Prospects of the Heterotic String

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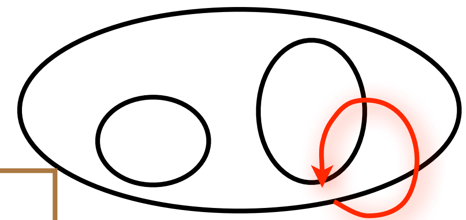
Strings at the LHC and in the Early Universe

KITP, Santa Barbara 2010

Smooth Heterotic Compactifications

$D = 10, \quad g_{MN}, \quad A^a_M, E_8$



$\mathbb{R}^4$

$N = 1 \; SUSY$

$H = [E_8, G]$

$\mathcal{H} = [H, F]$

$$H^1(V)^F \Rightarrow matter$$
$$H^1(V^*)^F \Rightarrow conjugate\ matter$$
$$H^1(\wedge^2 V)^F \Rightarrow Higgs$$
$$H^1(V \otimes V^*)^F \Rightarrow Bundle\ Moduli$$

- Heterotic Standard Model:

  $V\ , G = SU(4),\ W\ , F = \mathbb{Z}_3 \times \mathbb{Z}_3$

  Braun, He, Ovrut, Pantev 2006

  Anderson, Gray, He, Lukas 2009

$V, G = SU(5)$, $W, F = \mathbb{Z}_2$

Bouchard, Donagi 2006

These theories all contain the exact matter spectrum of the MSSM with one Higgs pair and no exotics. In addition

- must have three right-handed neutrino chiral multiplets
- must have matter (R-) parity

I) $G = SU(5)$ models:

a) Leray spectral "grading" of vector bundle cohomology-
Identifies a subset of bundle moduli as right-handed neutrinos
Disallows B and L violating cubic interactions in superpotential
But- wrong number, disallows most Yukawa terms, only SUSY cubic terms (not a discrete symmetry)

b) $G = SU(5) \longrightarrow S[U(4) \times U(1)] \Rightarrow H = SU(5) \times U(1)_{\text{anomalous}}$

$U(1)_{\text{anomalous}}$ identifies a subset of bundle moduli as r-h neutrinos

Texture disallows B and L violating cubic terms in superpotential

But- not matter parity $\Rightarrow$ large dim 5 operators, not realized in a heterotic vacuum

Kuriyama, Nakajima, Watari 2008

Blumenhagen, Moster, Weigand 2006

c) Discrete $\mathbb{Z}_2$ isometry with fixed points-

Identifies a subset of bundle moduli as right-handed neutrinos

Exactly matter parity

But- not realized in a realistic heterotic vacuum

Anderson, Ovrut 2010

2) $G = SU(4)$ models:

$\mathbb{R}^4$ Theory Gauge Group:

Gauge connection $G = SU(4) \Rightarrow E_8 \rightarrow H = Spin(10)$

Wilson line $F = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow$

$$Spin(10) \rightarrow \mathcal{H} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

rank Spin(10)=5 plus F Abelian ⇒ extra gauged $U(1)_{B-L}$.

By construction $U(1)_{B-L}$ is anomaly free. Note that

$$\mathbb{Z}_2 \ (R - \text{parity}) \subset U(1)_{B-L}$$

⇒ no rapid proton decay. But must be spontaneously broken above the scale of weak interactions.

$\mathbb{R}^4$ Theory Spectrum:

$$E_8 \xrightarrow{V} Spin(10) \Rightarrow$$

$$248 = (1, 45) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 10) \oplus (15, 1)$$

The Spin(10) spectrum is determined from $n_R = h^1(X, U_R(V))$.

$$Spin(10) \xrightarrow{F} SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \Rightarrow$$

The $3 \times 2 \times 1_Y \times 1_{B-L}$ spectrum is determined from $n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3}$. Tensoring and taking invariant subspace gives 3 families of quarks/leptons each transforming as

$$Q_L = (3, 2, 1, 1), \quad u_R = (\bar{3}, 1, -4, -1), \quad d_R = (\bar{3}, 1, 2, -1)$$

$$L_L = (1, 2, -3, -3), e_R = (1, 1, 6, 3), \qquad \nu_R = (1, 1, 0, 3)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.

Similarly we get 1 pair of Higgs-Higgs conjugate fields

$$H = (1, 2, 3, 0), \quad \bar{H} = (1, \bar{2}, -3, 0)$$

That is, we get exactly the matter spectrum of the MSSM

with 3 right-handed neutrinos! In addition, there are

$n_1 = h^1(X, V \times V^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli $\phi = (1, 1, 0, 0)$

Supersymmetric Interactions:

The most general superpotential is

$$W = \sum_{i=1}^{3} (\lambda_{u,i} Q_i H u_i + \lambda_{d,i} Q_i \bar{H} d_i + \lambda_{\nu,i} L_i H \nu_i + \lambda_{e,i} L_i \bar{H} e_i)$$

Note B-L symmetry forbids dangerous B and L violating terms

$$LLe, \quad LQd, \quad udd$$

Supersymmetry Breaking, the Renormalization Group and the LHC

Soft Supersymmetry Breaking:

Ambroso, Ovrut 2009

N=1 Supersymmetry is spontaneously broken by the moduli during compactification ⇒ soft supersymmetry breaking interactions. The relevant ones are

$$V_{2s} = m_{\nu_3}^2 |\nu_3|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 - (BH\bar{H} + hc) + \ldots$$

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \ldots$$

At the compactification scale $M_C \simeq 10^{16} GeV$ these parameters are fixed by the vacuum values of the moduli. For example

$$m_{\nu_3}^2 = m_{\nu_3}^2(\langle \phi \rangle)$$

But- as of yet no complete theory of moduli stabilization!

However, at a lower scale μ measured by $t = ln(\frac{\mu}{M_C})$ these parameters change under the renormalization group.

For example,

$$16\pi^2 \frac{dm_{\nu_3}^2}{dt} \simeq \frac{3}{4} {g_4}^2 \sum_{i=1}^{3} (m_{\nu_i}^2 + \dots)$$

Solving these assuming

$$m_H(0)^2 = m_{\bar{H}}(0)^2, \quad m_{Q_i}(0)^2 = m_{u_j}(0)^2 = m_{d_k}(0)^2$$

$$m_{L_i}(0)^2 = m_{e_j}(0)^2 \neq m_{\nu_k}(0)^2$$

$\Rightarrow$ at scale $\mu \simeq 10^4 GeV \Rightarrow t_{B-L} \simeq -25$

$$m_{\nu_3}(t_{B-L})^2 = m_\nu(0)^2 - 5m_\nu(0)^2 = -4m_\nu(0)^2$$

Therefore, we expect the spontaneous breaking of B-L by the third family right-handed sneutrino at t_{B-L}.

But- can such initial conditions arise after moduli stabilization?

The vacuum expectation value at t_{B-L} is

$$\langle \nu_3 \rangle = \frac{2m_\nu(0)}{\sqrt{\frac{3}{4}}g_4}$$

$\Rightarrow$ a B-L vector boson mass of

$$M_{A_{B-L}} = 2\sqrt{2}m_\nu(0)$$

Similarly, at the electroweak scale $\mu \simeq 10^2 GeV \Rightarrow t_{EW} \simeq -29.6$

$$m_{H'}(t_{EW})^2 \simeq -\frac{\Delta^2}{tan\beta^2} m_H(0)^2 \quad , \quad m_{\bar{H}'}(t_{EW})^2 \simeq m_H(0)^2$$

where $tan\beta = \frac{\langle H \rangle}{\langle \bar{H} \rangle}$ and $0 < \Delta^2 < 1$ is related to $M_3(0)$. $\Rightarrow$ at t_{EW} electroweak symmetry is broken by the expectation value

$$\langle H'^0 \rangle = \frac{2\Delta\ m_H(0)}{tan\beta\sqrt{\frac{3}{5}g_1^2 + g_2^2}}$$

$\Rightarrow$ a Z-boson mass of

$$M_Z = \frac{\sqrt{2}\Delta\ m_H(0)}{tan\beta} \simeq 91 GeV$$

It follows that there is a B-L/EW gauge hierarchy given by

$$\frac{M_{A_{B-L}}}{M_Z} \simeq \frac{tan\beta}{\Delta}$$

Our approximations are valid for the range $6.32 \leq tan\beta \leq 40$.
For $\Delta = \frac{1}{2.5}$, the B-L/EW hierarchy in this range is

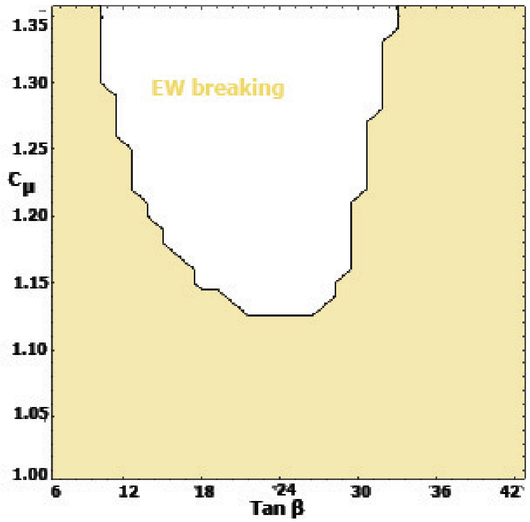
$$15.8 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 100$$

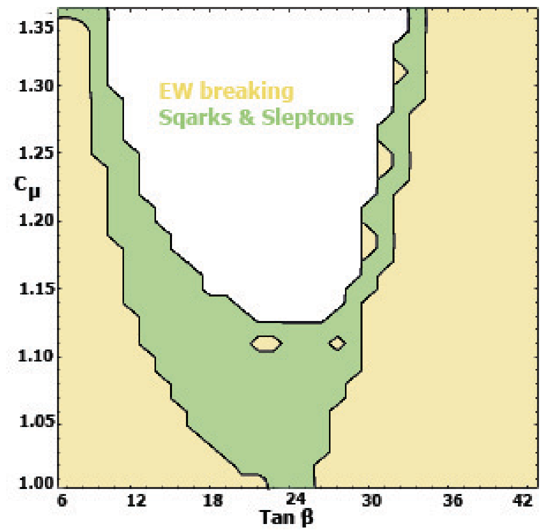
We conclude that this vacuum exhibits a natural hierarchy of $\mathcal{O}(10)$ to $\mathcal{O}(100)$ $\Rightarrow$

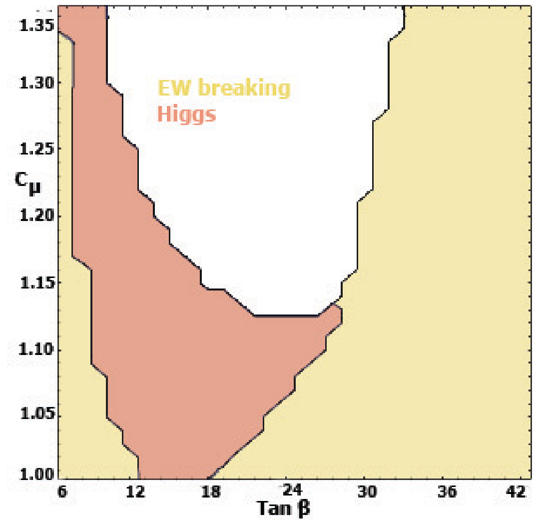
$$1.42 \times 10^3 GeV \lesssim M_{A_{B-L}} \lesssim 0.91 \times 10^4 GeV$$

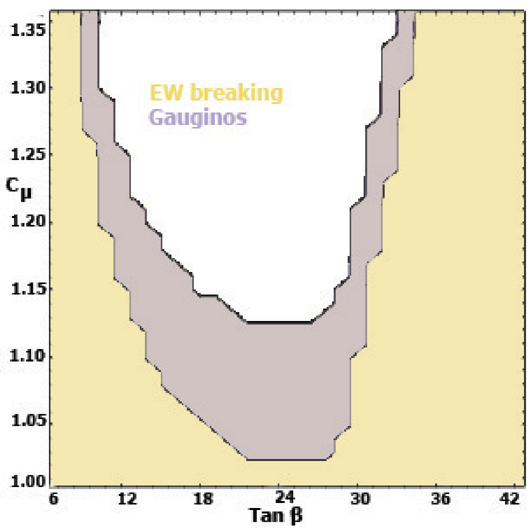
All super-partner masses are related through intertwined renormalization group equations. ⇒ Measuring some masses predicts the rest!

For a representative choice of initial parameters, the allowed values of $\mu, tan\beta$ are

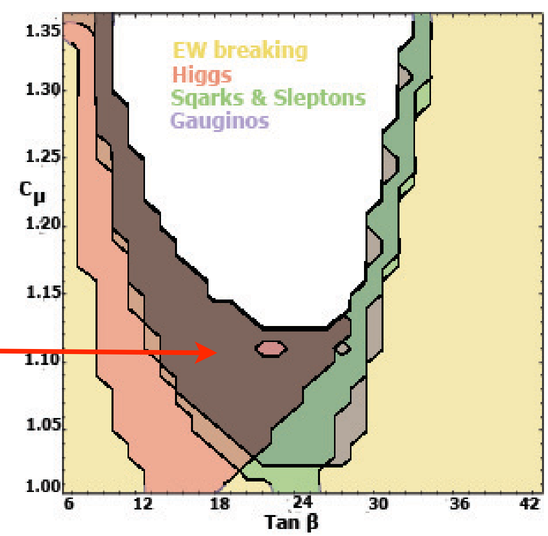








consistent with all present bounds on squark/slepton, Higgs, gaugino masses



For a representative choice of initial parameters, $\mu = 1.12, \tan\beta = 17$

Ambroso, Ovrut 2010

Particle Masses					
Class	Particle	Mass [GeV]	Class	Particle	Mass [GeV]
Higgs	h^0	92	Sleptons	$\tilde{\tau}_{eff2}$	1030
	H^0	466		$\tilde{\tau}_{eff1}$	984
	A^0	481		$\tilde{e}_{eff1,2}$	1018
	$H^{\pm}$	473		$\tilde{L}_{eff1,2}$	1045
Gauginos	$\tilde{N}_1$	86	Squarks	$\tilde{t}_{eff2}$	805
	$\tilde{N}_2$	126		$\tilde{t}_{eff1}$	622
	$\tilde{N}_3$	246		$\tilde{q}_{eff1,2}$	957
	$\tilde{N}_4$	504		$\tilde{b}_{eff2}$	937
	$\tilde{N}_5$	516		$\tilde{b}_{eff1}$	782
	$\tilde{C}_1^{\pm}$	246		$\tilde{t}_{eff1,2}$	900
	$\tilde{C}_2^{\pm}$	517		$\tilde{b}_{eff1,2}$	1007

- What are the LHC predictions of heterotic vacua?
- New approaches to heterotic standard models- example:

$SU(6) \to S[U(5) \times U(1)] \Rightarrow$ MSSM without Wilson lines

Blumenhagen, Moster, Weigand 2006

Evaluating Yukawa Couplings

Can we evaluate Yukawa couplings from first principles? Yes!

1) Texture:

a) Leray spectral "grading" of vector bundle cohomology-

$$W = \ldots \lambda LHr + \ldots$$ Braun, He, Ovrut 2006

$\Rightarrow$ a Yukawa coupling is the triple product

$$H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \otimes H^1(X,\wedge^2 V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \otimes H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \longrightarrow \mathbb{C}$$

Internal super-geometry (X elliptically fibered over dP9 base) $\Rightarrow$ in flavor diagonal basis for each of u, d, ν, e

$$\lambda_1 = 0, \quad \lambda_2, \lambda_3 \neq 0$$

b) Anomalous U(1) gauge factors

Stability Walls in Heterotic Theory

Anderson, Gray, Lukas, Ovrut 2009

Consider a CY threefold X defined by

$$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{array} \right]$$

with 2 Kahler moduli $T^k = t^k + 2i\chi^k, \qquad k = 1, 2$

and an SU(3) vector bundle V given by

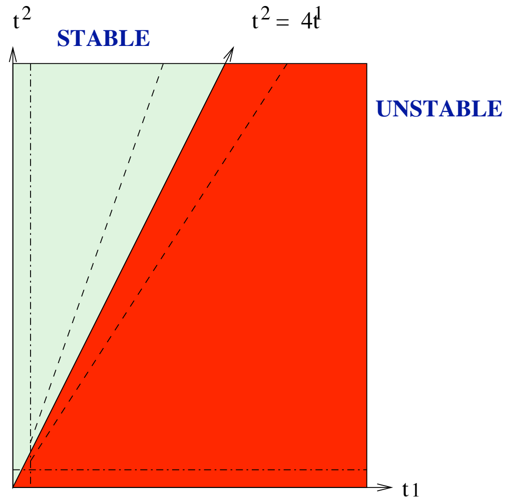
$$0 \to V \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(1,-1) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \xrightarrow{f} \mathcal{O}_X(2,1) \to 0$$

The gauge connection is supersymmetric if and only if V is "slope-stable" - that is, has no destabilizing sub-bundle.

There is a "maximal" sub-bundle $\mathcal{F} \subset V$ with slope

$$\mu(\mathcal{F}) = 4t^1t^2 - (t^2)^2$$

$\Rightarrow$ the stability regions of V are given by



In the stable region ($\mu(\mathcal{F}) < 0$) the structure group SU(3) $\Rightarrow$

$$G = E_6$$

with spectrum $n_{\mathbf{27}} = 2,\ n_{\overline{\mathbf{27}}} = 0,\ n_{\mathbf{1}} = 22$. On the boundary ($\mu(\mathcal{F}) = 0$) the SU(3) group factors to $S[U(2) \times U(1)]$ $\Rightarrow$

$$G \to E_6 \times U(1)$$

with spectrum

Fields	$E_6 \times U(1)$ charges	number of fields
ψ^β	1_0	7
B^I	$27_{-1/2}$	2
C^L	$1_{-3/2}$	16

charged bundle moduli

$7 + 16 - 1 = 22$

The anomalous U(1) induces a potential given by $\mathcal{V} = \frac{1}{2}D^2$ where

$$D = f(t^i) + \frac{3}{2}G_{L\bar{M}}C^L\bar{C}^{\bar{M}}, \quad f(t^i) \propto \mu(\mathcal{F}) = 4t^1t^2 - (t^2)^2$$

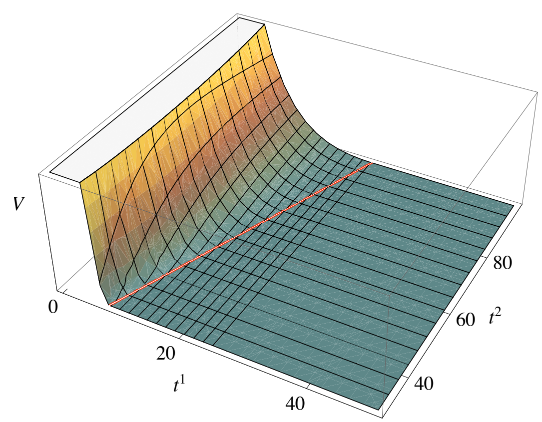
and $G_{L\bar{M}} > 0$. Note that $f(t^i) < 0$ in the stable region. This allows

$$\langle C_L \rangle \neq 0 \ \Rightarrow D = 0$$

$\Rightarrow \mathcal{V} = 0$ in the stable region. Note that $f(t^i) > 0$ in the unstable region. Here

$$\langle C_L \rangle = 0 \ \Rightarrow D \neq 0$$

$\Rightarrow \mathcal{V} > 0$ in the unstable region. Graphically



$\Longrightarrow$ Supersymmetry minimizes the energy!!

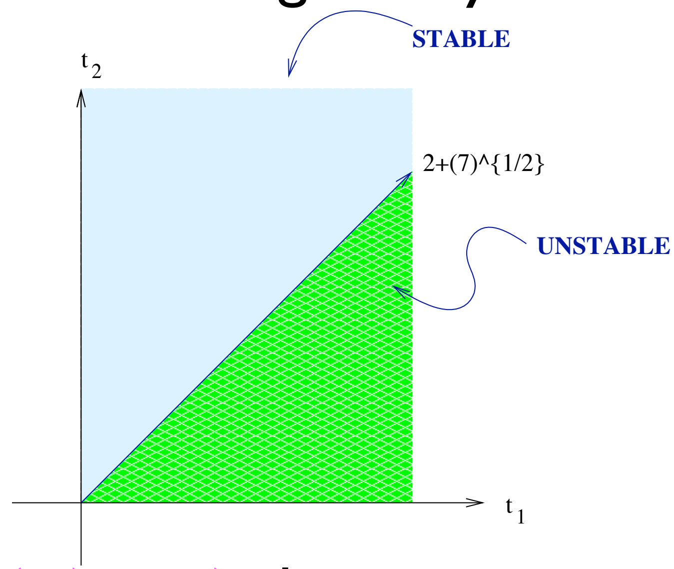
This "Wall of Stability" bounds the stable region.

Yukawa Textures from Stability Walls

In previous example all U(1) charges negative $\Rightarrow W = 0$.

Consider a more general CY and bundle V. Anderson, Gray, Ovrut 2010

$\Rightarrow$ the stability regions of V are given by



In the stable region ($\mu(\mathcal{F}) < 0$) the structure group SU(3) $\Rightarrow$

with spectrum $G = E_6$.

Representation	Field Name	Cohomology	Multiplicity
$(\mathbf{8}, \mathbf{1})$	ϕ	$h^1(X, V \otimes V^*)$	87
$(\mathbf{27}, \mathbf{3})$	F^I	$h^1(X, V)$	39
$(\overline{\mathbf{27}}, \overline{\mathbf{3}})$	$\overline{F}^A$	$h^1(X, V^*)$	0

On the boundary ($\mu(\mathcal{F}) = 0$) the SU(3) group factors to

$S[U(2)\times U(1)]$ $\Rightarrow$

$$G \to E_6 \times U(1)$$

with spectrum

Representation	Field Name	Cohomology	Multiplicity
$(\mathbf{1},\mathbf{2})_3$	C_1^p	$h^1(X,\mathcal{F}^*\otimes\mathcal{K})$	0
$(\mathbf{1},\mathbf{2})_{-3}$	C_2^Q	$h^1(X,\mathcal{F}\otimes\mathcal{K}^*)$	21
$(\mathbf{1},\mathbf{3})_0$	ψ	$h^1(X,\mathcal{K}\otimes\mathcal{K}^*)$	67
$(\mathbf{27},\mathbf{1})_{-2}$	F_1^i	$h^1(X,\mathcal{F})$	3
$(\mathbf{27},\mathbf{2})_1$	F_2^η	$h^1(X,\mathcal{K})$	36
$(\overline{\mathbf{27}},\overline{\mathbf{1}})_2$	$\overline{F}_1^a$	$h^1(X,\mathcal{F}^*)$	0
$(\overline{\mathbf{27}},\overline{\mathbf{2}})_{-1}$	$\overline{F}_2^\alpha$	$h^1(X,\mathcal{K}^*)$	0

The anomalous U(1) induces a potential given by $\mathcal{V} = \frac{1}{2}D^2$ where

$$D = \frac{3}{16}\frac{\epsilon_S\epsilon_R^2}{\kappa_4^2}\frac{\mu(\mathcal{F})}{\mathcal{V}} + \frac{3}{2}\sum_{P,\overline{Q}} G_{P\overline{Q}} C_2^P \overline{C}_2^{\overline{Q}}$$

and $G_{L\bar{M}} > 0.$ Negative slope in the stable region allows

$$\langle C_L\rangle \neq 0 \;\Rightarrow D = 0 \;\Rightarrow\; \mathcal{V} = 0$$

Positive slope in the unstable region restricts

$$\langle C_L\rangle = 0 \;\Rightarrow D \neq 0 \;\Rightarrow\; \mathcal{V} > 0$$

In general, a vacuum can have both $C_1^p,\ C_2^P$ charged moduli.

$\Rightarrow$ in addition to an enlarged D-term, the potential contains F-terms from the superpotential

$$W = \lambda_0 (C_1 C_2)^2 + \lambda_1 F_1^3 C_1^2 + \overline{\lambda}_1 \overline{F}_1^3 C_2^2 + \lambda_2 F_1^2 F_2 C_1 + \overline{\lambda}_2 \overline{F}_1^2 \overline{F}_2 C_2$$
$$+ \lambda_3 F_1 F_2^2 + \overline{\lambda}_3 \overline{F}_1 \overline{F}_2^2 + \lambda_4 F_2^3 C_2 + \overline{\lambda}_4 \overline{F}_2^3 C_1$$

Note: restricted by the U(1) symmetry.

On the stability wall $\langle C_1 \rangle = \langle C_2 \rangle = 0 \ \Rightarrow$

$$W_{\rm Yukawa}^{\rm wall} = \lambda_3 F_1 F_2^2 + \overline{\lambda}_3 \overline{F}_1 \overline{F}_2^2$$

Yukawa texture severely constrained.

Near the wall $\langle C_1 \rangle = 0,\ \langle C_2 \rangle \neq 0 \ \Rightarrow$

$$W_{\rm Yukawa}^{\rm near\ wall} = \lambda_3 F_1 F_2^2 + \overline{\lambda}_3 \overline{F}_1 \overline{F}_2^2 + \overline{\lambda}_1 \langle \mathbf{C_2^2} \rangle \overline{\mathbf{F}}_1^3 + \overline{\lambda}_2 \langle \mathbf{C_2} \rangle \overline{\mathbf{F}}_1^2 \overline{\mathbf{F}}_2 + \lambda_4 \langle \mathbf{C_2} \rangle \mathbf{F_2^3}$$

Some Yukawa couplings "grow back". Note several, such as $F_1^3,\ \overline{F}_2^3$, disallowed despite U(1) breaking.

What happens deep in the stability region? Note that each Yukawa coupling is a holomorphic function of the form

$$\lambda = \lambda(\langle C_i \rangle, \langle \mathfrak{z}^a \rangle, \langle \phi \rangle)$$

where $\mathfrak{z}^a$ are the complex structure moduli. Theorem:

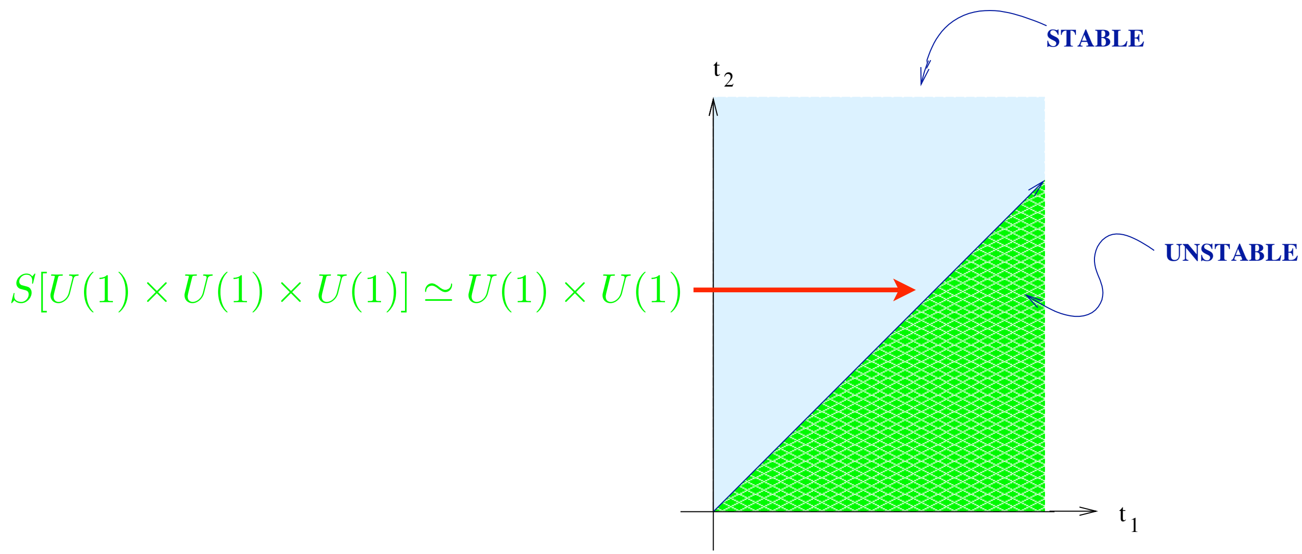
- *If a multivariate holomorphic function with domain $U \subset \mathbb{C}^n$ vanishes on an open subset $B \subset U$, then it vanishes everywhere on U*

Therefore the texture in the interior of the stable region is identical to the texture near the stability wall!

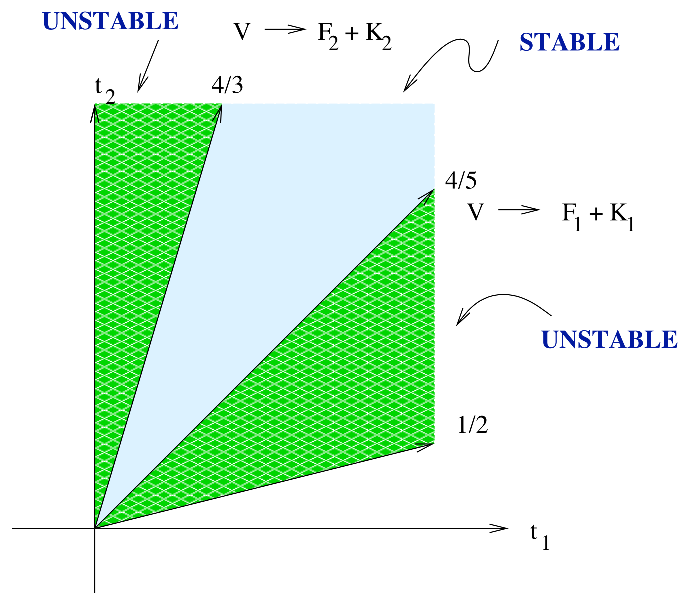
Conclusion: The existence of one stability wall with 1 anomalous U(1) can lead to a natural hierarchy of textures for Yukawa couplings

These results can be extended to

1) One stability wall with 2 U(1)'s

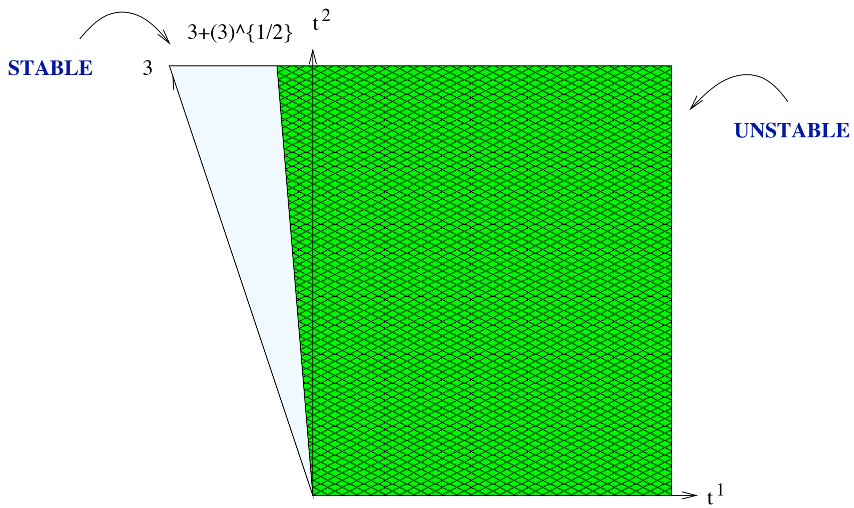


2) Two stability walls each with 1 U(1)



3) SU(4) and SU(5) bundles and equivariance under isometries.

Example: SU(4) under $\mathbb{Z}_3 \times \mathbb{Z}_3$



$\Rightarrow$ 3 family MSSM-like model with naturally heavy third family

4) Textures of vector-like pairs

$$W = \cdots + C_1 F_1 \bar{F}_2 + C_2 F_2 \bar{F}_1 + C_1 C_2 F_1 \bar{F}_1 + C_1 C_2 F_2 \bar{F}_2$$

$\langle C_1 \rangle = \langle C_2 \rangle = 0 \;\Rightarrow\;$ $W^{\mathrm{wall}}_{\mathrm{vec-like\ pairs}} = 0$

$\langle C_1 \rangle = 0,\ \langle C_2 \rangle \neq 0 \;\Rightarrow\;$ $W_{\mathrm{vec-like\ pairs}} = \mathbf{F_2 \bar{F}_1}$ only

2) Explicit Calculation:

The triple product ⇒

$$\lambda = \int_X \sqrt{g_{\mu\nu}} \psi_L^a \psi_H^{[b,c]} \psi_r^d \epsilon_{abcd} d^6 x$$

where

$$\nabla^2_{**} \psi^* = \lambda \psi^* \ , \lambda = 0$$

⇒ need to calculate the metric and eigenfunctions of the Laplacian. Unfortunately, a Calabi-Yau manifold does not admit a continuous symmetry. ⇒ the metric, gauge connection and, hence, the Laplacian are unknown! Remarkably, these can be well-approximated by numerical methods.

Ricci-Flat Metrics, Scalar Laplacians and Gauge Connections on Calabi-Yau Threefolds

1) Calabi-Yau Metrics: Braun, Brelidze, Douglas, Ovrut 2008

Example: Quintics are CY threefolds $\tilde{Q} \subset \mathbb{P}^4$

A Kahler metric is given by $g_{i\bar{j}}(z,\bar{z}) = \partial_i \partial_{\bar{j}} K(z,\bar{z})$

The unique SU(5) invariant Kahler metric on $\mathbb{P}^4$ comes from

$$K_{FS} = \frac{1}{\pi} \ln \sum_{\alpha,\bar{\beta}}^{4} h^{\alpha\bar{\beta}} z_\alpha \bar{z}_{\bar{\beta}}$$

But, the restriction of the Fubini-Study metric to $\tilde{Q}$ is not Ricci-flat.

Donaldson's Algorithm-

a) Generalize the Fubini-Study Kahler potential on $\mathbb{P}^4$ to

$$K(z,\bar{z})_{h,k} = \frac{1}{k\pi} \ln \sum_{\substack{i_1,\ldots,i_k=0 \\ \bar{j}_1,\ldots,\bar{j}_k=0}}^{4} h^{(i_1,\ldots,i_k),(\bar{j}_1,\ldots,\bar{j}_k)} \underbrace{z_{i_1}\cdots z_{i_k}}_{\text{degree } k} \underbrace{\bar{z}_{\bar{j}_1}\cdots \bar{z}_{\bar{j}_k}}_{\text{degree } k}$$

Now restrict to $\tilde{Q}$. We must pick a basis of the quotient $\mathbb{C}[z_0, \ldots, z_4]_k \Big/ \langle \tilde{Q}(z) \rangle$. Denote the basis by $s_\alpha, \alpha = 0, \ldots, N_k - 1$

Given any $\tilde{Q}$ and k, computing this basis is straightforward.

The generalized potential can be written on $\tilde{Q}$

$$K_{h,k} = \frac{1}{k\pi} \ln \sum_{\alpha, \bar{\beta}=0}^{N_k - 1} h^{\alpha\bar{\beta}} s_\alpha \bar{s}_{\bar{\beta}}$$

b) Define an inner product on the space of sections by

$$\langle s_\alpha, s_\beta \rangle = \frac{N_k}{Vol_{CY}(\tilde{Q})} \int_{\tilde{Q}} \frac{s_\alpha \bar{s}_{\bar{\beta}}}{\sum_{\gamma\bar{\delta}} h^{\gamma\bar{\delta}} s_\gamma \bar{s}_{\bar{\delta}}} dVol_{CY}$$

where $dVol_{CY} = \Omega \wedge \Omega$. Find the matrix $h_{\text{bal}}^{\alpha\bar{\beta}}$ such that

$$h_{\text{bal}}^{\alpha\bar{\beta}} = \left(\langle s_\alpha, s_\beta \rangle_{h_{\text{bal}}} \right)^{-1}$$

To do this, choose an initial $h_0^{\alpha\bar{\beta}}$ and iterate. The sequence quickly converges to a unique fixed point.

c) Defining

$$g^{(k)}_{(\mathrm{bal})i\bar{j}} = \frac{1}{k\pi}\partial_i\partial_{\bar{j}} \ln \sum_{\alpha,\bar{\beta}=0}^{N_k-1} h^{\alpha\bar{\beta}}_{\mathrm{bal}} s_\alpha \bar{s}_{\bar{\beta}}$$

then

$$g^{(k)}_{(\mathrm{bal})i\bar{j}} \xrightarrow{k\to\infty} g^{CY}_{i\bar{j}}$$

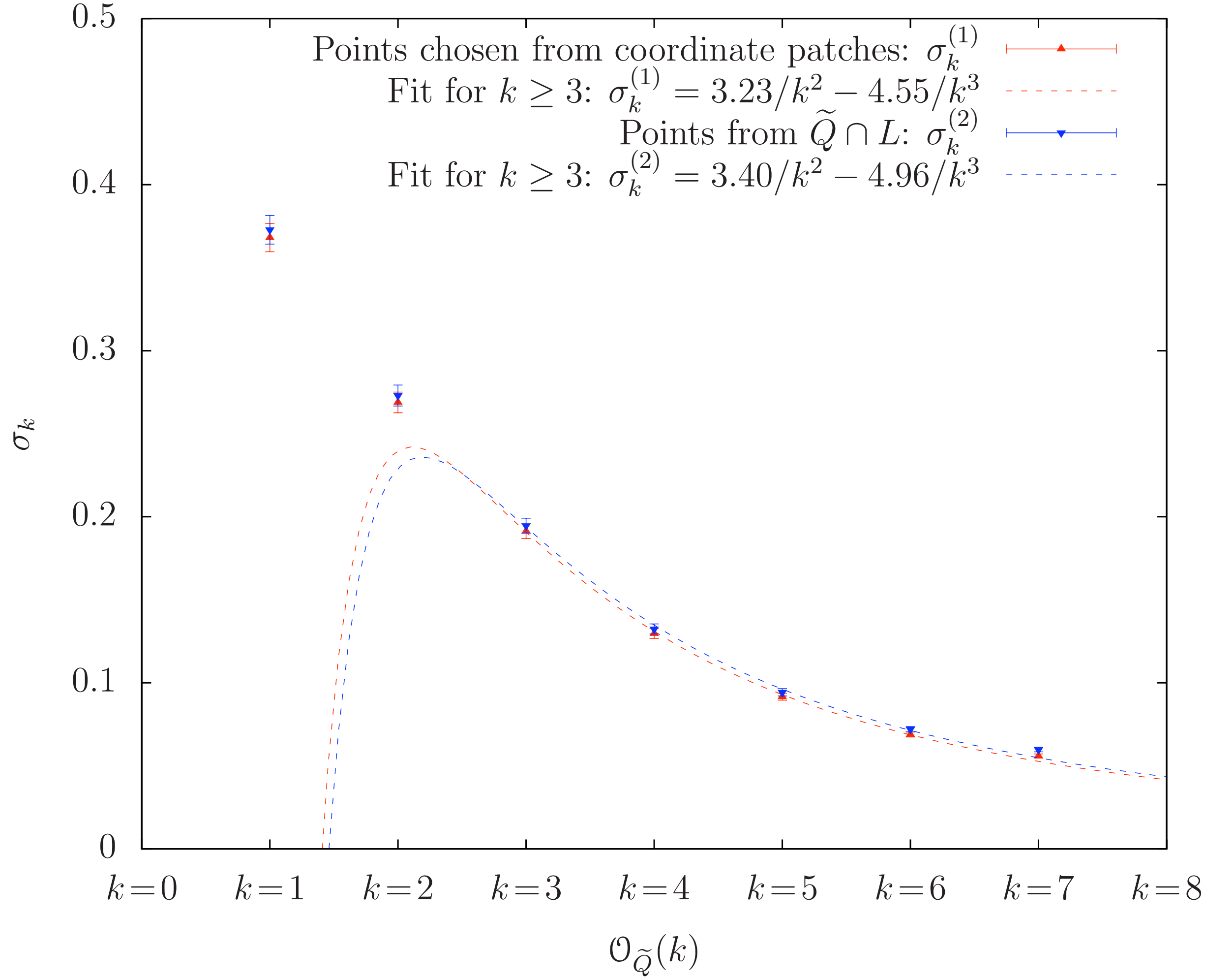
The form of $g^{(k)}_{(\mathrm{bal})i\bar{j}}$ at any finite k is not enlightening. How closely they approach $g^{CY}_{i\bar{j}}$ for large k can be estimated using

$$\sigma_k(\tilde{Q}) = \frac{1}{Vol_{CY}(\tilde{Q})}\int_{\tilde{Q}}\left|1 - \frac{\omega_k^3 \big/ Vol_K(\tilde{Q})}{\Omega\wedge\bar{\Omega}\big/ Vol_{CY}(\tilde{Q})}\right| dVol_{CY}$$

where

$$\omega_k = \frac{i}{2} g^{(k)}_{(\mathrm{bal})\mathrm{i}\bar{\mathrm{j}}} dz_i \wedge d\bar{z}_{\bar{j}}$$

Fermat quintic:



The error measure σ_k for the metric on the Fermat quintic, computed with the two different point generation algorithms

2) SU(N) Gauge Connections:

Anderson, Braun, Karp, Ovrut 2010

Let $z_\alpha^a, \ \alpha = 0, \ldots, N_{k_H} - 1$ be degree-k_H polynomials on the CY carrying the N-representation of U(N) and $H_{\rm bal}^{\alpha\bar{\beta}}$ a specific matrix. Defining an SU(N) connection

$$A^{(k_H)a\bar{b}}_{(\mathrm{bal})\mathrm{i}} = \partial_i(ln \sum_{\alpha,\bar{\beta}}^{N_{k_H}-1} H_{\rm bal}^{\alpha\bar{\beta}} z_\alpha^a \bar{z}_{\bar{\beta}}^{\bar{b}} - g^{a\bar{b}} ln \sum_{\alpha,\bar{\beta}}^{N_{k_H}-1} h_{\rm bal}^{\alpha\bar{\beta}} s_\alpha \bar{s}_{\bar{\beta}})$$

then

$$A^{k_H}_{(\mathrm{bal})\mathrm{i}} \xrightarrow{k_H \to \infty} A_i^H$$

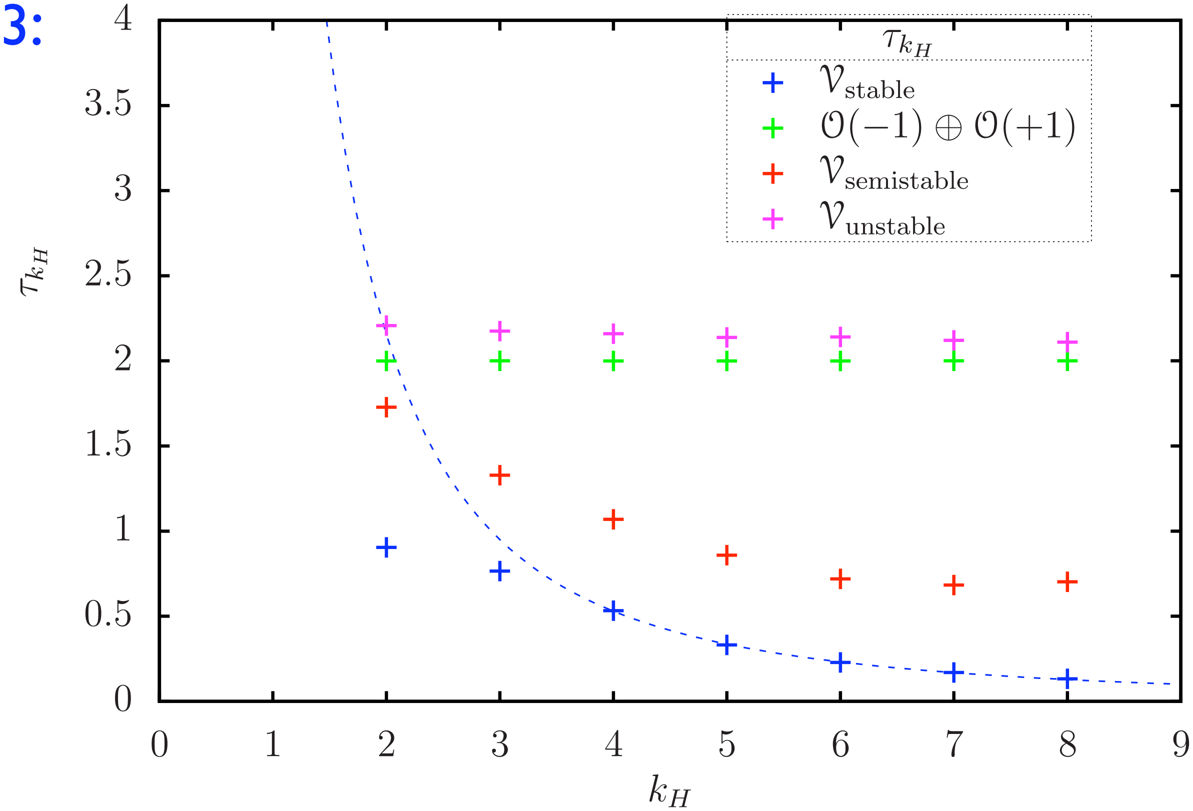
where A_i^H satisfies the Hermitian Yang-Mills equations. That is

$$\omega^{i\bar{j}} F^{(k_H)}_{(\mathrm{bal})\mathrm{i}\bar{\mathrm{j}}} = \omega^{i\bar{j}} \partial_{\bar{j}} A^{(k_H)}_{(\mathrm{bal})\mathrm{i}} \xrightarrow{k_H \to \infty} 0$$

Expressed this way $A^{k_H}_{(\mathrm{bal})\mathrm{i}}$ at any finite k_H is not enlightening. More interesting is how closely they approach A_i^H for large k_H. This can be estimated using

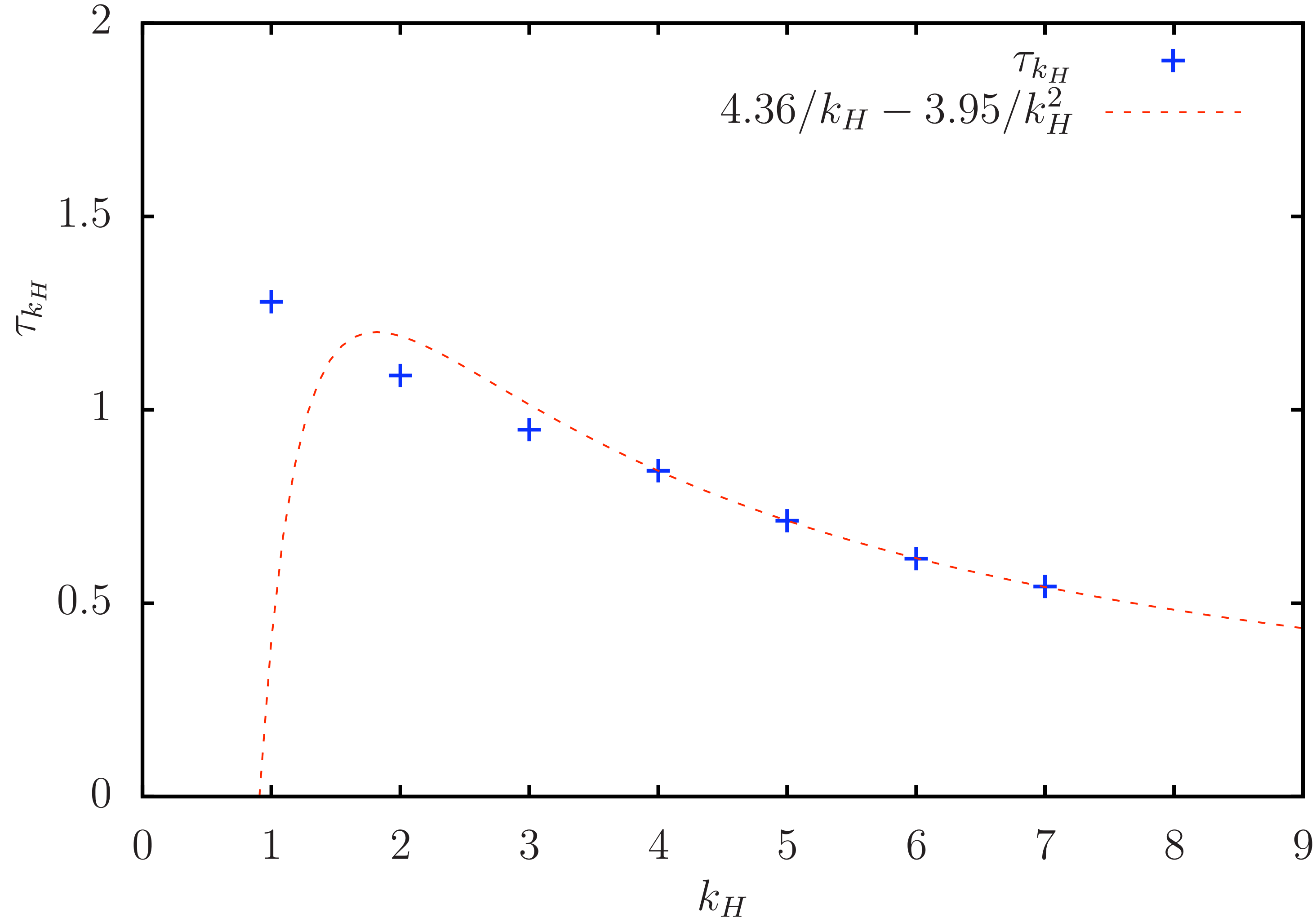
$$\tau_{k_H}(A) = \frac{1}{2\pi V_{CY}(\tilde{Q})} \int_{\tilde{Q}} \sum_{a=1}^{N} |\lambda_a| dVol_{CY} \quad \text{where} \quad \omega^{i\bar{j}} F^{(k_H)}_{(\mathrm{bal})i\bar{j}} = diag(\lambda_1, \ldots, \lambda_N)$$

quartic K3:



The integrated error, τ_{k_H}, for $SU(n)$ bundles on the Quartic $K3$. Shown above are the results for 1) a stable, $SU(2)$ bundle, $\mathcal{V}_{stable}$, 2) A semistable, $SU(3)$ bundle, $\mathcal{V}_{semistable}$, 3) An unstable $SU(2)$ bundle, $\mathcal{V}_{unstable}$ and a sum of line bundles $\mathcal{O}(-1) \oplus \mathcal{O}(1)$ for comparison to the Harder-Narasimhan filtration of $\mathcal{V}_{unstable}$.

quintic threefold:



The integrated error measure τ_{k_H} on the quintic threefold for a stable bundle.

3) Scalar Laplacians:

Given a metric $g_{\mu\nu}$ $\Rightarrow$

$$\Delta = -\frac{1}{\sqrt{g}}\partial_\mu(g^{\mu\nu}\sqrt{g}\partial_\nu)$$

Solve the eigen-equation

$$\Delta\phi_{m,i} = \lambda_m\phi_{m,i}\ ,\ i = 1, \ldots \mu_m$$

where μ_m is the multiplicity from continuous/finite symmetry.

Choose a basis $\{f_a | a = 1, \ldots, k\}$ $\Rightarrow$ the eigen-equation becomes

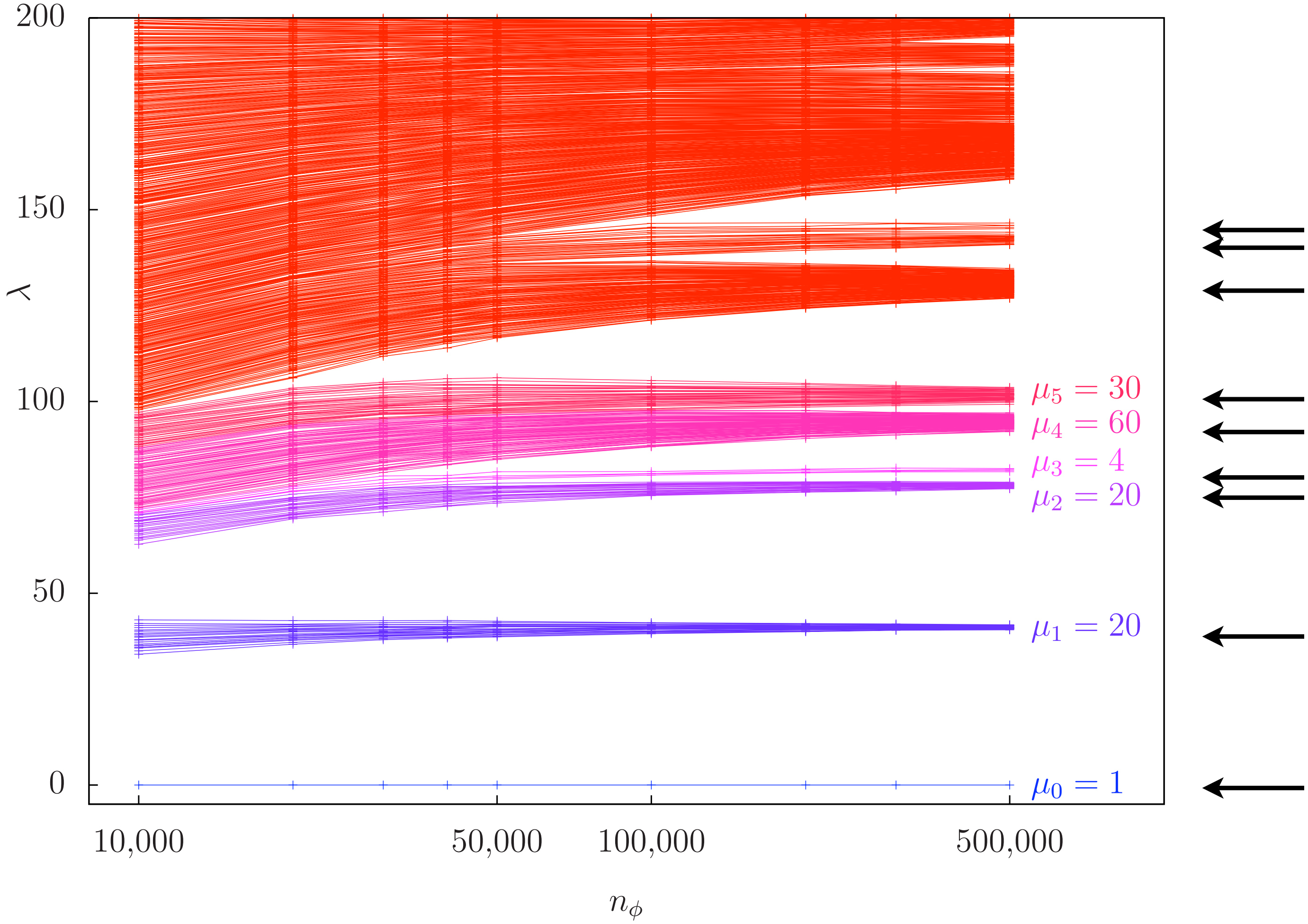
$$\sum_b \langle f_a|\Delta|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle = \sum_b \lambda_m\langle f_a|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle$$

Numerical Solution:

1) Solve numerically for λ_n and ϕ_n

2) For fixed k let $n_\phi \to \infty$

Fermat quintic:



Eigenvalues of the scalar Laplace operator on the Fermat quintic. The metric is computed at degree $k_h = 8$, using $n_h = 2{,}166{,}000$ points. The Laplace operator is evaluated at degree $k_\phi = 3$ using a varying number n_ϕ of points.

- But- implement parallel algorithms to Singular, Macaulay, String Vacua

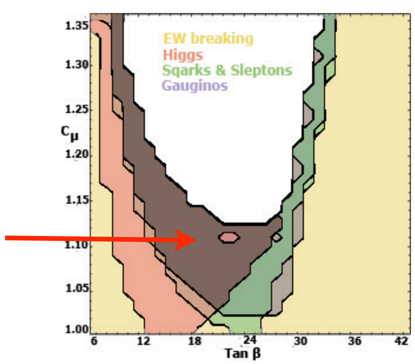
Heterotic Cosmology

1) B-L MSSM Cosmic Strings

$$\nu_{\mathbf{3}} = e^{in\theta}\langle \nu_3 \rangle f(r) \ , \qquad \mathbf{A_{B-Lr}} = 0, \ \ \mathbf{A_{B-L\theta}} = \frac{n}{g_{B-L} r}\alpha(r)$$

All effective slepton/squark masses positive at $\langle \nu_3 \rangle$. For example,

$$m_{e_3}^2|_{\nu_{\mathbf{3}}} = m_{e_3}^2 + g_{B-L}^2 {\nu_{\mathbf{3}}}^2$$



$\Rightarrow$ all soft slepton/squark masses positive except

$$m_{e_3}^2 < 0$$

$\Rightarrow$ possible charge breaking condensate at the core of the cosmic string.

To decide this, consider a small fluctuation $\mathbf{e}_3 = e^{i\omega t}\sigma_0(r)$

Satisfies

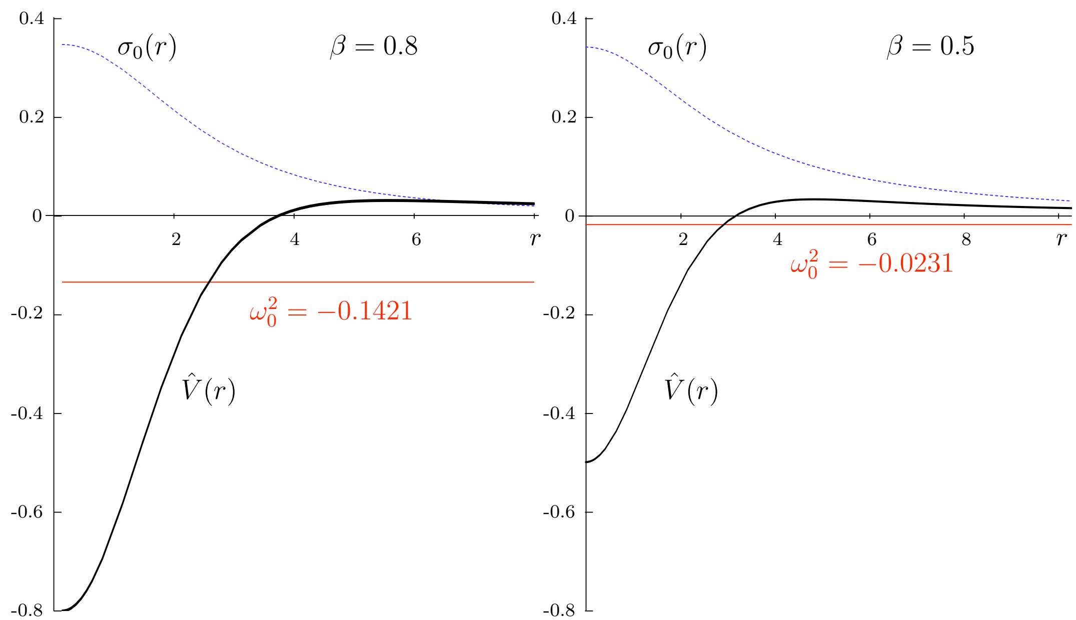
$$(-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r})\sigma_0 + \hat{V}\sigma_0 = \omega^2\sigma_0$$

where

$$\hat{V}(r) = \beta\langle\nu_3\rangle^2(f(r)^2 - 1) + \frac{\alpha(r)^2}{r^2}$$

Condensate $\Leftrightarrow$ bosonic superconductivity $\Leftrightarrow$ ground state with

$$\omega^2 < 0$$



$\Rightarrow$ boson superconductivity iff $\beta \gtrsim .42$

But- for above case $\beta \simeq .10$

EW phase transition removes fermion zero modes.

- Can have bound state current up to $\sim 10^{12} A$.
- Possible source of EW baryogenesis

2) Moduli Potentials

- D-term "uplifting", explicit calculation of worldsheet instanton superpotential on heterotic standard model vacua with torsion, Fixing complex structure moduli through F-terms.
- Discussing inflation and ekpyrotic cosmology in this context.
- Computation of multi-point functions and predictions for non-Gaussianity.