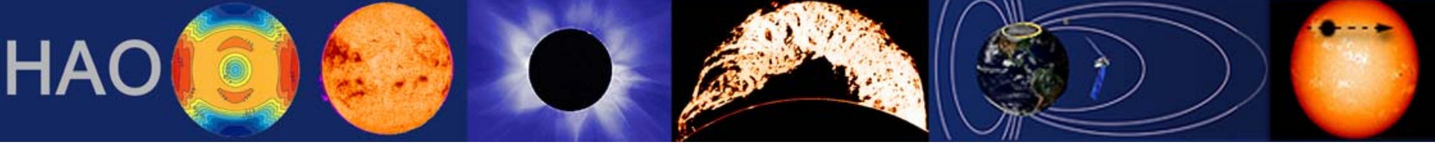

Solar Dynamo Models

Keith MacGregor

High Altitude Observatory

Observational Challenges for the Next Decade of Solar Magnetohydrodynamics

16-18 January 2002

NCAR

High Altitude Observatory (HAO) – National Center for Atmospheric Research (NCAR)

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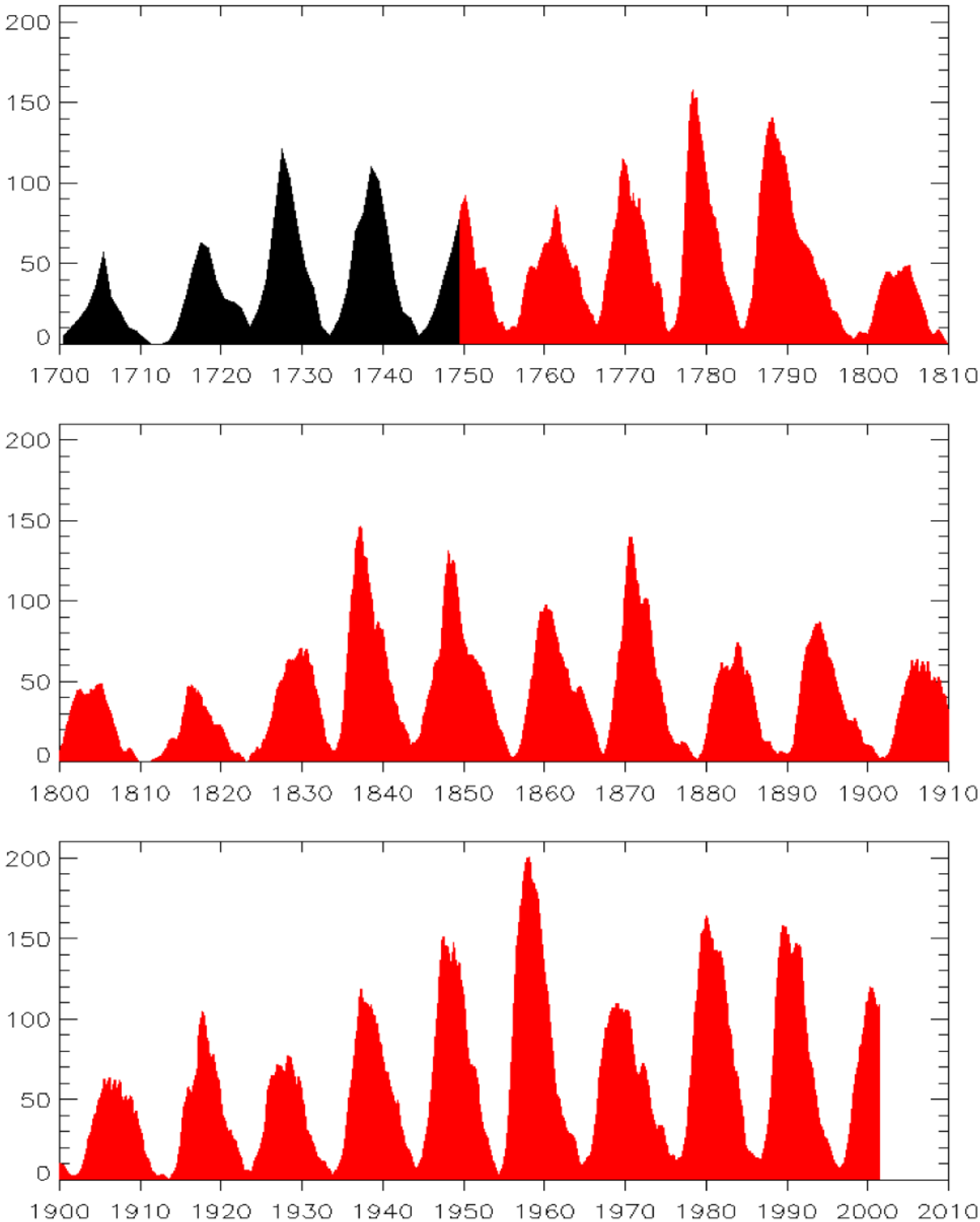
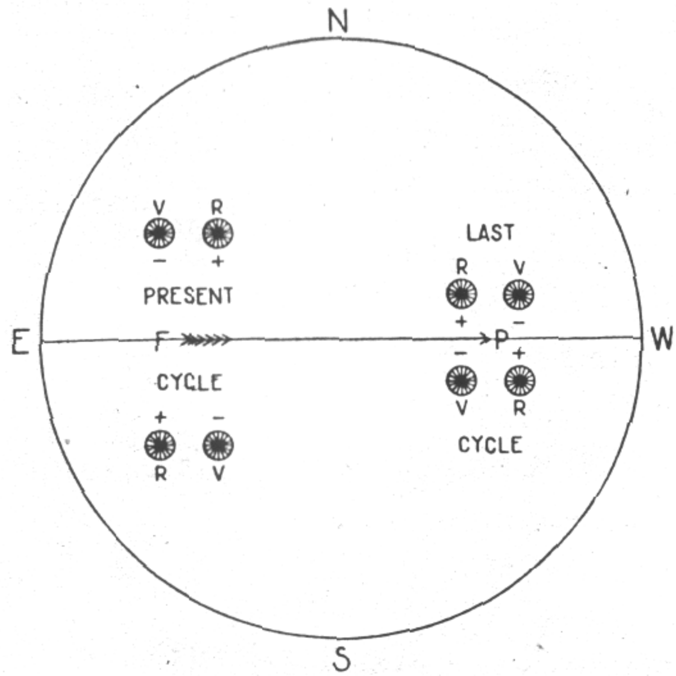
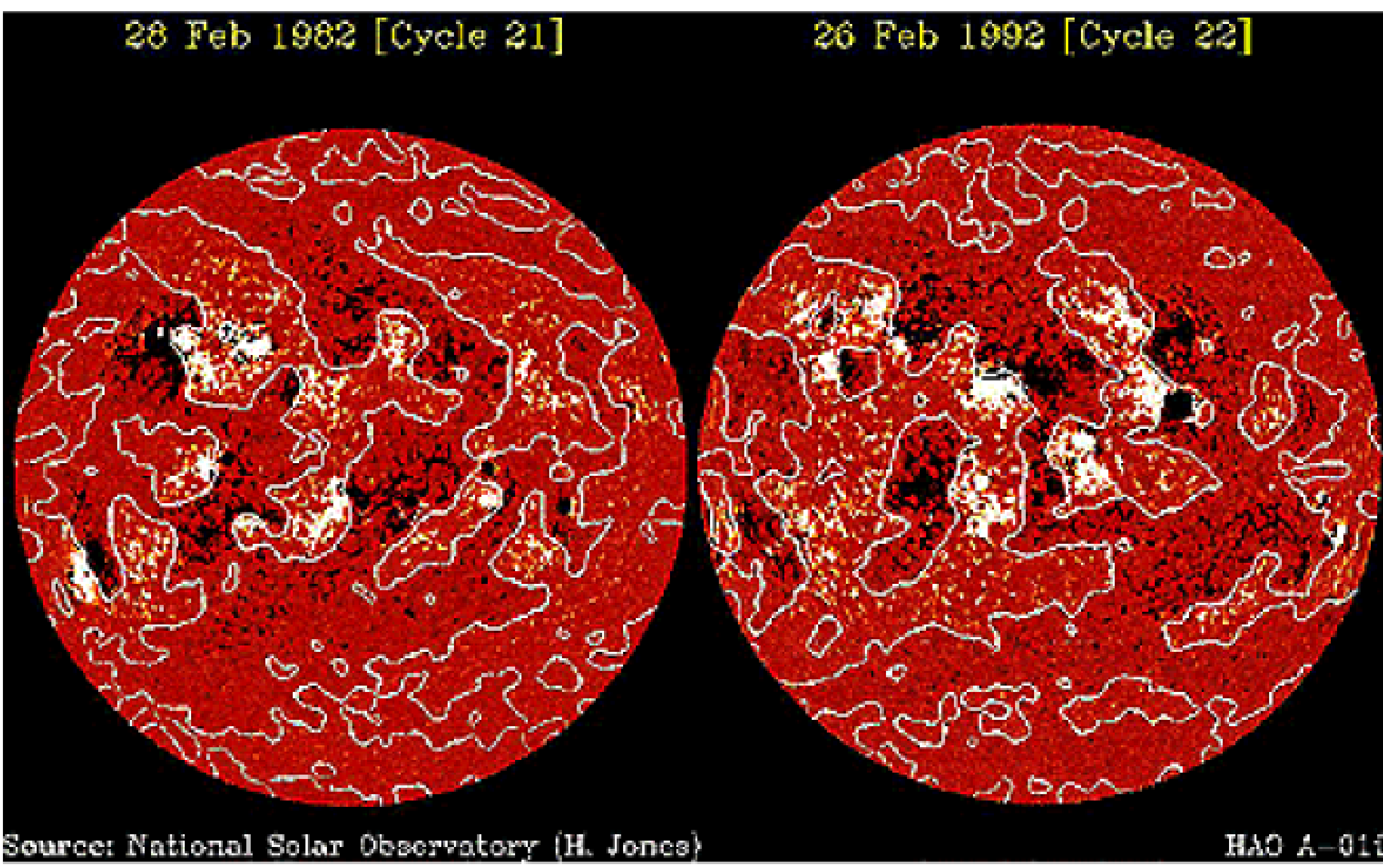
Sunspot Number Variation



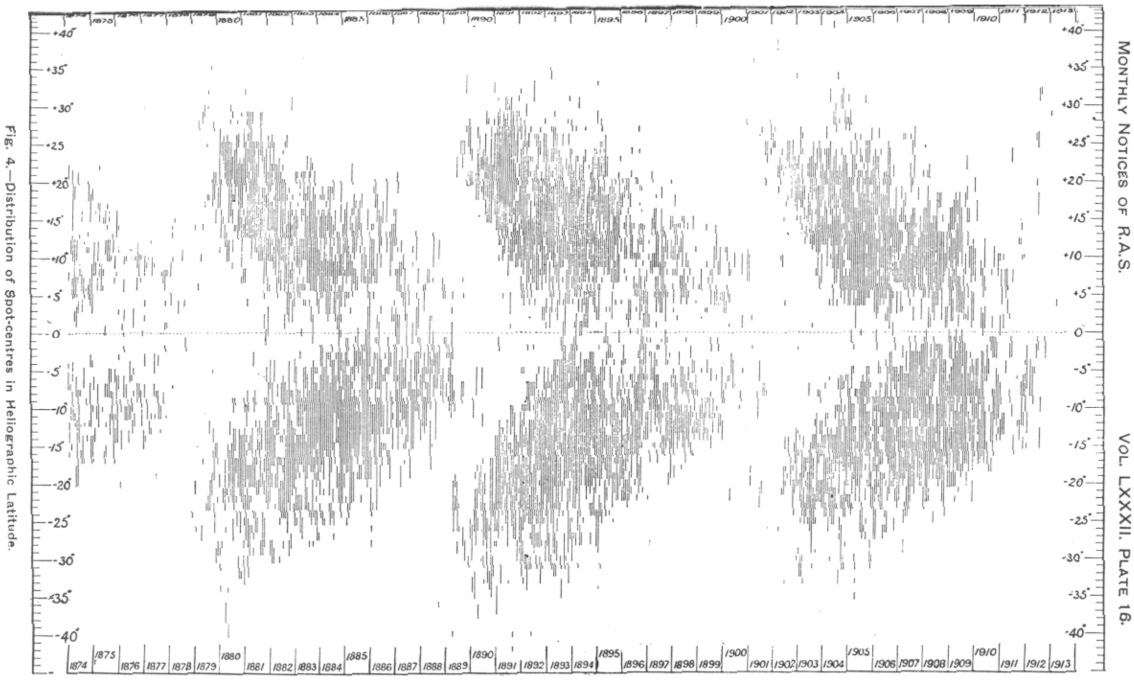
Hale's Polarity Laws



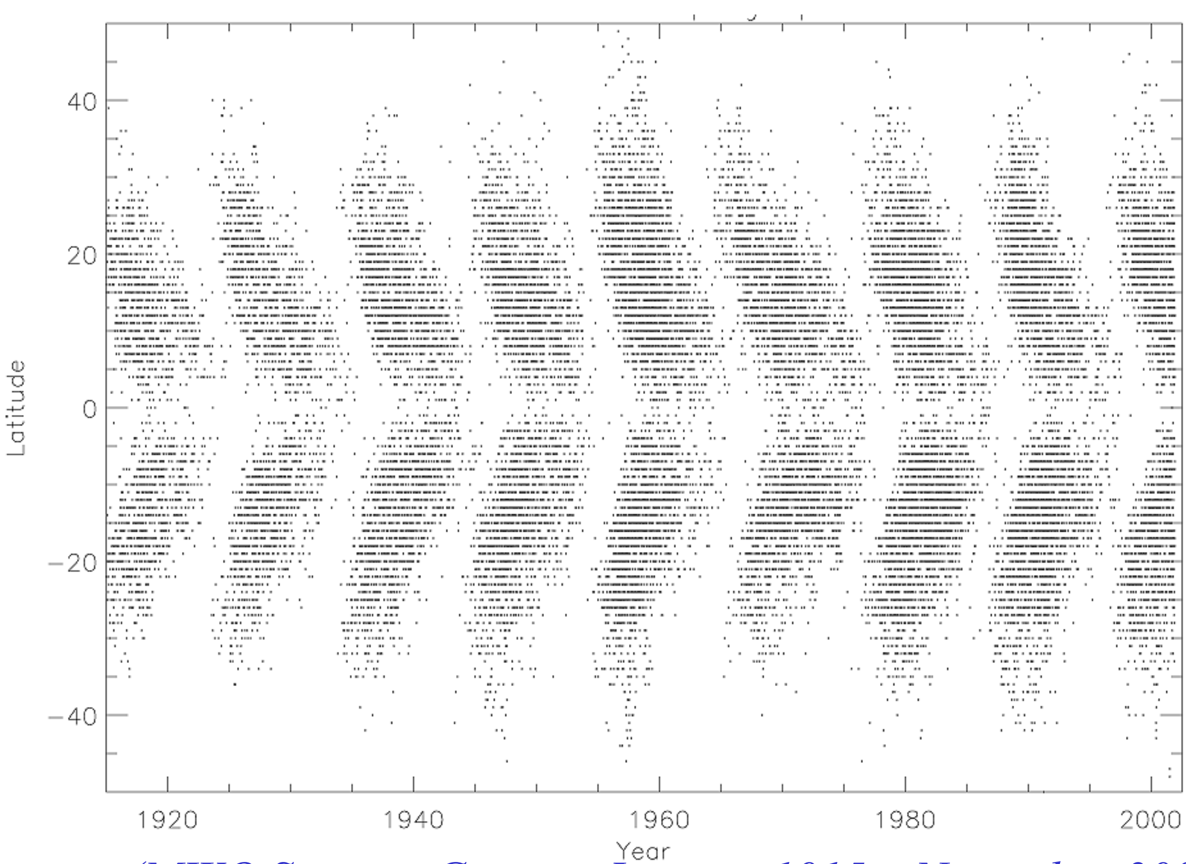
(Hale, Ellerman, Nicholson, & Joy, 1919)



Sunspot Migration

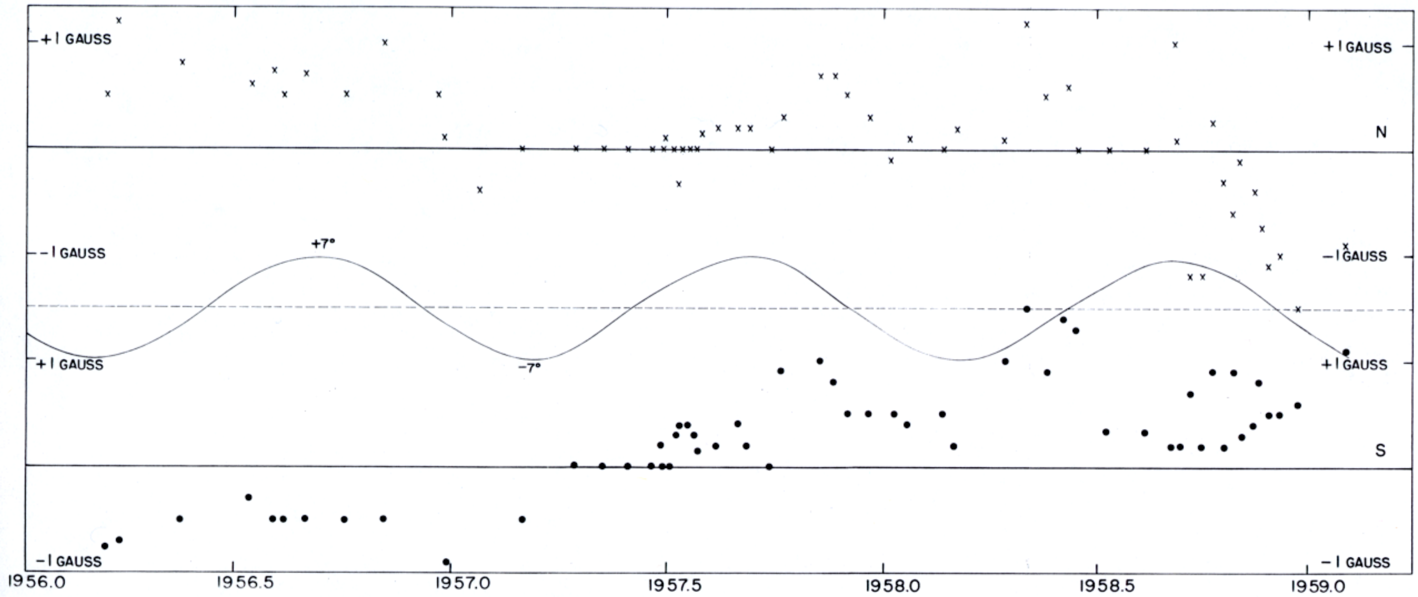


(Maunder, 1913)

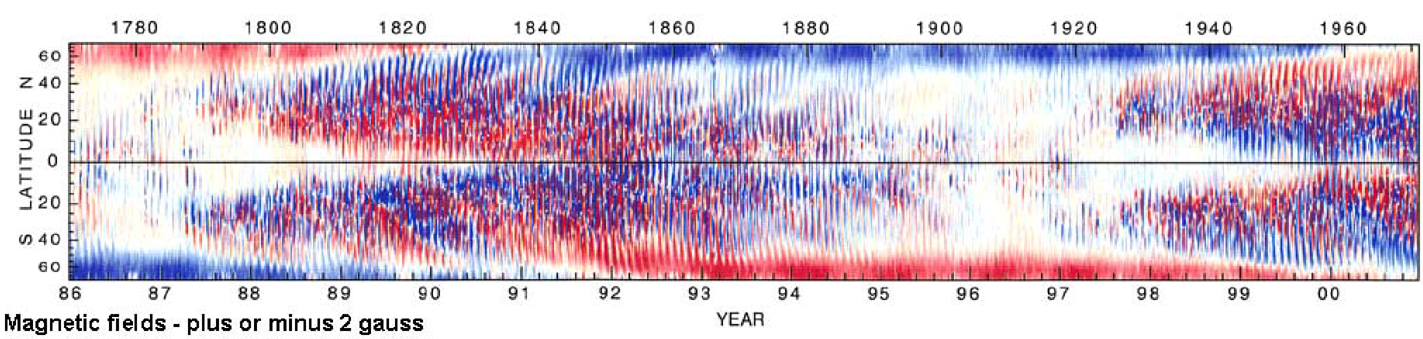


(MWO Sunspot Groups, January 1915 – November 2001)

Polar Field Reversal



(H. D. Babcock, 1959)

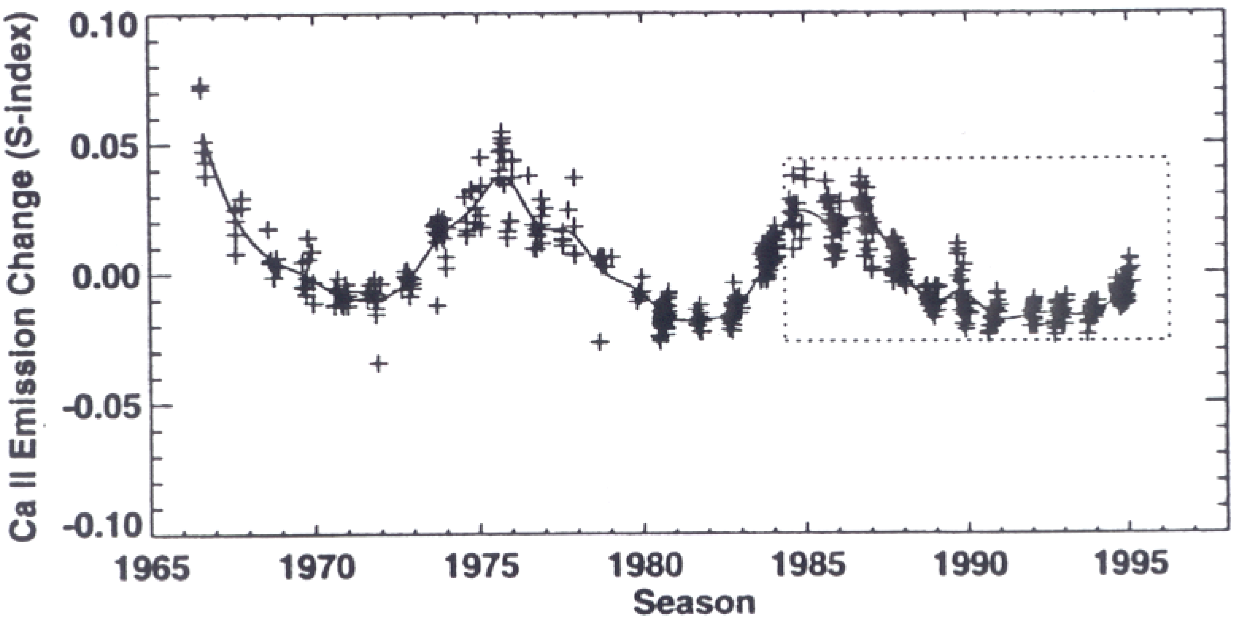


(MWO)

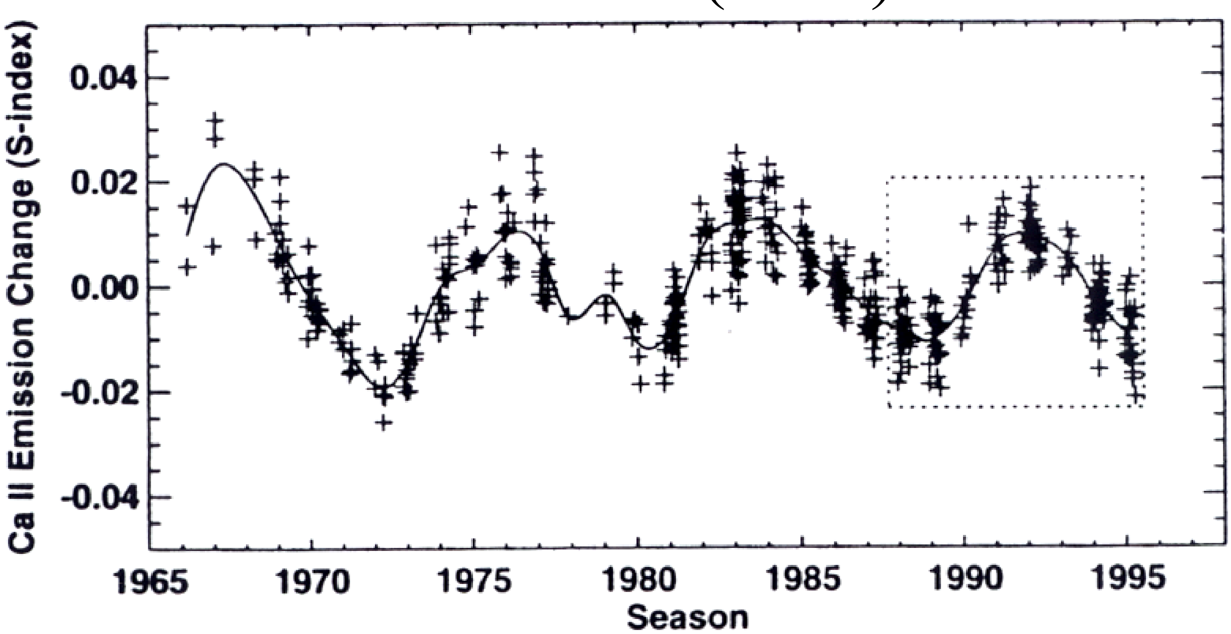
Stellar Cycles

(Radick et al. 1998; Baliunas et al. 1995)

HD 10476 (K1V)



HD 81809 (G2V)



Generation of the Solar Magnetic Field

MHD Induction Equation:

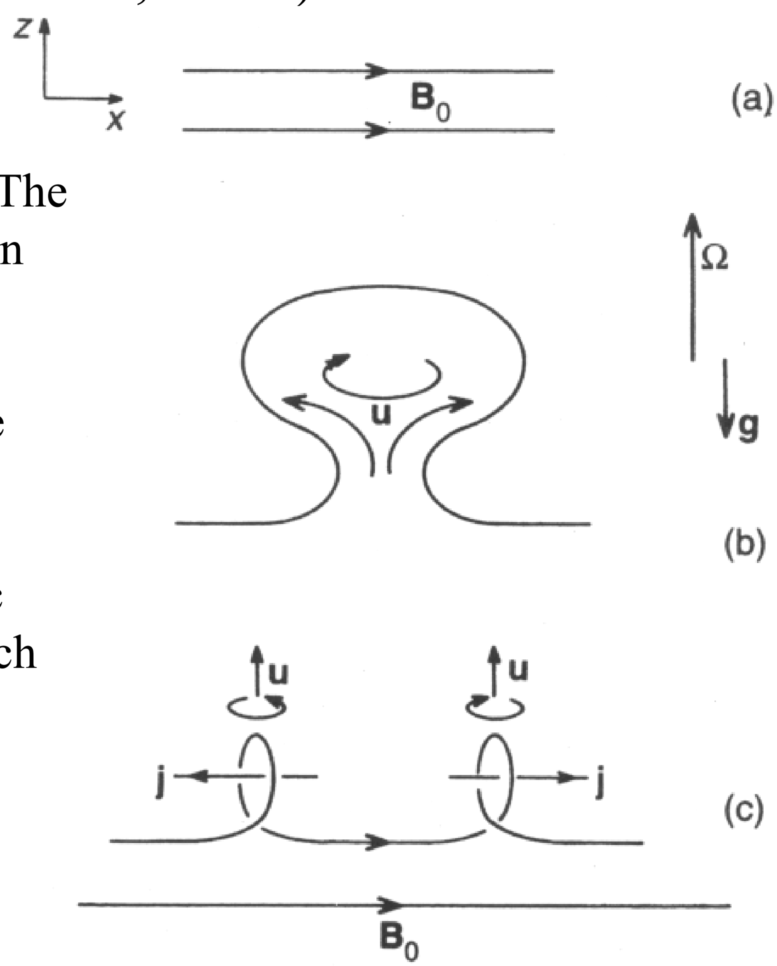
$$\frac{\partial \vec{B}}{\partial t} = -c\nabla \times \vec{E} = \nabla \times \left(\vec{u} \times \vec{B} - \eta \nabla \times \vec{B}\right)$$

Toroidal Field:

$$\vec{u} = \Omega(r,\theta) r \sin\theta \vec{e}_\phi \rightarrow \frac{\partial B_\phi}{\partial t} \sim r \sin\theta (\vec{B}_p \cdot \nabla)\Omega$$

Poloidal Field: (Parker 1955, 1970)

"…..The mechanism is simple. The Coriolis forces on the convection cause it to be cyclonic, with a rising cell of fluid rotating and carrying the lines of force of the azimuthal field into loops with nonvanishing projection on the meridional plane (see schematic drawing). A large number of such loops coalesce to regenerate the dipole field ….."



(Mestel, 1999)

Mean – Field Electrodynamics

(Moffatt 1978; Parker 1979; Krause & Rädler 1980)

MHD Induction Equation:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) - \nabla \times (\eta \nabla \times \vec{B})$$

Separate fields into mean (L) & fluctuating (ℓ«L) parts:

$$\vec{B} = \langle \vec{B} \rangle + \delta \vec{B}, \quad \vec{u} = \delta \vec{u}, \quad \langle \delta \vec{B} \rangle = \langle \delta \vec{u} \rangle = 0$$

Obtain:

$$\frac{\partial \langle \vec{B} \rangle}{\partial t} = \nabla \times \varepsilon - \nabla \times (\eta \nabla \times \langle \vec{B} \rangle)$$

$$\frac{\partial \ \delta \vec{B}}{\partial t} = \nabla \times (\delta \vec{u} \times \langle \vec{B} \rangle) + \nabla \times \vec{G} - \nabla \times (\eta \nabla \times \delta \vec{B})$$

$$\varepsilon = \langle \delta \vec{u} \times \delta \vec{B} \rangle, \quad \vec{G} = \delta \vec{u} \times \delta \vec{B} - \varepsilon$$

Homogeneity, isotropy, non-mirror symmetry:

$$\varepsilon = \alpha \langle \vec{B} \rangle - \beta \nabla \times \langle \vec{B} \rangle$$

Mean-Field Dynamo Equation:

$$\frac{\partial \langle \vec{B} \rangle}{\partial t} = \nabla \times \left[\langle \vec{u} \rangle \times \langle \vec{B} \rangle + \alpha \langle \vec{B} \rangle - (\eta + \beta) \nabla \times \langle \vec{B} \rangle \right]$$

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A Simple Example

One-Dimensional αΩ Dynamo:

$$(x, y, z) \leftrightarrow (\theta, \phi, r)$$

$$\vec{B} = \left[0, B_y(x,t), B_z(x,t)\right] = \left(0, B_y, \partial A / \partial x\right), \; \vec{A} = A(x,t)\vec{e}_y$$

$$\vec{u} = \left[0, u_y(z), 0\right], \qquad \Omega \equiv \partial u_y / \partial z = \text{constant}$$

$$C_\alpha = \frac{\alpha L}{\eta} \ll C_\Omega = \frac{\Omega L^2}{\eta}$$

Dynamo Equations:

$$\frac{\partial B_y}{\partial t} = \eta \frac{\partial^2 B_y}{\partial x^2} + \Omega \frac{\partial A}{\partial x}$$

$$\frac{\partial A}{\partial t} = \eta \frac{\partial^2 A}{\partial x^2} + \alpha B_y$$

Solve on interval $0 \le x \le \frac{1}{2} L \left(= \pi R_\odot\right)$ with :

$$B_y(0,t) = B_y(L/2, t) = A(0,t) = \frac{\partial A(L/2, t)}{\partial x} = 0$$

$$\alpha = \alpha_0 \cos(\pi x / L)$$

$$N_D = C_\alpha \cdot C_\Omega = -3130$$

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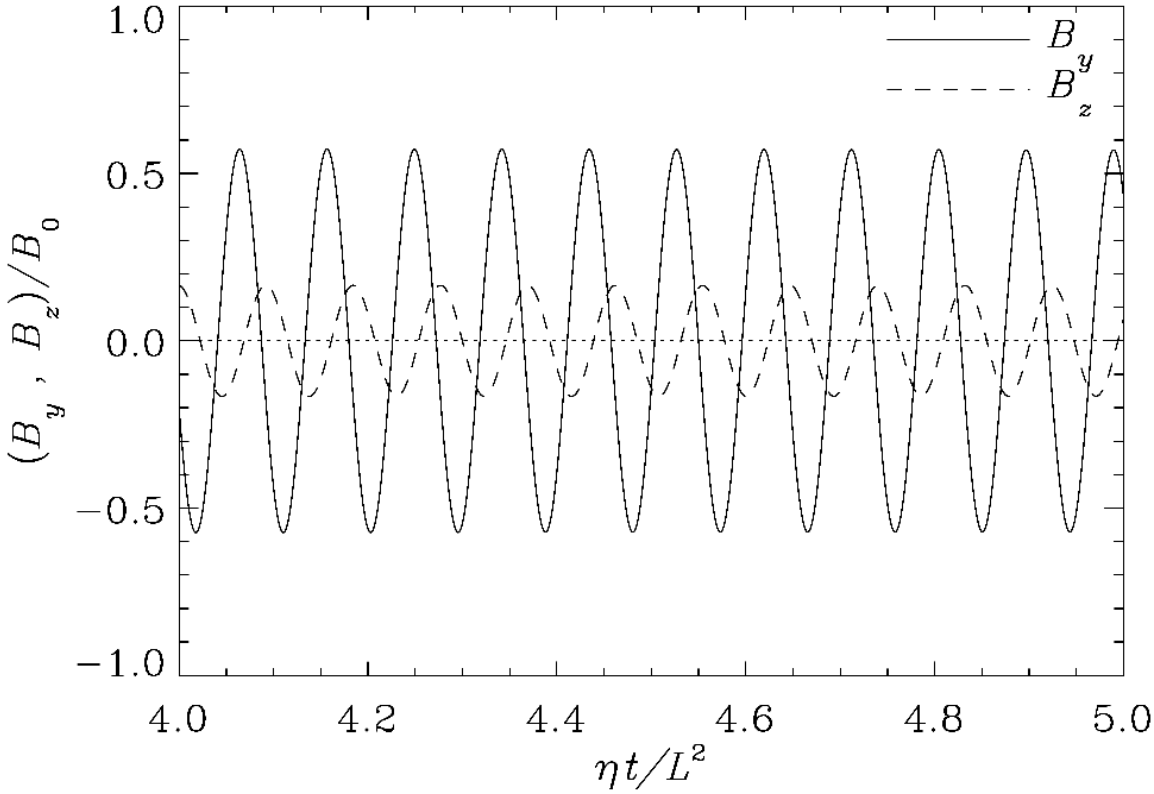
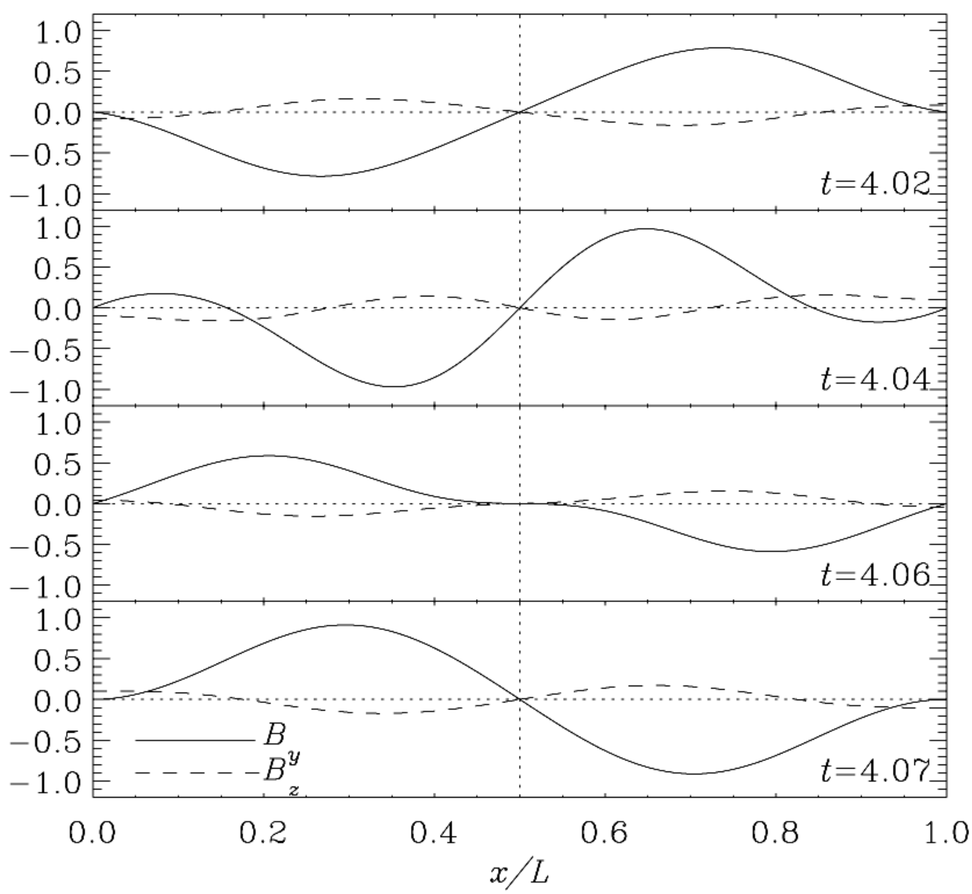
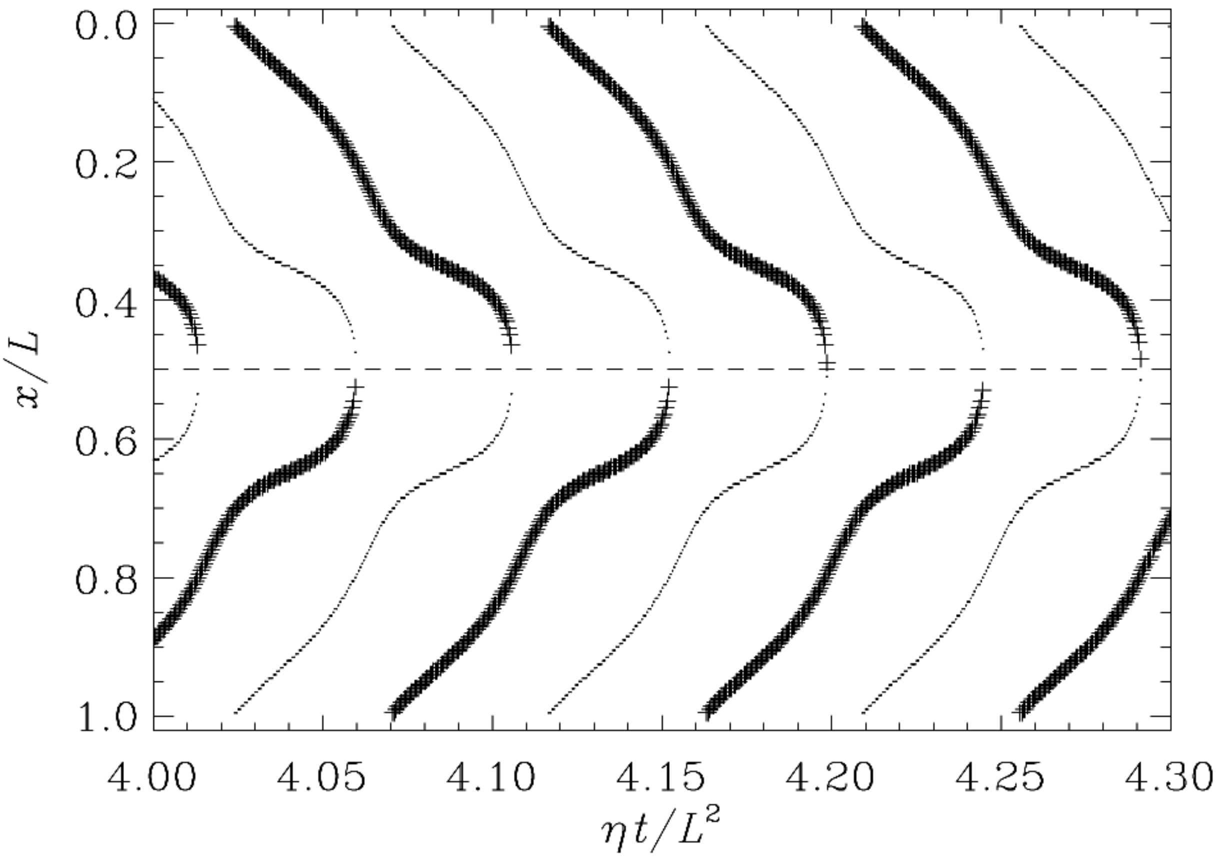
Cartesian αΩ-Dynamo



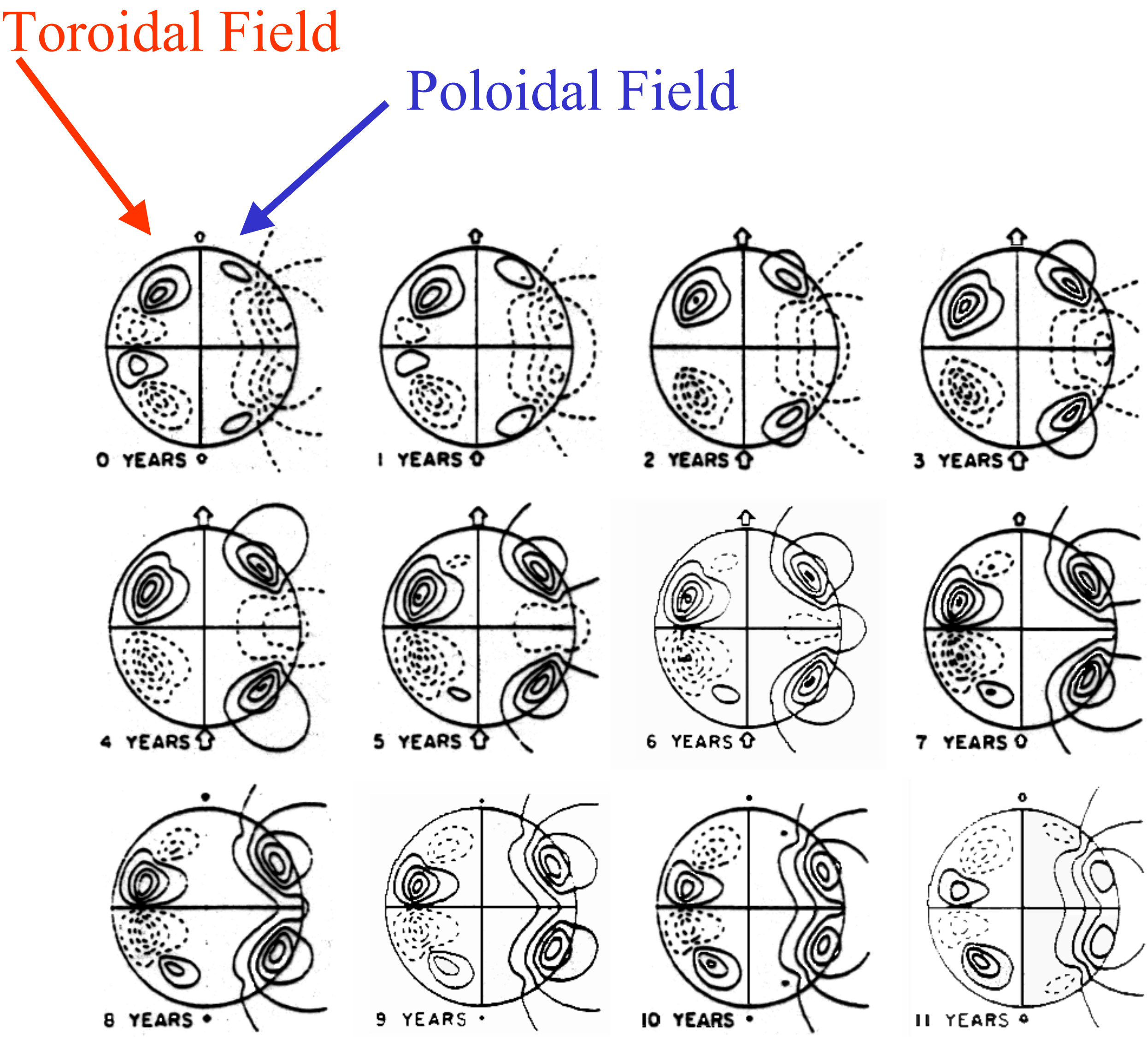


Cartesian αΩ-Dynamo



An αΩ-Dynamo Model for the Sun

(Stix 1976)



$$\frac{\partial \Omega}{\partial r} < 0, \qquad \alpha \sim \cos\theta$$

Propagation direction determined by sign of $\alpha \dfrac{\partial \Omega}{\partial r}$

Factors Affecting the Recent Development of Mean-Field Dynamo Models

- Magnetic buoyancy and the retention of fields in flux tube form (Parker 1975)

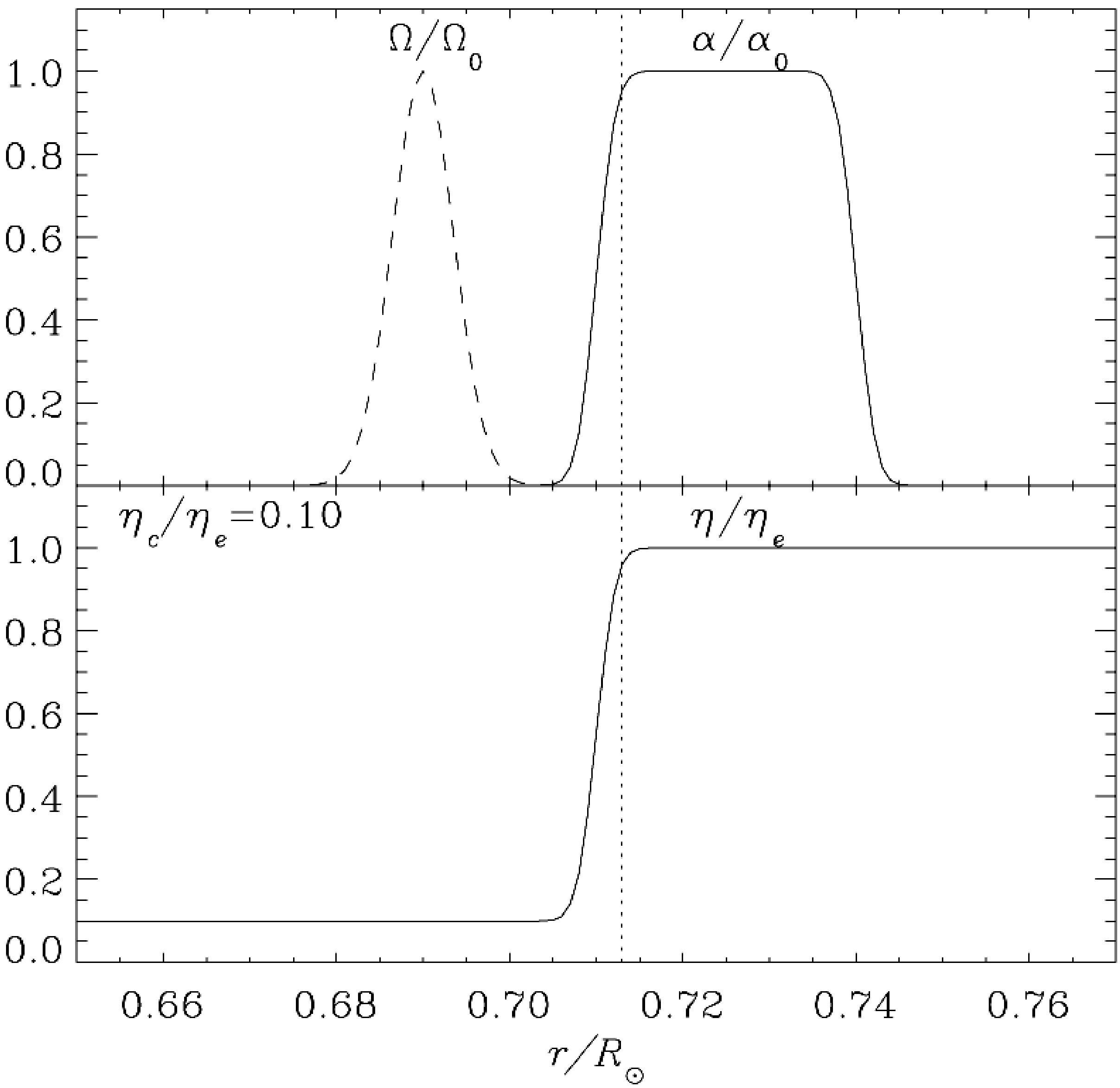
- The internal solar rotation angular velocity distribution, as inferred from helioseismology (e.g., Tomczyk et al. 1995; Charbonneau et al. 1999)

- Strong ($\sim 10^5$ G > B_{eq}) toroidal fields, as inferred from studies of flux tube dynamics (e.g., Fan et al. 1993; D'Silva & Choudhuri 1993; Caligari et al. 1995)

- The existence of meridional, circulatory flow in the convection zone (e.g., Hathaway et al. 1996; Braun & Fan 1998; Miesch et al. 2000)

Interface Dynamo Models

(Parker 1993)



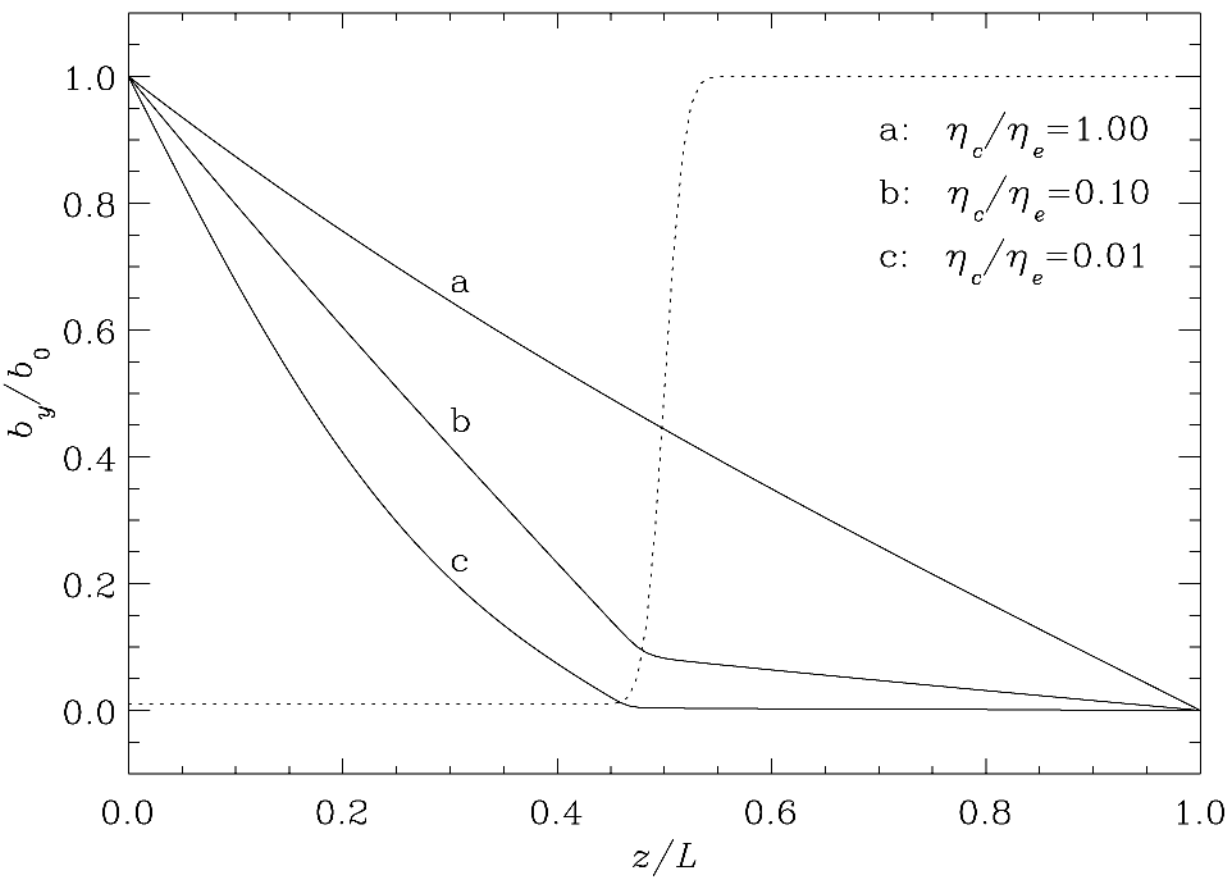
Rationale: Diffusion in a Composite Medium

Solve

$$\frac{\partial B_y}{\partial t} = \nabla \cdot \left(\eta \nabla B_y \right)$$

Assuming

$$\mathrm{B}_y(x,z,t) = b_y(z,t)e^{ikx}, \quad \eta = \eta(z)$$



For $\eta_c/\eta_e \neq 1$,

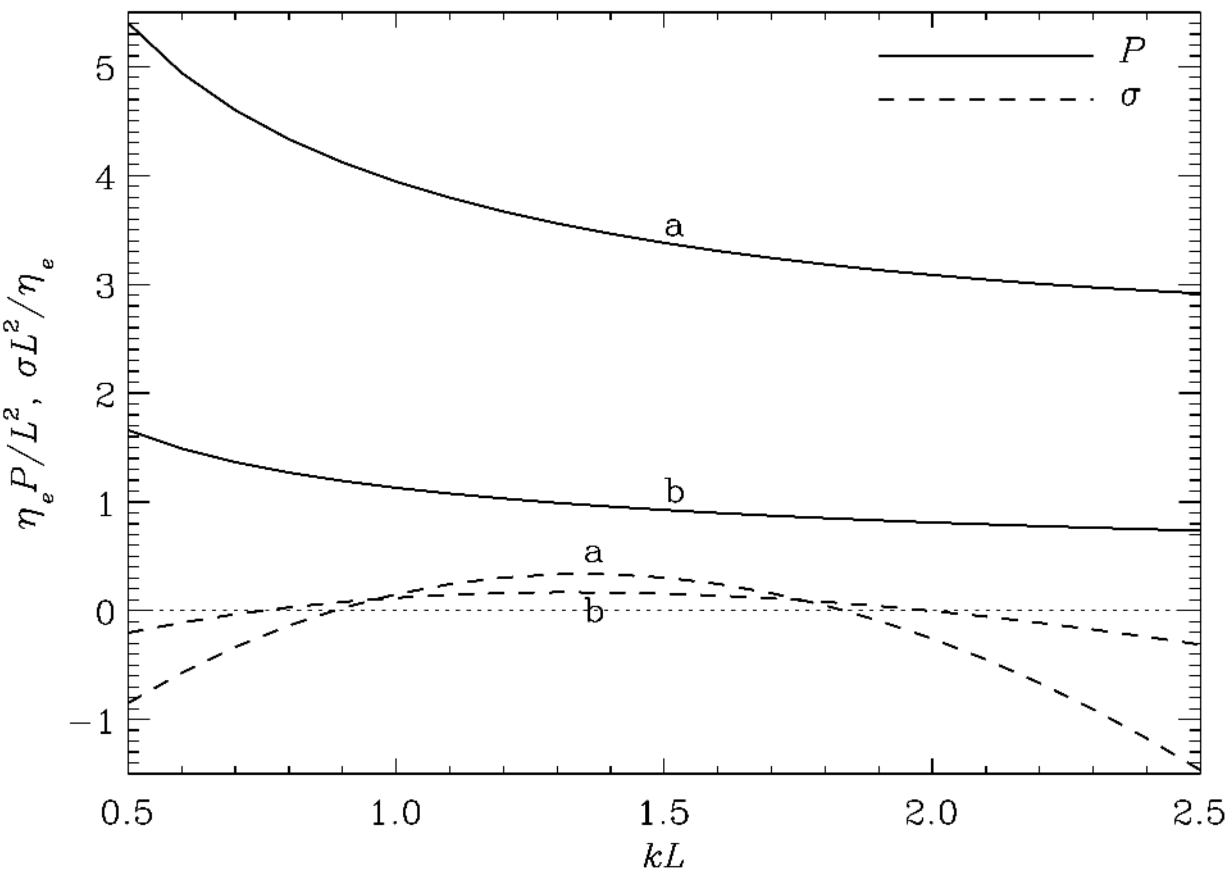
$$\frac{b_y(d,t)}{b_y(0,t)} \sim \frac{\eta_c}{\eta_e}$$

Cartesian Interface Dynamo Model

$$\vec{\mathrm{B}} = \nabla \times \left(A \vec{\mathrm{e}}_y \right) + B_y \vec{\mathrm{e}}_y$$

$$\frac{\partial B_y}{\partial t} = \eta \nabla^2 B_y + \Omega \frac{\partial A}{\partial x} + \frac{d\eta}{dz} \frac{\partial B_y}{\partial z}$$

$$\frac{\partial A}{\partial t} = \eta \nabla^2 A + \alpha B_y$$



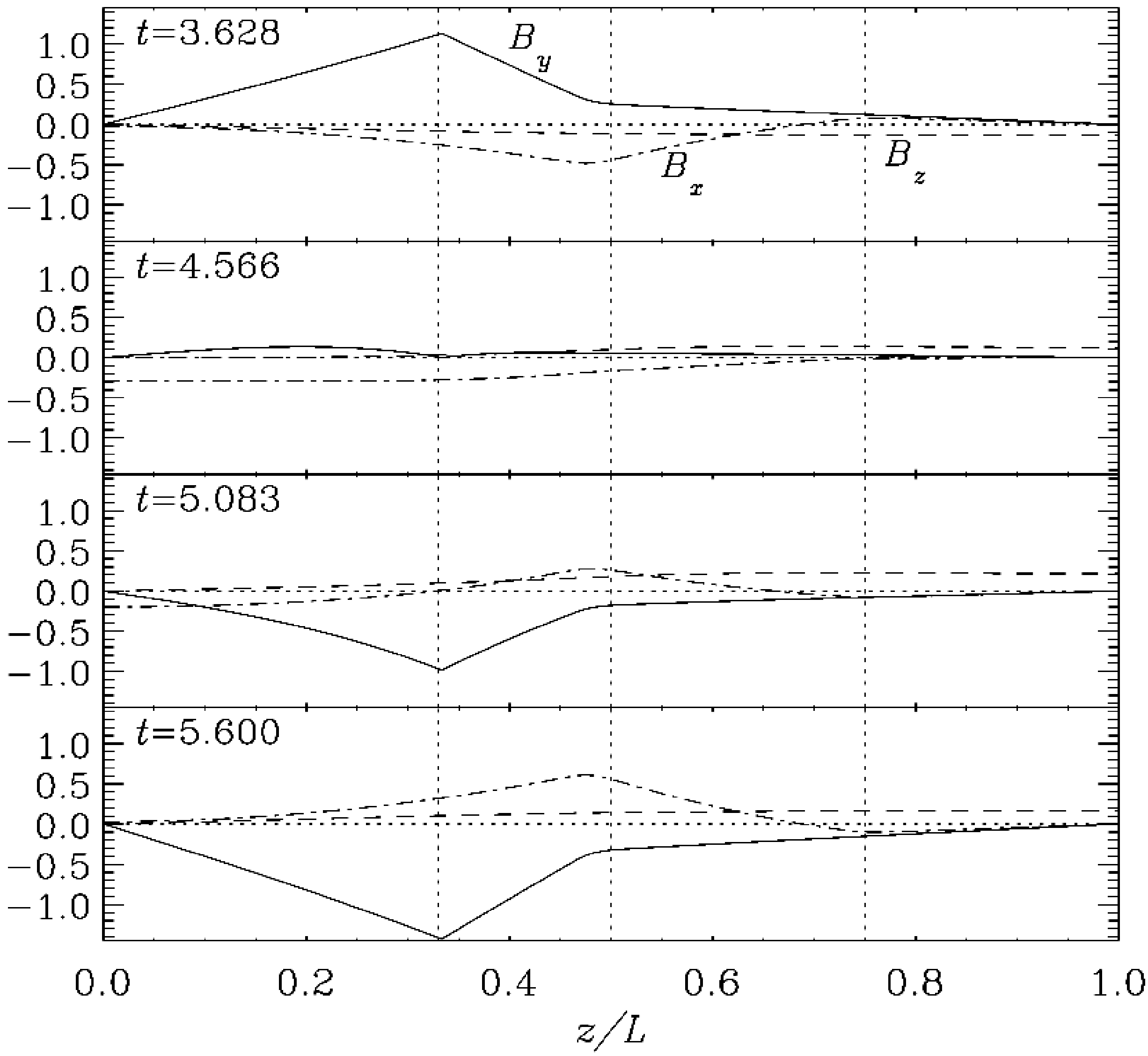
$$(\mathrm{a}): \eta_c / \eta_e = 1.00, \quad N_D = -3.8 \times 10^4$$

$$(\mathrm{b}): \eta_c / \eta_e = 0.10, \quad N_D = -1.8 \times 10^4$$

Interface Dynamo Fields

$$\eta_c / \eta_e = 0.1, \quad N_D = C_\alpha \cdot C_\Omega = -1.8 \times 10^4$$

$$C_\alpha = \frac{\alpha L}{\eta_e} > 0, \quad C_\Omega = \frac{\Omega L^2}{\eta_e} < 0, \quad |C_\alpha| = 10^{-2} |C_\Omega|$$



$$\frac{\eta_e P}{L^2} = 3.943, \quad \frac{\sigma L^2}{\eta_e} = 0.118$$

An Interface Dynamo Model for the Sun

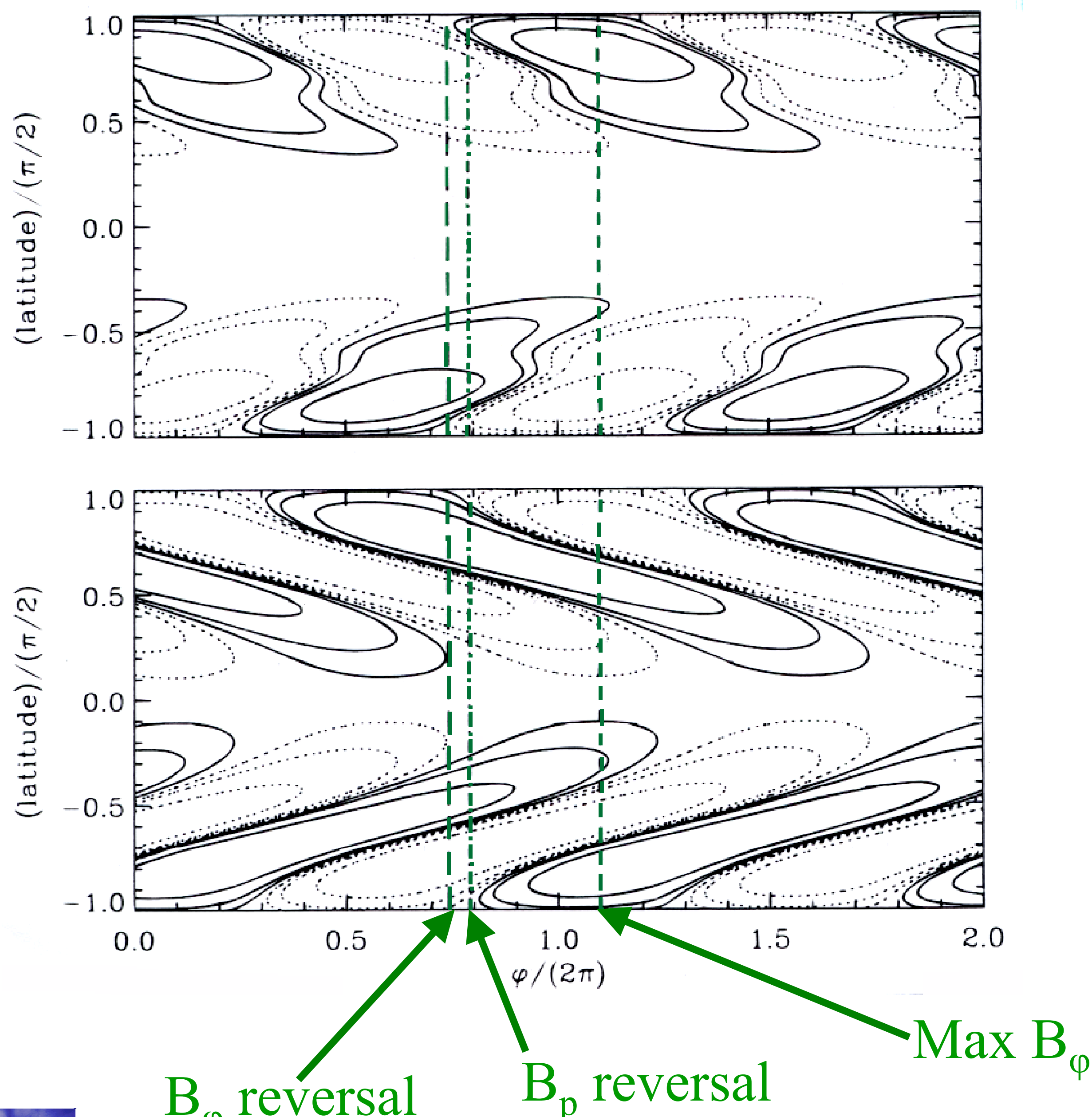
(Charbonneau & MacGregor 1997)

Solar-like differential rotation

$$\eta_c/\eta_e = 0.1, \quad C_\alpha = 27.5, \quad C_\Omega = 10^5$$

$$\alpha \sim \cos\theta$$

$$P = 10.5 \text{ years for } \eta_e = 10^{12} \text{ cm}^2 \text{ s}^{-1}$$



An Interface Dynamo Model for the Sun

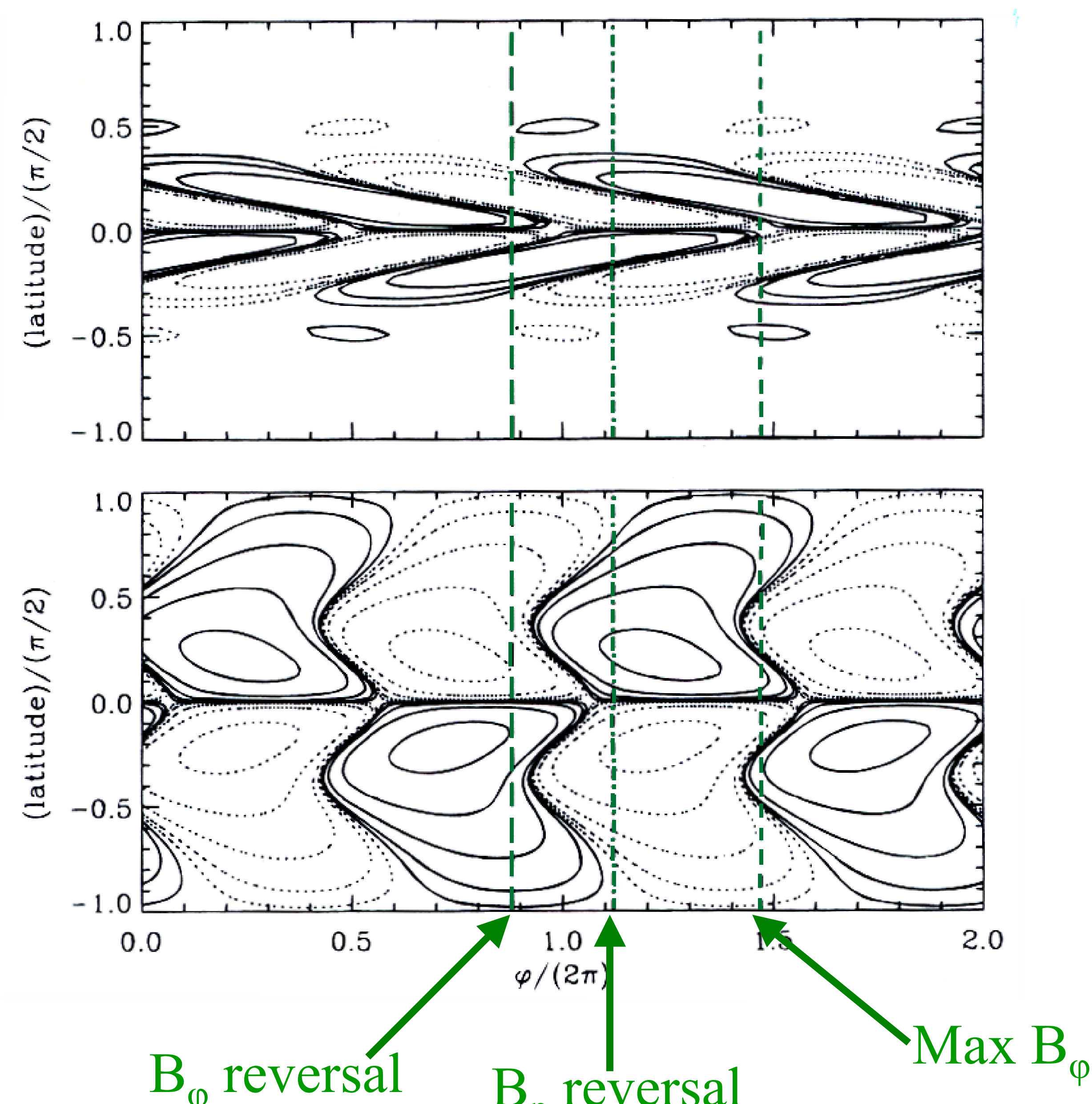
(Charbonneau & MacGregor 1997)

Solar-like differential rotation

$$\eta_c/\eta_e = 0.01, \quad C_\alpha = -5.0, \quad C_\Omega = 10^5$$

$$\alpha \sim -\sin 4\theta \quad (\pi/4 \le \theta \le 3\pi/4)$$

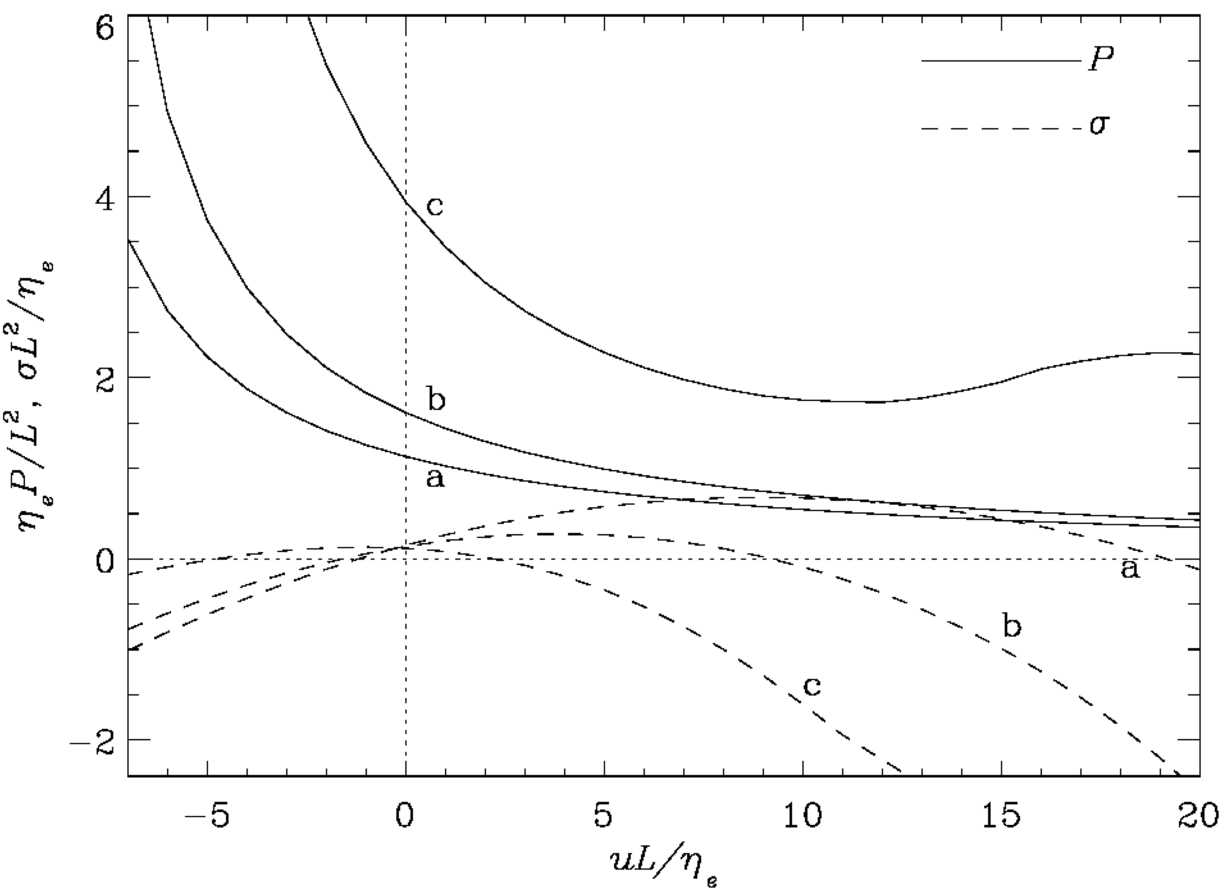
$$P = 22.3 \text{ years for } \eta_e = 10^{12} \text{ cm}^2 \text{ s}^{-1}$$



Effect of Horizontal Flow

$$\vec{u} = u_x(z)\vec{e}_x = u_P\left[\frac{\eta(z)-\eta_c}{\eta_e-\eta_c}\right]$$

$$kL = 1$$



$$(a): \eta_c/\eta_e = 1.0, \quad N_D = -3.8\times10^4$$

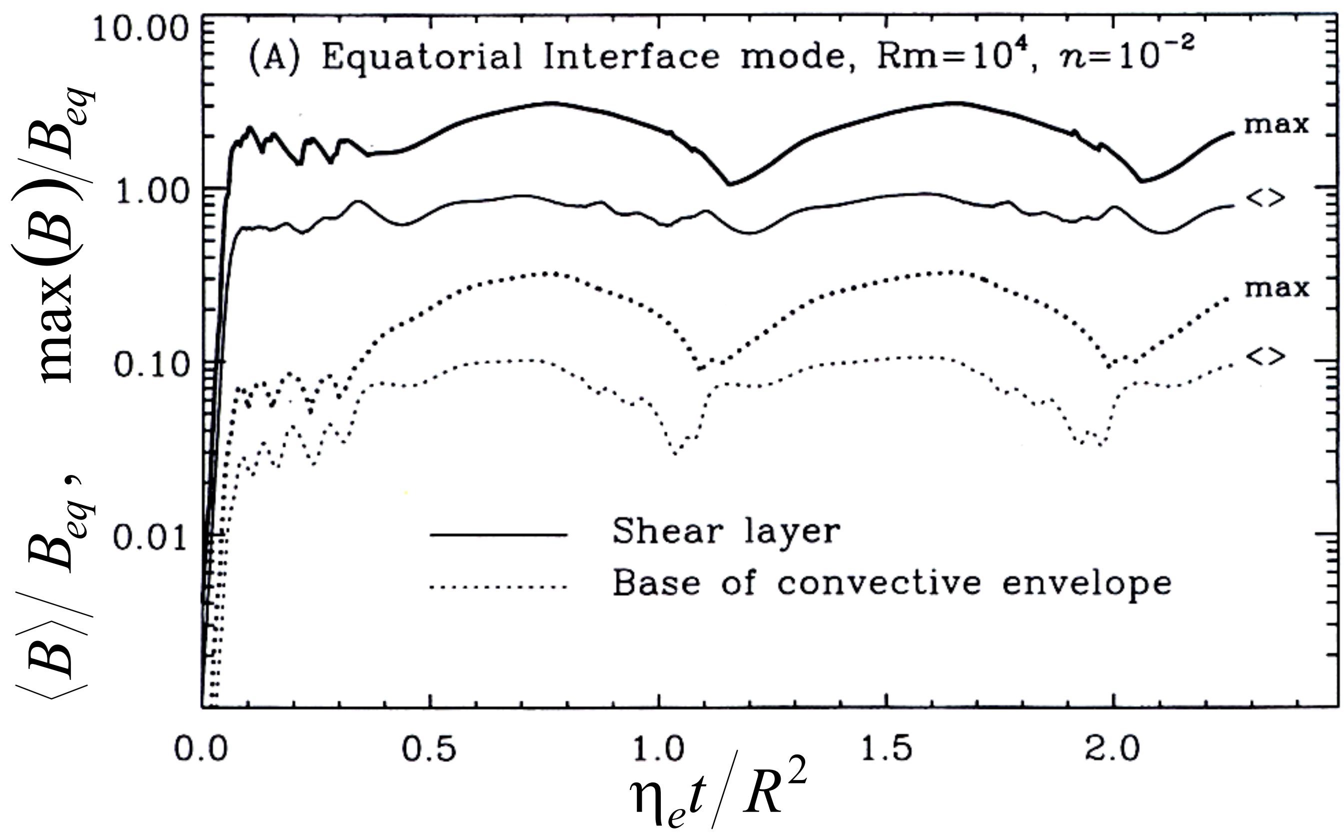
$$(b): \eta_c/\eta_e = 0.5, \quad N_D = -2.5\times10^4$$

$$(c): \eta_c/\eta_e = 0.1, \quad N_D = -1.8\times10^4$$

Nonlinear Interface Dynamo Solutions

$$\alpha = \frac{\alpha_0}{1 + C\left(\left|\langle\vec{\mathrm{B}}\rangle\right|/B_{eq}\right)^2}, \quad C = 1, \quad R_m$$

$$\eta_c/\eta_e = 0.01, \quad C_\alpha = -75, \quad C_\Omega = 10^5$$

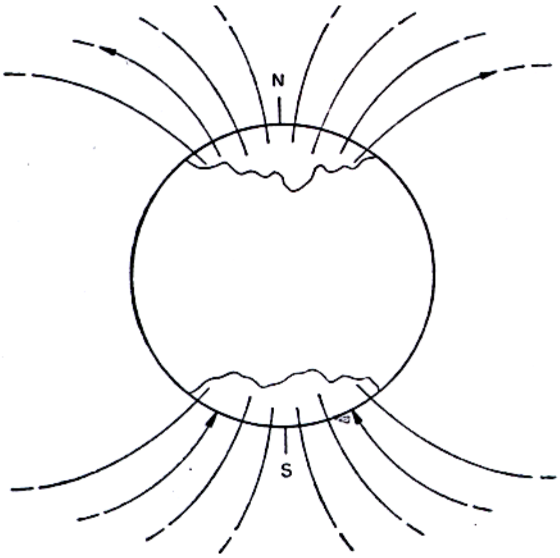


$$\langle B \rangle = \left[\frac{1}{V}\int dV\, B^2\right]^{1/2}, \quad B_{\max} = \max(|B|)_V$$

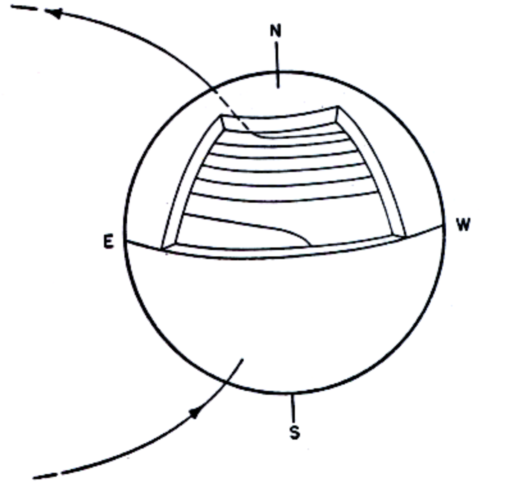
Babcock-Leighton-type Solar Dynamo Models

(H. W. Babcock 1961; Leighton 1964, 1969)

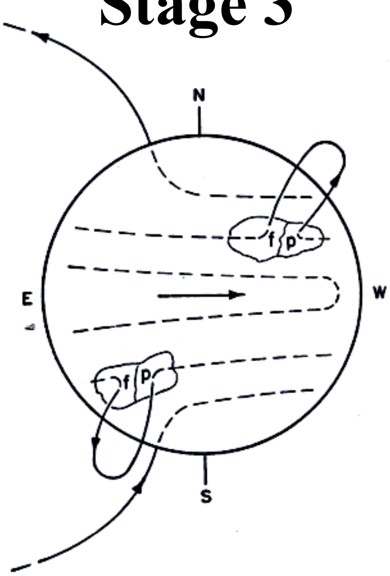
Stage 1



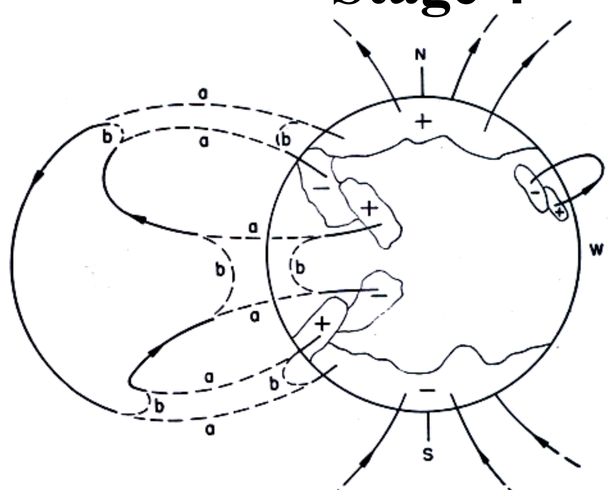
Stage 2



Stage 3



Stage 4



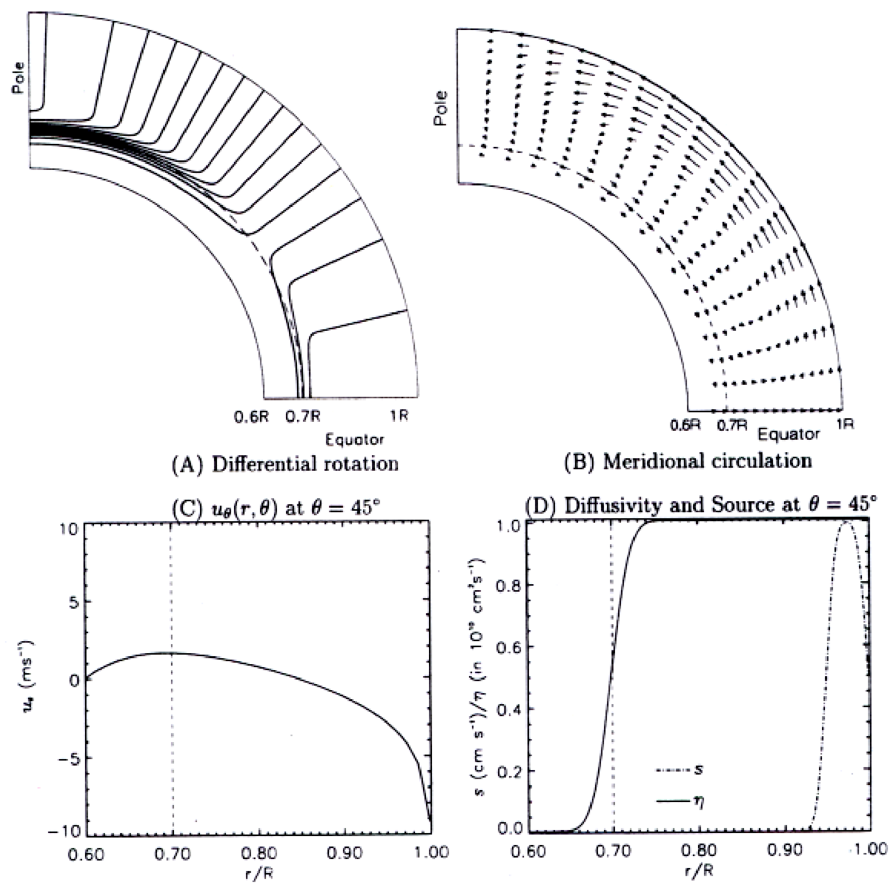
A BL-type Model with Meridional Circulation

(Dikpati & Charbonneau 1999)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{u} \times \vec{B} - \eta \nabla \times \vec{B} \right)$$

$$\vec{u} = \vec{u}_P(r,\theta) + \Omega(r,\theta) r \sin\theta \, e_\phi$$

$$\vec{B} = B_\phi(r,\theta,t)\vec{e}_\phi + \nabla \times \left[A(r,\theta,t)\vec{e}_\phi \right]$$



Source term for poloidal field:

$$S(r,\theta;B_\phi) = s_0 \; f(r,\theta) \frac{B_\phi(r_c,\theta,t)}{1+\left[B_\phi(r_c,\theta,t)/10^5 \text{ G}\right]^2}$$

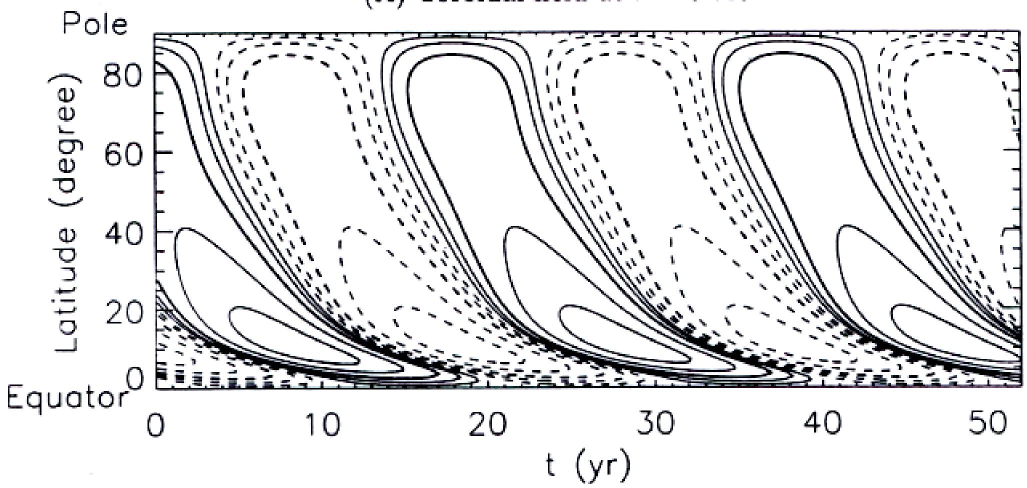
A BL-type Model with Meridional Circulation

(Dikpati & Charbonneau 1999)

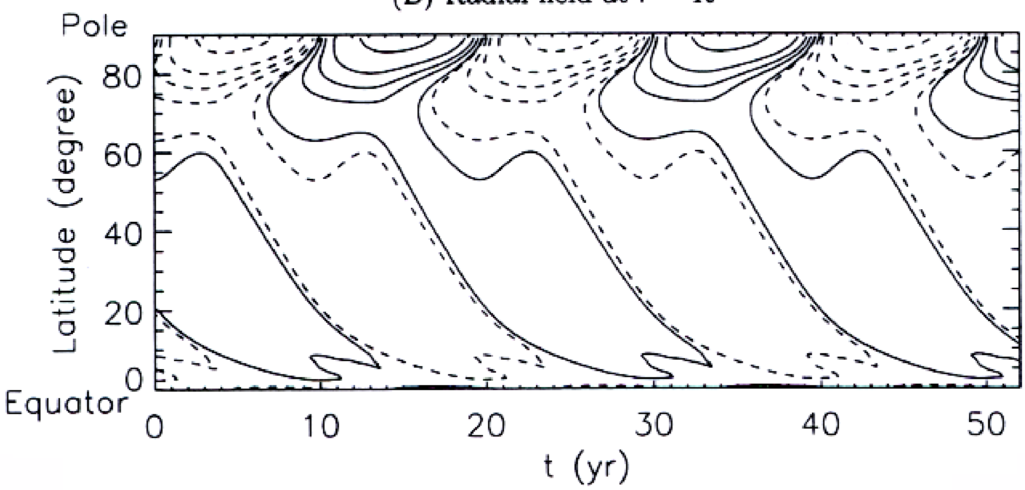
$$\eta_c/\eta_e = 0.003, \quad u_P = 1500\,\text{cm s}^{-1}, \quad s_0 = 20\,\text{cm s}^{-1}$$

$$C_s = \frac{s_0 R_\odot}{\eta_e} = 4.64, \quad C_\Omega = \frac{\Omega_{eq} R_\odot^2}{\eta_e} = 4.7\times 10^4$$

(A) Toroidal field at $r = 0.7R$



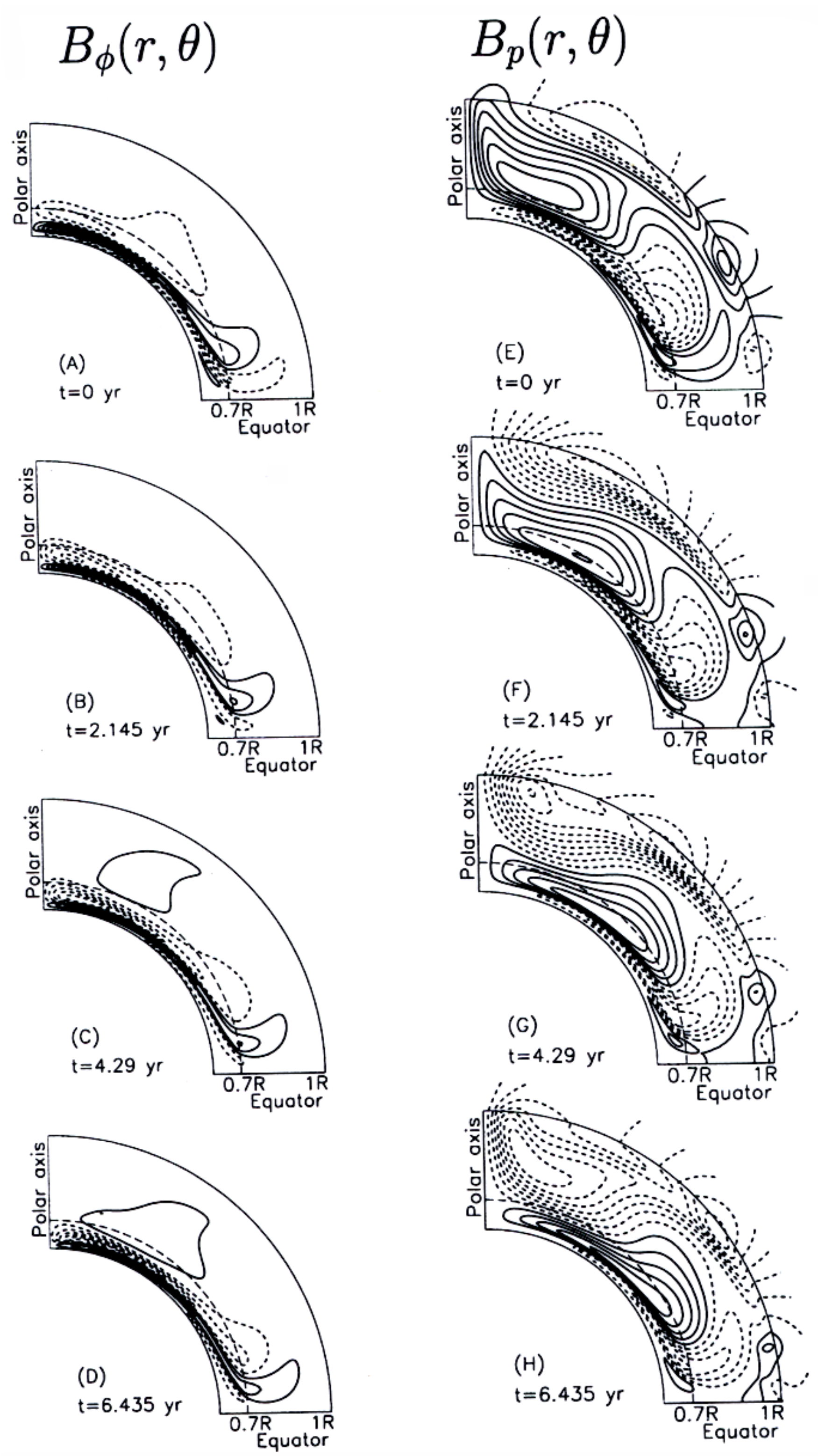
(B) Radial field at $r = R$



$$P = 19.8\ \text{years} \approx 56.8\, u_0^{-0.89}\ s_0^{-0.13}\ \eta_e^{0.22}\ \text{years}$$

A BL-type Model with Meridional Circulation

(Dikpati & Charbonneau 1999)



Conclusion: Comparison of Models

Interface	Babcock-Leighton
Merits	
Can account for basic solar cycle features	Can account for basic solar cycle features
Able to operate when $B > B_{eq}$	Strong fields required
Variety of sources for α-effect	Robust-period set by circulation
	Correct phase between toroidal/poloidal fields
Demerits	
Kinematic	Kinematic
Applicability of mean-field theory	Not self-excited
Require fine-tuning of input parameters	Strong polar poloidal fields
Easily disrupted by nonlinearities	Schematic description of poloidal source