Anomalous transport in spin-orbit coupled systems: AHE, SHE, miscellaneous thoughts and calculations

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KITP, June 6th 2005

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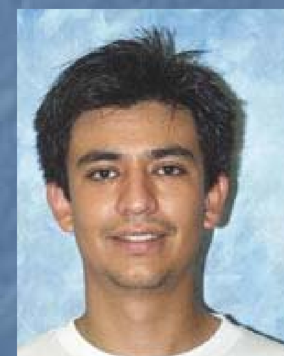

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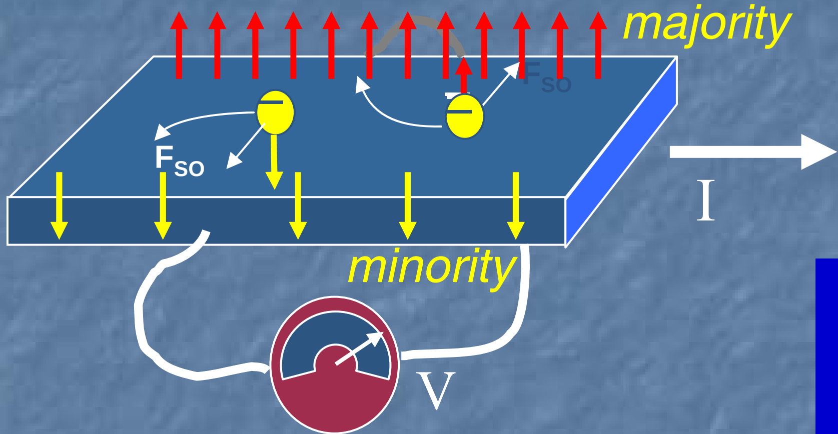
OUTLINE

- Intro and short history and origin of the SHE frenzy
- Hall effect (long ago) and Spin Hall effect (not so long ago)
  - Parsing the AHE
    - Perturbation theory treatment
    - Semiclassical vs. Kubo
  - Parsing the SHE (example: 2D systems)
    - Perturbation theory treatment
  - Resolution of some of the controversy:
    A community that works together, SHE workshop
- Ladder diagrams in 2D SO coupled systems
- Laughlin argument for transport:
  - What and how is the argument used
  - Why does it work in the QHE case
  - Why it does not work in metallic systems
- AHE in graphene (time permitting)

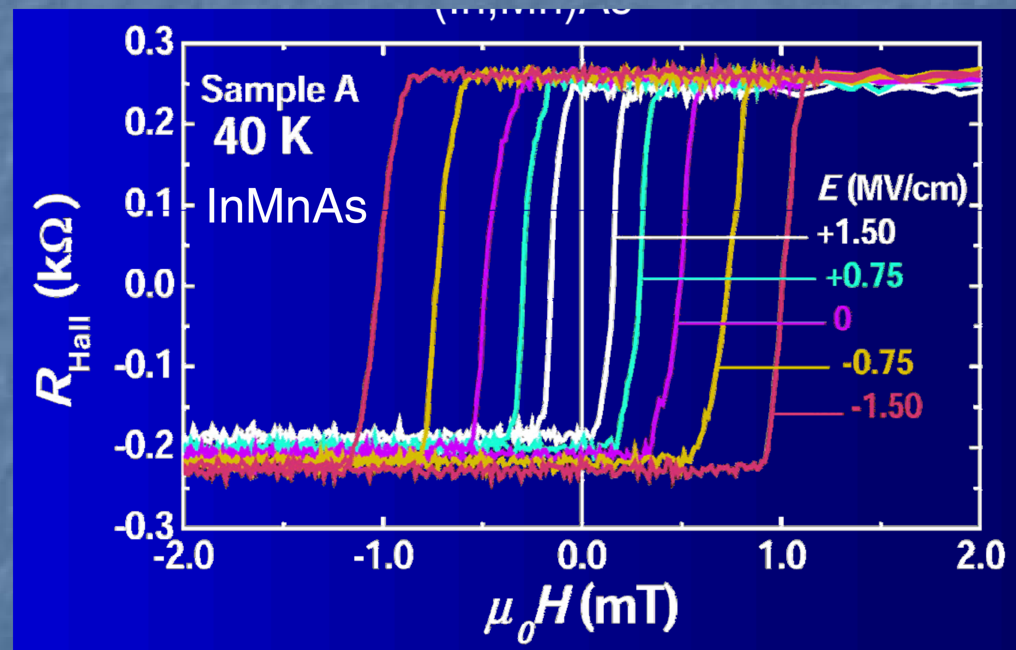
Anomalous Hall effect: where things started, the unresolved problem

Spin-orbit coupling "force" deflects ***like-spin*** *particles*



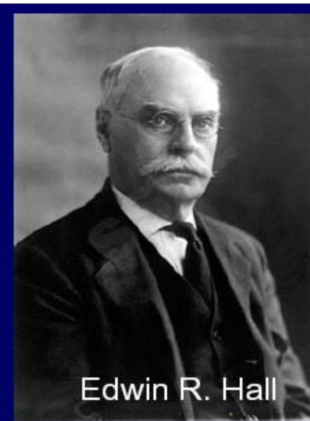
$$\rho_H = R_0 B + 4\pi R_s M$$

Simple electrical measurement of magnetization

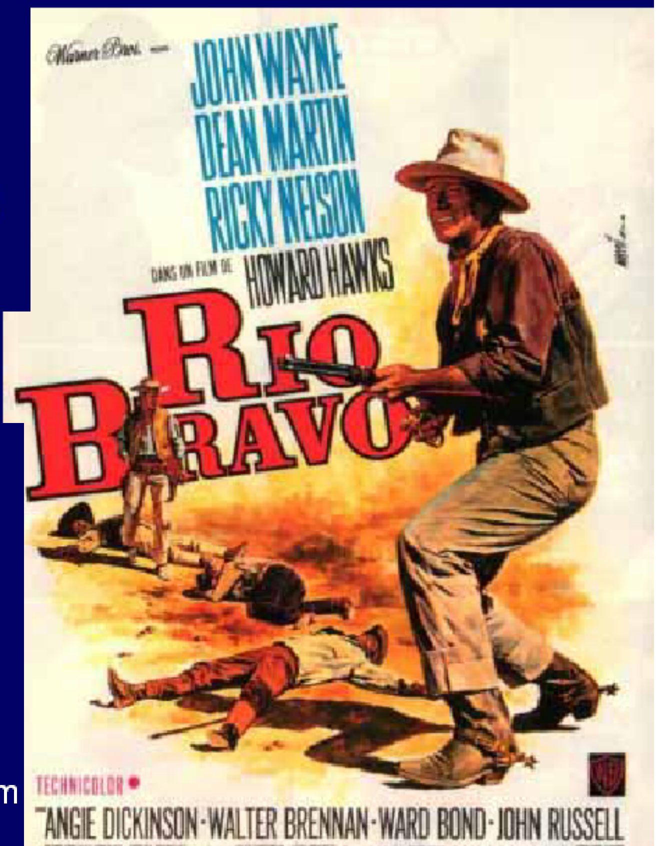


D. Chiba *et al.*, Science **301**, 943 (2003)

1. Brief historic sketch

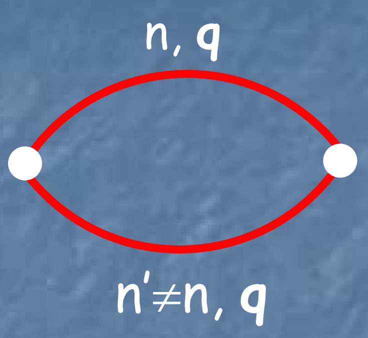

Edwin R. Hall

- 1880: discovery of the normal Hall effect (E.R. Hall)
- 1881: discovery of the anomalous Hall effect (E.R. Hall)
- 1893 – 1950: numerous experimental studies of AHE (A. Kundt, A.W. Smith, A. Perrier, E.M. Pugh, J. Smit, ...)
- 1954: first quantitative theory of the AHE (Karplus and Luttinger): anomalous velocity current due to the spin-orbit coupling
- 1955: skew scattering contribution (J. Smit)
- 1957 - 1958: systematic theory of AHE with effect of scattering (Kohn, Luttinger)
- 1970: side-jump mechanism (Berger)
- 1970 – 1980: battles of comments and replies between Smit and others (Berger, Lyo and Holstein, ...)
- 1999: AHE due to the spin chirality Berry phase in textured magnets (Ye *et al.*)
- since 2000: further theoretical work on the spin chirality induced mechanism (Lyanda-Geller *et al.,* Nagaosa *et al.,* Tatara *et al.,* PB *et al.*)
- since 2000: experimental work on AHE due to spin chirality in pyrochlores, manganates, oxides, ... (Taguchi *et al.,* Salamon *et al.*)
- 2002: interpretation of the Karplus-Luttinger term as a Berry phase in momentum space (Jungwirth, Niu, MacDonald)
- since 2002: *ab initio* calculations of the Karplus-Luttinger (Berry phase) term (Fang *et al.,* Yao *et al.,* ...)



(thanks to P. Bruno– CESAM talk)

INTRINSIC AHE: semiclassical and Kubo



Kubo:

$$\mathrm{Re}[\sigma_{xy}] = -\frac{e^2\hbar}{V}\sum_{\vec{k}\; n\neq n'}\left(f_{n'k} - f_{nk}\right)\frac{\mathrm{Im}\left[\langle n'\vec{k}|\hat{v}_x|n\vec{k}\rangle\langle n\vec{k}|\hat{v}_y|n'\vec{k}\rangle\right]}{(E_{n\vec{k}} - E_{n'\vec{k}})^2}$$

Semiclassical approach in the “clean limit”

$$\dot{x}_c = \frac{\partial\epsilon}{\hbar\partial\vec{k}} + (e/\hbar)\vec{E}\times\vec{\Omega}.$$

$$\mathbf{\Omega}_n(\mathbf{k}) = -\,\mathrm{Im}\,\langle\nabla_{\mathbf{k}}u_{n\mathbf{k}}|\times|\nabla_{\mathbf{k}}u_{n\mathbf{k}}\rangle$$

$$\mathrm{Re}[\sigma_{xy}] = -\frac{e^2}{V\hbar}\sum_{\vec{k}\; n} f_{n'k}\Omega_n(\vec{k})$$

K. Ohgushi, et al PRB 62, R6065 (2000); T. Jungwirth et al PRL 88, 7208 (2002);
T. Jungwirth et al. Appl. Phys. Lett. 83, 320 (2003); M. Onoda et al J. Phys. Soc. Jpn. 71, 19 (2002); Z. Fang, et al, Science 302, 92 (2003).

Success of intrinsic AHE approach

- DMS systems (Jungwirth et al PRL 2002, APL 03)
- Fe (Yao et al PRL 04)
- layered 2D ferromagnets such as SrRuO3 and pyrochlore ferromagnets [Onoda and Nagaosa, J. Phys. Soc. Jap. 71, 19 (2001),Taguchi et al., Science 291, 2573 (2001), Fang et al Science 302, 92 (2003), Shindou and Nagaosa, Phys. Rev. Lett. 87, 116801 (2001)]
- colossal magnetoresistance of manganites, Ye et~al Phys. Rev. Lett. 83, 3737 (1999).
- CuCrSeBr compounts, Lee et al, Science 303, 1647 (2004)

Experiment
$\sigma_{AH} \sim 1000\ (\Omega\ \mathrm{cm})^{-1}$

Theroy
$\sigma_{AH} \sim 750\ (\Omega\ \mathrm{cm})^{-1}$

Berry's phase based AHE effect is quantitative-successful in many instances BUT still not a theory that treats systematically intrinsic and extrinsic contribution in an equal footing.

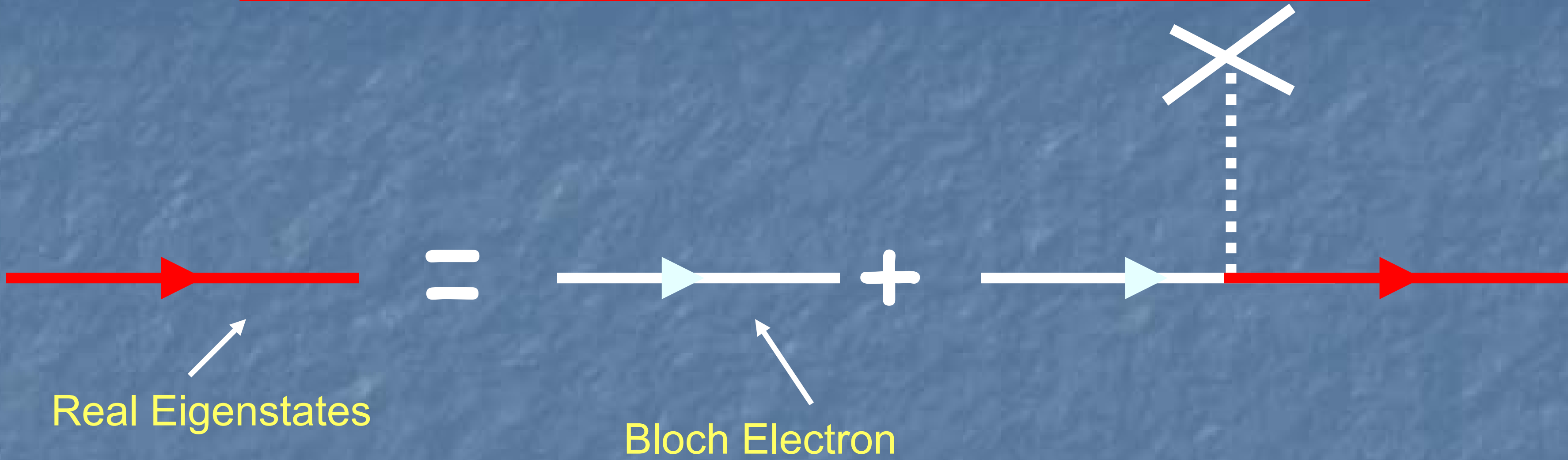
Anomalous Hall Effect

Linear response Kubo theory:

$$\sigma_H(z) = \frac{e^2\hbar}{L^2}\sum_{\alpha,\beta}\frac{f_\alpha - f_\beta}{(E_\alpha - E_\beta)(z + E_\alpha - E_\beta)}\ \mathrm{Im}\left[\langle\alpha|v_x|\beta\rangle\langle\beta|v_y|\alpha\rangle\right]$$

$$\sigma_H^{dc} = \lim_{z\to 0}\left[\lim_{L\to\infty}\sigma_H(z)\right]$$

Perturbation Theory



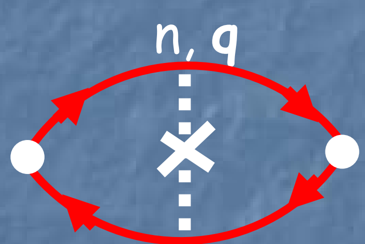
Averaging procedures:

$= \tau^{-1} / \nu_0$

$= \nu_0 \tau$

Perturbation Theory: diagonal conductivity

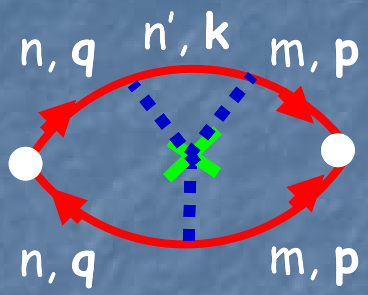
Drude Conductivity
$\sigma = ne^2\tau/m^*$



Vertex Corrections
$\sim 1-\cos(\theta)$

● $= j_\nu = -ev_\nu$

AHE conductivity– Perturbation Theory



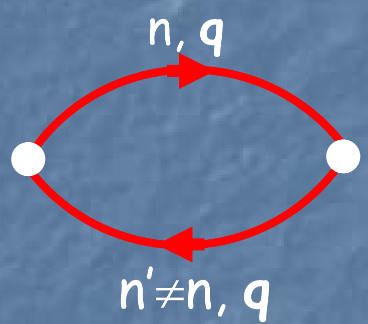
Skew

$\sigma^{H}_{Skew} \sim (\tau_{skew})^{-1} \quad \tau^2 \sim \sigma_0\, S$

where

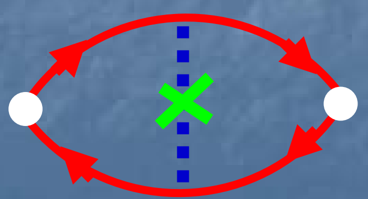
$\mathbf{S} = Q(\mathbf{k},\mathbf{p})/Q(\mathbf{p},\mathbf{k}) - 1 \sim$

$V\nu_0\ \mathbf{Im}[\langle \mathbf{k}|\mathbf{q}\rangle\langle \mathbf{q}|\mathbf{p}\rangle\langle \mathbf{p}|\mathbf{k}\rangle]$



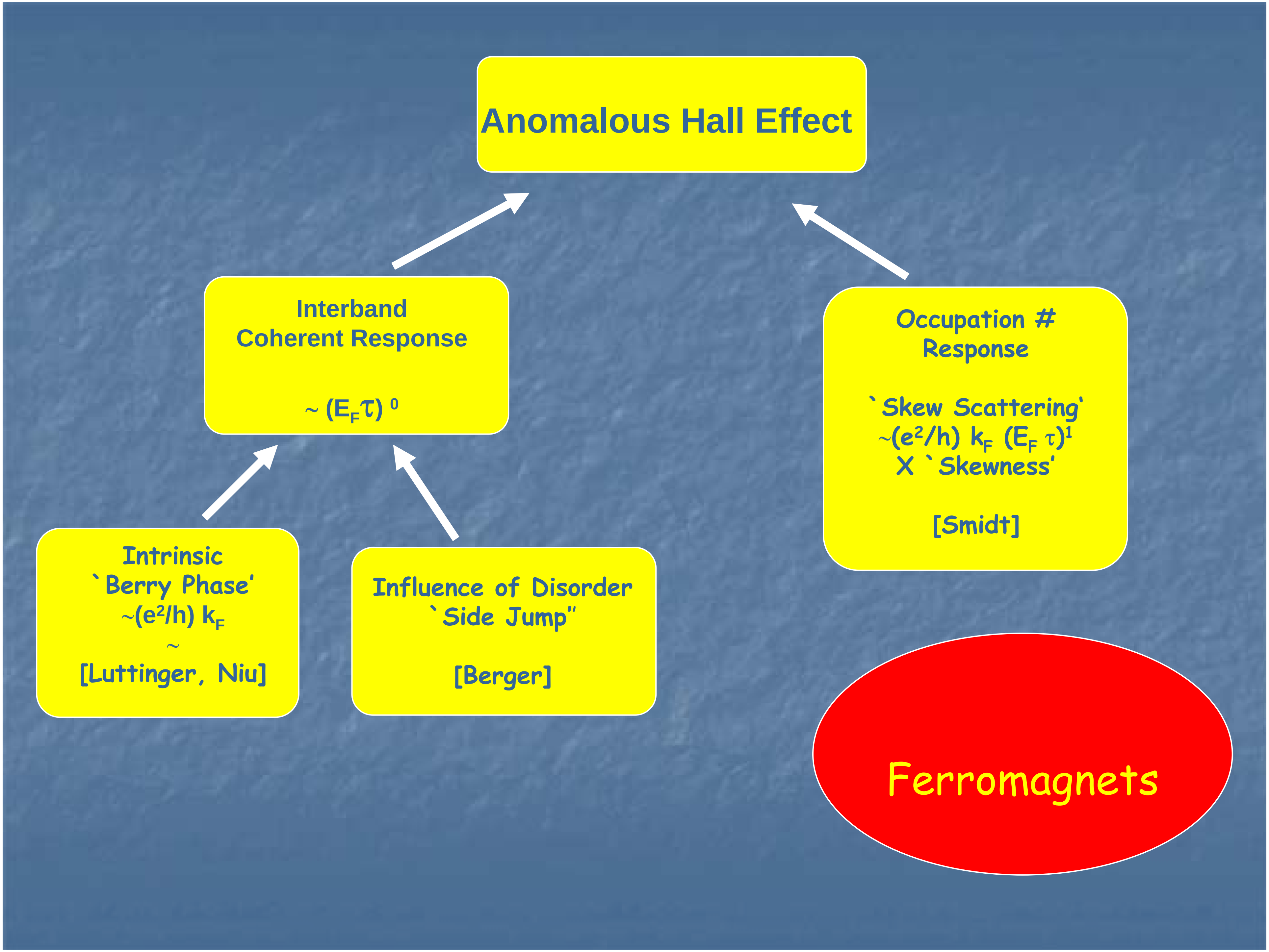
Intrinsic

$\sim\sigma_0 / \varepsilon_F \tau$



Vertex Corrections

$\sim \sigma_{Intrinsic}$


Anomalous Hall Effect
Interband
Coherent Response
~ $(E_F\tau)^0$
Occupation #
Response
`Skew Scattering'
~$(e^2/h)\, k_F\, (E_F\, \tau)^1$
X `Skewness'
[Smidt]
Intrinsic
`Berry Phase'
~$(e^2/h)\, k_F$
~
[Luttinger, Niu]
Influence of Disorder
`Side Jump"
[Berger]
Ferromagnets

INTRINSIC+EXTRINSIC: STILL CONTROVERSIAL!!

AHE in Rashba systems with disorder:

Dugaev et al PRB 05
Sinitsyn et al PRB 05
Inoue et al (cond-mat May 06)
Nagaosa et al (cond-mat June 06)
Sinova et al (unpublished 06)

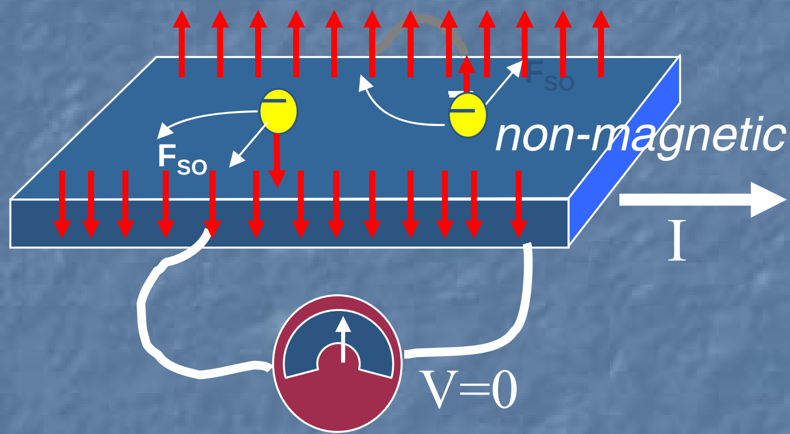
All are done using same or equivalent linear response formulation– ALL have different answers!!!

The only way to create consensus is to show (IN DETAIL) agreement between the different equivalent linear response theories both in AHE and SHE

Only known success so far is Sinitsyn et al cond-mat/0602598 in graphene

Spin Hall effect

Take now a PARAMAGNET instead of a FERROMAGNET:
Spin-orbit coupling "force" deflects ***like-spin*** *particles*



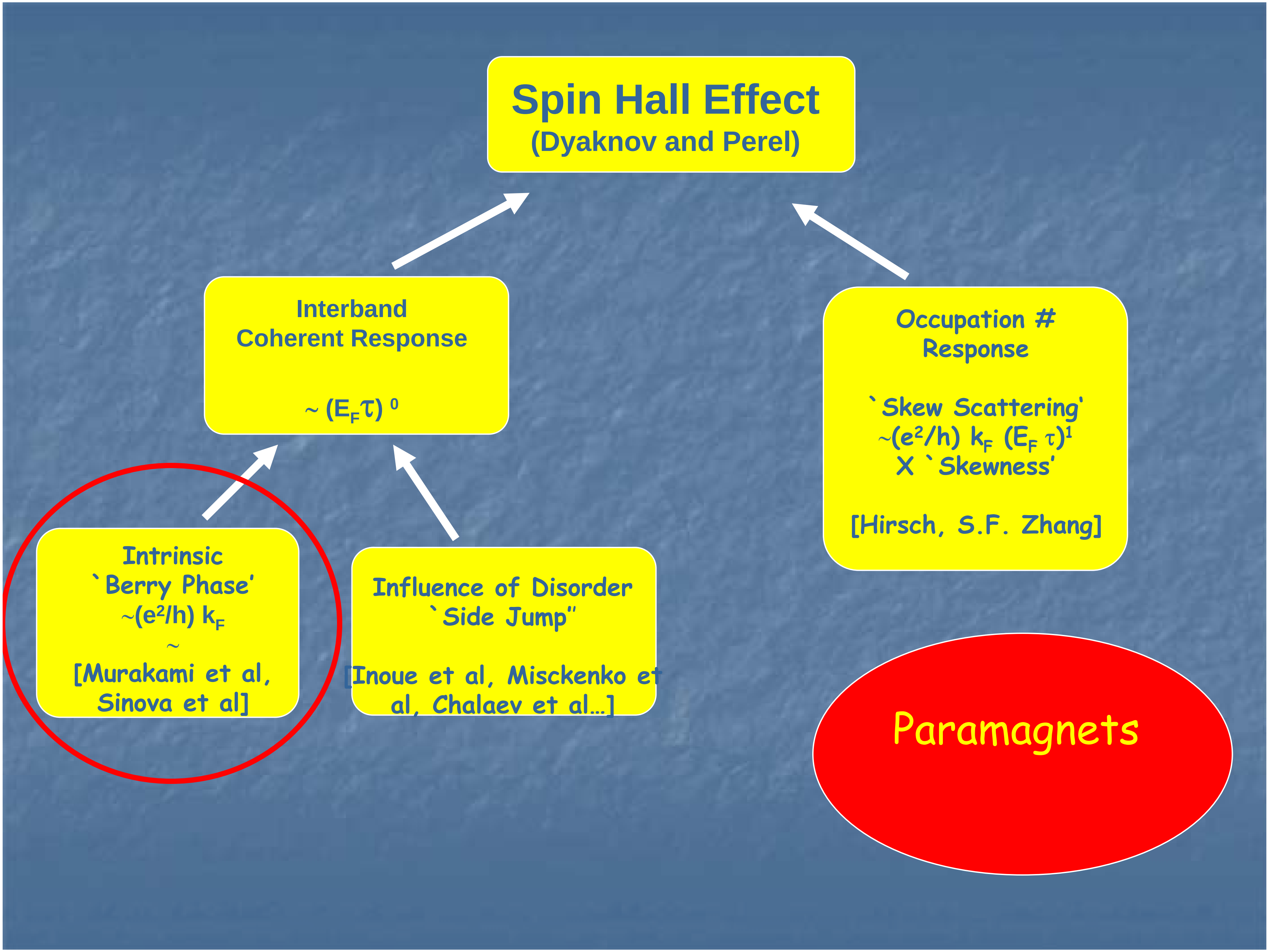
Carriers with same charge but opposite spin are deflected by the spin-orbit coupling to opposite sides.

Spin-current generation in non-magnetic systems without applying external magnetic fields

Spin accumulation without charge accumulation excludes simple electrical detection

SHE: how we got here; a quick overview

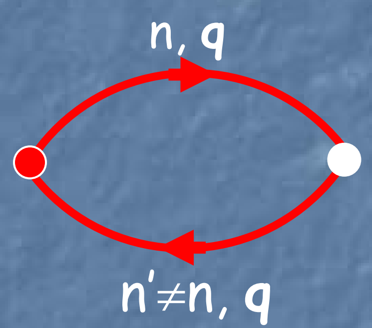
- 1971 Dyakonov and Perel propose the SHE based on the phenomenology of the AHE idea is believed but unnoticed in the west
- 1999 Hirsch proposes again the SHE based on the phenomenology of AHE: initial experimental attempts fail and some theory interest
- 2003 Murakami et al and Sinova et al propose the idea of the intrinsic SHE based on their work on intrinsic AHE: theoretical interest explodes. D&K contribution resurfaces.
- 2004 SHE is first observed through optical techniques in 3D electron systems (Kato et al) and 2DHG systems (Wunderlich et al)
- 2003-06: SHE inherits the controversy (some of it now resolved– SHE workshop 05) and confusion reigning in the AHE theory: What is its origin? is it edge or is it bulk? Extrinsic vs. intrinsic? Etc. ~250 theory papers since 03!
- 2005-06: Ratio of theory to experimental papers improves (~1/40): SHE observed in 2DEG (Sih et al 05); first observation in transport (Saitoh et al 06; Valenzuela et al 06); 3DEG experiments point to a bulk origin (Sih et al 06)


Spin Hall Effect
(Dyaknov and Perel)
Interband
Coherent Response
~ (E_F τ) ^0
Occupation #
Response
`Skew Scattering'
~(e²/h) k_F (E_F τ)¹
X `Skewness'
[Hirsch, S.F. Zhang]
Intrinsic
`Berry Phase'
~(e²/h) k_F
~
[Murakami et al,
Sinova et al]
Influence of Disorder
`Side Jump"
[Inoue et al, Misckenko et
al, Chalaev et al…]
Paramagnets

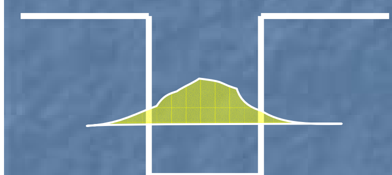
INTRINSIC SPIN-HALL EFFECT:

Murakami et al Science 2003 (cond-mat/0308167)
Sinova et al PRL 2004 (cont-mat/0307663)

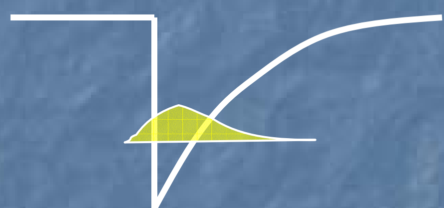
as there is an intrinsic AHE (e.g. Diluted magnetic semiconductors), there should be an intrinsic spin-Hall effect!!!



(differences: spin is a non-conserved quantity, define spin current as the gradient term of the continuity equation. Spin-Hall conductivity: linear response of this operator)

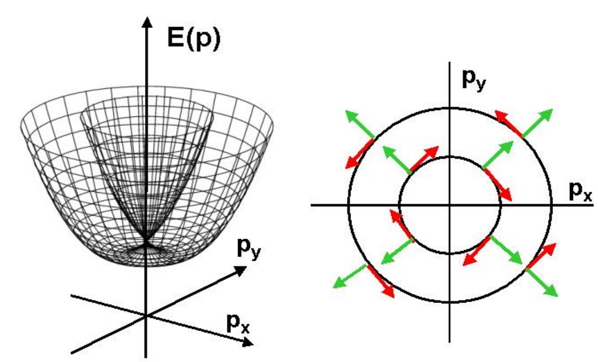


Inversion symmetry
⇒ no R-SO



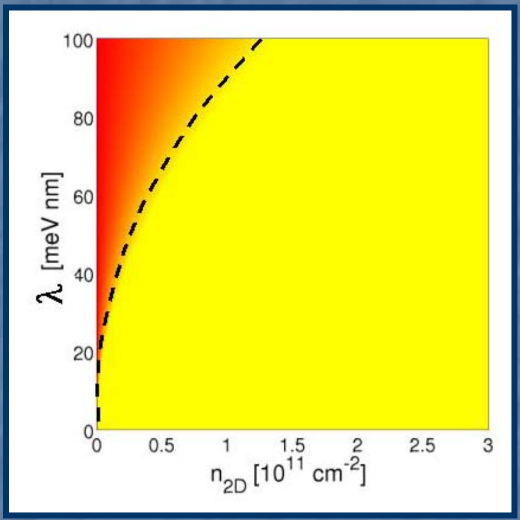
Broken inversion symmetry
⇒ R-SO

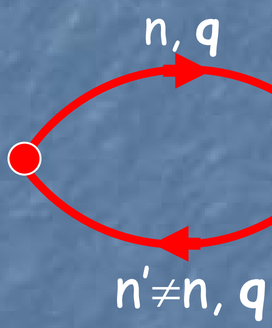
$$H_k = \frac{\hbar^2 k^2}{2m}\sigma_0 + \lambda(k_x\sigma_y - k_y\sigma_x) = \frac{\hbar^2 k^2}{2m}\sigma_0 + \lambda\vec{\sigma}\times\vec{k}$$



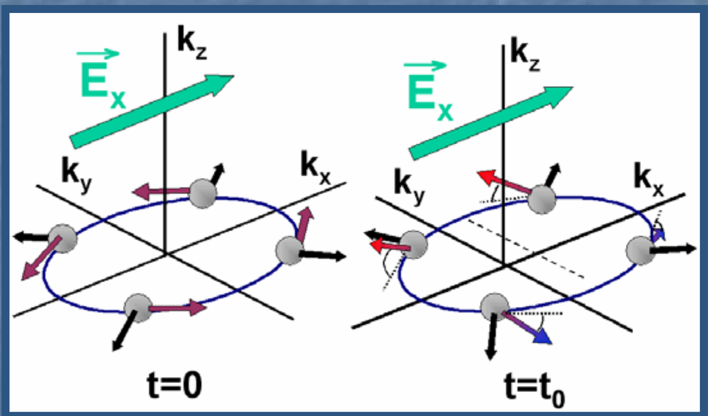
Bychkov and Rashba (1984)

‘Universal’ spin-Hall conductivity





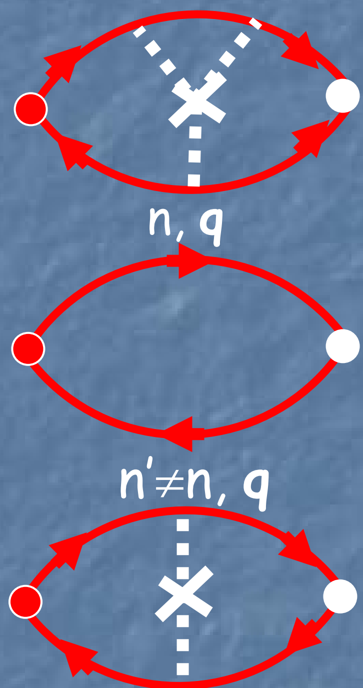
$$\sigma_{xy}^{\mathrm{sH}} = \frac{e\hbar}{mV}\sum_{\mathbf{k},n\neq n'}(f_{n',k}-f_{n,k}) \times \frac{\mathrm{Im}[\langle n'k|\hat{j}^{z}_{\text{spin x}}|nk\rangle\langle nk|\hat{p}_y|n'k\rangle]}{(E_{nk}-E_{n'k})^2}$$



$$\sigma_{xy}^{\mathrm{sH}} = \begin{cases} \dfrac{e}{8\pi} \text{ for } & n_{2D} > n_{2D}^{*} = \dfrac{m^2\lambda^2}{\pi\hbar^4} \\ \dfrac{e}{8\pi}\dfrac{n_{2D}}{n_{2D}^{*}} \text{ for } & n_{2D} < n_{2D}^{*} \end{cases}$$

Color plot of spin-Hall conductivity:
yellow=e/8π and red=0

SHE conductivity– Perturbation Theory



Skew
$\sim\sigma_0\ S$

Intrinsic
$\sim\sigma_0\ /\varepsilon_F\tau$

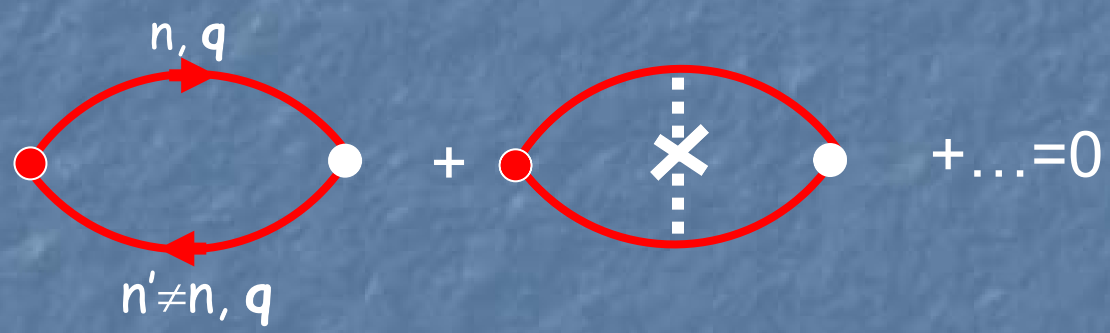
Vertex Corrections
$\sim\ \sigma_{Intrinsic}$

$= j_\nu = -e\ v_\nu$

$= j_\nu^z = \{v_\nu, s^z\}$

Disorder effects: beyond the finite lifetime approximation for Rashba 2DEG

Question: Are there any other major effects beyond the finite life time broadening? Does side jump contribute significantly?



Inoue et al PRB 04
Raimondi et al PRB 04
Mishchenko et al PRL 04
Loss et al, PRB 05

$$\sigma_{\alpha x}^{\sigma_i} = \frac{\hbar}{2\pi L^2} \mathrm{Tr} J_{\alpha}^{\sigma_i} \langle G J_x G \rangle_{AV}$$

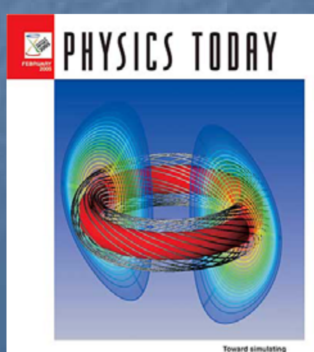
$$J_x = e\{(\hbar k/m)\mathbf{1} - \lambda \sigma_y\}$$

Ladder partial sum vertex correction: $\lambda \rightarrow \tilde{\lambda}$

the vertex corrections are zero for 3D hole systems (Murakami 04) and 2DHG (Bernevig and Zhang 05)

SHE controversy

- Does the SHE conductivity vanish due to scattering?
  Seems to be the case in 2DRG+Rashba,
  does not for any other system studied
- Dissipationless vs. dissipative transport
- Is the SHE non-zero in the mesoscopic regime?
- What is the best definition of spin-current to relate spin-conductivity to spin accumulation
- ……



Sankar Das Sarma of the University of Maryland in College Park has been closely following the controversy as its unfolds. "The theoretical situation is a complete mess," he says.

A COMMUNITY WILLING TO WORK TOGETHER

APCTP Workshop on Semiconductor
Nano-Spintronics: Spin-Hall Effect and Related Issues
August 8-11, 2005 APCTP, Pohang, Korea

http://faculty.physics.tamu.edu/sinova/SHE_workshop_APCTP_05.html

Semantics agreement:

The intrinsic contribution to the spin Hall conductivity is the the spin Hall conductivity in the limit of strong spin orbit coupling and $\omega\tau$>>1. This is equivalent to the single bubble contribution to the Hall conductivity in the weakly scattering regime.

General agreement

- The spin Hall conductivity in a 2DEG with Rashba coupling vanishes in the absence of a magnetic field and spin-dependent scattering. The intrinsic contribution to the spin Hall conductivity is identically cancelled by scattering (even weak scattering). This unique feature of this model can be traced back to the specific spin dynamics relating the rate of change of the spin and the spin current directly induced, forcing such a spin current to vanish in a steady non-equilibrium situation.

- The cancellation observed in the 2DEG Rashba model is particular to this model and in general the intrinsic and extrinsic contributions are non-zero in all the other models studied so far. In particular, the vertex corrections to the spin-Hall conductivity vanish for p-doped models.

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Condensed Matter Highlight

Spin-Hall effect: Back to the beginning at a higher level

Jairo Sinova [a,*], Shuichi Murakami [b], Shun-Qing Shen [c], Mahn-Soo Choi [d]

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- Hall effect (long ago) and Spin Hall effect (not so long ago)
  - Parsing the AHE
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    - Semiclassical vs. Kubo
  - Parsing the SHE (example: 2D systems)
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  - Resolution of some of the controversy:
    A community that works together, SHE workshop
- Ladder diagrams in 2D SO coupled systems
- Laughlin argument for transport:
  - What and how is the argument used
  - Why does it work in the QHE case
  - Why it does not work in metallic systems
- AHE in graphene (time permitting)

FOLLOWING THAT SPIRIT: KITP WORKSHOP

AHE in Rashba systems with disorder:

Dugaev et al PRB 05
Sinitsyn et al PRB 05
Inoue et al (cond-mat May 06)
Nagaosa et al (cond-mat June 06)
Sinova et al (unpublished 06)

PREDICTION: BY THE END OF THE WORKSHOP ALL RESULTS WILL AGREE

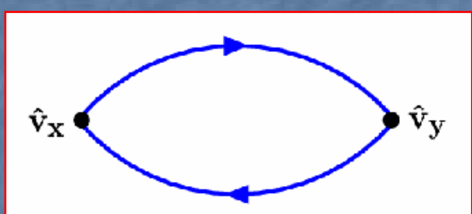
EXAMPLE OF A SUCCESS STORY: AHE IN GRAPHENE

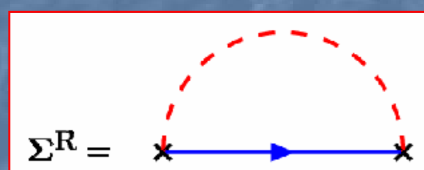
Kubo-Streda formula summary

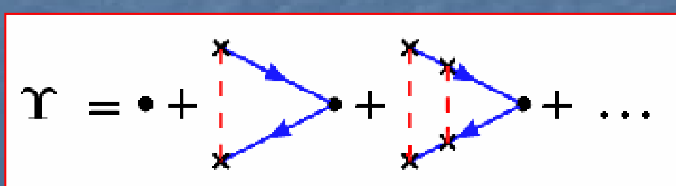
$$\sigma_{xy} = \sigma^{I}_{xy} + \sigma^{II}_{xy}$$

$$\sigma^{I}_{xy} = -\frac{e^2}{4\pi}\int_{-\infty}^{+\infty} d\varepsilon \frac{df(\varepsilon)}{d\varepsilon} Tr[v_x (G^R - G^A) v_y G^A - -v_x G^R v_y (G^R - G^A)]$$

$$\sigma^{II}_{xy} = \frac{e^2}{4\pi}\int_{-\infty}^{+\infty} d\varepsilon f(\varepsilon) Tr[v_x G^R v_y \frac{dG^R}{d\varepsilon} - -v_x \frac{dG^R}{d\varepsilon} v_y G^R - v_x G^A v_y \frac{dG^A}{d\varepsilon} + v_x \frac{dG^A}{d\varepsilon} v_y G^A]$$







Semiclassical Boltzmann equation

$$\frac{\partial f_l}{\partial t} + e\vec{E}\frac{\partial f_l}{\partial \vec{k}} = -\sum_{l'} \omega_{l'l}(f_l - f_{l'})$$

Golden rule:

$$\omega_{l'l} = \frac{2\pi}{\hbar} | T_{l'l} |^2 \delta(\varepsilon_{l'} - \varepsilon_l)$$

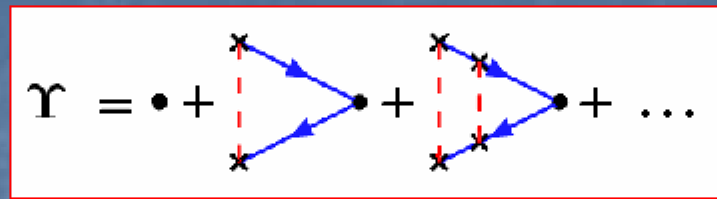
In metallic regime:

$$T_{l'l} = V_{l'l} + \frac{V_{l'l''} V_{l''l}}{\varepsilon_{l'} - \varepsilon_{l''} + i\eta} + \ldots$$

J. Smit (1956): Skew Scattering

$$\omega_{l'l} \neq \omega_{ll'}$$

LADDER DIAGRAMS FOR A GENERAL 2D SYSTEM



$$H_0 = \epsilon(\mathbf{k})\sigma_0 - B_i(\mathbf{k})\sigma_i$$

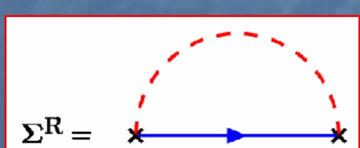
$$B_i(\mathbf{k}) = -B_i(-\mathbf{k})$$

$$E_\pm(k) = \epsilon(k) \mp \lambda(k) \text{ with } \lambda(k) \equiv \sqrt{B_i(\mathbf{k})B_i(\mathbf{k})}$$

White noise disorder: $W = \sum_i V_0 \delta(r - r_i)$

SELF ENERGY:

$$\Sigma = \langle T \rangle_c = \langle W \rangle_c + \langle W G_0 W \rangle_c + \langle W G_0 W G_0 W \rangle_c + \ldots$$



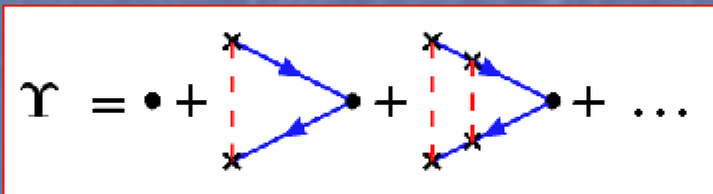
$$= \quad \Sigma(\epsilon + i\delta) = -i\frac{\pi}{2} n_i V_0^2 [(\nu_+ + \nu_-)\sigma_0 + h\left(\frac{\nu_+}{\lambda_+} - \frac{\nu_-}{\lambda_-}\right)\sigma_z \equiv -i\Gamma\sigma_0 - i\tilde{\Gamma}\sigma_z$$

LADDER DIAGRAMS FOR A GENERAL 2D SYSTEM: CONTINUATION

Born approximation Green's function

$$G^{\mathrm{ret/adv}}(\mathbf{k},\epsilon) = \frac{\epsilon - \epsilon(k) \pm i\Gamma - B_x(\mathbf{k})\sigma_x - B_y(\mathbf{k})\sigma_y - (h \pm i\tilde{\Gamma})\sigma_z}{(\epsilon - E_+(k) \pm i\Gamma_+)(\epsilon - E_-(k) \pm i\Gamma_-)}$$

Ladder diagrams



$$= \quad \tilde{J}_\alpha(z,z') \equiv J_\alpha + \delta\tilde{J}_\alpha(z,z')$$

$$\delta\tilde{J}_\alpha(z,z') = \frac{n_i V_0^2}{V}\sum_{\mathbf{k}} G(z)J_\alpha G(z') + \frac{n_i V_0^2}{V}\sum_{\mathbf{k}} G(z)\delta\tilde{J}_\alpha(z,z')G(z')$$

$$\delta\tilde{J}_\alpha = \delta\tilde{J}_0^\alpha\sigma_0 + \delta\tilde{J}_i^\alpha\sigma_i, \quad G = G_0\sigma_0 + G_i\sigma_i$$

To do the calculation we use the following expressions rather liberally:

$$\mathrm{Tr}[\sigma_i\sigma_{i'}\sigma_j\sigma_{j'}] = \mathrm{Tr}[(\delta_{ii'} + i\epsilon_{ii'k}\sigma_k)(\delta_{jj'} + i\epsilon_{jj'k'}\sigma'_k)] = 2(\delta_{ii'}\delta_{jj'} - \delta_{ij}\delta_{j'i'} + \delta_{ij'}\delta_{ji'})$$

$$\mathrm{Tr}[\sigma_i\sigma_j\sigma'_j] = \mathrm{Tr}[i\epsilon_{ijk}\sigma_k\sigma_{j'}] = 2i\epsilon_{ijj'}$$

RESULT FOR 2DEG+RASHBA

$$\tilde{J}_0^y = \frac{\hbar k_y}{m}, \quad \tilde{J}_x^y = \left(\frac{\alpha}{\hbar}\right)\left(1 + \frac{a+a'}{1-a}\right) + O(\Gamma^2) = \left(\frac{\alpha}{\hbar}\right)\left(\frac{1+a'}{1-a}\right) + O(\Gamma^2),$$

$$\tilde{J}_y^y = \left(\frac{\alpha}{\hbar}\right)\frac{-(b+b')+b'a-ba'}{(1-a)^2} + O(\Gamma^2), \quad \tilde{J}_z^y = 0$$

Easy to miss

$$a = \frac{n_i V_0^2}{V}\sum_{\mathbf{k}}\frac{(\epsilon_F-\epsilon(k))^2-h^2}{D_+D_-}, \quad a' = \frac{n_i V_0^2}{V}\sum_{\mathbf{k}}\frac{2\epsilon(k)(\epsilon_F-\epsilon(k))}{D_+D_-},$$

$$b = \frac{2n_i V_0^2}{V}\sum_{\mathbf{k}}\frac{\Gamma h - \tilde{\Gamma}(\epsilon_F-\epsilon(k))}{D_+D_-}, \quad b' = -\frac{2n_i V_0^2}{V}\sum_{\mathbf{k}}\frac{\epsilon(k)\tilde{\Gamma}}{D_+D_-}$$

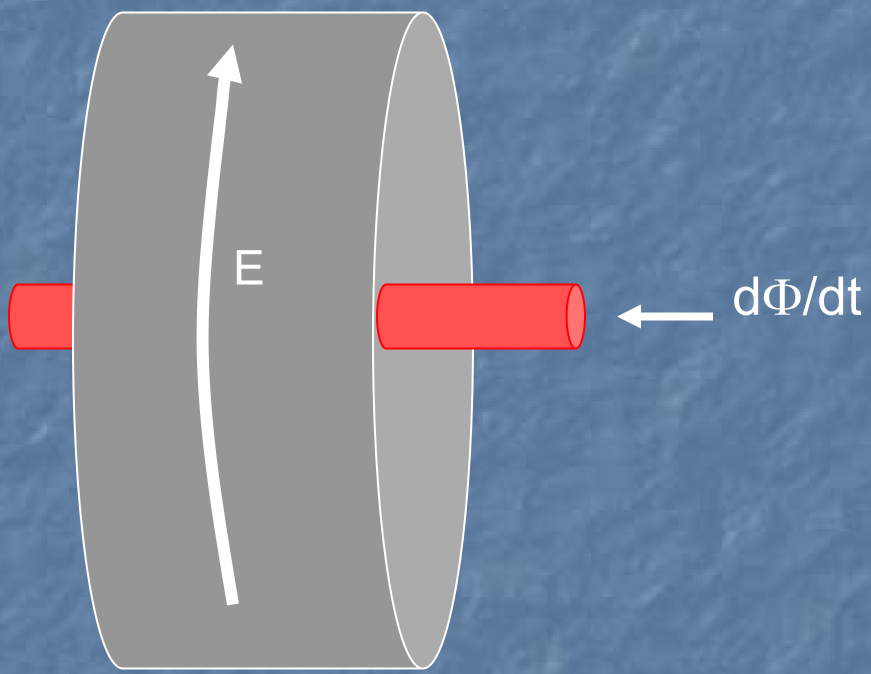
$$D_\pm = (\epsilon_F - E_\pm(k))^2 + \Gamma_\pm^2)$$

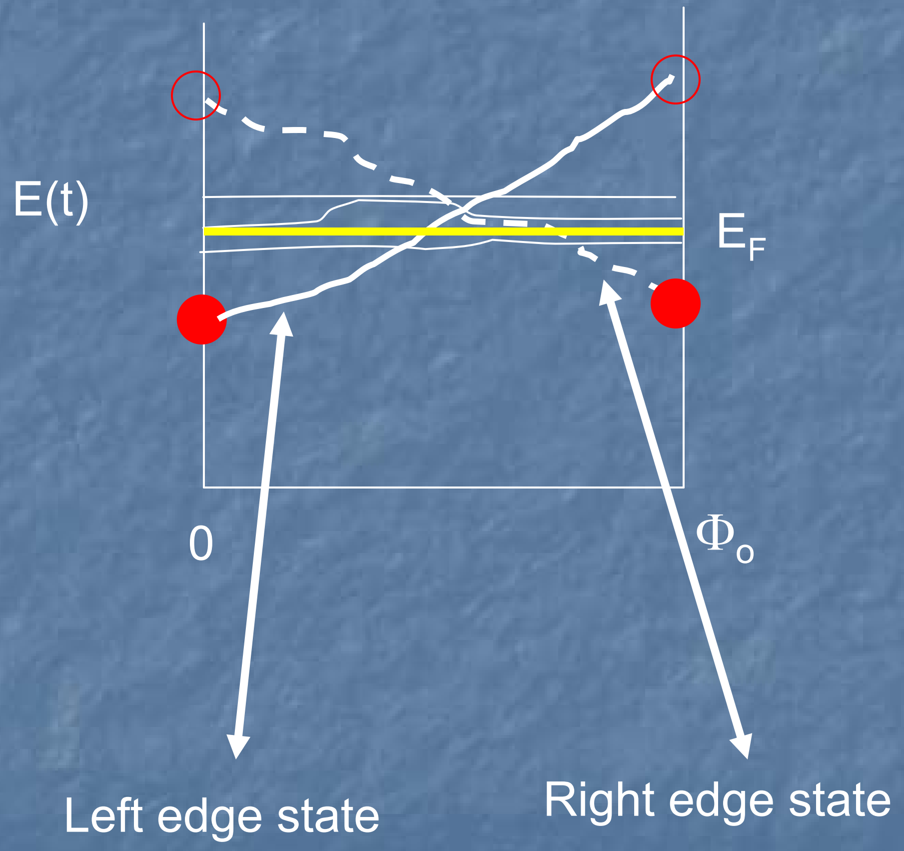
$$\sigma_{xy}^{I(a)} = \frac{e^2\hbar}{2\pi V}\sum_{\mathbf{k}}\mathrm{Tr}[v\sigma_x G\tilde{J}_y G] = \frac{e^2}{2\pi\hbar}\epsilon_\alpha\left(\left(\frac{1+a'}{1-a}\right)b - \frac{(-(b+b')+b'a-ba')}{(1-a)^2}a\right)$$

OUTLINE

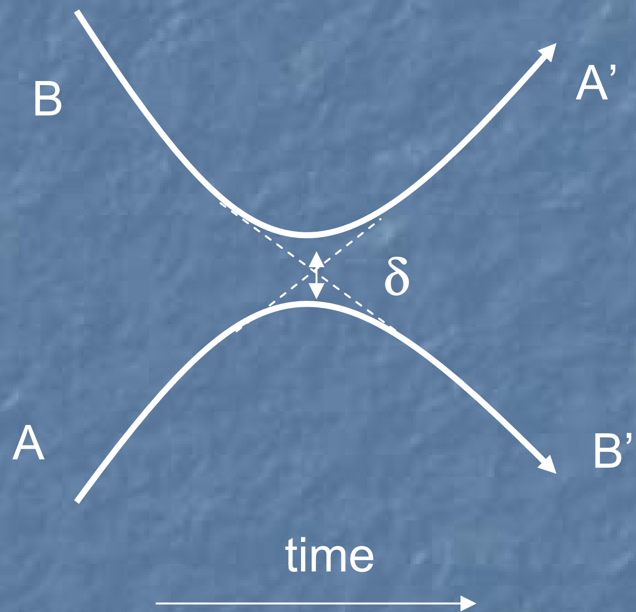
- Intro and short history and origin of the SHE frenzy
- Hall effect (long ago) and Spin Hall effect (not so long ago)
  - Parsing the AHE
    - Perturbation theory treatment
    - Semiclassical vs. Kubo
  - Parsing the SHE (example: 2D systems)
    - Perturbation theory treatment
  - Resolution of some of the controversy:
    A community that works together, SHE workshop
- Ladder diagrams in 2D SO coupled systems
- Laughlin argument for transport:
  - What and how is the argument used
  - Why does it work in the QHE case
  - Why it does not work in metallic systems
- AHE in graphene (time permitting)

Laughlin argument for transport: how I understand it





Laughlin argument for transport: how I understand it



Instantaneous eigenstates evolve with time $dE/dt=\Delta$

Probability of A going into A’ is $\exp(-\pi\delta^2/\hbar\Delta)$

For metals $dE/dt=dE/dk\ dk/dt=v\ eE/\hbar$

Probability of A going into A’ is $\exp(-\pi\delta^2/evE)$

For crossing we need $\delta^2/evE \ll 1$

This condition is satisfied both for metals and the QHE problem:

Metal: $\delta\sim/L^d$

QHE: $\delta\sim\exp(-L/l)$ for the edge states

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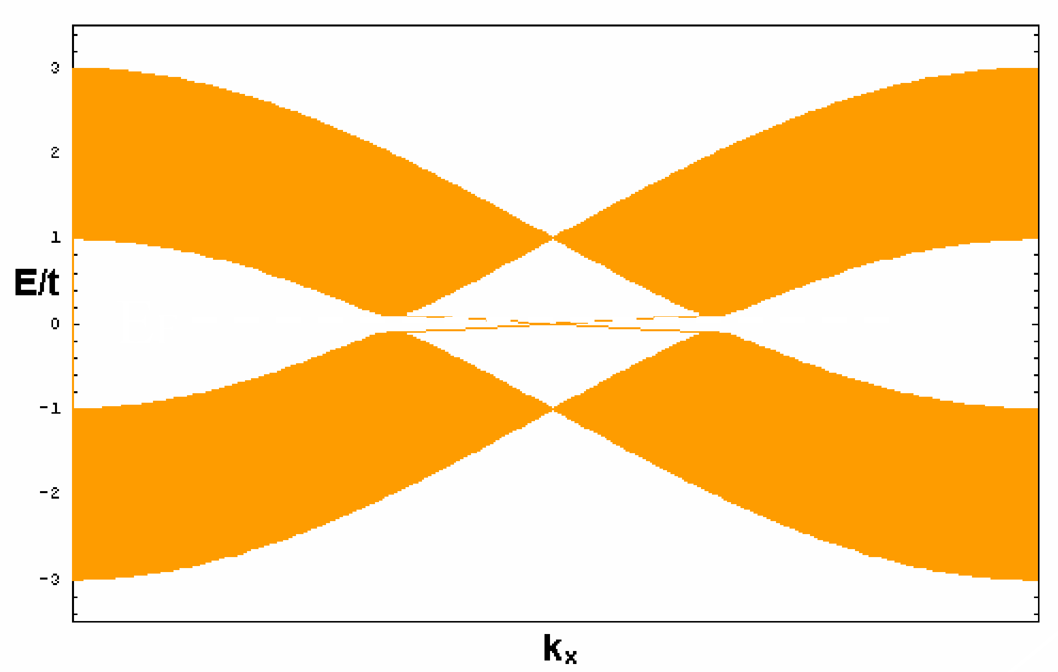
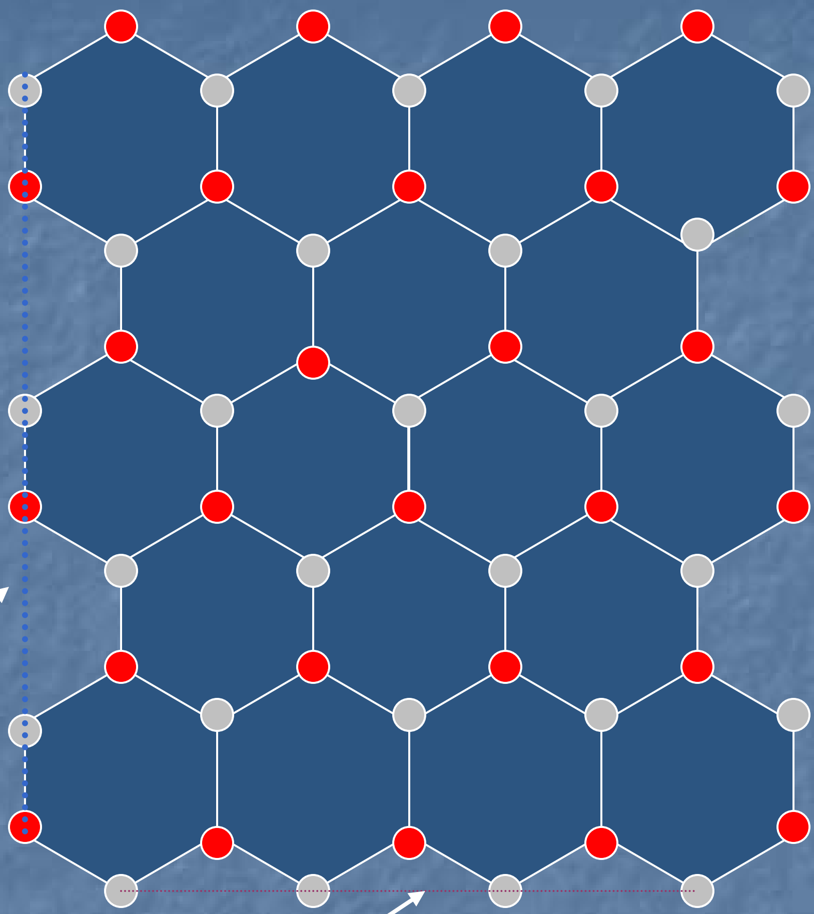
Some success in graphene



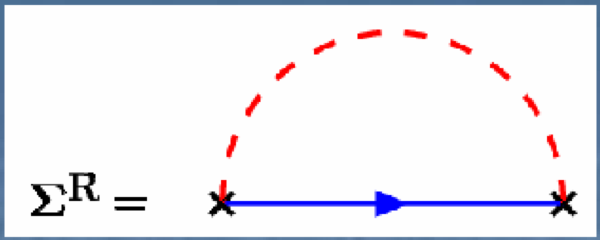


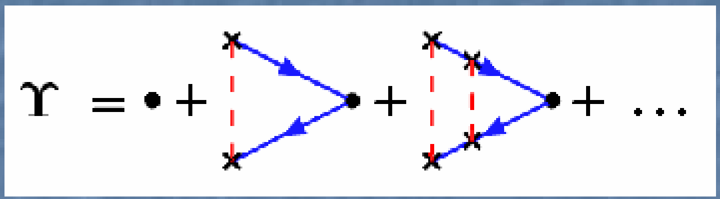
Armchair edge

Zigzag edge

Single K-band with spin up

$$H_{K\uparrow} = v(k_x\sigma_x + k_y\sigma_y) + \Delta_{so}\sigma_z$$





Kubo-Streda formula:

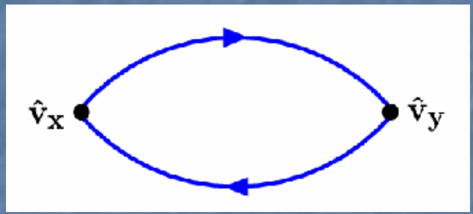
$$\sigma_{xy} = \sigma^{I}_{xy} + \sigma^{II}_{xy}$$

$$\sigma^{I}_{xy} = -\frac{e^2}{4\pi}\int_{-\infty}^{+\infty} d\varepsilon \frac{df(\varepsilon)}{d\varepsilon} Tr[v_x(G^R - G^A)v_y G^A - v_x G^R v_y (G^R - G^A)]$$

$$\sigma^{II}_{xy} = \frac{e^2}{4\pi}\int_{-\infty}^{+\infty} d\varepsilon f(\varepsilon) Tr[v_x G^R v_y \frac{dG^R}{d\varepsilon} - v_x \frac{dG^R}{d\varepsilon} v_y G^R - v_x G^A v_y \frac{dG^A}{d\varepsilon} + v_x \frac{dG^A}{d\varepsilon} v_y G^A]$$

In metallic regime:

$$\sigma^{II}_{xy} = 0$$



$$\sigma^{I}_{xy} = \frac{-e^2\Delta_{so}}{4\pi\hbar\sqrt{(vk_F)^2 + \Delta_{so}^2}}\left(1 + \frac{4(vk_F)^2}{(vk_F)^2 + 4\Delta_{so}^2} + \frac{3(vk_F)^4}{\left((vk_F)^2 + 4\Delta_{so}^2\right)^2}\right) - \frac{e^2\langle V^3\rangle}{2\pi n\langle V^2\rangle}\frac{\Delta_{so}(vk_F)^4}{\left((vk_F)^2 + 4\Delta_{so}^2\right)^2}$$

Sinitsyn et al cond-mat/0602598

Semiclassical approach II: Sinitsyn et al PRB 06

Golden Rule: $$\omega_{l'l} = \frac{2\pi}{\hbar} |V_{l'l}|^2 \, \delta(\varepsilon_{l'} - \varepsilon_l)$$

$$l = (\mu, \vec{k})$$

$$V_{l'l} \to T_{l'l}$$

Coordinate shift: $$\delta \vec{r}_{l'l} = \left\langle u_{l'} \middle| i \frac{\partial}{\partial \vec{k}'} \middle| u_{l'} \right\rangle - \left\langle u_l \middle| i \frac{\partial}{\partial \vec{k}} \middle| u_l \right\rangle - \hat{D}_{\vec{k}',\vec{k}} \arg\left(V_{l'l}\right)$$

Modified Boltzmann Equation: $$\frac{\partial f_l}{\partial t} + e\vec{E}\vec{\mathrm{v}}_l \frac{\partial f_0(\varepsilon_l)}{\partial \varepsilon_l} + \sum_{l'} \omega_{l'l} \frac{\partial f_0(\varepsilon_l)}{\partial \varepsilon_l} e\vec{E}\delta\vec{r}_{l'l} = -\sum_{l'} \omega_{l'l}(f_l - f_{l'})$$

Sinitsyn et al PRB 06

Berry curvature: $$F_z^l \equiv \mathrm{Im}\left(\left\langle \frac{\partial u_l}{\partial k_y} \middle| \frac{\partial u_l}{\partial k_x} \right\rangle - \left\langle \frac{\partial u_l}{\partial k_x} \middle| \frac{\partial u_l}{\partial k_y} \right\rangle \right)$$

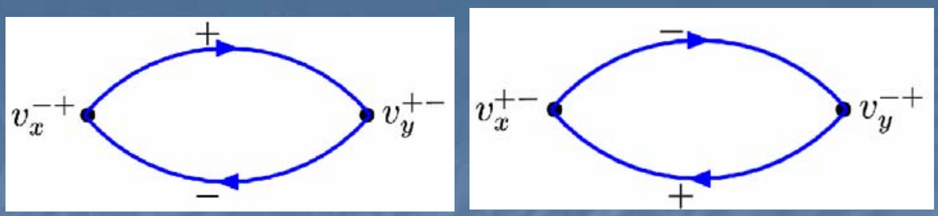
velocity: $$\vec{v}_l = \frac{\partial \varepsilon_l}{\partial \vec{k}} - \vec{F}^l \times e\vec{E} + \sum_{l'} \omega_{l'l} \delta \vec{r}_{l'l}$$

current: $$\vec{J} = e \sum_l f_l \vec{v}_l$$

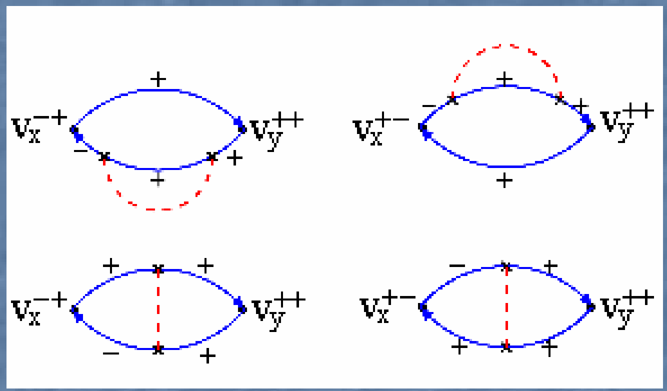
$$\sigma_{xy} = \sigma^{I}_{xy} + \sigma^{II}_{xy}$$

$$\sigma^{I}_{xy} = -\frac{e^2}{4\pi}\int_{-\infty}^{+\infty} d\varepsilon \frac{df(\varepsilon)}{d\varepsilon} Tr[v_x (G^R - G^A) v_y G^A - -v_x G^R v_y (G^R - G^A)]$$

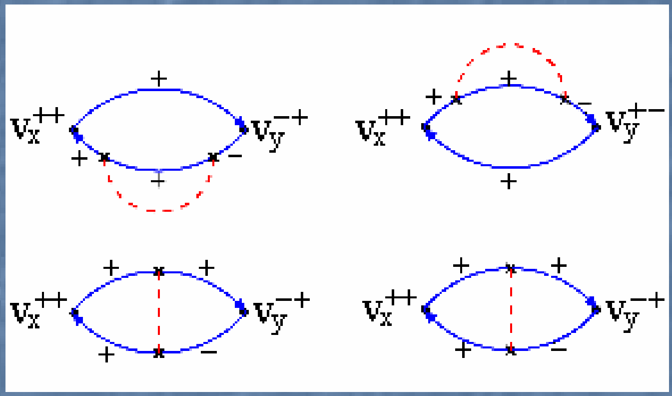
Intrinsic contribution:



Side-jump velocity:



Anomalous distribution:



Skew scattering: