Spin Qubits in Single and Double Quantum Dots and Decoherence Due to Nuclear Spins

Daniel Loss
Department of Physics & Astronomy
University of Basel

Collaborators:

Hansres Engel (Harvard)
Vitaly Golovach
Sasha Khaetskii (Chernogolovka)
Bill Coish
Daniel Klauser

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D. Bulaev

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N. Bonesteel
M.S. Choi
L. Glazman
K. Ensslin
A. Imamoglu
J. Lehmann
A. MacDonald
D. Saraga
J. Schliemann
C. Schönenberger
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…

Outline

A. Quantum computing with spin qubits

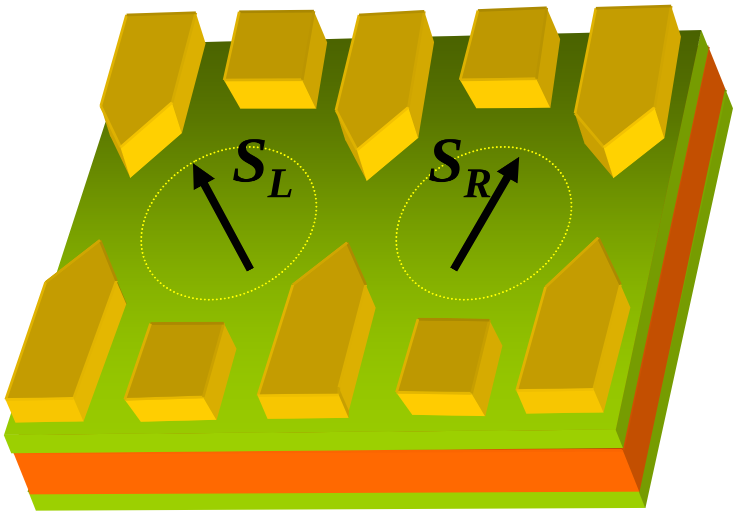
1. interaction based
2. measurement based (Bell state)

B. Spin decoherence in GaAs quantum dots

1. Spin-orbit & phonon: T_1 ~ 1ms and $T_2 = 2T_1$
2. Nuclear spins and hyperfine interaction:
   - spin ½ in single dot
   - singlet-triplet in double dot, effective H, decay, state narrowing

Quantum Computing with Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)

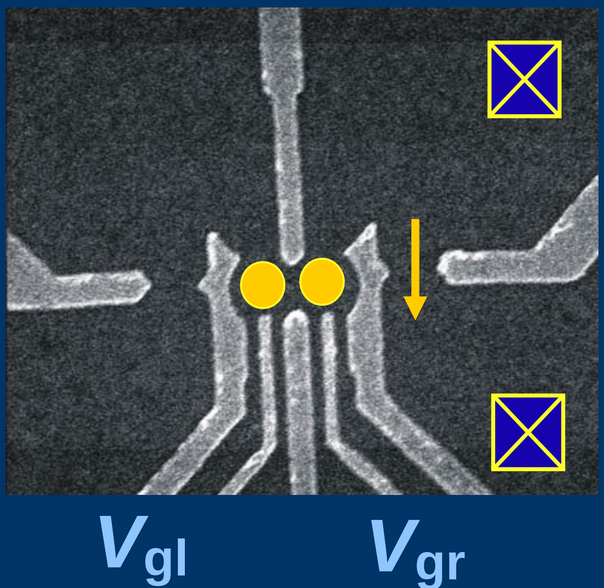


Spin 1/2 of electron = qubit

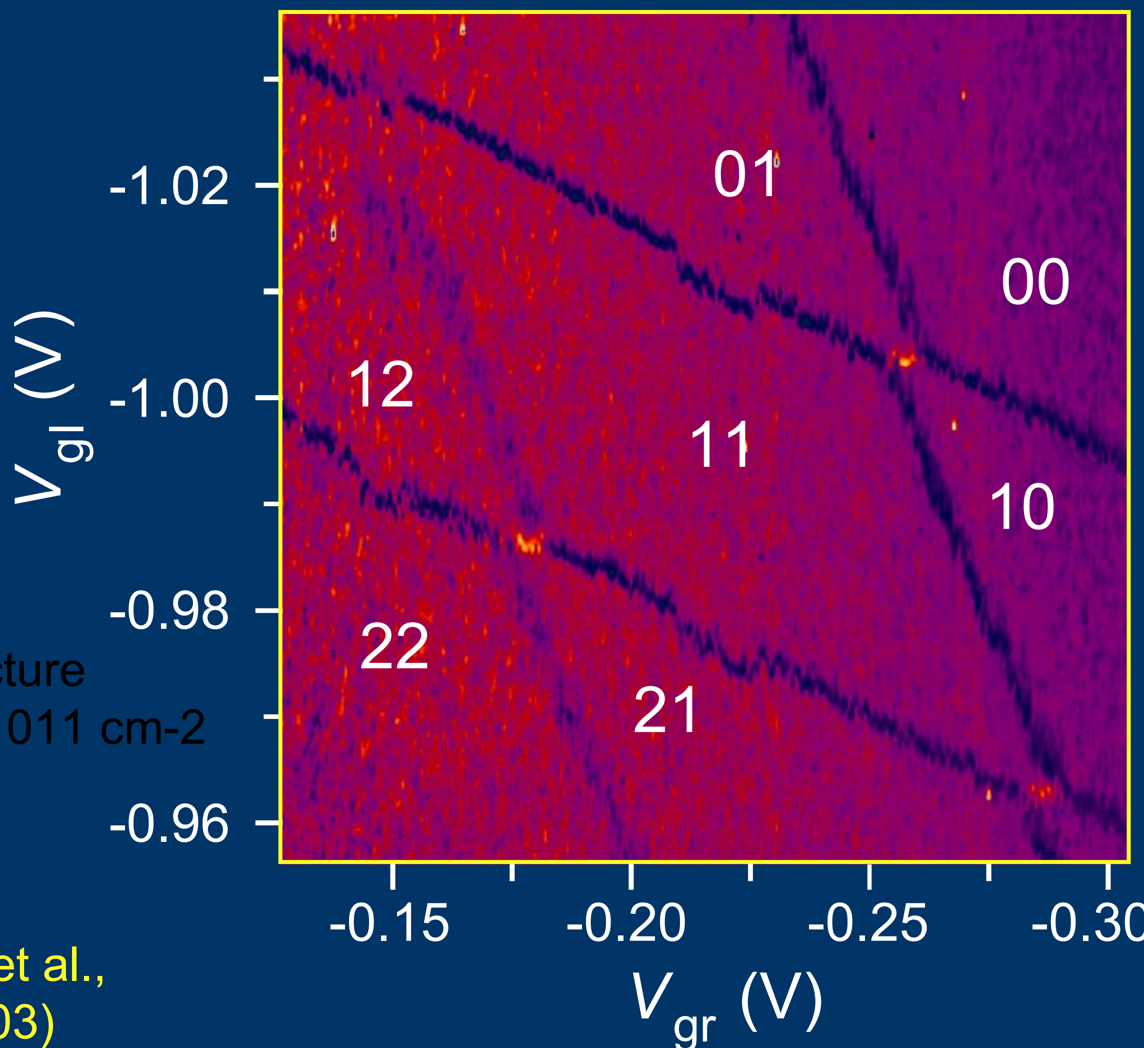
1. Quantum gates based on exchange interaction:

$$H(t) = J(t)\, \mathbf{S}_L \cdot \mathbf{S}_R$$

GaAs Double Dot and Stability Diagram



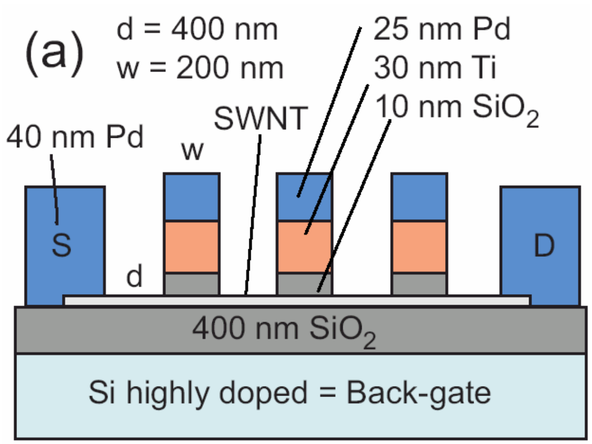
GaAs/AlGaAs heterostructure
2DEG 90 nm deep, ns = 2.9 x 1011 cm-2

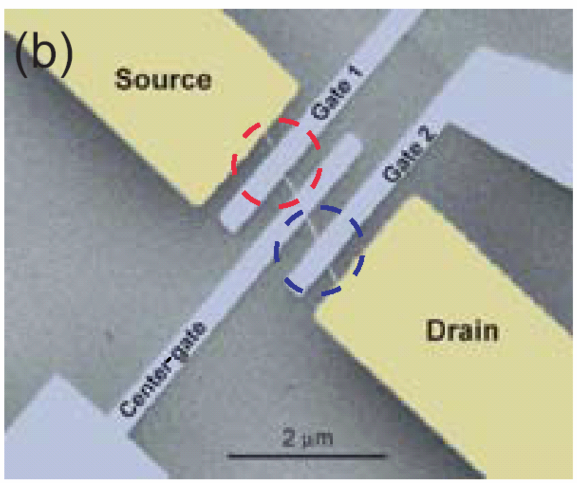


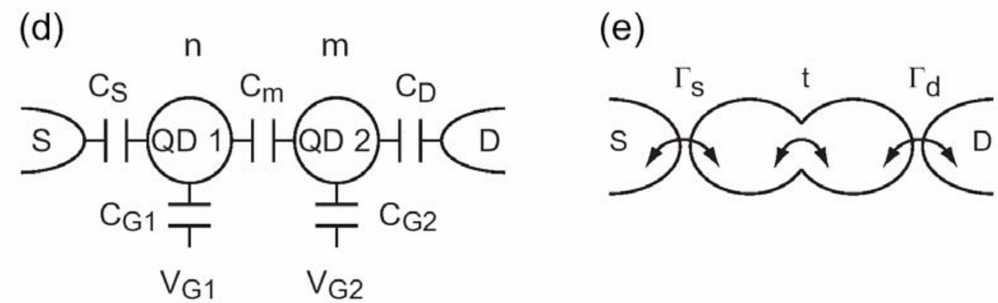
Kouwenhoven et al. & Tarucha et al.,
Phys. Rev. B **67,** 161308 (2003)

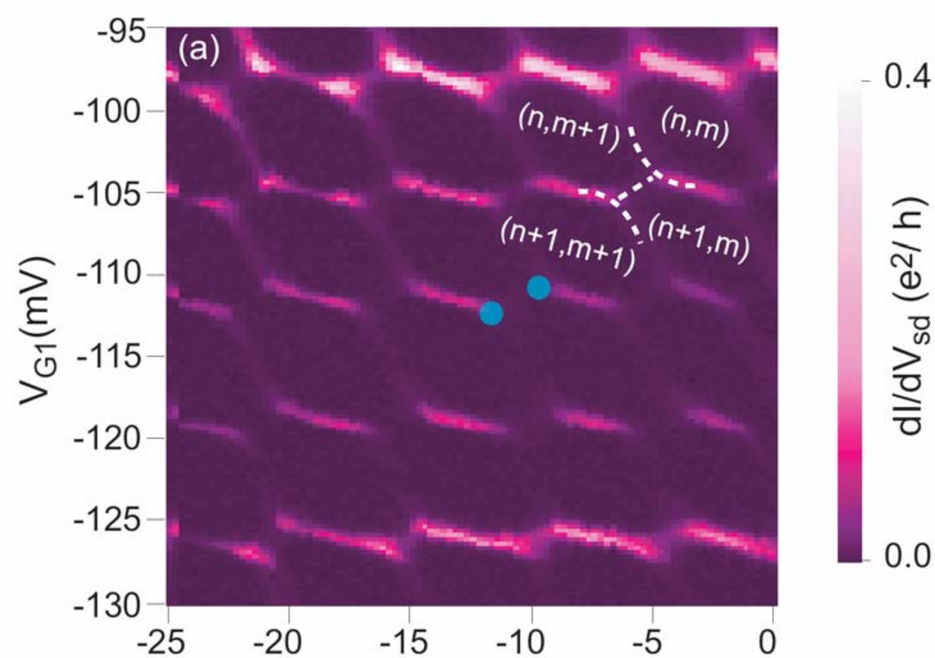
Double Dot in Carbon Nanotube (SW)

Schönenberger group (Gräber *et al.*, cond-mat/0603367)



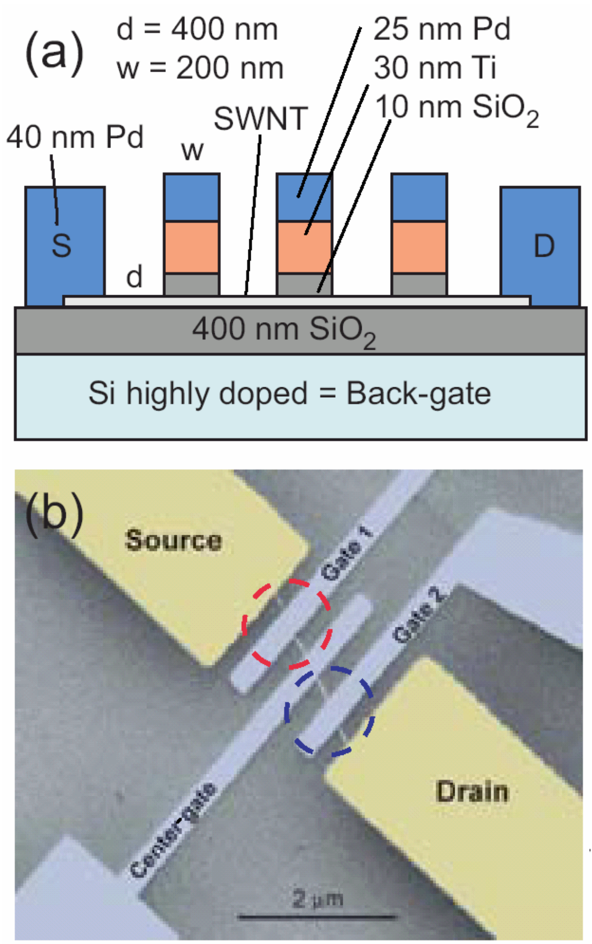




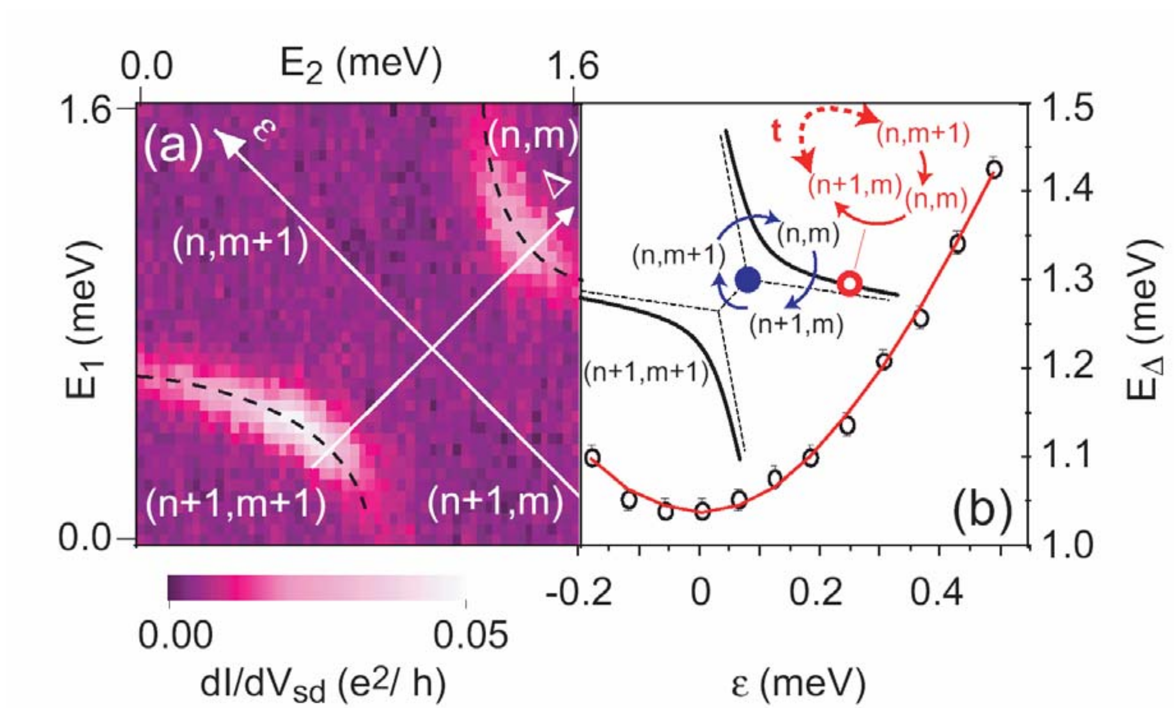


Double Dot in Carbon Nanotube (SW)

Gräber *et al.*, cond-mat/0603367



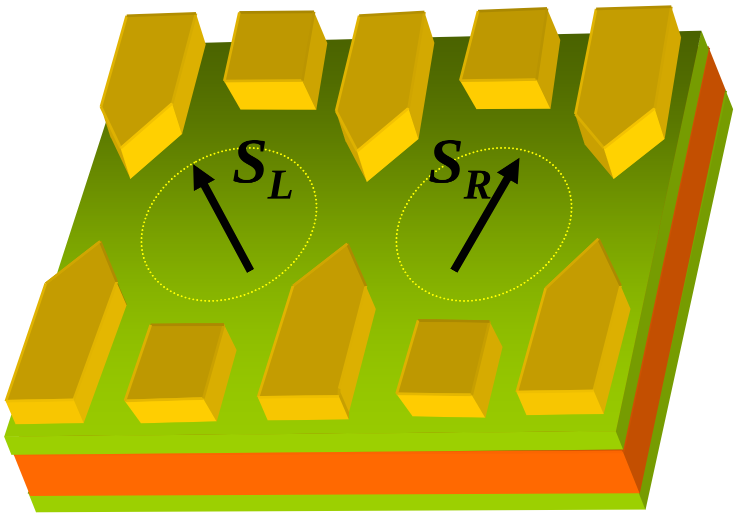
Clear evidence for coherent state:



$$E_{\Delta} = \sqrt{2}U' + \sqrt{4\varepsilon^2 + 8t^2} \quad \Rightarrow \quad t \approx 350\mu eV$$

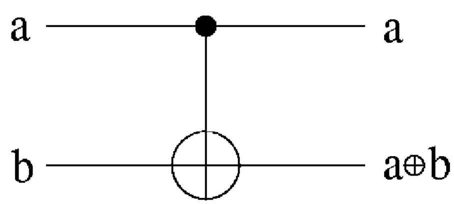
Quantum Computing with Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\, \mathbf{S}_L \cdot \mathbf{S}_R$$

➔ CNOT (XOR) gate

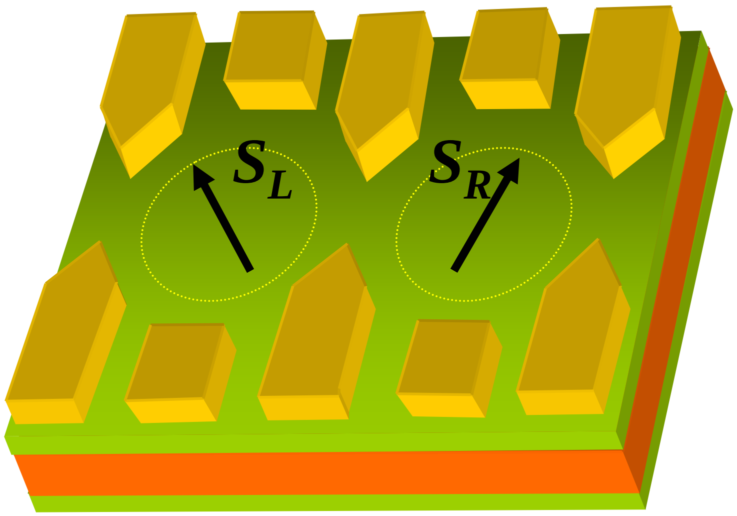


$$\Leftrightarrow \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_x \end{pmatrix} \overset{a\,b}{\begin{Bmatrix} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{Bmatrix}} = \begin{Bmatrix} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\downarrow \\ \downarrow\uparrow \end{Bmatrix}$$

$$U(\tau_s) = T\, e^{-i\int_0^{\tau_s} H(t)\,dt}, \qquad J \neq 0 \;\; during \;\; \tau_s$$

Quantum Computing with Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\, \mathbf{S_L} \cdot \mathbf{S_R}$$

➔ CNOT (XOR) gate

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

$$U_{SW} : \uparrow\downarrow \;\Rightarrow\; \downarrow\uparrow$$

$$U_{SW}^{1/2} : \uparrow\downarrow \;\Rightarrow\; \uparrow\downarrow + e^{i\alpha} \downarrow\uparrow$$

switching time: 180 ps

Petta *et al.*, Science, 2005

Note: Control of exchange interaction J and switching time needs to be very precise (1:10^4)
➔ experimental challenge

➔ CNOT gate without interaction?

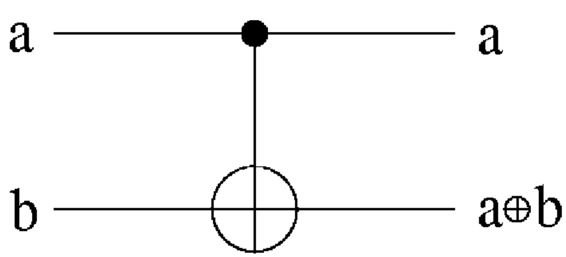
Yes: CNOT gates based on measurement:

Linear optics & single-photon detection	→	conditional sign flip (non-deterministic) [1]
Full Bell state analyzer & GHZ state	→	deterministic quantum computing [2]
Partial Bell-state (parity) measurements	→	deterministic quantum computing [3]

[1] E. Knill, R. Laflamme and G. J. Milburn, Nature 409, **46** (2001).
[2] D. Gottesman and I.L. Chuang, Nature **402**, 390 (1999).
[3] C.W.J. Beenakker *et al*., Phys. Rev. Lett. **93**, 020501 (2004).

CNOT gate can be implemented with two parity gates

Beenakker, Kindermann & DiVincenzo, PRL '04



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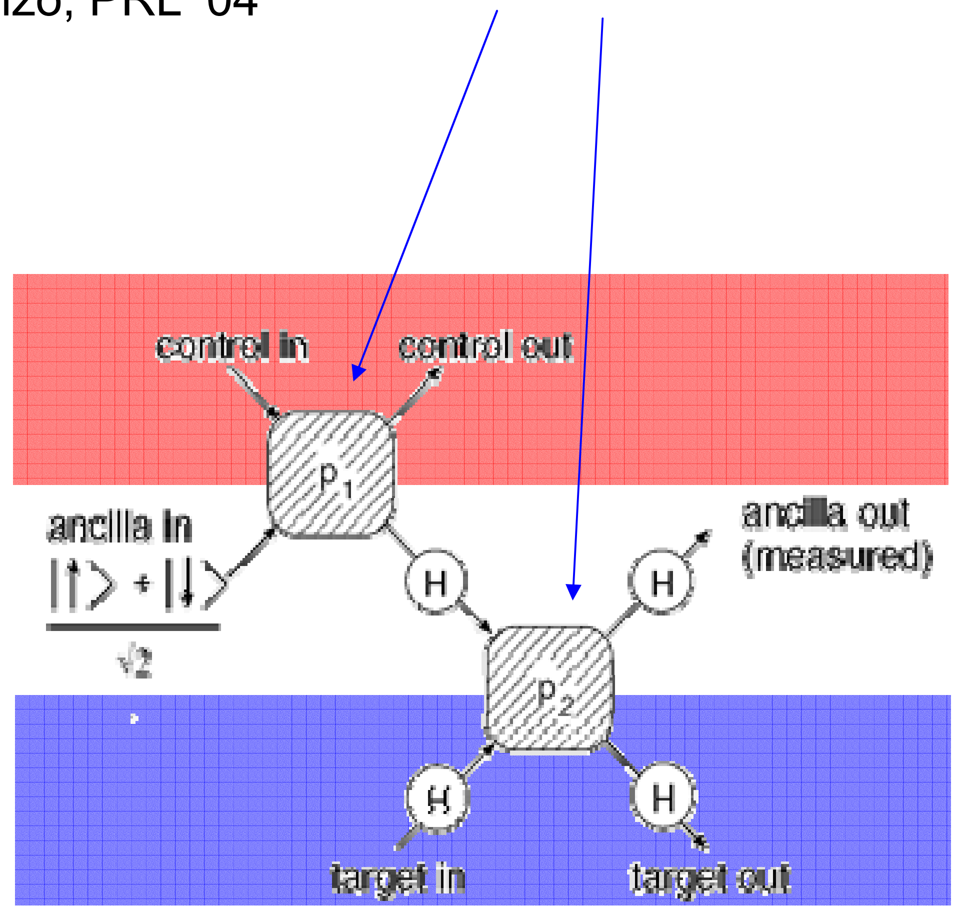
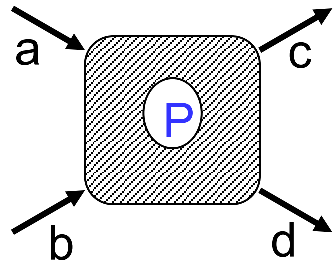


Deterministic entangler:



a,b: input arms
c,d: output arms

$$\underbrace{(\alpha\uparrow_a+\beta\downarrow_a)}_{\text{input state in arm a}}\underbrace{(\uparrow_b+\downarrow_b)}_{\text{input state in arm b (ancilla)}}=(\alpha\uparrow_a\uparrow_b+\beta\downarrow_a\downarrow_b)+(\alpha\uparrow_a\downarrow_b+\beta\downarrow_a\uparrow_b)$$

$$\uparrow_a\downarrow_b\equiv|\uparrow\rangle_a\otimes|\downarrow\rangle_b$$

$$\Rightarrow\begin{cases}\alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d\ ,\ if\ \ p=1\\ \alpha\uparrow_c\downarrow_d+\beta\downarrow_c\uparrow_d\ ,\ if\ \ p=0,\ \Rightarrow\ \ \alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d\end{cases}$$

Projective measurement: measurement of parity p projects input state into either parallel output state ($p=1$) or antiparallel output state ($p=0$). If $p=0$, then apply $\sigma_x^{(d)}$ on output state ➔ get always same final output state in arms c and d.

Thus, we get: $$\alpha\uparrow_a+\beta\downarrow_a\ \Rightarrow\ \alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d$$

Beenakker et al., 2004

Measurement-based quantum computing with spin qubits

Engel & DL, Science **309**, 586 (2005)

$$\left.\begin{array}{l} |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \\ |\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle \end{array}\right\}$$ even parity Bell state: parallel spins

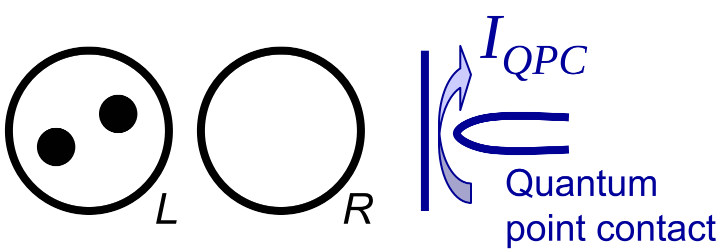
$$\left.\begin{array}{l} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \end{array}\right\}$$ odd parity Bell state: antiparallel spins

Advantage:
parity measurement is digital (0 or 1) ➔ quantum gate is digital

Q: Does scheme exist for electron spins to measure parity of Bell states non-destructively?

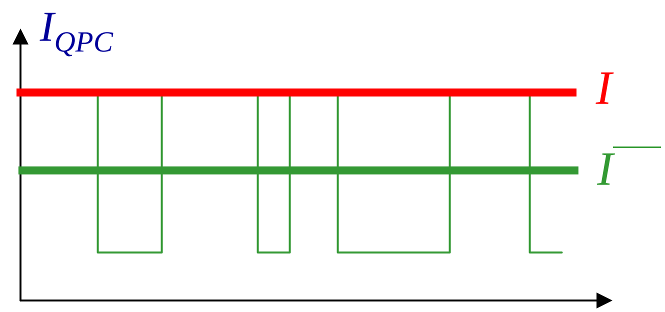
Double Quantum Dot and QPC

- Current I_{QPC} depends on charge state[1]



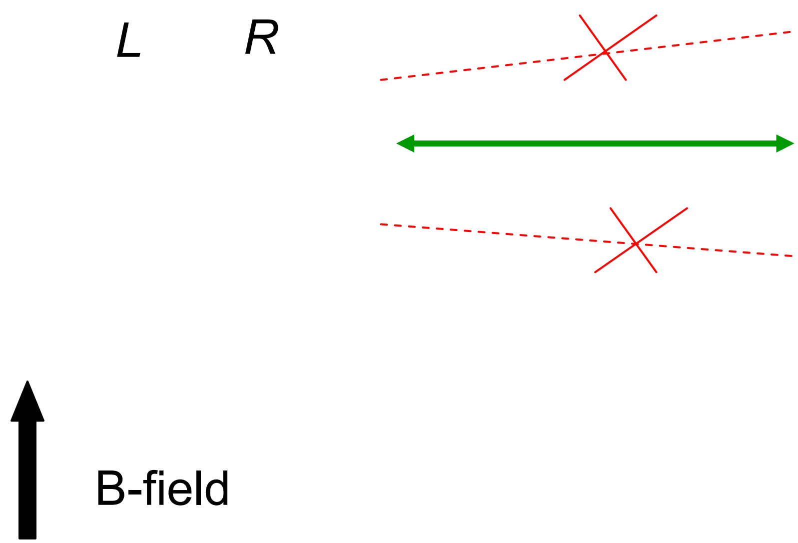
odd parity: tunneling

even parity: no tunneling



[1] J.M. Elzerman *et al.*, Nature **430**, 431 (2004)

Convert spin parity to charge info



Different Zeeman splittings

$\Delta^z{}_L \neq \Delta^z{}_R$

12 dim. Hilbert space

Bloch-Redfield eq.

$$W_{L \leftrightarrow R} = \frac{2t_d^4}{U^2} \frac{\Gamma_{d2}}{\varepsilon^2 + \Gamma_{d2}^2}$$

- resonant tunneling ($\varepsilon=0$) for antiparallel spins
- but NOT ($\varepsilon >> \Gamma_{d2}$) for parallel spins

➔

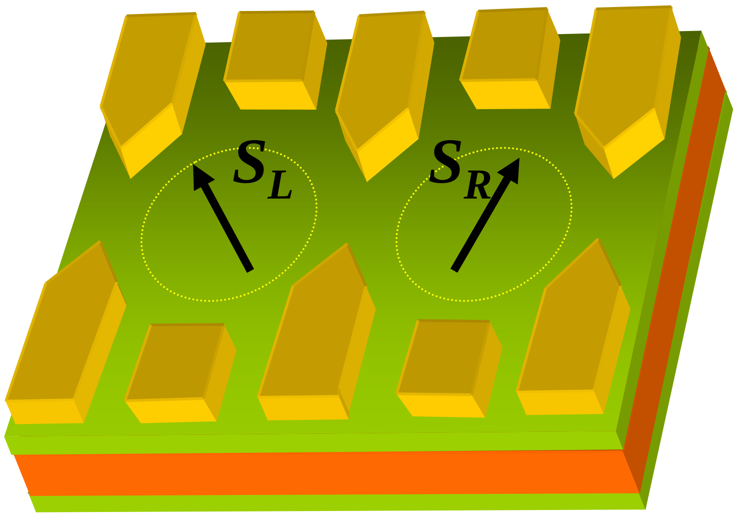
QPC detects charge on right dot ➔ parity of Bell state

Parity detection is robust against imperfections:

high fidelity

- initial state e.g. $\frac{1}{\sqrt{2}}\left|\uparrow\downarrow\right\rangle + \frac{1}{2}\left(\left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle\right)$
- simulation in 144-dimensional Liouville space
- compare ideal result
- quantify with the Uhlmann (square-root) fidelity

Quantum Computing with Spin-Qubits



CNOT (XOR) gate based on entanglement such as

$$\uparrow\downarrow + e^{i\alpha} \downarrow\uparrow \quad or \quad \uparrow\uparrow + e^{i\alpha} \downarrow\downarrow$$

i.e. **phase coherence** is crucial

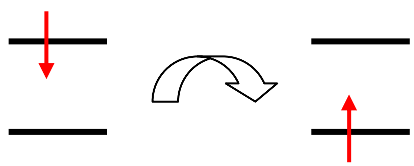
➔ Need to understand the dynamics and **decoherence** mechanisms for electron spins in quantum dots

Spin decoherence in GaAs quantum dots

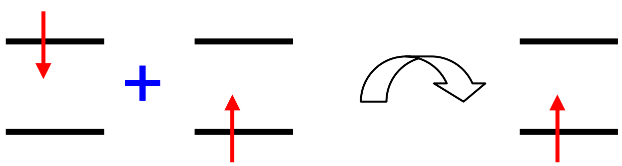
Two important sources of spin decay in GaAs:

1) Spin-orbit coupling (Dresselhaus & Rashba)

➔ interaction between spin and charge fluctuations



Relaxation with rate $1/T_1$



Decoherence with rate $1/T_2$

= decay of coherent superposition

2) Hyperfine interaction between electron spin and nuclear spins leads to non-exponential decay

Spin orbit interaction in GaAs quantum dots

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) + \beta\left(-p_x\sigma_x + p_y\sigma_y\right) + O(p^3)$$

Rashba SOI   Dresselhaus SOI

- interaction between spin and charge fluctuations
- SOI is weaker in quantum dots than in bulk since $\langle H_{so}\rangle_{dot} = 0$
  (Khaetskii & Nazarov, `00; Halperin et al., `01; Aleiner & Falko, `01)

e.g. spin-phonon: Khaetskii & Nazarov, PRB 64 (2001)
Golovach, Khaetskii & Loss, PRL 93 (2004)
Fal'ko, Altshuler, Tsyplyatev, PRL (2005)
Bulaev & Loss, PRL `05 (hole spin)

Spin orbit interaction in GaAs quantum dots

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) + \beta\left(-p_x\sigma_x + p_y\sigma_y\right) + O(p^3)$$

Rashba SOI   Dresselhaus SOI

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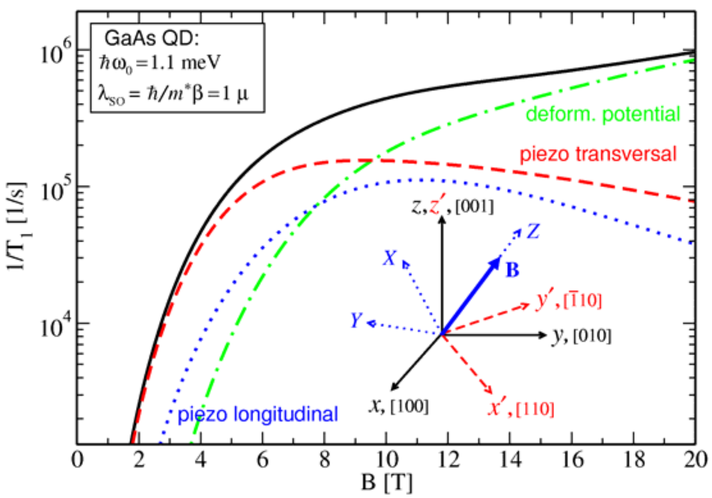
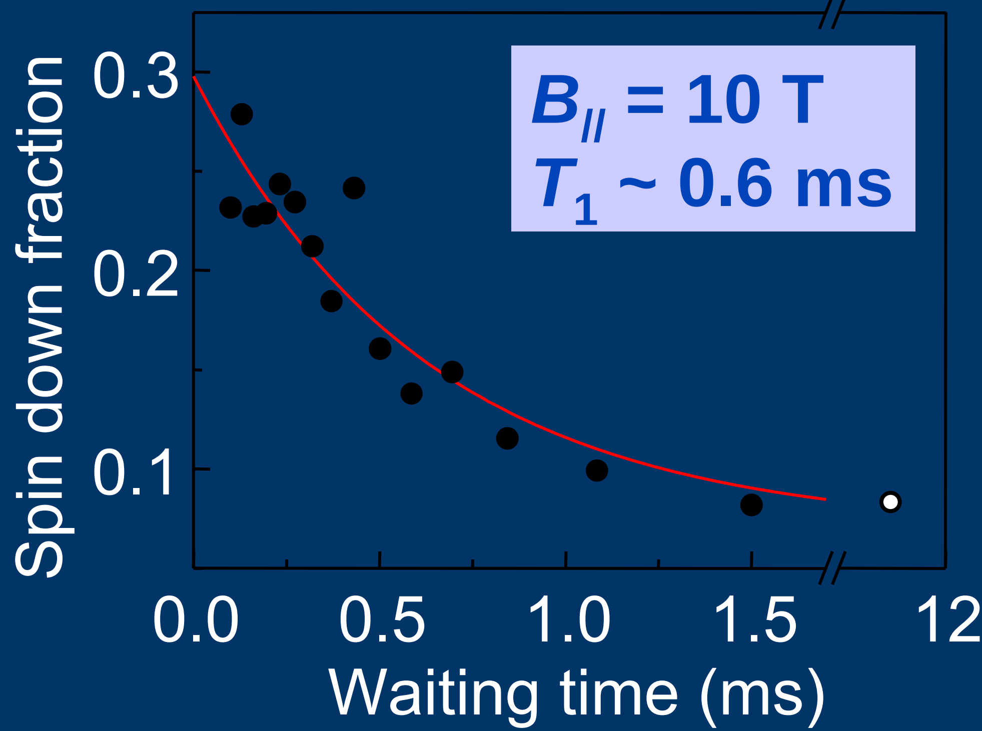
  (Khaetskii & Nazarov, \`00; Halperin et al., \`01; Aleiner & Falko, \`01)

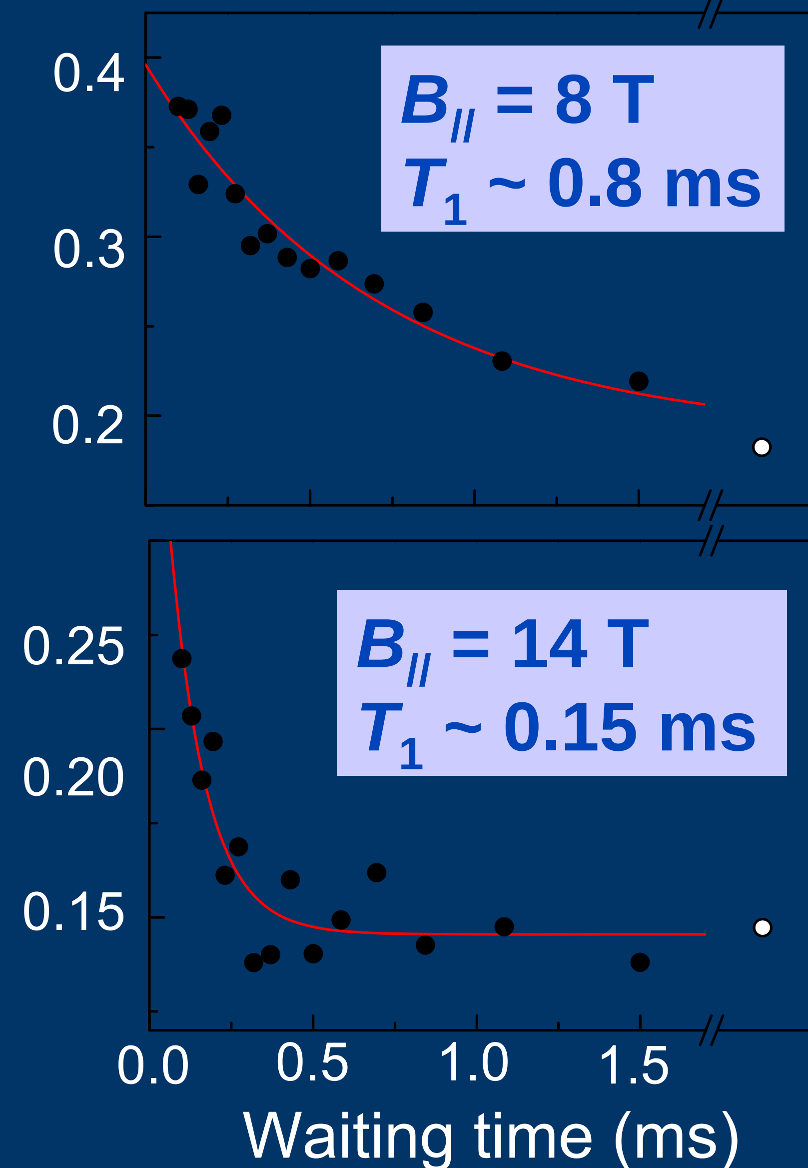
- $T_2 = 2T_1$, i.e. no pure "dephasing" -- for ALL charge fluct.
- T_1 ~ 1ms (B=8T)
  Golovach, Khaetskii & DL, PRL 93 (2004)



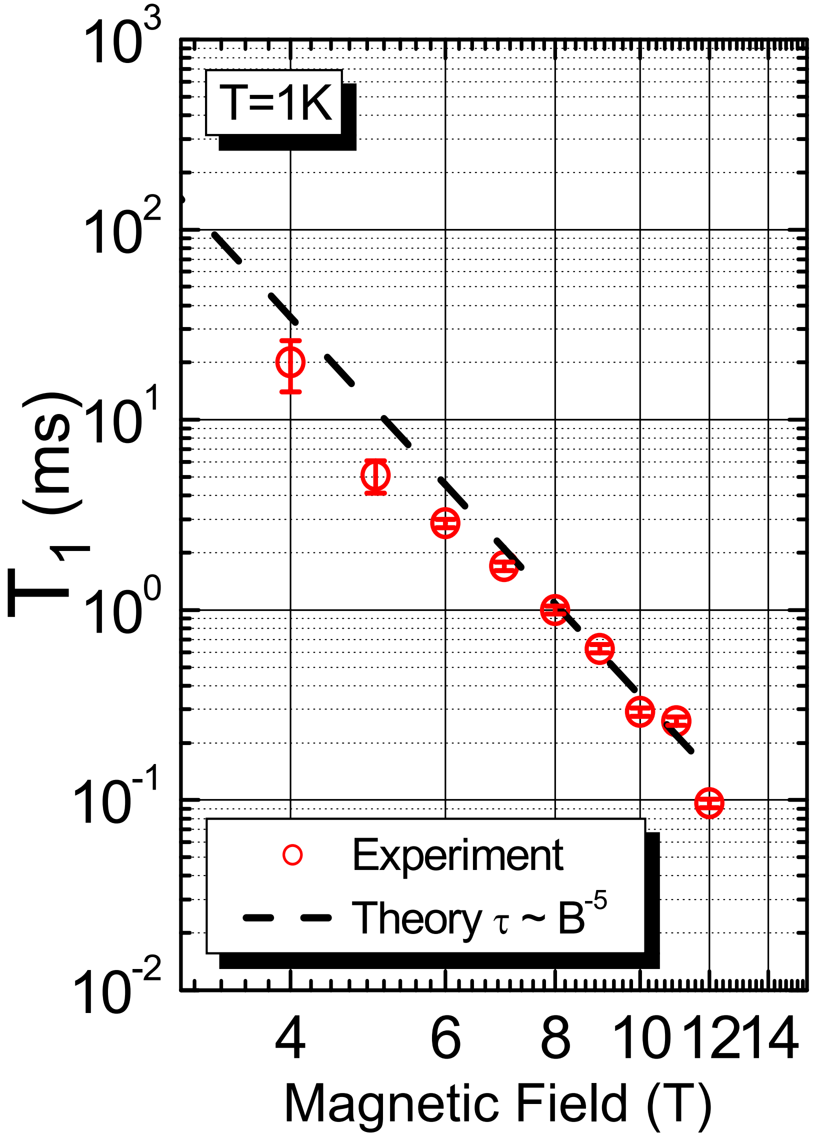
Single-spin energy relaxation



Fit to: $C \exp(-t/T_1) + A$



Elzerman *et al.*, Nature 430, 431 (2004)



Magnetic Field Dependence
Kroutvar *et al*., Nature 432, 81 (2004)

Ultra long spin lifetimes
→ Maximum T_1=20ms for B=4T, T=1K

$T_1 \propto B^5$ dependence
→ Spin-flip mediated by one phonon processes at low temperature in agreement with theory *)

*) Khaetskii & Nazarov, PRB 64 (2001)
Golovach, Khaetskii, DL, PRL 93 (2004)

Result

Decay of spin due to spin-orbit & phonons:

1. T_1 = 1ms @ 8T: confirmed by Delft exp. on GaAs dot
2. $T_2 = 2T_1$ to test this need ESR for single spin*)
   (➔ Delft group reports now ESR!)
3. But: nuclear effects occur on shorter time scale (see next)

Thus, from theory we can conclude:

SOI is not limiting factor for spin coherence

*) Engel & DL, PRL '01

Hyperfine Interaction in *Single* Quantum Dot

Burkard, DL, DiVincenzo, PRB ’99; Khaetskii, DL, Glazman, PRL ’02 & PRB ’03;
Schliemann, Khaetskii, DL, PRB ‘02; Coish &DL, PRB ’04 & PRB ’05

Theory work on nuclear spins in dots (incomplete list):

I.A. Merkulov, Al.L. Efros, M. Rosen, Phys. Rev. B **65**, 205309 (2002)
S.I. Erlingsson, Y.V. Nazarov, and V.I. Falko, Phys. Rev. B **64**, 195306 (2001)
V.V. Dobrovitski, H.A. De Raedt, M.I. Katsnelson, and B.N. Harmon, quant-ph/0112053
D. Mozyrsky, S. Kogan, G.P. Berman , cond-mat/0112135; V. Privman, cond-mat/0203039
R.de Sousa, S. Das Sarma, , Phys. Rev B **67,** 033301 (2003) and cond-mat/0211567
S.I. Erlingsson and Y.V. Nazarov, cond-mat/0202237
E.A. Yuzbashyan, B.L. Altshuler, V.B. Kuznetsov, and V.Z. Enolskii, cond-mat/0407501
N. Shenvi, R. de Sousa, K.B. Whaley, PRB 71, 224411 (2005)
J.M. Taylor et al., PRL '05
Wang Yao, Ren-Bao Liu, L. J. Sham, cond-mat/0508441
G. Giedke et al., cond-mat/0508144
Stepanenko, Burkard, Giedke, Imamoglu, cond-mat/0512044
W. Yao, R.-B. Liu, L. J. Sham, cond-mat/0508441

Experimental work:

.
Single dot (optically): Gammon et al., PRL `05

Double dots: Ono & Tarucha, PRL `04, transport and spin blockade
Petta et al., Nature `05 & Science`05: isolated dots & QPC ➔ dephasing
Koppens et al., Science `05, transport and spin blockade

Hyperfine Interaction in *Single* Quantum Dot

$$H = \sum_i A_i \vec{S} \cdot \vec{I}_i + g\mu_B B S^z + H_{dd}$$

(The H_{dd} term is crossed out on the slide.)

hyperfine interaction is non-uniform:

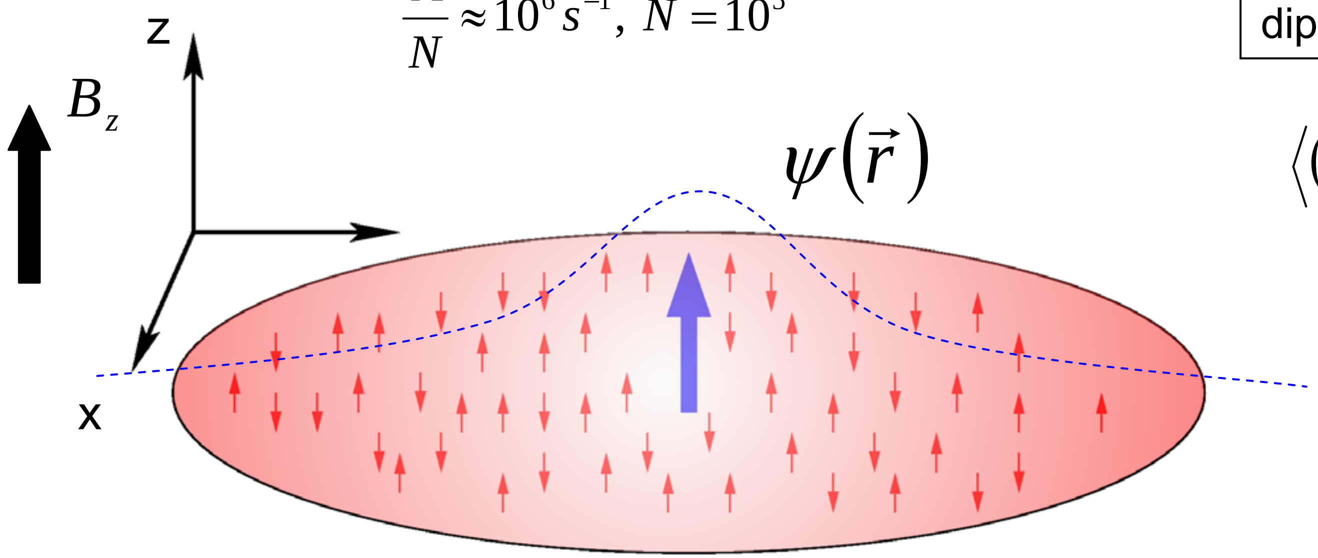
$$A_i \propto A \left|\psi(\vec{r}_i)\right|^2$$

$$\frac{A}{N} \approx 10^6 s^{-1}, \; N = 10^5$$

electron Zeeman energy

nuclear spin dipole-dipole interaction

$$\left\langle \left(\delta H_{dd}\right)^2 \right\rangle^{1/2} \approx 10^4 s^{-1}$$



Burkard, DL, DiVincenzo, PRB '99; Khaetskii, DL, Glazman, PRL '02 & PRB '03; Schliemann, Khaetskii, DL, PRB '02; Coish &DL, PRB '04 & PRB '05

Separation of the Hyperfine Hamiltonian

Hamiltonian: $$H = g\mu_B B S_z + \vec{S}\cdot\vec{h} = H_0 + V$$

Note: nuclear field $\vec{h} = \sum_i A_i \vec{I}_i$ is a quantum operator

Separation:

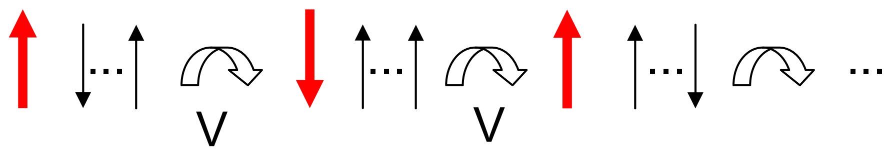
$$H_0 = (g\mu_B B + h_z) S_z$$

longitudinal component

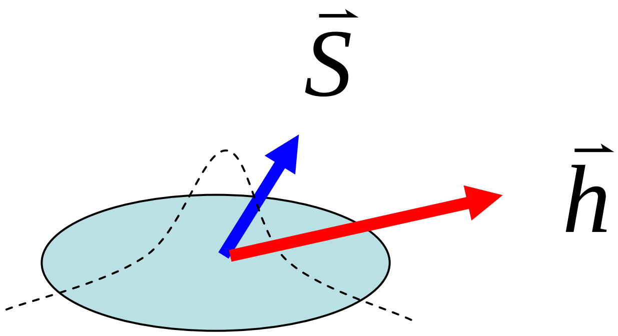
$$V = \frac{1}{2}\left(h_+ S_- + h_- S_+\right)$$

flip-flop terms

$$h_\pm = h_x \pm i h_y$$



Nuclear spins provide hyperfine field *h* with quantum fluctuations seen by electron spin:



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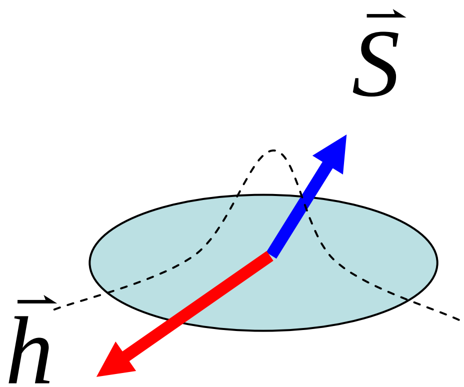
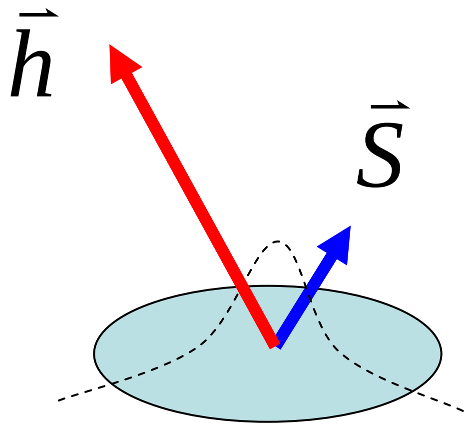


Nuclear spins provide hyperfine field *h* with quantum fluctuations seen by electron spin:



With mean <*h*>=0 and quantum variance *δh*:

$$\delta h = \sqrt{\langle h^2 \rangle_{nucl}} = \sqrt{\left\langle \left(\sum_{k=1}^{N} A_k \vec{I}_k \right)^2 \right\rangle_{nucl}} = A/\sqrt{N} = 5mT = (10ns)^{-1}$$

what state?

Initial conditions for nuclear spins

Coish &DL, PRB 70, 195340 (2004)

1. $\rho_I^{(1)}(0) = |\psi_I\rangle\langle\psi_I|$, $|\psi_I\rangle = \bigotimes_{k=1}^{N}\left(\sqrt{f_\uparrow}|\uparrow_k\rangle + e^{i\phi_k}\sqrt{1-f_\uparrow}|\downarrow_k\rangle\right)$

2. $\rho_I^{(2)}(0) = \sum_{N_\uparrow}\binom{N}{N_\uparrow} f_\uparrow^{N_\uparrow}(1-f_\uparrow)^{N-N_\uparrow}|N_\uparrow\rangle\langle N_\uparrow|$

3. $\rho_I^{(3)}(0) = |n\rangle\langle n|,\quad h_z|n\rangle = \sum_k A_k I_k^z|n\rangle = [h_z]_{nn}|n\rangle$

Spin dynamics for V=0: $\langle S_+\rangle_t = \langle S_+\rangle_0 \mathrm{Tr}_I[e^{i(g\mu_B B+h_z)t}\rho_I(0)]$

- Superposition (1) or mixture (2) of h_z-eigenstates:

$$\langle S_+\rangle_t^{(1,2)} \approx \langle S_+\rangle_0 e^{i\omega t}e^{-t^2/2t_c^2},\quad t_c = \frac{2\hbar}{A}\sqrt{\frac{N}{1-p^2}}$$

Rapid Gaussian decay!

$t_c \approx 5\,\mathrm{ns},\quad (\mathrm{GaAs}, \mathrm{N} = 10^5)$

- But: Single h_z eigenstate (3):

$$\langle S_+\rangle_t^{(3)} \approx \langle S_+\rangle_0 e^{i\omega t},\ \omega = g\mu_B B + [h_z]_{nn}$$

No decay! (if flip-flop V is neglected)

Initial conditions for nuclear spins

Coish &DL, PRB 70, 195340 (2004)

- Superposition or mixture of h_z-eigenstates:

$$\langle S_+ \rangle_t^{(1,2)} \approx \langle S_+ \rangle_0 e^{i\omega t} e^{-t^2/2t_c^2}, \quad t_c = \frac{2\hbar}{A}\sqrt{\frac{N}{1-p^2}}$$

Rapid Gaussian decay!

$t_c \approx 5\,\mathrm{ns}, \quad (\mathrm{GaAs}, \mathrm{N} = 10^5)$

- But: Single h_z eigenstate $|n\rangle$:

$$\langle S_+ \rangle_t^{(3)} \approx \langle S_+ \rangle_0 e^{i\omega t}, \quad \omega = g\mu_B B + [h_z]_{nn}$$

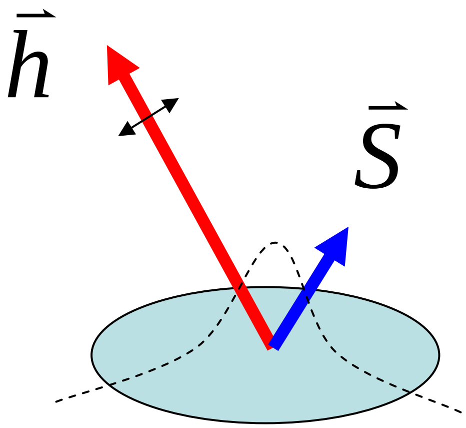
No decay! (if flip-flop V is neglected)

➔ It is advantageous to prepare the nuclear spin system with a von Neumann measurement on the Overhauser field (operator!):

$$h_z |n\rangle = [h_z]_{nn} |n\rangle \quad ➔ \quad \delta h = 0$$

[e.g. via ESR, see Klauser, Coish & DL, cond-mat/0510177]

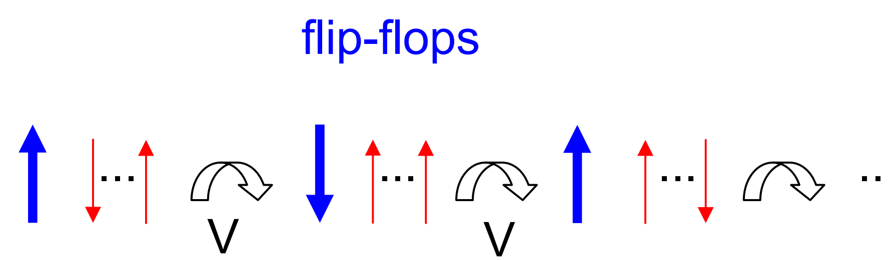
Sharp initial nuclear spin state: δh=0 at t=0



$$\vec{S} + \sum_k \vec{I}_k = const.$$

➔ back action of ***S*** on ***h***

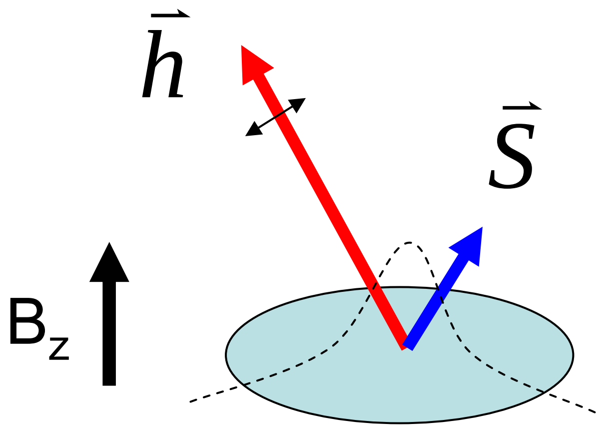
t>0: quantum dynamics



changes hyperfine field in time by 1/N ➔ spin precesses in fluctuating hyperfine field ➔ spin dephases (power law decay)

Khaetskii, DL, Glazman, PRL ’02 & PRB ’03
Coish &DL, PRB 70, 195340 (2004)

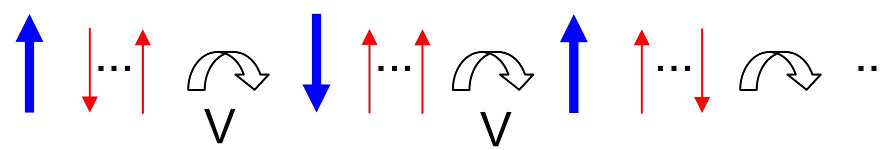
Sharp initial nuclear spin state ➔ $\delta h=0$ at $t=0$



$$\vec{S} + \sum_k \vec{I}_k = const.$$

➔ back action of ***S*** on ***h***

Dynamics (flip-flops):



E.g.

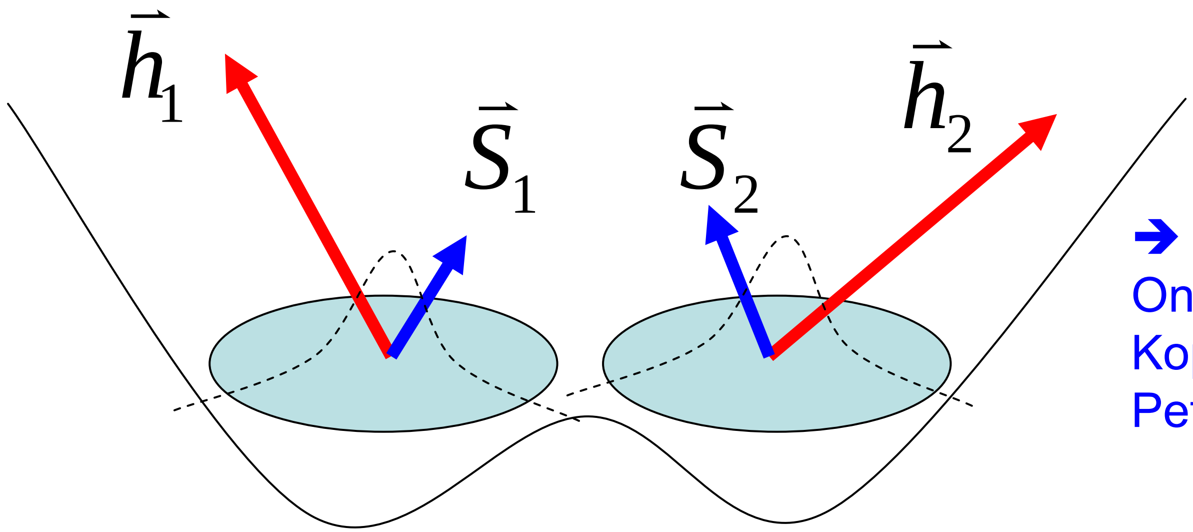
$$S_z(t) - S_z(0) \propto \frac{A^2}{4N(b+pIA)^2} \frac{e^{itA/N}}{(At/N)^{3/2}}$$

power law decay

Time scale is N/A = 1μs (GaAs) and **decay is bounded**

Hyperfine Interaction in *Double* Dots

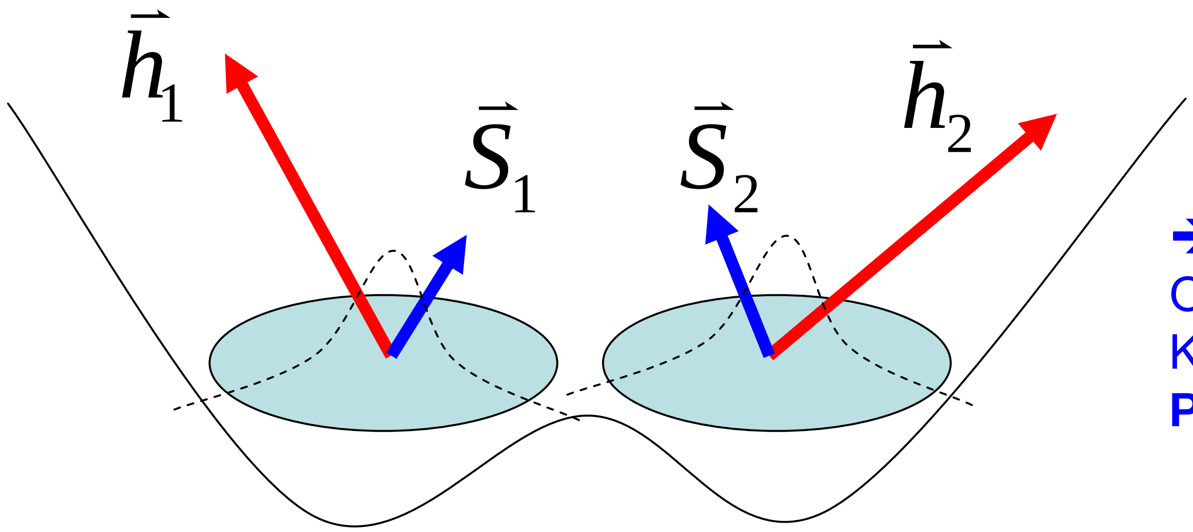
Coish & Loss, Phys. Rev. B 72, 125337 (2005)



➔ experiments by
Ono&Tarucha, PRL `04,
Koppens *et al.,* Science`05
Petta *et al.,* Science `05

Hyperfine Interaction in *Double* Dots

Coish & DL, Phys. Rev. B 72, 125337 (2005)



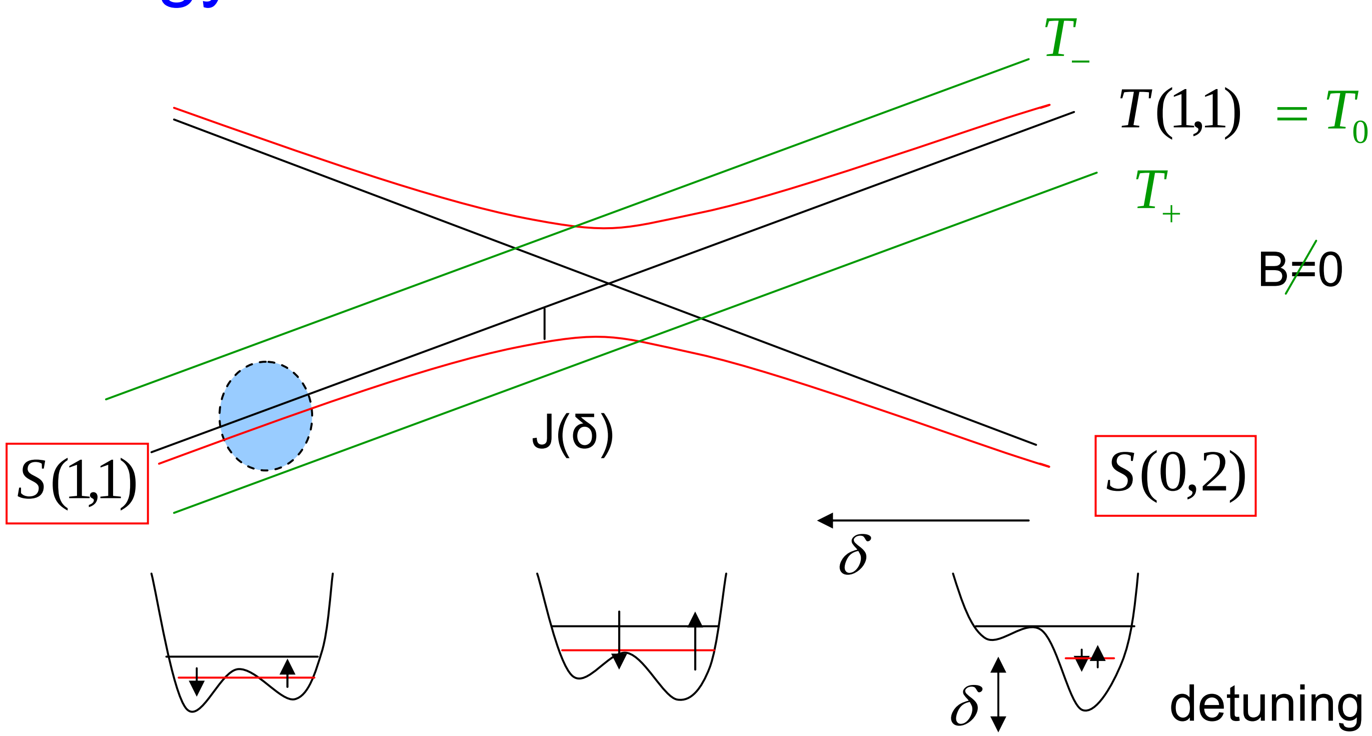
➔ experiments by
Ono&Tarucha `04,
Koppens *et al.*, `05
Petta *et al.*, `05

$$H = g\mu_B B\left(S_1^z + S_2^z\right) + \vec{h}_1\cdot\vec{S}_1 + \vec{h}_2\cdot\vec{S}_2 + J\vec{S}_1\cdot\vec{S}_2$$
$$= g\mu_B BS^z + \vec{h}\cdot\vec{S} + \delta\vec{h}\cdot\delta\vec{S} + \frac{J}{2}\vec{S}\cdot\vec{S} + \text{const}\ .$$

$$\delta\vec{h} = \frac{1}{2}\left(\vec{h}_1 - \vec{h}_2\right)$$
$$\vec{h} = \frac{1}{2}\left(\vec{h}_1 + \vec{h}_2\right)$$
$$\delta\vec{S} = \vec{S}_1 - \vec{S}_2$$
$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

mixes singlet with triplets

Energy levels and effective Hamiltonians



Note: Singlets S(0,2) and S(1,1) mix ➔ anticrossing ➔ J
but: S(0,2) and triplet T(1,1) do not mix ➔ crossing

Hyperfine interaction mixes S(1,1) and T(1,1)

Effective Hamiltonian for double dot

$$H=\sum_{ls}V_{gl}d_{ls}^{+}d_{ls}+U\sum_{l}n_{l\uparrow}n_{l\downarrow}+U'(n_{1\uparrow}+n_{1\downarrow})(n_{2\uparrow}+n_{2\downarrow})+t_{12}\sum_{s}(d_{1s}^{+}d_{2s}+h.c.)$$

$$+\frac{\varepsilon_z}{2}\sum_{l}(n_{l\uparrow}-n_{l\downarrow})+\sum_{l}\vec{S}_l\cdot\vec{h}_l\ ,\qquad \vec{S}_l=\frac{1}{2}\sum_{ss'}d_{ls}^{+}\,\vec{\sigma}_{ss'}\,d_{ls'}\ ,\quad \vec{h}_l=Av\sum_{k}\left|\psi_0^l(\vec{r}_k)^2\right|\vec{I}_k$$

Use:

$$H_{eff}=PHP+PHQ\frac{1}{E-QHQ}QHP\qquad (+)$$

(+) ➔

$$H_{eff}=\frac{\varepsilon_z}{2}\sum_{l}S_l^z+\sum_{l}\vec{S}_l\cdot\vec{h}_l+J\vec{S}_1\cdot\vec{S}_2\ ,\quad J=-2t_{12}^2\left(\frac{1}{\delta}-\frac{1}{\delta+U+U'}\right)$$

S-T space 4-dim.

(+) ➔

$$H_{\text{eff}}=\frac{J}{2}\left(1+\tau^z\right)+\delta h^z\tau^x$$

$$+\frac{1}{2\varepsilon_z}([h^-,h^+]-[\delta h^-,\delta h^+])\tau^z+\frac{1}{\varepsilon_z}(\delta h^+h^-+\delta h^-h^+)\tau^++h.c.+\frac{1}{4\varepsilon_z}([h^-,h^+]+[\delta h^-,\delta h^+])$$

S-T_0 space: 2-dim.

$$O(A^2/\varepsilon_z)\sim O(1/10^{-6}\,s)\quad for\quad B=1T$$

Dynamics in the S-T_0 subspace

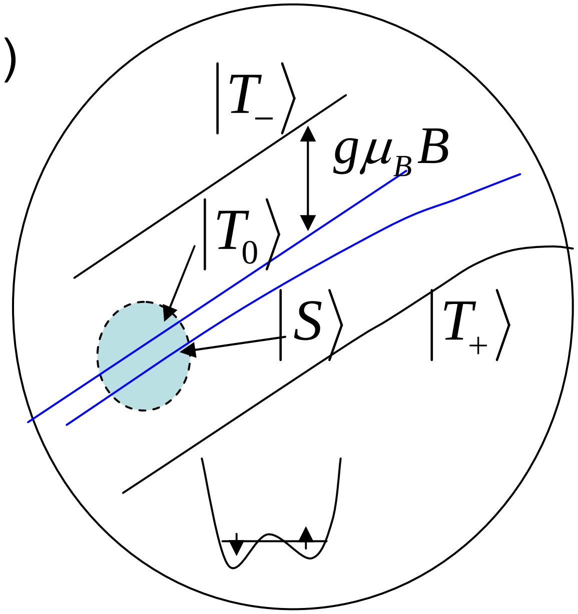
Effective Hamiltonian:

$$H_{\text{eff}} = \frac{J}{2}\left(1+\tau^z\right) + \delta h^z \tau^x$$

$$\left|T_0\right\rangle \rightarrow \left|\tau^z = +1\right\rangle$$

$$\left|S\right\rangle \rightarrow \left|\tau^z = -1\right\rangle$$

*)



valid for times $t < g\mu_B B/(A/N)^2 \sim 1\ \mu s$

Eigenvalues and eigenvectors:

$$E_n^{\pm} = \frac{J}{2} \pm \frac{1}{2}\sqrt{J^2 + 4\left(\delta h_n^z\right)^2}$$

$$\left|E_n^{\pm}\right\rangle = \frac{\delta h_n^z \left|S\right\rangle + E_n^{\pm}\left|T_0\right\rangle}{\sqrt{\left(E_n^{\pm}\right)^2 + \left(\delta h_n^z\right)^2}} \otimes \left|n\right\rangle, \quad \delta h^z \left|n\right\rangle = \delta h_n^z \left|n\right\rangle$$

i.e. eigenstates *remain* a product of electron and nuclear spin states

*) pseudo-spin ½ = qubit: J. Levy, PRL ´02; J. Taylor *et al.,* Nature Physics '05

Dynamics in the S-T_0 subspace

Arbitrary initial nuclear state: $$|\psi(t=0)\rangle = |S\rangle \otimes \sum_n a_n |n\rangle$$

Probability to be in T_0: $$C_{T_0}(t) = \sum_n |a_n|^2 \left| \langle n| \otimes \langle T_0| e^{-iHt} |S\rangle \otimes |n\rangle \right|^2$$

Even without ensemble averaging, the initial nuclear state will be a superposition of δh^z eigenstates – this leads to entanglement & decay. However, this decay is reversible via standard `Hahn echo technique'

Note: **quantum** nature of hyperfine field leads to entanglement between electron spin and and nuclear spin system
(no such entanglement if hyperfine field were classical field)

Formation of Entanglement between Electron and Nuclear Spins

Schliemann, Khaetskii & DL, Phys. Rev. B**66**, 245303 (2002)

Measure of entanglement (bipartite): von Neumann entropy E of reduced density matrix (see C.H. Bennett et al., Phys. Rev. A53 2046 (1996))

i.e. trace out pure-state density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ of electron-nuclear spin system over nuclei ➔

$$\rho_{el}(t) = Tr_{nuclei}\, |\Psi(t)\rangle\langle\Psi(t)| = \begin{pmatrix} 1/2+\langle S_z(t)\rangle & \langle S_+(t)\rangle \\ \langle S_-(t)\rangle & 1/2-\langle S_z(t)\rangle \end{pmatrix}$$

eigenvalues:

$$\lambda_\pm = 1/2 \pm \left|\langle \vec{S}(t)\rangle\right|$$

➔

$$E(|\Psi(t)\rangle) = -\lambda_+ \log\lambda_+ - \lambda_- \log\lambda_-$$

i.e. entanglement *E* reaches **maximum** (log 2) for completely decayed spin of electron, i.e. for $\left|\langle \vec{S}(t)\rangle\right| = 0$

Note: if nuclear field were classical ➔ E=0, i.e. no entanglement

Spin dynamics in the S-T_0 subspace

Arbitrary initial nuclear state: $$|\psi(t=0)\rangle = |S\rangle \otimes \sum_n a_n |n\rangle$$

Probability to be in T_0: $$C_{T_0}(t) = \sum_n |a_n|^2 \left| \langle n| \otimes \langle T_0 | e^{-iHt} | S\rangle \otimes |n\rangle \right|^2$$

➔ $$C_{T_0}(t) = \sum_n |a_n|^2 C_n \left(1 - \cos\left(\left[E_n^+ - E_n^-\right]t\right)\right)$$

$$C_n = \frac{2\left(\delta h_n^z\right)^2}{J^2 + 4\left(\delta h_n^z\right)^2} = f(J/\delta h_n^z)$$

The long-time saturation value depends only on J and the distribution of δh^z:

Spin dynamics in the S-T_0 subspace

Arbitrary initial nuclear state: $$|\psi(t=0)\rangle = |S\rangle \otimes \sum_n a_n |n\rangle$$

Probability to be in T_0:

$$C_{T_0}(t) = \sum_n |a_n|^2 C_n \left(1-\cos\left(\left[E_n^+ - E_n^-\right]t\right)\right)$$

$$C_n = \frac{2\left(\delta h_n^z\right)^2}{J^2 + 4\left(\delta h_n^z\right)^2} = f(J/\delta h_n^z)$$

$$\overline{C}_n = \sum_n |a_n|^2 C_n \to \int dx\, P_{\sigma,\bar{x}}(x) C(x) \;,\; P_{\sigma,\bar{x}}(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\bar{x})^2/2\sigma^2} \;,\; x = \delta h_n^z$$

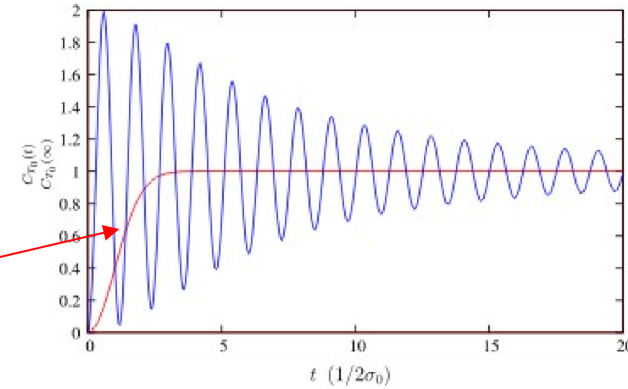
Gauss distribution for a_n

$$\rightarrow \quad \overline{C}_n \approx \begin{cases} \frac{1}{2} - \sqrt{\frac{\pi}{2}}\frac{J}{4\sigma_0}, & J \ll \sigma_0; \\ 2\frac{\sigma_0^2}{J^2}, & J \gg \sigma_0; \end{cases}$$

$$\overline{\delta h_n^z} = 0,$$

$$\overline{\left(\delta h_n^z\right)^2} = \sigma_0^2 = A^2/N$$

Correlator asymptotics



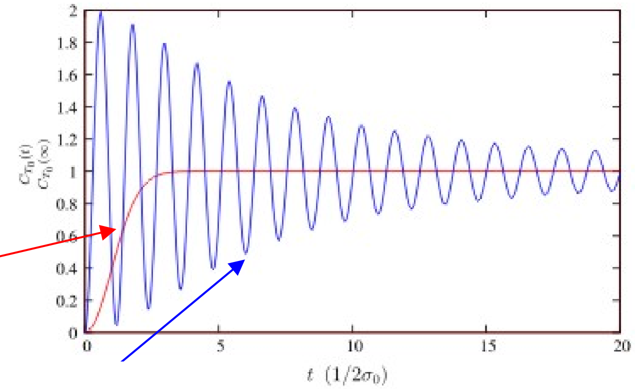
$$C_{T_0}(t) = C_{T_0}(\infty) + C_{T_0}^{\text{int}}(t)$$

Exchange J=0 ➔ rapid Gaussian decay:

$$C_{T_0}^{\text{int}}(t) = \frac{-e^{-t^2/2t_0^2}}{2} \quad , \qquad t_0 = 1/2\sigma_0 = \sqrt{N}/A,$$

Correlator asymptotics

$$C_{T_0}(t) = C_{T_0}(\infty) + C_{T_0}^{\text{int}}(t)$$



Exchange J=0 ➔ rapid Gaussian decay:

$$C_{T_0}^{\text{int}}(t) = \frac{-e^{-t^2/2t_0^2}}{2}, \quad t_0 = 1/2\sigma_0 = \sqrt{N}/A,$$

Non-zero exchange J ➔ power-law decay for t>1/J:

phase shift (universal)

$$C_{T_0}^{\text{int}}(t) \sim -\frac{\cos(Jt + 3\pi/4)}{4\sigma_0\sqrt{J}t^{3/2}}, \qquad J \neq 0.$$

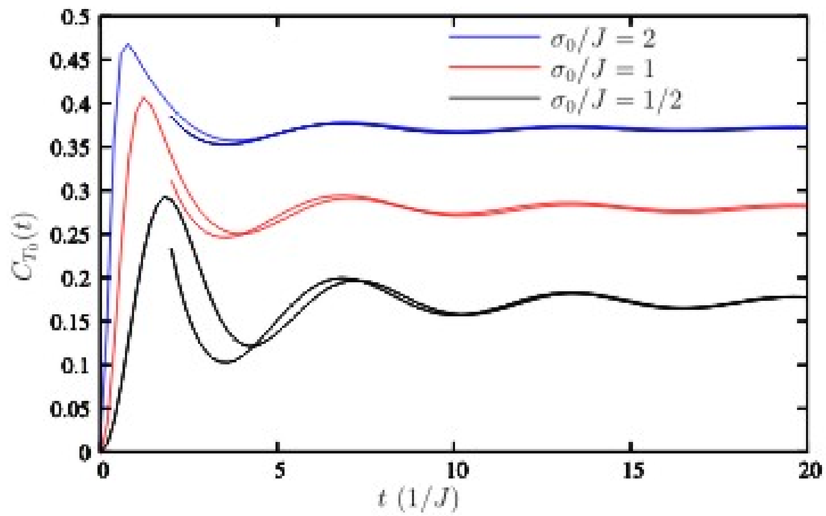
power-law decay!

Large Exchange (J) Limit

Experiment: can measure J and σ_0 independently from the period and time-dependent phase shift

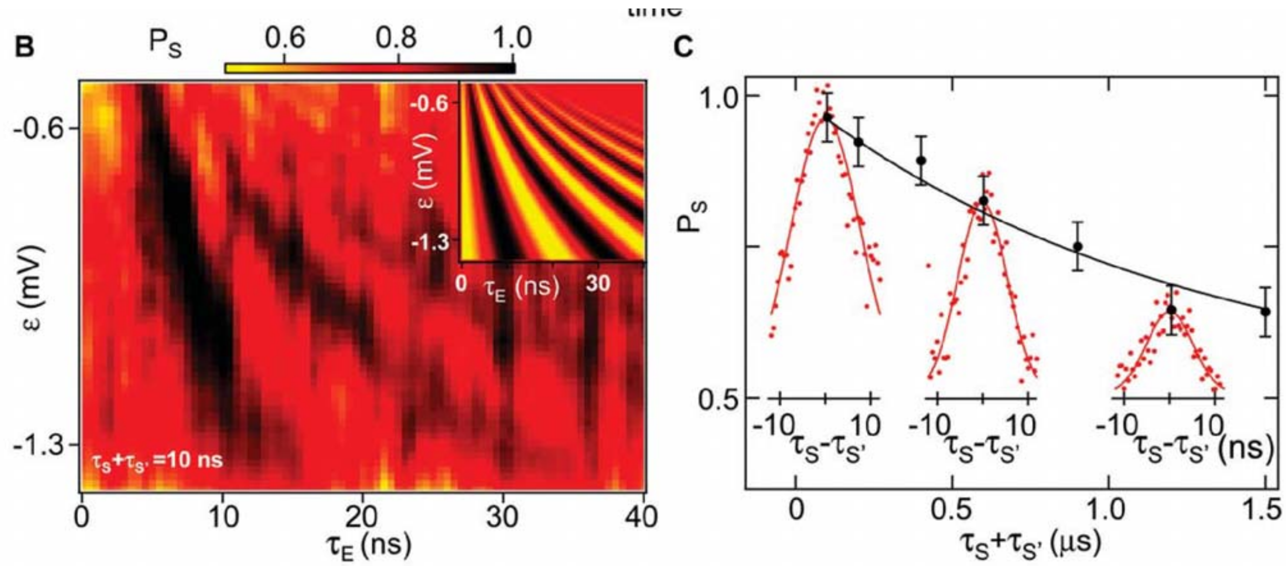
$$C_{T_0}^{\text{int}}(t) = -2\left(\frac{\sigma_0}{J}\right)^2 \frac{\cos\left(Jt + \frac{3}{2}\arctan\left(\frac{t}{t_0'}\right)\right)}{\left(1+\left(\frac{t}{t_0'}\right)^2\right)^{3/4}}, \qquad t_0' = \frac{J}{4\sigma_0^2}, \quad J >> \sigma_0$$

'dynamical narrowing'



Singlet Triplet Dephasing & Spin Echo

Petta *et al.*, Science, 2005



T_2*=10 ns ➔ T_2 =1-10 μs

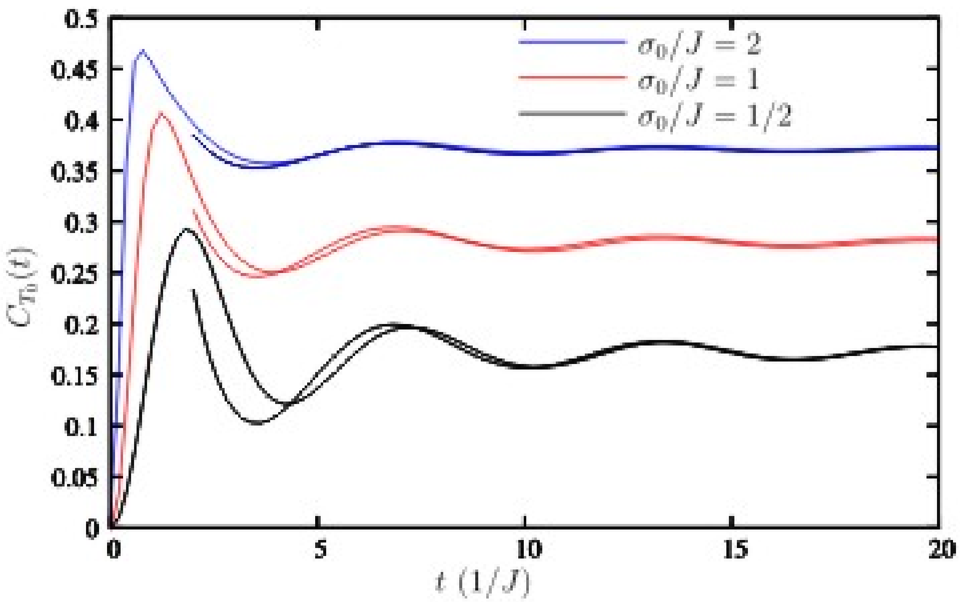
Square root of swap in 180 ps

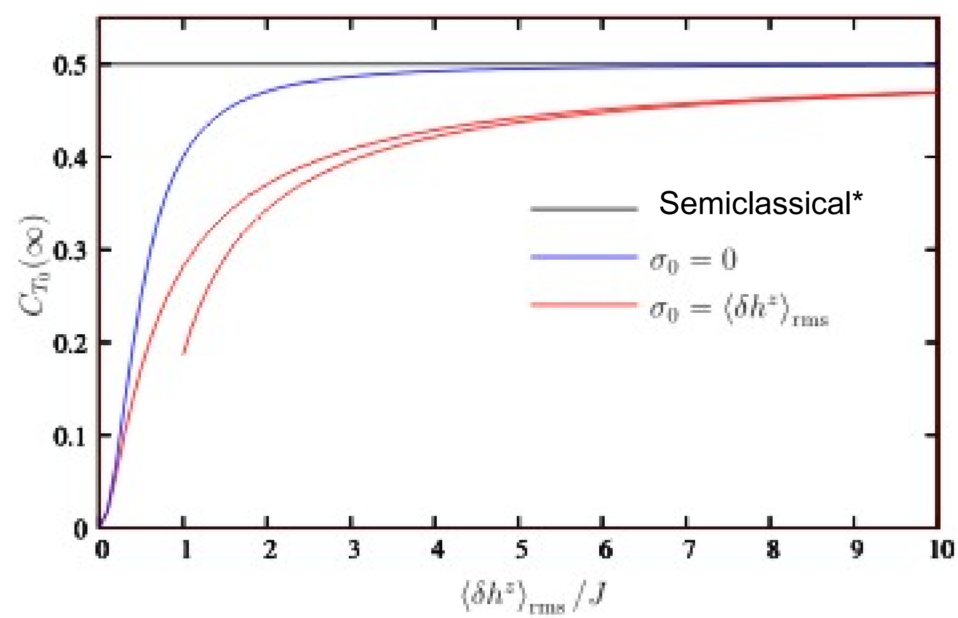
important step toward CNOT gate

Dynamics in the S-T_0 subspace

Bill Coish & DL, PRB 72, 125337 (2005)

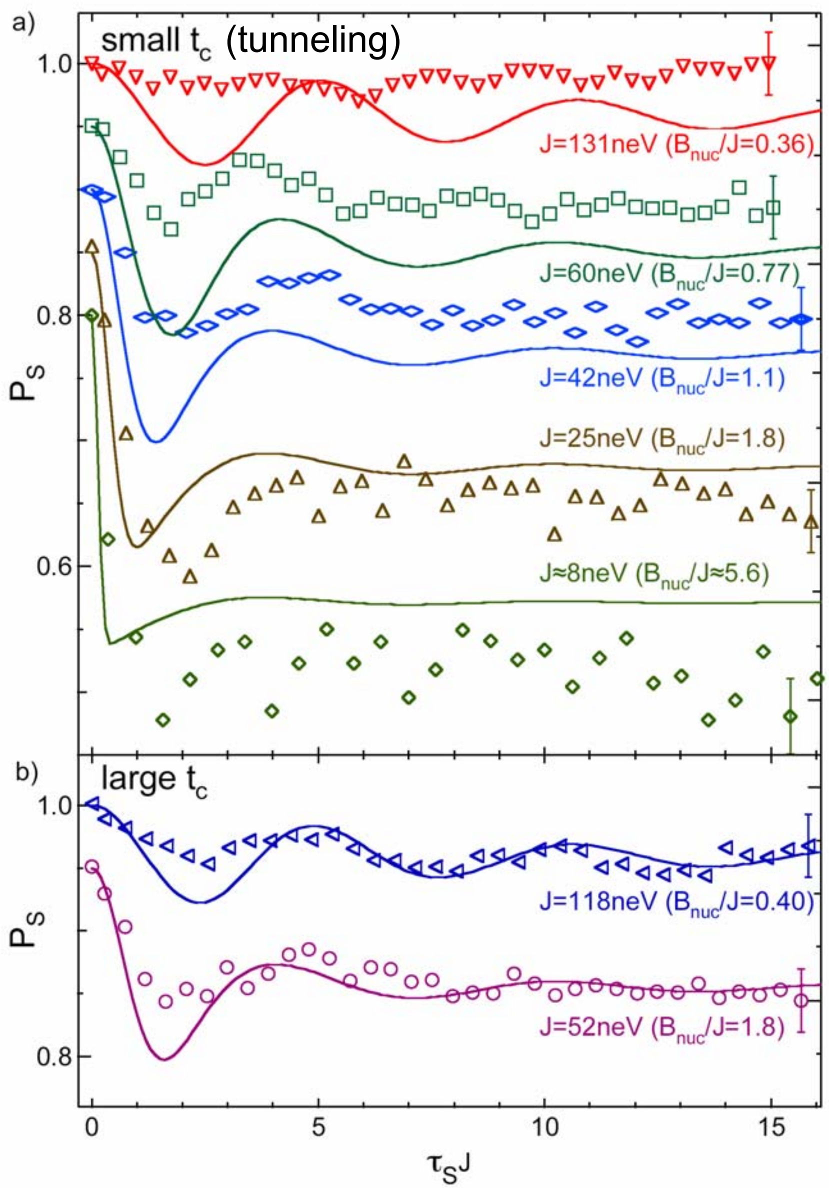
Gaussian distribution of δh_n^z with width σ_0:





*Semiclassical result: Schulten and Wolynes, J. Chem. Phys. **68**, 3292 (1978)

Experiment (Harvard group): E. Laird *et al.,* cond-mat/0512077



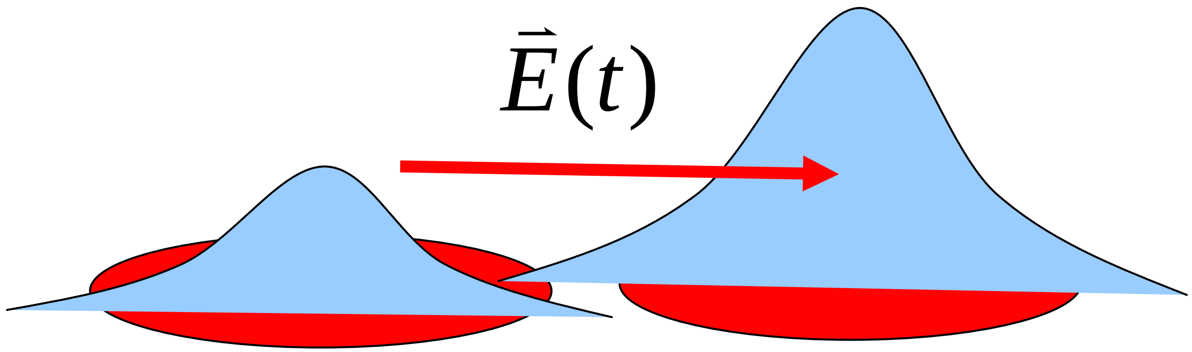
Theory (full lines): Coish & DL, PRB 72, 125337 (2005)

Coherent singlet- triplet oscillations due to hyperfine mixing ➔ reasonable agreement

Orbital Dephasing ➔ Spin Decoherence

Bill Coish & DL, PRB 72, 125337 (2005)

Fluctuating electric field due to charge fluctuators, leads, QPCs, etc.



Electric dipole moment for N=1 or N=2 electrons in the double dot: $\vec{p}_N$

Electric dipole coupling can lead to additional spin decoherence since singlet and triplet have different orbital states ψ_{orb} ➔ different dipole moments:

$$V_{orb}(t) = -\vec{p}_N \cdot \vec{E}(t)$$

Orbital Dephasing

Orbital dephasing rate for N electrons in a double dot:

$$\frac{1}{t_\phi^{(N)}} = \frac{\left|\Delta\vec{p}_N\right|^2}{4} \int_{-\infty}^{\infty} dt \left\langle E(t)E(0)\right\rangle$$

$\Delta\vec{p}$: difference in dipole moment of two quantum states.

➔ $$\frac{t_\phi^{(1)}}{t_\phi^{(2)}} = \left|\frac{\Delta\vec{p}_2}{\Delta\vec{p}_1}\right| \le D^2$$

D: Double occupancy of the singlet state

$t_\phi^{(1)} \approx 1\,\mathrm{ns}$, T. Hayashi et al., PRL 91, 225804 (2003)

$t_\phi^{(1)} \approx 400\,\mathrm{ps}$, J. Petta et al., PRL 93, 186802 (2004)

➔ Orbital dephasing can be comparable to hyperfine unless D<<1 !!

Orbital Dephasing

Generalize to dephasing due to *any* charge fluctuations:

Exchange: $J(t) = J + \delta J(t)$

Single-particle level spacing in the double-dot: $\varepsilon(t) = \varepsilon + \delta\varepsilon(t)$

$$\Rightarrow \quad \boxed{\frac{t_\phi^{(1)}}{t_\phi^{(2)}} = \left|\frac{\delta J}{\delta\varepsilon}\right|^2}$$

Optimal point: $\boxed{\frac{\delta J}{\delta\varepsilon} = 0}$ $\Rightarrow$ Orbital dephasing becomes *in*effective for two-electron states

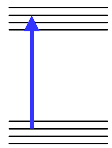
Narrowing of nuclear spins in double dots with ESR

Klauser, Coish & DL, cond-mat/0510177

- ESR: oscillating exchange $J(t)=J_0+j\cos(\omega t)$ leads to Rabi oscillations:

$$|\uparrow\downarrow\rangle = |-\rangle$$

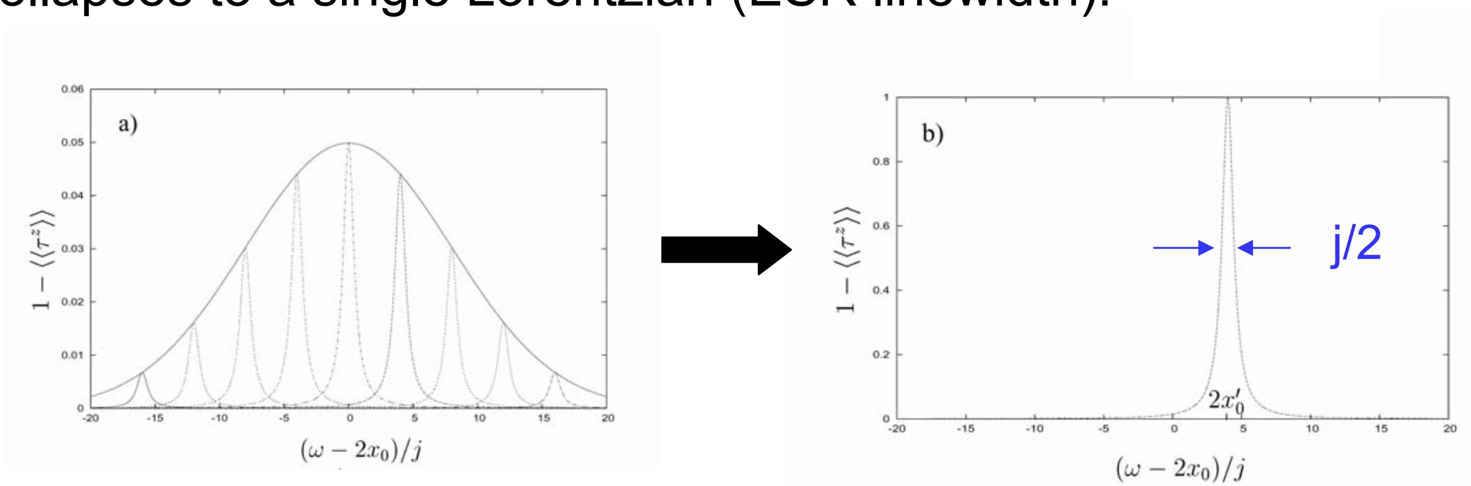
$$|\downarrow\uparrow\rangle = |+\rangle$$



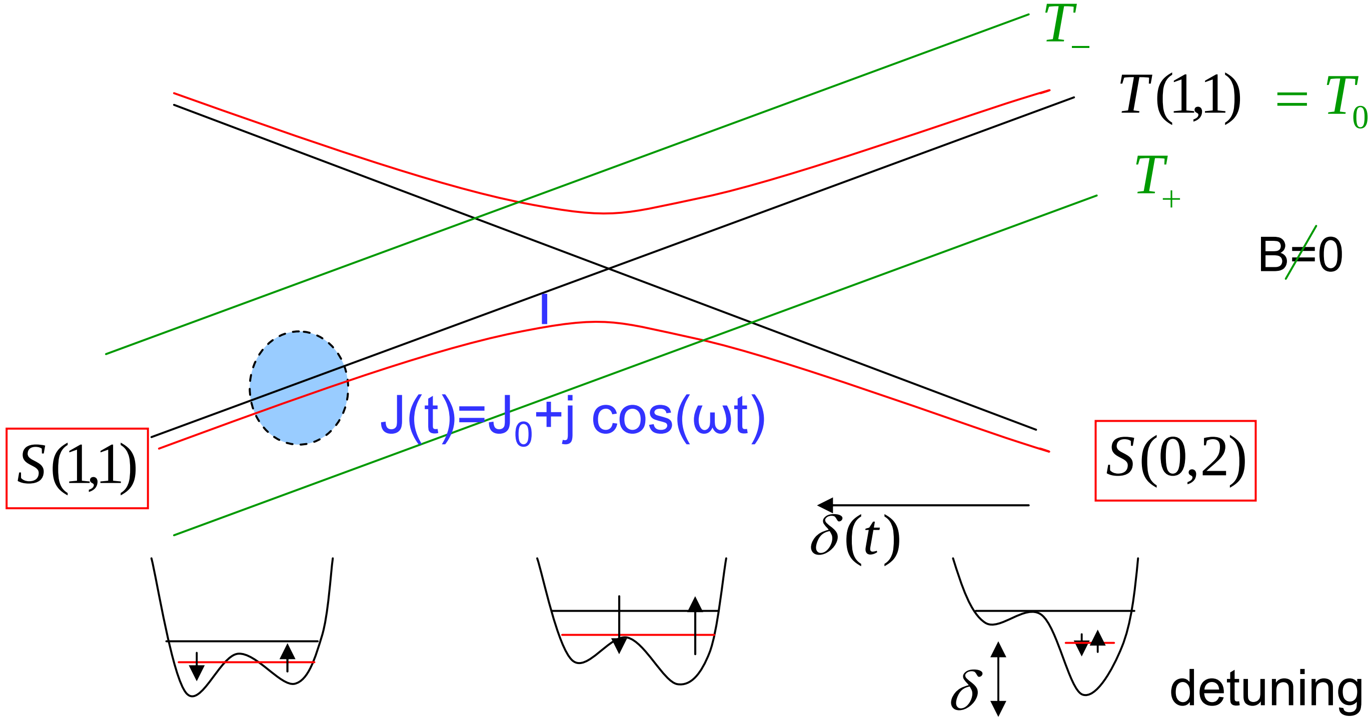
$$g\mu_B B + \delta h_n^z$$

ESR at frequency ω = $g\mu_B B + \delta h_n^z$ measures eigenvalue δh_n^z
➔ nuclear spins projected into corresponding eigenstate |n>

If quantum measurement is ideal, then Gaussian superposition collapses to a single Lorentzian (ESR linewidth):



ESR: oscillating exchange ➔ Rabi oscillations



Narrowing of nuclear spin state via measurements of electron spins

$$\rho(0) = \underbrace{|+\rangle\langle+|}_{\rho_e} \otimes \underbrace{\sum_i p_i |\psi_I^i\rangle\langle\psi_I^i|}_{\rho_I}, \qquad |\psi_I^i\rangle = \sum_n a_n^i |n\rangle, \qquad \delta h_z |n\rangle = \delta h_z^n |n\rangle$$

time evolution of electron-nuclear spin system (in S-T_0 subspace) ➔

$$\rho(t) = U(t)\left(\rho_e \otimes \sum_{i;n,l} p_i a_n^i a_l^{i*} |n\rangle\langle l|\right)U^+(t) = \sum_{n,l} U_n \rho_e U_l^+ \otimes \sum_i p_i a_n^i a_l^{i*} |n\rangle\langle l|$$

At time t_m perform measurement of the electronic state ➔ obtain state |+/-> with probability

$$P^{\pm} = Tr\left\{ |\pm\rangle\langle\pm| \rho(t_m) \right\}$$

➔ *after* measurement the entire system is in *new* state

$$\rho^{(1,\pm,\omega)}(t_m) = \frac{1}{P^{\pm}} |\pm\rangle\langle\pm| \rho(t_m) |\pm\rangle\langle\pm| \qquad \delta t_m \le 1/j$$

➔ in particular, the nuclear state $<n|\rho_I|n>$ gets changed into:

$$\rho_I^{(1,+,\omega)}(x) = \rho_I(x)(1 - L_\omega(x))\frac{1}{P_\omega^+}$$

$$\rho_I^{(1,-,\omega)}(x) = \rho_I(x)L_\omega(x)\frac{1}{P_\omega^-}$$

$$L_\omega(x) = \frac{1}{2}\frac{(j/4)^2}{(x-\omega/2)^2 + (j/4)^2}$$

$$x = \delta h_n^z + \delta b^z$$

Generalize to M consecutive measurements:

$$\rho_I^{(M,\alpha^-,\omega)}(x) = \frac{\rho_I(x)}{Q_\omega(M,\alpha^-)} L_\omega(x)^{\alpha^-}[1 - L_\omega(x)]^{M-\alpha^-}$$

e.g. $P_\omega^- = Q_\omega(1,1), \quad P_\omega^+ = Q_\omega(1,0)$

➔ in particular, the nuclear state $\langle n|\rho_I|n\rangle$ gets changed into:

$$\rho_I^{(1,+,\omega)}(x) = \rho_I(x)(1 - L_\omega(x))\frac{1}{P_\omega^+}$$

$$\rho_I^{(1,-,\omega)}(x) = \rho_I(x)L_\omega(x)\frac{1}{P_\omega^-}$$

$$L_\omega(x) = \frac{1}{2}\frac{(j/4)^2}{(x-\omega/2)^2 + (j/4)^2}$$

$$x = \delta h_n^z + \delta b^z$$

Generalize to M_1 consecutive measurements at frequency ω_1,
M_2 " " " ω_2,
...

$$\rho_I^{(\{M_i\},\{\alpha_i^-\},\{\omega_i\})}(x) = \rho_I(x)\prod_i^{m_f}\frac{L_{\omega_i}(x)^{\alpha_i^-}[1 - L_{\omega_i}(x)]^{M_i - \alpha_i^-}}{Q_{\omega_i}}$$

I. Unconditional scheme

- Allow nuclear spins to re-randomize between measurements ➔ time evolution of nuclear spins is independent of previous measurement outcome

- Measurement of electrons for single fixed ESR frequency ω:

  if outcome is $|+\rangle$ ➔ no narrowing; wait for the system to re-randomize

  if outcome is $|-\rangle$ ➔ initial distribution narrowed by a factor $j/4\sigma_0$ >>1

- Probability to measure $|-\rangle$ $\; is \;\; P^{-} \approx j/6\sigma_0$

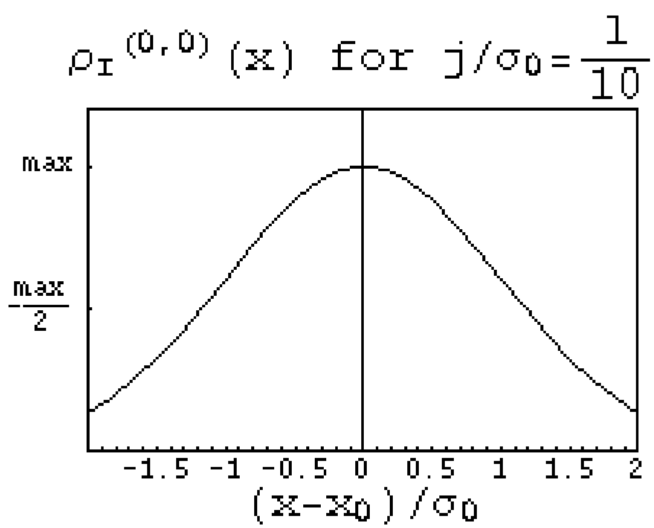
➔ $M \approx 6\sigma_0/j$ measurements are needed to narrow by $j/4\sigma_0$

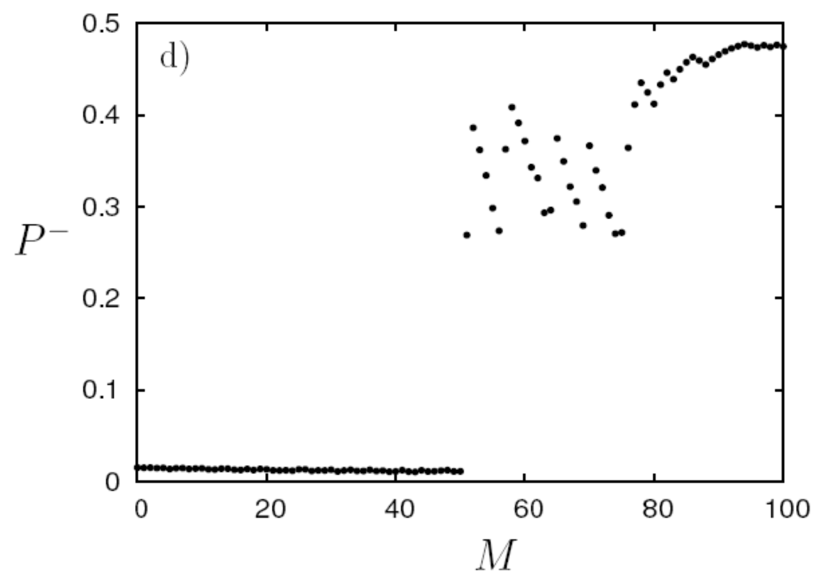
typically $M \approx 50$ ➔ simplest scheme for double dots

II. Conditional scheme

- Conditional evolution of the nuclear-spin density matrix between measurements (nuclear spins assumed to be static between meas.)

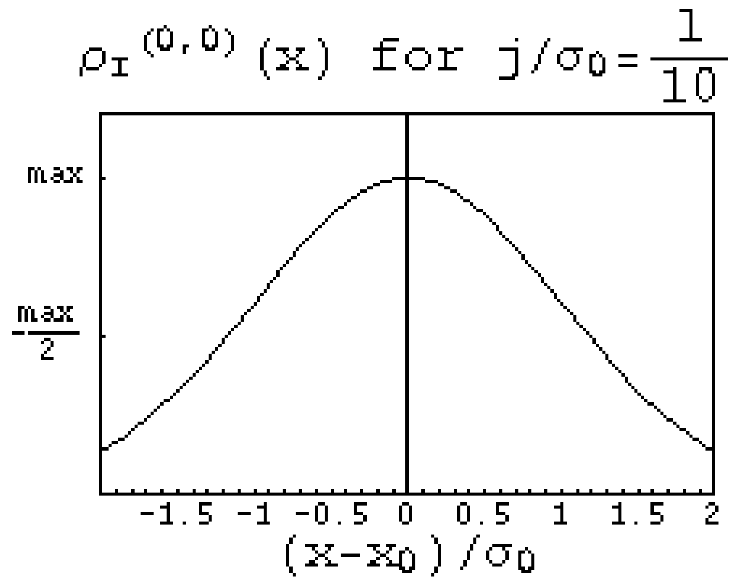
- Perform M measurements and 'adapt' frequency ω after each measurement*) to increase the probability P_{ω}^{-}

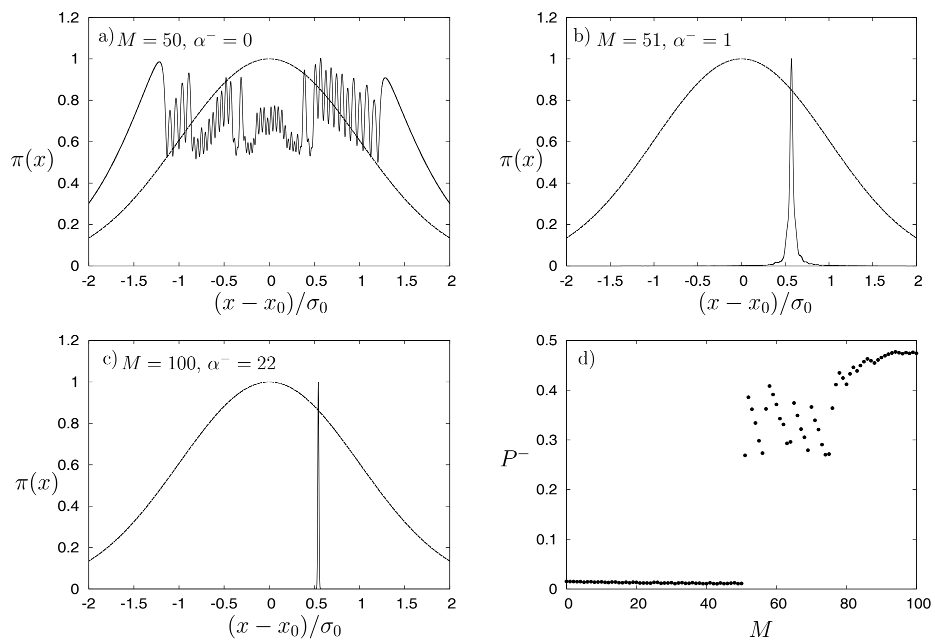




*) 'adaptive scheme', see all-optical setup by Stepanenko *et al.,* cond-mat/0512044

Theoretical prediction:
Projective measurement
of electron spin narrows
nuclear spin distrubution:





Conclusion

A. Quantum computing with spin qubits

1. interaction based
2. measurement based (Bell state & parity gates)

B. Spin decoherence in GaAs quantum dots is dominated by nuclear spins:

-- rapid Gaussian decay (~10 ns) due to quantum variance of hyperfine field ➔ narrowing of initial nuclear state ➔ expect ms range

-- non-exponential (non-Markovian) decay for hyperfine *eigenstate*; amount of decay bounded by large B and/or Overhauser field

-- quantitative agreement between theory and measurement in double dots