Global aspects of the space of 6D string vacua

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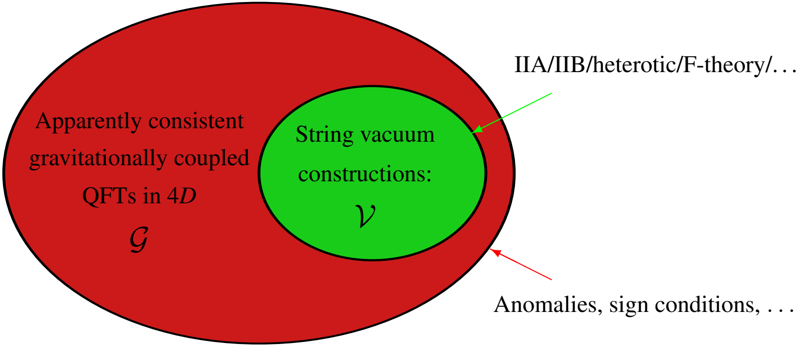
Based on:

arXiv: 0903.0386, 0906.0987, 0910.1586
V. Kumar, WT

arXiv: 0911.3393, 1005.nnnn, 10mm.nnnn
V. Kumar, D. Morrison, WT

Grappling with the landscape

- Many apparently consistent string vacua in 4D (tip of iceberg?)



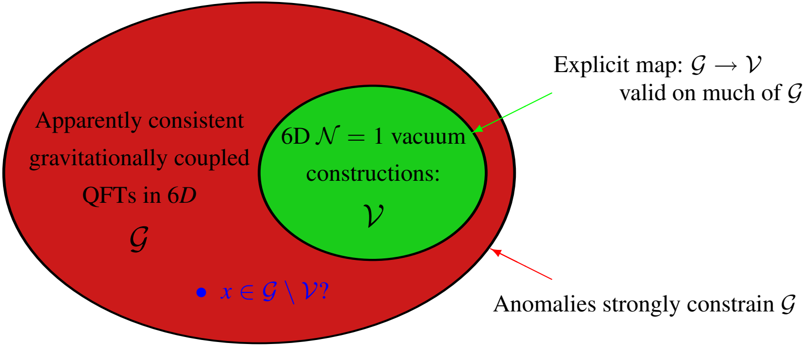
- Even more vast range of 4D QFT + gravity without known inconsistencies

Apparent 4D swampland $\mathcal{G} \setminus \mathcal{V}$ is vast [Vafa, Ooguri/Vafa]

Hard to attain global picture in 4D.

Point of this talk:

Global analysis, mapping landscape seems tractable for 6D $\mathcal{N} = 1$ SUGRA



Program: systematically analyze $\mathcal{G}$ for 6D $\mathcal{N} = 1$ SUGRA

Find map $\mathcal{G} \to \mathcal{V}$ where possible (uniqueness?), chart regions

If $x \in \mathcal{G} \setminus \mathcal{V}$, must indicate one of
- a) new string construction: $\mathcal{V}' \supset \mathcal{V}$
- b) new low-E constraint: $\mathcal{G}' \subset \mathcal{G}$
- c) true stringy constraint

Three approaches to understanding string vacua

Search for Standard Model-like vacua: string construction → EFT

(+) Useful for developing technology

(+) Understand components corresponding to SM features

(–) Doesn't give insight into constraints

Generate classes of vacua, do statistics, seek patterns

(+) Technology as above + constraints/distribution in class of constructions

(–) In 4D only piece of picture, connecting classes, swampland unclear

Study global structure of $\mathcal{G}, \mathcal{V}$

(–) Hard in 4D

(+) Understand global string constraints, map EFT → ST
May be possible in 6D, lessons for 4D.

Outline

1. Bounds and lattices for 6D supergravity

- Focus on 6D $\mathcal{N} = (1, 0)$ supersymmetric theories w/gravity
- Originally studied in mid 80's [Nishino/Sezgin ($T = 1$), Romans ($T > 1$)]
- (1, 0) 6D SUSY fields

Multiplet	Matter Content
SUGRA	$(g_{\mu\nu}, B^-_{\mu\nu}, \psi^-_\mu)$
Tensor (T)	$(B^+_{\mu\nu}, \phi, \chi^+)$
Vector (V)	(A_μ, λ^-)
Hyper (H)	$(4\varphi, \psi^+)$

- Semi-simple group $\mathcal{G} = G_1 \times G_2 \times \cdots \times G_k$ $(/\Gamma)$ [ignore $U(1)$'s]
- Matter $\mathcal{M}$ in (generally reducible) representation of G
- Tensors transform under $SO(1, T)$, ϕ's $\rightarrow j \in SO(1, T)/SO(T)$

Question: what combinations of $\mathcal{G}, \mathcal{M}, T$ possible?

Claim: For $T < 9$, only a finite set of possible $\mathcal{G}, \mathcal{M}$

[proven for $T = 1$ in arXiv:0910.1586; $T > 1$ in KMT paper *to appear*]

Key: anomaly cancellation [Green/Schwarz, G/S/West, Sagnotti, Sadov]

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta \qquad X_4^\alpha = \frac{1}{2}a^\alpha \mathrm{tr}R^2 + \sum_i b_i^\alpha \left(\frac{2}{\lambda_i}\mathrm{tr}F_i^2\right)$$

$\mathrm{sign}(\Omega) = (+,-,-,-,\ldots)$, $a^\alpha, b_i^\alpha \in \mathbb{R}^{1,T}$, $\mathrm{tr} \to \lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \ldots$

$T = 1$:

$$\Omega_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad a = (-2,-2), \qquad b = \frac{1}{2}(\alpha, \tilde{\alpha})$$

$$I_8 = X^1 X^2 = (\mathrm{tr}R^2 - \sum_i \alpha_i \mathrm{tr}F_i^2)(\mathrm{tr}R^2 - \sum_i \tilde{\alpha}_i \mathrm{tr}F_i^2)$$

Further constraint: physical kinetic terms [Sagnotti]

$\mathrm{SUSY} \to -j \cdot b\, \mathrm{tr}F^2 \to j \cdot b > 0 \qquad (j \cdot b = e^\phi \alpha + e^{-\phi}\tilde{\alpha}$ for $T = 1)$

Anomaly conditions for factorization:

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}\left(\frac{1}{2}a^\alpha \mathrm{tr}R^2 + 2b_i^\alpha/\lambda_i\ \mathrm{tr}F_i^2\right)\left(\frac{1}{2}a^\beta \mathrm{tr}R^2 + 2b_i^\beta/\lambda_i\ \mathrm{tr}F_i{}^2\right)$$

give relations:

R^4:	$H - V = 273 - 29T$	$H - V \sim$ partition bound
F^4:	$0 = B^i_{Adj} - \sum_R x^i_R B^i_R$	
$(R^2)^2$:	$a \cdot a = 9 - T$	A_R, B_R, C_R from expanding :
F^2R^2:	$a \cdot b_i = \frac{1}{6}\lambda_i \left(A^i_{Adj} - \sum_R x^i_R A^i_R\right)$	$\mathrm{tr}_R F^2 = A_R \mathrm{tr}F^2$
$(F^2)^2$:	$b_i \cdot b_i = \frac{1}{3}\lambda_i^2 \left(\sum_R x^i_R C^i_R - C^i_{Adj}\right)$	$\mathrm{tr}_R F^4 = B_R \mathrm{tr}F^4 + C_R (\mathrm{tr}F^2)^2$
$F_i^2 F_j^2$:	$b_i \cdot b_j = 2\sum_{R,S} x^{ij}_{RS} A^i_R A^j_S$	

Remarkable fact: $a \cdot a,\ a \cdot b_i,\ b_i \cdot b_i,\ b_i \cdot b_j \in \mathbb{Z}$ if no local/global anomalies.

Defines integral lattice $\Lambda \subset \mathbb{R}^{1,T}$ [Proof in upcoming KMT paper]

Proof of finite possible $\mathcal{G}, \mathcal{M}$ in consistent 6D SUGRA's with $T < 9$

For fixed $\mathcal{G}, T$, finite possible $\mathcal{M}$ from $H - V = 273 - 29T$.
So must have unbounded $\mathcal{G}$, prove impossible by contradiction

Case 1: $\{G = G_1 \times \cdots \times G_k\}, \ |G_i| \leq D, \ k \to \infty$

Use $(1, T)$ geometry, bound on matter: $H \leq 273 - 29T + kD$;

$$\text{classify} \begin{cases} b^2 > 0 : \mathrm{P} \\ b^2 = 0 : \mathrm{Z} \\ b^2 < 0 : \mathrm{N} \end{cases}$$

P: $b_i = (x_i, \vec{y}_i), |x_i| > |\vec{y}_i| \Rightarrow b_i \cdot b_j > 0 \ \Rightarrow$ at most $\mathcal{O}(\sqrt{k \ln k})$ type P's

N: At most T mutually orthogonal with $b_i^2 < 0, b_i \cdot b_j = 0$
Turán's theorem: graph on n nodes with $> (1 - 1/T)n^2/2$ edges $\supset T$-clique
$\Rightarrow$ at most $\mathcal{O}(\sqrt{k \ln k})$ type N's

Z: $b_i \cdot b_j > 0$ if not parallel, $\to \mathcal{O}(k)$ parallel (all but $\mathcal{O}(\ln k)$ Z's)
All Z's have positive $H - V$ (explicit check) $\Rightarrow$ exceed bound, contradiction $\forall T$.

Case 2: $|G_1| \to \infty$

Only limited possibilities for factors of unbounded dimension

- Schwarz: Two $T = 1$ families: $SU(N) \times SU(N)$, $SO(2N+8) \times Sp(N)$
- Also: $SU(N) \times SO(N+8), SU(N) \times SU(N+8), Sp(N) \times SU(2N+8)$
- $T > 1$: Three 3-factor families $\sim SU(N-8) \times SU(N) \times SU(N+8)$

e.g. $SU(N) \times SU(N)$,   matter = $2 \times (\square, \bar{\square})$

$$a \cdot b_1 = a \cdot b_2 = 0, \quad b_1^2 = b_2^2 = -2, \quad b_1 \cdot b_2 = 2$$

Families all have common problem for $a^2 > 0$ ($T < 9$)

$$a \cdot (b_1 + b_2) = 0 \;\; \& \;\; (b_1 + b_2)^2 = 0 \;\; \Rightarrow \;\; b_1 + b_2 = 0$$

$$\Rightarrow j \cdot b_1 = -j \cdot b_2 \;\; \Rightarrow \;\; \text{bad kinetic terms}$$

Proven finite models for $T < 9$;   Proof fails at $T \geq 9$

What happens at $T = 9$?

Infinite families with anomaly cancellation, ok kinetic terms

e.g. for $SU(N) \times SU(N)$ family, $\Omega = \text{diag}\,(+1, -1, -1, \ldots), j = (1, 0, 0 \ldots)$

$$
\begin{aligned}
a &= (2, 1, 1, 1, 1, 0, 0, 0, 0, 0) \\
b_1 &= (1, 1, 1, 0, 0, 1, 0, 0, 0, 0) \\
b_2 &= (1, 0, 0, 1, 1, -1, 0, 0, 0, 0)
\end{aligned}
$$

Summary so far:

- Each consistent 6D SUGRA $\Rightarrow$ integral lattice Λ
- Finite gauge group, matter combinations for $T < 9$
- $T = 1$ models can be enumerated; e.g. 16,418 with $\prod_i SU(N_i)$, matter $\square, \begin{array}{c}\square\\[-4pt]\square\end{array}$
- Infinite families at $T \geq 9$

Analysis so far independent of string theory.
Which models have string realizations?

2. Mapping supergravity theories to F-theory

F-theory: IIB with variable axiodilaton τ (7-branes $\rightarrow$ fiber singularities)

[Vafa, Morrison/Vafa]

F-theory in 6D:

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \\ & & B \end{array}$$

$X = \text{CY 3-fold}$

$B = \text{complex surface}$

Kodaira condition for $X = \text{CY}$ ($K_X = 0$)

$$-12K = \Delta = \sum_i \nu_i \xi_i + Y$$

$X =$ elliptic fibration over B

$K =$ canonical class of B

$\Delta =$ singularities; ξ_i divisors $\rightarrow G_i$

Intersection form on $B \Rightarrow$ unimodular lattice

e.g. $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for $\mathbb{F}_{2m}$, $\text{diag}(+1, -1, -1, \ldots)$ (for blow-up of $\mathbb{F}_n, \mathbf{P}^2$)

Identifying topological F-theory data from SUGRA structure

$T = h_{1,1}(B) - 1$; each factor G_i maps to singular divisor ξ_i

$$a \to K \qquad \Lambda \hookrightarrow H_2(B; \mathbb{Z})$$
$$b_i \to \xi_i \qquad j \to J$$

matches [Sadov, Grassi-Morrison]
$$-a \cdot b = -K \cdot \xi_i$$
$$b_i \cdot b_j = \xi_i \cdot \xi_j$$

F-theory construction only possible under certain conditions, such as

- unimodular embedding
- $b_i^2 < 0 \to b_i$ primitive ($b_i \neq n\tilde{b}_i$)
- $j \cdot a > 0$; $j \cdot Y = j \cdot (-12a - \sum_i \nu_i \xi_i) > 0$
- Weierstrass model $y^2 = x^3 + fx + g$, f, g sections of $-4K, -6K$

Question: how do these constraints appear in SUGRA?

Examples: 6D supergravity and F-theory images

- Looked at 16,418 $T = 1$ models with $G = \prod_i SU(N_i)$, matter $\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}$

–all seem topologically consistent with F-theory on $\mathbb{F}_{0,1,2}$

–identified some Weierstrass models

–# DOF $= 273 - 29 = 244$; imposing $\mathcal{G}, \mathcal{M} \Rightarrow H - V$ constraints
seems plausible all admit Weierstrass models

- Exotic matter representations $\rightarrow$ unknown F-theory singularities ($T = 1$)

– *e.g.* $\square + 3\,\square\!\square + 2\,\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}$, $\quad H - V = 243$

- Some $\mathcal{G}, \mathcal{M}$ topologically inconsistent w/F-theory (rare at $T = 1$)

– *e.g.* $SU(4)$ with 1 adjoint, $10 \times \square\!\square + 40 \times \square$:

$$\Lambda = \begin{pmatrix} 8 & 10 \\ 10 & 10 \end{pmatrix} \quad \text{admits no unimodular embedding}$$

– *e.g.* $SO(8) \times SU(24)$, embedding ok but J outside Kähler cone

Infinite families revisited

- Recall infinite $SU(N) \times SU(N)$ family at $T = 9$

$$a \cdot (b_1 + b_2) = 0 \ \ \& \ \ (b_1 + b_2)^2 = 0$$

When $a^2 = 0$, this implies $b_1 + b_2 = na$. a primitive $\rightarrow n \in \mathbb{Z}$.

Kodaira constraint $\rightarrow j \cdot Y = j \cdot (12a - N(b_1 + b_2)) > 0$

limits $N \leq 12$ in F-theory [$N = 8$: Dabholkar/Park].

- More work needed to rule out other infinite families

Family: $G = E_8^k, T = 8k, a \cdot b_i = -10, b_i^2 = -12, b_i \cdot b_j = 0, i \neq j$.

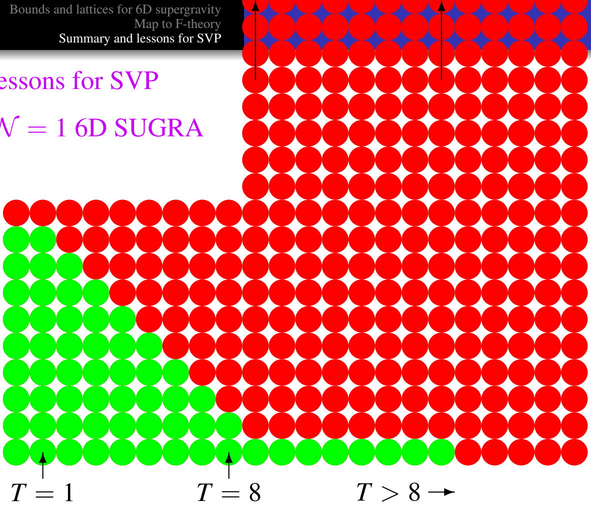
Can find acceptable supergravity lattice. Seiberg-Witten: $k = 2, T = 25$.

Does not violate case 1 as $T \rightarrow \infty$. F-theory??

3. Summary and lessons for SVP

Global picture of $\mathcal{N} = 1$ 6D SUGRA

(cartoon)



- Each $\mathcal{G}, \mathcal{M} \rightarrow$ continuous (quaternionic) moduli space
- Connected by Higgs/small instanton/massless string transitions
  [Witten, Duff/Minasian/Witten, Seiberg/Witten, Morrison/Vafa, KMT III]
- May be possible to chart regions, connect $\rightarrow$ one theory (string universality?)

Summary:

- Each consistent supergravity theory is associated with an integral lattice Λ
- Λ, vectors $a, b_i \rightarrow$ structure of F-theory compactification.
- $\mathcal{N} = 1$ 6D supergravity has finite groups, matter for $T < 9$.
- For $T > 8$ there are infinite families of SUGRA models.
  Some of these families are inconsistent in F-theory.

We have begun to map the 6D supergravity landscape.
? Can we map other string regions, perhaps using Λ?
Many other 6D string constructions from heterotic ($T = 1$), orientifolds ($T > 1$), etc.
Some evidence: most other constructions $\subset$ F-theory (need G-flux in F-theory)

? Can we identify new low-energy constraints based on properties like

— Unimodular lattice embedding?

— $j \cdot a > 0$, Kodaira condition? (may be related to $j \cdot a \;$ tr R^2 [∼AADNR?])

? Identify new phases of string theory (non-geometric F-theory?)

Qualitative lessons from 6D which may be relevant to 4D, SVP

- String theory $\rightarrow$ strong constraints on low E theory
- Many if not all constraints visible in LET
- Can map low E data $\rightarrow$ string theory
- String realizations of low E physics $\sim$ unique up to duality
  (though may involve all structure to high E scale if true in 4D)
- Generally F-theory $\supset$ other constructions (modulo G-flux)
- Moduli spaces of vacua generically connected