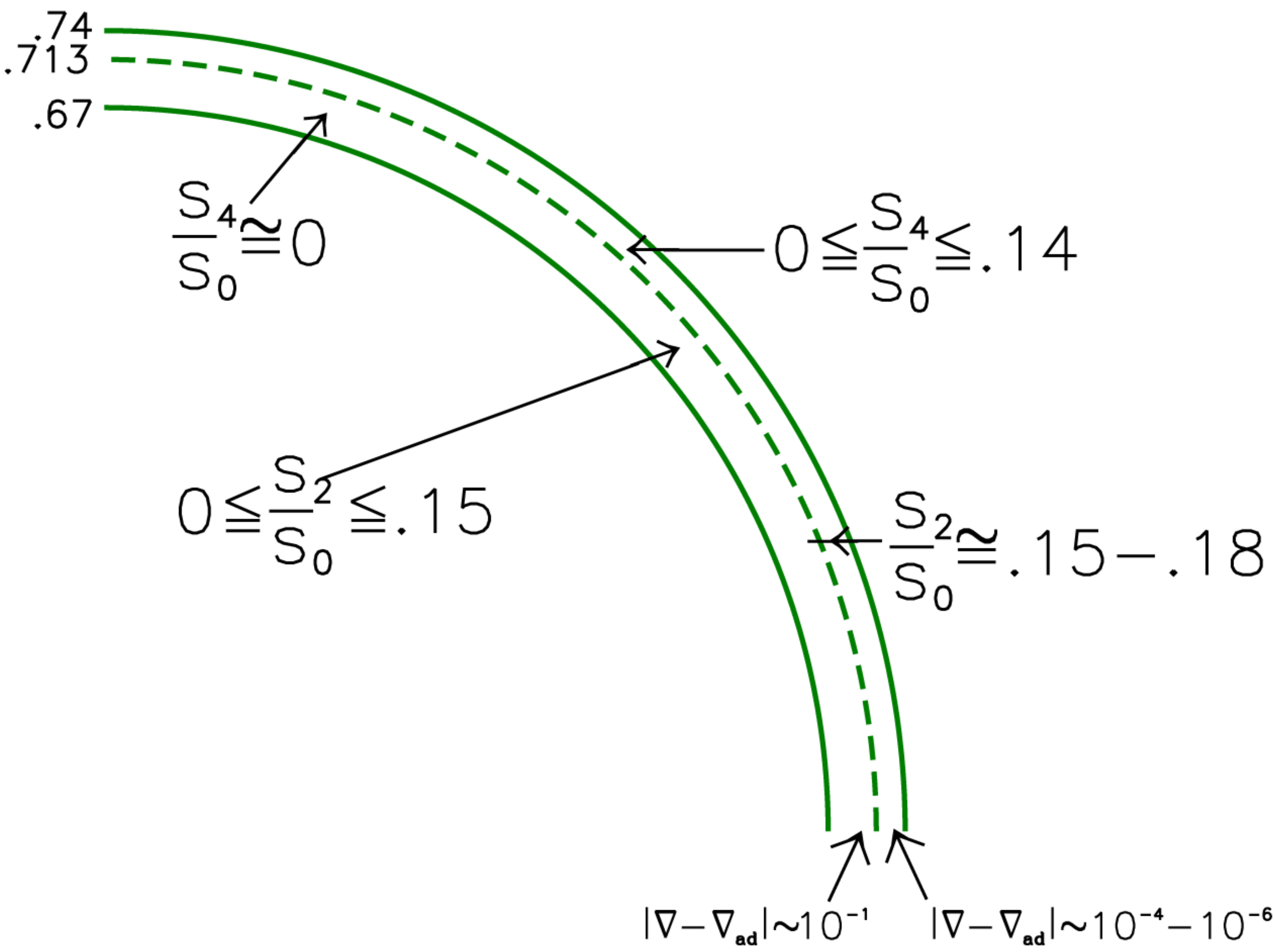
If bottom of overshoot defined as depth where temperature gradient becomes radiative, then tachocline contains both overshoot (~1/3) and radiative (~2/3) layers

Caveat: Even in radiative layer, can still have weak turbulence, so long as doesn't change temperature gradient very far from the radiative value (magnitude allowed?)

Can also have waves there, particularly gravity waves, excited by convection above

Rotation Detail Within Solar Tachocline

$$\frac{\Omega}{2\pi} = s_0(r) - s_2(r)\sin^2\phi - s_4(r)\sin^4\phi$$



Types of HD and MHD Relevant to the Tachocline

- Global HD and MHD Instabilities
- Gravity /Alfvén wave/mean flow interactions
- Magnetic Buoyancy
- Penetrative Convection and “Turbulence”
- Induction / Dynamo Effects
- Coupling Among the Above

Fast Maintenance versus Slow Maintenance of the Radiative Part of the Tachocline

Fast	Slow
Strong latitudinal mixing or transmission of angular momentum by Reynolds and/or Maxwell stresses	By very weak meridional circulation
Gravity waves may be present, transmitting angular momentum radially (with critical layers) and latitudinally. Interaction with toroidal field complex	Gravity waves not contributing to transmission or mixing of momentum
Toroidal field up to 10^5 gauss present	Toroidal field < 0.1 gauss?
Some poloidal field to complete interface dynamos ($\sim 10^{-3}$ toroidal field?)	Primordial poloidal field only
Small scale turbulent mixing $\leq 10^{-2}$ or 10^{-3} of convection zone values	Small scale turbulent mixing $\leq 10^2$ or 10^3 of molecular values?
Tachocline participates significantly in overall angular momentum "cycle" with convection zone above	Tachocline angular momentum transfer processes too weak to participate much in overall angular momentum cycle

Plausible Assumptions for Tachocline

- Differential rotation imposed by convection zone above
- Toroidal fields very strong (~ 10^5 gauss?)
- Thickness < 5% radius; spherical geometry important, but divergence of radii minor
- Density variations with radius small enough that anelastic approximation very good, Boussinesq approximation okay.
- Hydrostatics or magnetohydrostatics good for global problems (but filters out magnetic buoyancy)

Global, Quasi 2D MHD of the Solar Tachocline

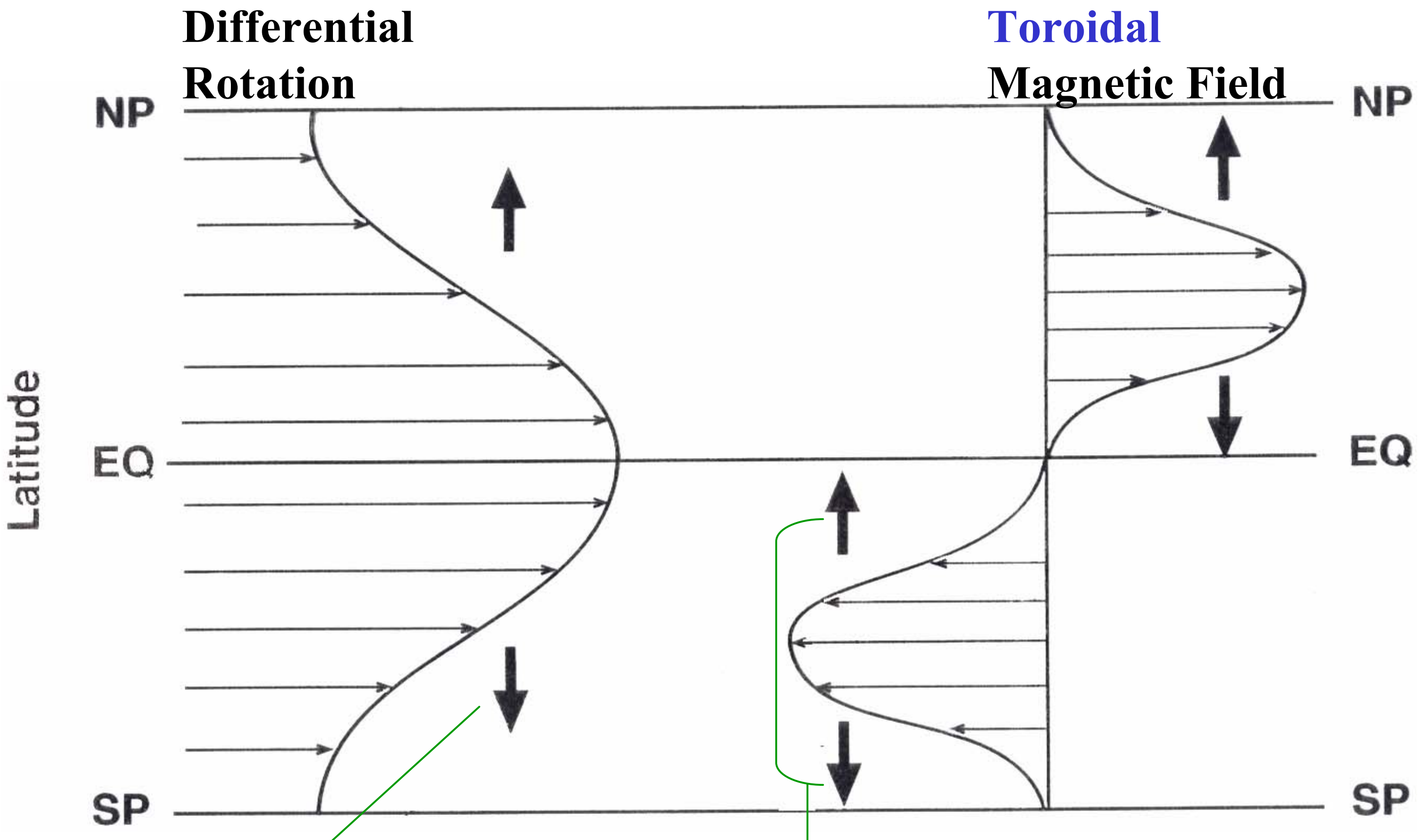
Ingredients: Differential Rotation

Subadiabatic Stratification

Strong Toroidal fields

- MHD analog to classical GFD problems of barotropic & baroclinic instability
- Magnetic field can make unstable differential rotations that are stable without it
- If allow some variation in radial direction, instability can generate kinetic helicity.
- Subject of continuing long term study by: Gilman, Fox, Dikpati, Cally, Miesch *(in order of first involvement)*
- Plus additional contributions by: deSterk (HAO Newkirk Fellow), Schecter (ASP), Boyd (Summer Student 2000), Whisker (Summer Student 2001)

Properties of 2D MHD Instability of Differential Rotation & Toroidal Magnetic Field


Differential Rotation

Toroidal Magnetic Field

Angular momentum transport toward the poles primarily by the Maxwell Stress (perturbations field lines tilt upstream away from equator)

Magnetic flux transport away from the peak toroidal field by the Mixed Stress (phase difference in longitude between perturbation velocities & magnetic fields)

Nonlinear 2D MHD Equations

Defining velocity & magnetic filed respectively as $\vec{V}=u\hat{\lambda}+v\hat{\phi}, \vec{B}=a\hat{\lambda}+b\hat{\phi},$ and using a modified pressure variable $(\pi=p/\rho),$ we can write,

Continuity Equations:

$$\frac{\partial u}{\partial\lambda}+\frac{\partial}{\partial\phi}(v\cos\phi)=0,$$

$$\frac{\partial a}{\partial\lambda}+\frac{\partial}{\partial\phi}(b\cos\phi)=0,$$

Equations of Motion:

$$\frac{\partial u}{\partial t}+\frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{u^2+v^2}{2}\right)-\frac{v}{\cos\phi}\left[\frac{\partial v}{\partial\lambda}-\frac{\partial}{\partial\phi}(u\cos\phi)\right]$$

$$=-\frac{1}{\cos\phi}\frac{\partial\pi}{\partial\lambda}-\frac{b}{\cos\phi}\left[\frac{\partial b}{\partial\lambda}-\frac{\partial}{\partial\phi}(a\cos\phi)\right],$$

$$\frac{\partial v}{\partial t}+\frac{\partial}{\partial\phi}\left(\frac{u^2+v^2}{2}\right)+\frac{u}{\cos\phi}\left[\frac{\partial v}{\partial\lambda}-\frac{\partial}{\partial\phi}(u\cos\phi)\right]$$

$$=-\frac{\partial\pi}{\partial\phi}+\frac{a}{\cos\phi}\left[\frac{\partial b}{\partial\lambda}-\frac{\partial}{\partial\phi}(a\cos\phi)\right],$$

Induction Equations:

$$\frac{\partial a}{\partial t}-\frac{\partial}{\partial\phi}(ub-va)=0,$$

$$\frac{\partial b}{\partial t}+\frac{1}{\cos\phi}\frac{\partial}{\partial\lambda}(ub-va)=0.$$

Quasi-2D Global MHD Instability of Tachocline

(Gilman, Fox, Dikpati, Cally)

- DR and TF generally unstable to global waves, particularly longitudinal wave number m=1, sometimes also m=2 or higher.
- efolding Growth Times: few months – few years.
- Longitudinal Propagation Speeds: between minimum and maximum rotation rates (➔ max for strong fields)
- Nonlinear growth leads to “tipping” of toroidal field rings *(can be same or opposite in NH and SH)*
- Allowing even weak radial motions leads to unstable modes with kinetic helicity ➔ α-effect
- Global disturbances in tachocline could set “template” for surface magnetic features.

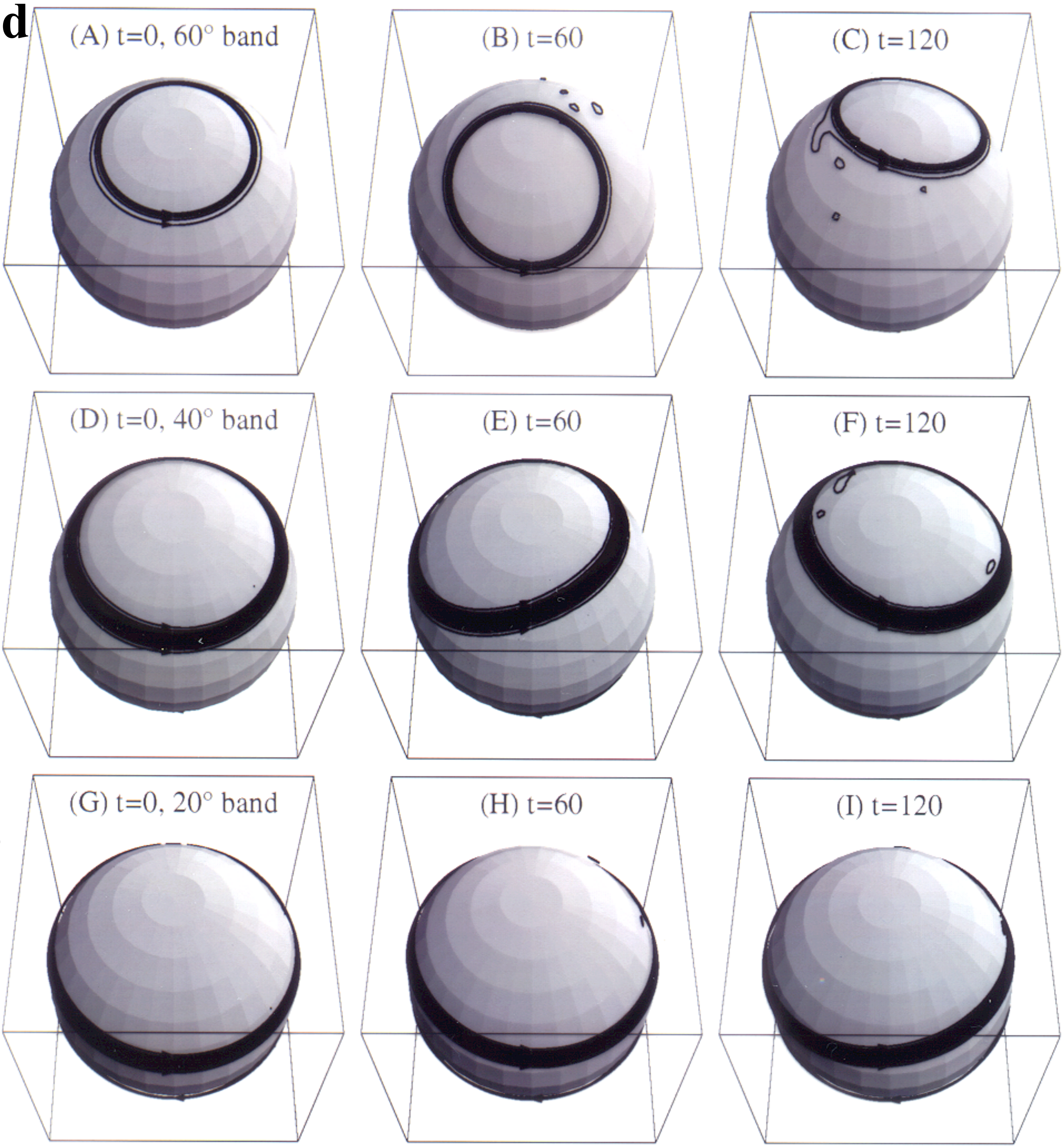
Global Instabilities of Solar Tachocline

Assume differential rotation from helioseismology

Hydrostatic Models	Result
2D HD	Stable
2D MHD	Unstable for wide range of toroidal fields
“Shallow Water” HD	Overshoot part Unstable Radiative part Stable
Shallow Water” MHD	Both Parts Unstable
	SW HD Instabilities suppressed for peak fields ≥ 10 kG
Multi-layer SW HD, MHD	Expect Instability
3D HD, MHD	Expect Instability
3D Nonhydrostatic HD, MHD	More modes of Instability Magnetic buoyancy enters

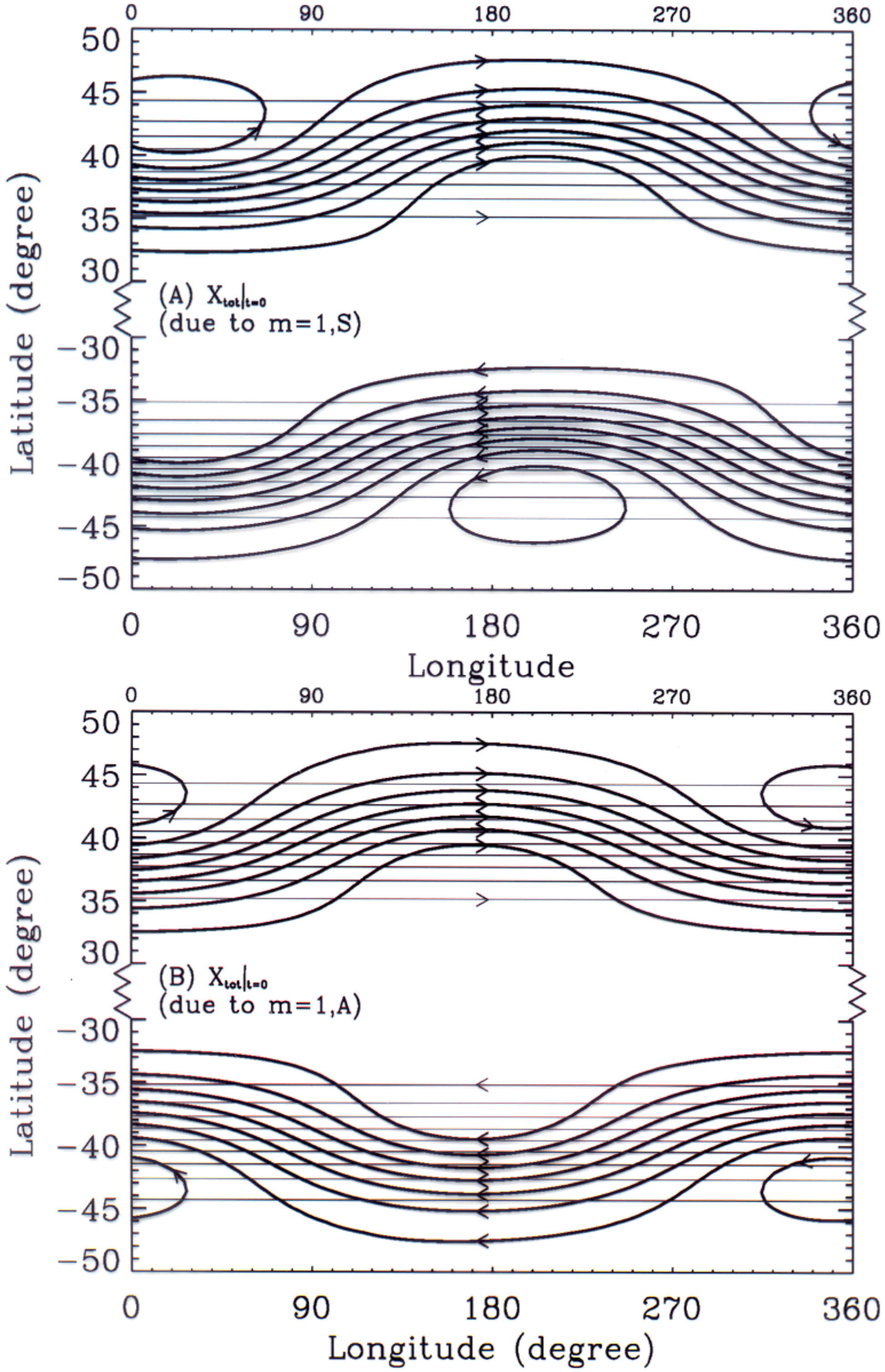
Dynamo Potential

Nonlinear Evolution of Tip of Toroidal Rings Due to 2D MHD Instability



(Cally, Dikpati, & Gilman, 2002 in preparation)

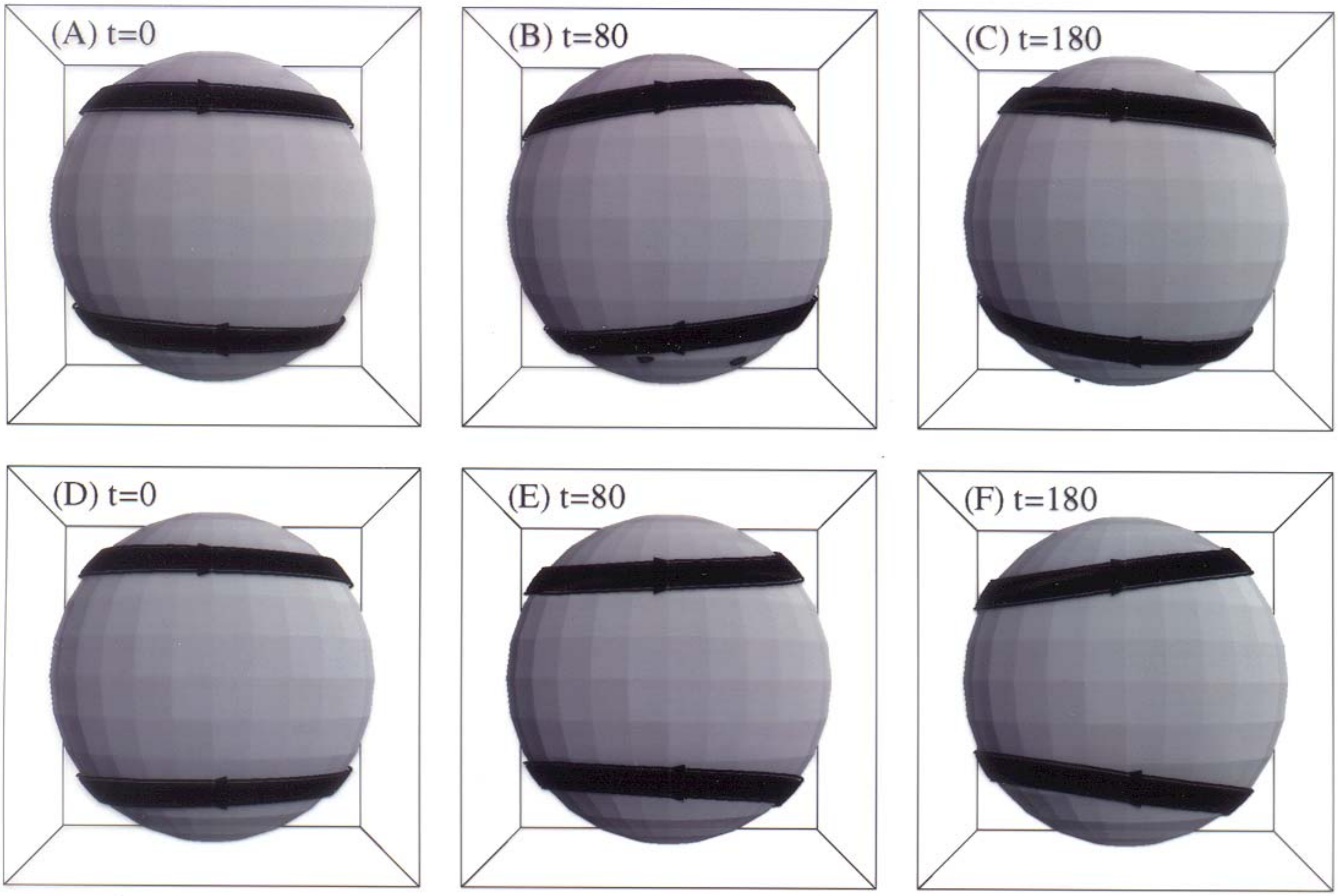
Tipped Toroidal Ring in Longitude-latitude Coordinates Linear Solutions with Two Possible Symmetries



(Cally, Dikpati, & Gilman, 2002 in preparation)

Nonlinear Evolution of Toroidal Bands at 40°

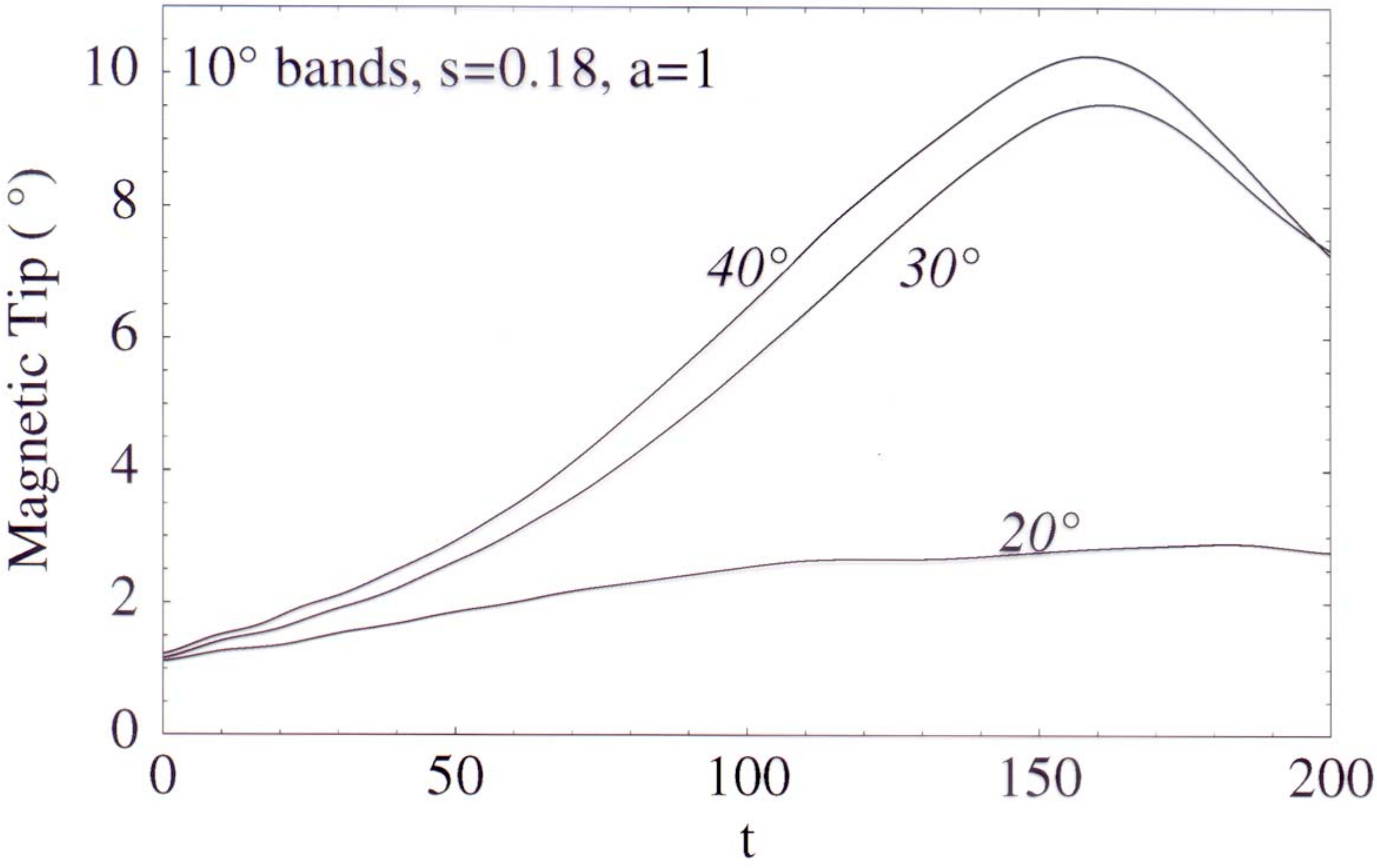
(opposite symmetries in initial state)



Time ➔

(Cally, Dikpati, & Gilman, 2002 in preparation)

Tip Angle As Function of Time



(Cally, Dikpati, & Gilman, 2002 in preparation)

What Do We Want to Know About Tachocline?

- Thickness, Shape, Depth with Latitude
- Meridional Circulation
- Toroidal fields
- Other Global Motions (m = 1 or higher)
- Radial Differential Rotation Profile